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Topological Spaces F_1 And F_2

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ABSTRACT: In This work, we study F_1 space and F_2 space and about these two spaces, we proved various theorems and properties.

Keywords: F 1 space and F 2 space

1. INTRODUCTION

separation qualities are a standout amongst the most vital and fascinating concepts in topology. In 1963, N.levin [1] proposed concept of a semi-open set. S.N Maheshwari and R. Prasad [2], used semi-open sets to characterize and investigare new partition aphorisms known as semi-detachment aphorisms. In 1975, N Levine characterized the idea of new type of topological space called $T_{1/2}$ in 1970 [3] (i.e. the space where the closed sets and summed up sets classes meet). Maheshwari and et. al. [4] initiated the study of feebly open in 1978. Aaad Aziz Hussan Abdulla in [5] presented the idea of semi-feebly open (sf-open) set. "the goal of this study is to provide some characterizations of F_1 space and F_2 space ".

2. BASIC DEFINITION

Definition 2.1 [6]

A subset A of a topological space $(X,)$ is called feebly open (f-open) set if there exists an open set U such that $U \subseteq A \subseteq \overline{U}^s$.

Definition 2.2 [7]

Let $(X,)$ be a topological space. A subset A of X is said to be g-closed

if $\bar{A} \subseteq U$ whenever $A \subseteq U$ and U is open set.

Definition 2.3 [3]

Let $(X,)$ be a topological space. A subset A of X is said to $T_{1/2}$ space
if each g-closed set is closed set.

3. CHARACTERIZATION OF F_1 SPACE AND F_2 SPACE

Definition 3.1

Let (X, t) be a topological space. A subset A of X is said to be

(1) F^*-closed set if $\bar{A} \subseteq U$ whenever $A \subseteq U$ and U is f-open set.

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.(2) F^*-open set if the complement of A in X is F^*-closed set.

Example 3.2

Let $X = \{1, 2, 3\}, - = \{\emptyset, X, \{1\}, \{1, 2\}\}$ be a topology defined on X.
Let $A = \{2, 3\}$, then $\overline{A} = \{2, 3\}$, thus the f-open sets contain A is only X.
Hence $\overline{A} \subseteq X$, then A is F^*-closed set.

Proposition 3.3

Every closed set is F^*-closed set.

Remark 3.4

The converse [**Proposition (3.3)**] is not necessarily true as shown by the following example.

Example 3.5

Let $X = \{1, 2, 3\}, \ = \{X, \emptyset, \{1, 3\}\}$ be a topology defined on X.
Let $A = \{1, 2\}$, then $\overline{A} = X$, thus the f-open sets contain A is only X
It is clear A is F^*-closed set but not closed.

Proposition 3.6

Every F^*-closed set is g-closed set.

Remark 3.7

The converse [**Proposition (3.6)**] is not necessarily true as shown by the following example.

Example 3.8

Let $X = \{1, 2, 3\}, \ = \{X, \emptyset, \{1\}\}$ be a topology defined on X.
Let $A = \{1, 3\}$, then $\overline{A} = X$, implies the f-open sets contain A is X and $\{1, 3\}$. It is clear $\overline{A} \nsubseteq \{1, 3\}$
Hence A is not F^*-closed set but A is g-closed set since the open sets contain A is only X, $\overline{A} \subseteq X$.

Remark 3.9

g-closed set is F^*-closed set if every f-open set is open set.

Lemma 3.10

Let $(X, \ )$ is a topological space, if every closed set is open set then
(1) every f-open set is f-closed set.
(2) every f-open (f-closed) set is open set (closed) set.

Theorem 3.11

Let $(X, \ )$ be a topological space , then $\ = f$ if and only if every subset of X is F^*-closed set, where f is the family of closed sets in X.

Proof.

$\Longrightarrow$ Let $\ = f$ and $A \subseteq X$ and $A \subseteq O$, where O is f-open set in X.
Since $A \subseteq O$, then $\overline{A} \subseteq \overline{O}$, but $\overline{O} = O$.
Then $\overline{A} \subseteq O$, implies A is F^*- closed set.
$\Longleftarrow$ Let every subset of X is F^*- closed set
Assume that $O \in$ then O is f-open set
Since $O \subseteq O$, O is F^*- closed set
Hence $\overline{O} \subseteq O$ implies $\overline{O} = O$
Therefore $O \in f$

$$Thus \ \subset f \tag{1}$$

Assume that $F \in f$, then $F^c \in$
Hence F^c is f-open set
Since $F^c \subset X$, implies F^c is F^*- closed set
But $F^c \subseteq F^c$ and F^c is f-open set
So, $\overline{F^c} \subseteq F^c$, consequently $F^c \in f$
Therefore $F \in -$

$$Thus \ f \subset \tag{2}$$

Then by (1) and (2), we have $= f$.

Remark 3.12

Let $(X, -)$ is a topological space. If $= f$ then every g-closed set is F^*-closed set.

Theorem 3.13

Let $(X, \ )$ be a topological space and $A \subseteq Y \subseteq X$ and Y open set in X. If A Is F^*- closed set in X then A is F^*- closed set in Y

Proof.

Let $A \subseteq O$, O is f-open in X
Then $A \cap Y \subseteq O \cap Y$
Since Y is open set in X.
Hence $O \cap Y$ is f-open in Y.
Since A is F^*- closed set in X.
So, $\bar{A} \subseteq O$.
Therefore $\bar{A} \cap Y \subseteq O \cap Y$.
But $\bar{A}_Y = \bar{A} \cap Y$, such that $\bar{A}_Y$ is closure of A in Y.
Thus A is F^*- closed set in Y

Theorem 3.14

If $A \subseteq Y \subseteq X$ such that Y is open and closed in X, A is F^*- closed set in Y then A is F^*- closed set in X

Proof.

Let $A \subseteq O$ and O is f-open in X
Then $A \cap Y \subseteq O \cap Y$
Since $A \subseteq Y$ implies $A \subseteq O \cap Y$
But Y is open set in X
Hence $O \cap Y$ is f-open in Y
Since A is F^*- closed set in Y
Then $\bar{A}_Y \subseteq O \cap Y$
We know $\bar{A}_Y = \bar{A} \cap Y$
Since $A \subseteq Y implies \bar{A} \subseteq \bar{Y}$
But Y is closed set in X
Therefore $\bar{A} \subseteq Y$
So, $\bar{A} \subset O \cap Y$
Then $\bar{A} \subseteq O$
Thus A is F^*- closed set in X.

Definition 3.15

A topological space X is called F_1 space if and only if every g-closed set in X is F^*- closed set.

Example 3.16

Let $X = \{1, 2, 3\}$, $- = \{\emptyset, X, \{1\}, \{3\}, \{1, 3\}\}$ be a topology defined on X.
Then g-closed on X are $\{\emptyset, X, \{2\}, \{1, 2\}, \{2, 3\}\}$,
F^*- closed set on X are $\{\emptyset, X, \{2\}, \{1, 2\}, \{2, 3\}\}$.
It is clear every g-closed set is F^*-closed set.

Proposition 3.17

Let X be a topological space, if X is $T_{\frac{1}{2}}$ space, then X is F_1 space.

Proof.

Assume that A is g-closed set in X.
Since X is $T_{\frac{1}{2}}$ space, implies A is closed set.
Therefore, $\bar{A}$ is F^*- closed set [**Proposition (3.3)**].
Then X is F_1 space.

Remark 3.18

The converse [**Proposition (3.17)**] is not necessarily true as shown in the Following example.

Example 3.19

Let $X = \{1, 2, 3, 4\}$, $-$ = $\{\varnothing, X, \{3\}, \{1, 2\}, \{1, 2, 3\}\}$ be a topology defined on X.

The closed sets are $\{\varnothing, X, \{1, 2, 3\}, \{3, 4\}, \{4\}\}$

g-closed sets are $\{\varnothing, X, \{2,3,4\}, \{1, 3, 4\}, \{2, 4\}, \{1, 4\}, \{1, 2, 4\}, \{3, 4\}, \{4\}\}$.

F^*- closed sets are $\{\varnothing, X, \{2,3,4\}, \{1, 3, 4\}, \{2, 4\}, \{1, 4\}, \{1, 2, 4\}, \{3,4\}, \{4\}\}$.

It is clear X is F_1 space since every g-closed set is F^*-closed set but not $T_{\frac{1}{2}}$ space since $\{2, 4\}$ is g-closed set but not closed set.

Lemma 3.20

If $A \subseteq Y \subseteq X$ such that Y is closed in X, A is g- closed set in Y then A is g- closed set in X.

Theorem 3.21

Let X be a F_1 topology space. If Y is an open and closed subspace then Y is F_1 space.

Proof.

Assume that A is g-closed set in Y

Since Y is a closed subspace, implies A is g-closed in X [**Lemma (3.20)**].

But X is F_1 space. Then A is F^*- closed set in X

Since Y is an open subspace in X

Thus A is F^*- closed in Y [**Theorem (3.13)**].

Then Y is F_1 space.

Remark 3.22

F_1 space does not hereditary property.

Theorem 3.23

If $(X, \ )$ be F_1 space then every singleton is either F^*- closed *or* F^*- open.

Proof.

Let X be F_1 space

Assume that $x \in X$ and $\{x\}$ is not F^*- closed set

Since X is the only open set contain $\{x\}^c$ then $\overline{\{x\}^c} \subseteq X$

Hence $\{x\}^c$ g-closed set.

Since X is F_1 space then $\{x\}^c$ is F^*- closed set.

Therefore, $\{x\}$ is F^*- open.

Definition 3.24

A topological space X is called F_2 space if and only if every f-open set in X is open set.

Example 3.25

Let $X = \{1, 2, 3, 4\}$, $-$ = $\{\varnothing, X, \{1\}, \{2\}, \{1, 2\}, \{3, 4\}, \{1, 3, 4\}, \{2, 3, 4\}\}$ be a topology defined on X.

The f-open sets are $\{\varnothing, X, \{1\}, \{2\}, \{1,2\}, \{3, 4\}, \{1, 3, 4\}, \{2, 3, 4\}$.

It is clear every f-open set in X is open set.

Then X is F_2 space.

Proposition 3.26

Every F_2 space is F_1 space.

Proof.

Assume that X is F_2 space

Then every f-open set is open set .

Therefore, every g-closed set is F^*- closed set [**Remark (3.9)**].

Thus X is F_1 space.

Remark 3.27

Every F_1 space is F_2 if $\ = f$.

Theorem 3.28

If $(X, \ )$ be F_2 space then every f-closed set is f-open set.

Proof.

Assume that X is F_2 space

Then $-$ **=** f

Therefore, f-open set = f-closed set [**Lemma (3.10)**].

Hence every f-closed set is f-open set.

Theorem 3.29

Let X be F_2 topological space, if Y is an open subspace then Y is F_2 space.

Proof.

Assume that A is f-open set in Y

Since Y is an open subspace, implies$A$ is f-open set in$X$

But X is F_2 space, then A is open set in X

Therefore, A is open set in Y

Then Y is F_2 space.

Theorem 3.30

If Y is F_2 space and $f : X \longrightarrow Y$ be continuous function, open and surjective, then X is F_2 space.

Proof.

Assume that B is f-open set in X

Since f is continuous, open and surjective, implies $f(B)$ is f-open set in Y

But Y is F_2 space, then (B) is open set in Y

Since f is continuous, then $f^{-1}(f(B))$ is open set in X.

But f is surjective, then $f^{-1}(f(B)) = B$

Therefore, X is F_2 space.

4. CONCLUSION

In this work, several properties of these two space were studied, and these properties, a relationship was drawn between $T_{1/2}$ space and F_1space, there is also relationship between F_1 space and F_2 space.

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CONFLICTS OF INTEREST

The author declares no conflict of interest.

REFERENCES

[1] N. Levine, “Semi “open sets and semi continuity in Topological spaces”,” *Amer. Math. Monthly*, vol. 70, pp. 36–41, 1963.
[2] S. N. Msheshwari and R. Prasad, “Some new separation axioms,” *Ann. Soc., Sci. Bruxelles*, vol. 89, pp. 395–402, 1975.
[3] N. Levine, “Generalized closed sets in topology,” *Rend. Circ. Mat. Palermo*, vol. 19, no. 2, pp. 89–96, 1970.
[4] S. N. Msheshwari and U. Tapi, “Feebly space,” *Ann. Univ., Timisoara S. sti. Mat*, vol. 16, pp. 395–7402, 1978.
[5] “On Semi Feebly open set and its properties,” *AlQadisiyah Journal of pure scince*, vol. 25, no. 3, pp. 35–45, 2002.
[6] D. S. Jankovic and I. L. Reidly, “On Semi-Separation Properties”,” *Indian J. Pure Appl. Math*, vol. 16, no. 9, pp. 957–964, 1985.
[7] S. G. Crossley, “Semi-closure,” *Texas J. Sci*, vol. 22, pp. 99–112, 1971.