Fuzzy α-Proper Mapping

Habeeb Kareem Abdullah
University of Kufa
College of Education for Girls
Department of Mathematics

Abstract

The purpose of this paper is introduce the concept of fuzzy α-proper mapping in fuzzy topological spaces. We give some characterization of fuzzy α-compact mapping and fuzzy α-coercive mapping. We study the relation of fuzzy proper mapping, fuzzy α-compact mapping and fuzzy α-coercive mapping and we obtained several properties.

Keywords: Fuzzy topology, Fuzzy α-Proper, fuzzy α-compact

الخلاصة

الهدف من هذا البحث تقديم مفهوم تطبيق α السديد في الفضاءات التبولوجية الضبابية. نحن اعطينا بعض مميزات تطبيق α المرصوص الضبابي وتطبيق α القاسي الضبابي. و درسنا العلاقة بين التطبيق السديد الضبابي و التطبيق α المرصوص الضبابي والتطبيق α القاسي الضبابي وحصلنا على عدة خصائص.

1. Introduction and Preliminaries.

The concept of a fuzzy set and fuzzy set operations were first introduced by Zadeh [9] in 1965. Several other authors applied fuzzy sets to various branches of mathematics. One of these objects is a topological space. At the first time in 1968, Chang in [2] formulated the natural definition of fuzzy topology on a set and investigated how some of the basic ideas and theorems of point-set topology behave in generalized setting. Chang's definition on a fuzzy topology is very similar to the general topology by exchange all subsets of a universal set by fuzzy subset but this definition is not investigate some properties if we comparison with the general topology. For example in ageneral topology, any constant mapping is continuous while this idea is not true in Chang's definition on fuzzy topology. One of the very important concepts in fuzzy topology is the concept of mapping. There are several types of mapping.The purpose of this paper is to introduced and study the concept of fuzzy α-proper mapping in fuzzy setting. Throughout this paper (X, T) (or simply X), we shall mean a fuzzy topological space (fts, for short) in Chang's [2] sense. A fuzzy point [3] with support $x \in X$ and value r ($0< r \leq 1$) at $x \in X$ will be denoted by x_r, and for fuzzy set A, $x_r \in A$ iff $r \leq A(x)$. Two fuzzy points x_r and y_s are said to be distinct iff their supports are distinct. For two fuzzy sets A and B, we shall write AqB to mean that A is quasi-coincident (q-coincident, for short) with B, i.e., there exists $x \in X$ such that $A(x) + B(x) > 1$, and B is said to be q-neighborhood (q-nbd, for short) of x_r if there is a fuzzy open set A with $x_r qA \leq B$, the family of all fuzzy q-nbds of x_r is called the system of fuzzy q-nbds of x_r and it is denoted by $N^Q_{x_r}$. If A is not q-coincident with B, then we write $A\tilde{q}B$. Also, we shall write $A\backslash B = A \wedge B^c$[7], to mean the different between A and B. For a fuzzy set A in an fts X, $\overline{A}, A^{\circ}$ and $A^c = 1_X - A$ denote the fuzzy closure, fuzzy interior and fuzzy complement of A, respectively. By 0_X and 1_X we mean the constant fuzzy sets taking on the value 0 and 1 on X, respectively.

Definition 1.1 [2]Let X and Y be two non-empty sets, $f: X \longrightarrow Y$ be a mapping. For a fuzzy set B in Y with membership function $B(y)$. Then the inverse image of B under f is the fuzzy set $f^{-1}(B)$ in X whose membership function is defined by:
$f^{-1}(B)(x) = B(f(x))$ for all $x \in X$. (i.e., $f^{-1}(B) = B \circ f$)

Conversely, let A be a fuzzy set in X with membership function $A(x)$. The image of A under f is the fuzzy set $f(A)$ in Y whose membership function $f(A)(y)$, $y \in Y$ is given by:

$$f(A)(y) = \begin{cases} \sup\limits_{z \in f^{-1}(y)} A(z), & \text{if } f^{-1}(y) \neq \emptyset \\ 0, & \text{if } f^{-1}(y) = \emptyset \end{cases}$$

for all $y \in Y$, where $f^{-1}(y) = \{x: f(x) = y\}$.

Theorem 1.2 [2,8]

Let X, Y and Z be non-empty sets and $f: X \longrightarrow Y$, $g: Y \longrightarrow Z$ be mappings, then the following statements are holds:

(a) $f^{-1}(B^c) = (f^{-1}(B))^c$, for any fuzzy set B in Y.

(b) $(f(A))^c \leq f(A^c)$, for any fuzzy set A in X.

(c) If $B_1 \leq B_2$, then $f^{-1}(B_1) \leq f^{-1}(B_2)$, where B_1 and B_2 are fuzzy sets in Y.

(d) If $A_1 \leq A_2$, then $f(A_1) \leq f(A_2)$, where A_1 and A_2 are fuzzy sets in X.

(e) For any fuzzy set A in X:

(1) $A \leq f^{-1}(f(A))$;

(2) $A = f^{-1}(f(A))$, if f is an injective mapping.

(f) For any fuzzy set B in Y:

(1) $f(f^{-1}(B)) \leq B$;

(2) $f(f^{-1}(B)) = B$, if f is a surjective mapping.

(g) If $g \circ f: X \longrightarrow Z$ is the composition mapping between g and f, then:

(1) $(g \circ f)(A) = g(f(A))$, for any fuzzy set A in X.

(2) $(g \circ f)^{-1}(C) = f^{-1}(g^{-1}(C))$, for any fuzzy set C in Z.

(h) $f(f^{-1}(B) \wedge A) = B \wedge f(A)$.

Definition 1.3 [3]An fts(X, T) is called fuzzy Hausdorff or fuzzy T_2-space if and only if for any pair of distinct fuzzy points x_r, y_s in X, there exists $A \in N^Q_{x_r}$, $B \in N^Q_{y_s}$ such that $A \wedge B = 0_X$.

Definition 1.4 [2] A mapping f from anyfts (X, T) into anyfts (Y, T) is called fuzzy continuous if and only if the inverse image of each fuzzy open set in Y is a fuzzy open set in X.

Proposition 1.5 [2,6] A composition of fuzzy continuous mappings is fuzzy continuous.

Proposition 1.6 [5] Let X and Y be fts's, and $f: X \longrightarrow Y$ be a mapping, then the following statements are equivalent:

(a) f is fuzzy continuous.

(b) The inverse image of each fuzzy closed set in Y is fuzzy closed in X.

(c) For each fuzzy set B in Y, $\overline{f^{-1}(B)} \leq f^{-1}(\overline{B})$.

(d) For each fuzzy set A in X, $f(\overline{A}) \leq \overline{f(A)}$.

(e) For each fuzzy set B in Y, $f^{-1}(B^\circ) \leq (f^{-1}(B))^\circ$.

Definition 1.7 [6] A mapping f from an fts (X, T) into an fts (Y, T') is called fuzzy closed if $f(A)$ is fuzzy closed in Y for each fuzzy closed set A in X.

2. Fuzzy α-Topological Concepts

In this section, we study and investigate some definitions, examples, remarks and propositions fuzzy α-open set

Definition 2.1 [4] A fuzzy subset A of an fts X is called fuzzy α-open if $A \leq \overline{A^{\circ}}^{\circ}$. The complement of a fuzzy α-open set is defined to be a fuzzy α-closed set.

Remark2.2 Every fuzzy open (resp. closed) set of an ftsX is fuzzy α-open (α-closed).

Proposition 2.3 [4]Let A be a fuzzy set of an fts X, then the following statements are equivalent:
(a) A is a fuzzy α-open set.
(b) $O \leq A \leq \overline{O}^{\circ}$, for some fuzzy open set O.
(c) $O \leq A \leq \overline{O^{\circ}}^{s}$, for some fuzzy open set O.
(d) $A \leq \overline{A^{\circ}}^{s}$.

Proposition 2.4 [4] Let A be a fuzzy set of an fts X, then the following statements are equivalent:
(a) A is a fuzzy α-closed set.
(b) $\overline{\overline{A}^{\circ}} \leq A$.
(c) $\overline{F^{\circ}} \leq A \leq F$, for some fuzzy closed set.

Definition 2.5 A fuzzy set A in an fts X is called a fuzzy α- quasi-neighborhood of a fuzzy point x_r if there exists a fuzzy α-open set B in X such that $x_r qB \leq A$.

The family of all fuzzy α-q-nbds of x_r is called the system of fuzzy α-q-nbds of x_r and it is denoted by $N_{x_r}^{Q\alpha}$.

Definition 2.6 Let A be a fuzzy set of an fts X, Then the fuzzy α-closure of A, denoted by $\overline{A}^{\alpha}$, is the intersection of all fuzzy α-closed sets in X which containing A.It is evident that A is fuzzy α-closed if and only if $\overline{A}^{\alpha} = A$.

Proposition 2.7 Let x_r be a fuzzy point in X and A be any fuzzy set in an fts X, then $x_r \in \overline{A}^{\alpha}$ if and only if BqA for every $B \in N_{y_s}^{Q\alpha}$.

Proof:

⟹Let $x_r \in \overline{A}^{\alpha}$ and there exists a fuzzy α-q-nbd B of x_r, $B\tilde{q}A$. Then there exists a fuzzy α-open set $C \leq B$ in X such that $x_r qC$, which implies $C\tilde{q}A$ and hence $A \leq C^c$. Since C^c is a fuzzy α-closed set, then $\overline{A}^{\alpha} \leq C^c$, then $x_r \notin \overline{A}^{\alpha}$, which is a contradiction.

⟸Let $x_r \notin \overline{A}^{\alpha} = \wedge\{B: B$ is fuzzy α-closed in X, $A \leq B\}$. Then there exists a fuzzy α-closed set $A \leq B$ such that $x_r \notin B$. Hence $x_r qB^c$, where B^c is a fuzzy α-open set in X and $B^c\tilde{q}A$. Then there exists a fuzzy α-q-nbd B^c of x_r with $B^c\tilde{q}A$. Hence the result.

Definition 2.8 [4] A mapping f from any fts(X,T) into any fts (Y,T) is calledfuzzy α-closed if $f(A)$ is a fuzzy α- closed set in Y for each fuzzy closed set A in X.

Proposition 2.9 If $f: X \longrightarrow Y$ be a fuzzy closed mapping and $g: Y \longrightarrow Z$ be a fuzzy α-closed mapping, then $g \circ f: X \longrightarrow Z$ is a fuzzy α- closed mapping.

Proof :

Let A be a fuzzy closed subset of X, then $f(A)$ is a fuzzy closed set in Y. But g is a fuzzy α- closed mapping, then $g(f(A))$ is a fuzzy α-closed set in Z, hence $(g \circ f)(A)$ is a fuzzy α-closed set in Z. Thus $g \circ f: X \longrightarrow Z$ is a fuzzy α-closed mapping.

Definition 2.10 [4] A mapping f from anyfts(X, T) into anyfts(Y, T) is called fuzzy α-irresolute if $f^{-1}(A)$ is a fuzzy α-open set in X for each fuzzy α-open set A in Y.

Proposition 2.11 Let X and Y be fuzzy spaces, and$f: X \longrightarrow Y$ be a mapping. Then f is a fuzzy α-irresolute mapping if and only if $f^{-1}(A)$ is a fuzzy α-closed set in X, for every fuzzy α-closed set A in Y.

Proof:

Let A be a fuzzy α-closed set in Y. Then$A^c = 1 - A$ is fuzzy α-open in Y, so $f^{-1}(A^c)$ is fuzzy α-open in X, i.e., $(f^{-1}(A))^c$ is fuzzy α-open in X. Thus$f^{-1}(A)$ is a fuzzy α-open set in X. The proof of the converse is obvious.

Definition 2.12 [3]A mapping $\mathfrak{F}: D \longrightarrow FP(X)$ is called a fuzzy net in X and is denoted by $\{\mathcal{S}(n): n \in D\}$, where D is a directed set. If $\mathcal{S}(n) = x^n_{r_n}$ for each $n \in D$, where $x \in X$, $n \in D$ and $r_n \in (0,1]$, then the fuzzy net $\mathfrak{F}$ is denoted as $\{x^n_{r_n}: n \in D\}$ or simply $\{x^n_{r_n}\}$.

Definition 2.13[3]A fuzzy net $\mathfrak{L} = \{y^m_{r_m}: m \in E\}$ in X is called a fuzzy subnet of a fuzzy net $\mathfrak{F} = \{x^n_{r_n}: n \in D\}$ if and only if there exists a mapping $f: E \longrightarrow D$ such that :

1. $\mathfrak{L} = \mathfrak{F} \circ f$, that is, $\mathfrak{L}_i = \mathfrak{F}_{f(i)}$ for each $i \in E$.
2. For each $n \in D$ there exists some $m \in E$ such that, if $p \in E$ with $p \geq m$, then $f(p) \geq n$.

We shall denote a fuzzy subnet of a fuzzy net $\{x^n_{r_n}: n \in D\}$ by $\{x^{f(m)}_{r_{f(m)}}: m \in E\}$.

Definition 2.14 [3] Let (X, T) be an fts and $\mathfrak{F} = \{x^n_{r_n}: n \in D\}$ be a fuzzy net in X and A be a fuzzy set in X. Then $\mathfrak{F}$ is said to be:

(a) Q-eventually with A if $\exists\, m \in D$, such that $x^n_{r_n} qA$, $\forall\, n \geq m$.

(b) Q-frequently with A if $\forall\, n \in D$, $\exists\, m \in D$, with $m \geq n$, such that $x^m_{r_m} qA$.

Proposition 2.15 [1] Let (X, T) be an fts, $x_r \in FP(X)$ and A be a fuzzy set in X. Then $x_r \in \overline{A}$ if and only if there exists a fuzzy net $\mathfrak{F} = \{x^n_{r_n}: n \in D\}$in A, such that it's Q-convergent to x_r.

Definition2.16 Let (X, T) be an fts, $x_r \in FP(X)$ and let $\mathfrak{F} = \{x^n_{r_n}: n \in D\}$ be a fuzzy net in X and $x_r \in FP(X)$. Then $\mathfrak{F}$ is said to be:

(a) α-Q-convergent to x_r and x_r is called a α-Q-limit point of $\mathfrak{F}$, denoted by $\mathfrak{F} \overset{Q\alpha}{\longrightarrow} x_r$, if for each $A \in N^{Q\alpha}_{x_r}$, $\mathfrak{F}$ is Q-eventually with A.

(b) Has a α-Q-adherent point of a fuzzy net $\mathfrak{F}$, denoted by $\mathfrak{F} \propto^{Q\alpha} x_r$, if for each $A \in N^{Q\alpha}_{x_r}$, $\mathfrak{F}$ is Q-frequently with A.

Proposition 2.17A fuzzy point x_r is α-Q-adherent point of a fuzzy net $\{x^n_{r_n} : n \in D\}$ in an fts X if and only if it has a fuzzy subnet which α-Q-convergent to x_r.

Proof :

$\Longrightarrow$ Let x_r be an α-Q-adherent point of the fuzzy net $\{x^n_{r_n}: n \in D\}$. Then for each $V \in N^{Q\alpha}_{x_r}$ there exists $x^n_{r_n}$ such that $x^n_{r_n} qV$. Let E denote the set of all ordered pairs (n, V) with the above property, i.e., $n \in D$, $V \in N^{Q\alpha}_{x_r}$ and $x^n_{r_n} qV$. Let us define a relation " $\gtrsim$" on E given by $(m, U) \gtrsim (n, V)$ iff $m \geq n$ in D and $U \leq V$. Then $(E, \gtrsim)$ is a directed set and it is easy to see that $\mathfrak{L}: E \rightarrow FP(X)$ given by $\mathfrak{L}(m, U) = x^m_{r_m}$ is a fuzzy subnet of the given fuzzy net. Let V be any fuzzy α- q-nbd of x_r. Then there is an $n \in D$, such that $(n, V) \in E$ and hence $x^n_{r_n} qV$.

Now, $(m,U) \in E$ and $(m,U) \gtrsim (n,V)$, then $\mathfrak{L}(m,U) = x_{r_m}^{m}\, qU$ and $U \leq V$, so $x_{r_m}^{m}\, qV$. Hence $\mathfrak{L}$ is α-Q-convergent to x_r.
$\Leftarrow$ It is obvious.

Theorem 2.18 Let (X,T) be an fts, $x_r \in FP(X)$ and A be a fuzzy set in X. Then $x_r \in \overline{A}^{\alpha}$ if and only if there exists a fuzzy net $\mathfrak{F} =\{x_{r_n}^{n} : n \in D\}$in A, such that it's α-Q-convergent to x_r.
Proof :
$\Longrightarrow$ Let $x_r \in \overline{A}^{\alpha}$, then BqA for each $B \in N_{x_r}^{Q\alpha}$. That is, there exists $t_B \in (0,1]$, such that $x_{t_B}^{B} \in A$ and $x_{t_B}^{B} qB$. Let $D = N_{x_r}^{Q\alpha}$. Then $(D, \geqslant)$ is a directed set under the inclusion relation so we can define a fuzzy net $\mathfrak{F}: N_{x_r}^{Q\alpha} \longrightarrow FP(X)$ given by $\mathfrak{F} = x_{t_B}^{B}$, $\forall\, B \in N_{x_r}^{Q\alpha}$. Then $\mathfrak{F}$ is a fuzzy net in A. Now, let $P \in N_{x_r}^{Q\alpha}$, such that $P \geqslant B$ iff $P \leq B$, so there exists $x_{t_P}^{P} qP$. Then $x_{t_P}^{P} qB$. So $\mathfrak{F} \xrightarrow{Q\alpha} x_r$.
$\Leftarrow$ Let $\mathfrak{F}$ be a fuzzy net in A, such that $\mathfrak{F} \xrightarrow{Q\alpha} x_r$. If $B \in N_{x_r}^{Q\alpha}$, then $x_{r_n}^{n} qB$ for some n and so BqA. Thus by Proposition 2.7, $x_r \in \overline{A}^{\alpha}$.

3. Fuzzy α-Compact Space

The section contains the definition, properties and theorems about fuzzy compact space and we give a new results.

Definition3.1 [2]A family Ω of fuzzy sets is a cover of a fuzzy set A if and only if $A \leq \bigvee\{G_i : G_i \in \Omega\}$ and it is called a fuzzy open cover if and only if Ω is a cover of A and each member of Ω is a fuzzy open set. A subcover of Ω is a subfamily of Ω which is also a cover of A.

Definition 3.2 Let (X,T) be an fts and A be a fuzzy set in X. Then A is said to be fuzzy compact if for every open cover of A has a finite subcover of A.

Proposition 3.3 The fuzzy continuous image of a fuzzy compact set is fuzzy compact.
Proof:
It is clear.

Proposition3.4 [1] In any ftsX, the intersection of any fuzzy closed set with any fuzzy compact set is fuzzy compact.

Corollary 3.5 [1] A fuzzy closed subset of a fuzzy compact space is fuzzy compact.

Proposition 3.6 [1] A fuzzy compact subset of a fuzzy T_2-space is fuzzy closed.

Theorem 3.7 [1] An ftsX is fuzzy compact if and only if every fuzzy net in X has aQ-adherent point.

Definition 3.8 Let X be an fts. A fuzzy subset V of X is called fuzzy compactly closed if for every fuzzy compact set K in X, $V \wedge K$ is fuzzy compact.

Proposition 3.9 Every fuzzy closed subset of an ftsX is fuzzy compactly closed.

Proof :

Let A be a fuzzy closed subset of an fts X and K be a fuzzy compact set in X. Then by Proposition 3.4, $A \wedge K$ be a fuzzy compact set. Thus A is fuzzy compactly closed.

Theorem 3.10 Let X be a fuzzy T_2-space. A fuzzy subset A of X is fuzzy compactly closed if and only if A is fuzzy closed.

Proof :

$\Rightarrow$Let A be a fuzzy compactly closed set in X and let $x_r \in \overline{A}$. Then by Proposition 2.15, there exists a fuzzy net $\{x^n_{r_n}\}_{n\in D}$ in A, such that $x^n_{r_n} \overset{Q}{\rightarrow} x_r$. Then $F = \{x^n_{r_n}, x_r\}$ is a fuzzy compact set. Since A is fuzzy compactly closed, then $A \wedge F$ is a fuzzy compact set. But X is a fuzzy T_2-space, then by Proposition 3.6, $A \wedge F$ is a fuzzy closed set. Since $x^n_{r_n} \overset{Q}{\rightarrow} x_r$ and $x^n_{r_n} \in A \wedge F$, then by Proposition 2.15, $x_r \in A \wedge F$. Hence $\overline{A} \leq A$, therefore A is a fuzzy closed set.

$\Leftarrow$By Proposition 3.9.

Remark 3.11 Not every fuzzy singleton point of an ftsX is fuzzy compact in general. See the following example.

Example 3.12 Consider the fts$T = \{0_X, 1_X, A_n\}$ on a set $X = \{x\}$, where $A_n = 1 - \frac{1}{n}$, and let the fuzzy point $x_{0.5}$, then $x_{0.5} \leq \bigvee_{n=3}^{\infty} A_n$, but it has no a finite subcover.

Definition 3.13 An fts(X, T) is called fuzzy singleton compact space (fuzzy sc-space) if every fuzzy singleton point of X is fuzzy compact.

Definition 3.14A family Ω of fuzzy sets is called a fuzzyα-open cover if Ω is a cover of A and each member of Ω is a fuzzy α-open set. A subcover of Ω is a subfamily ofΩ which is also a cover of A.

Definition 3.15 Let (X, T) be an fts and A be a fuzzy set in X. Then A is said to be fuzzy α-compact if for every α-open cover of A has a finite subcover of A.

Proposition 3.16 Every fuzzy α-compact space is fuzzy compact.

Theorem 3.17 An fts is fuzzy α-compact if and only if each family of fuzzy α-closed sets which has the finite intersection property has a non-empty intersection.

Proof :

$\Rightarrow$Let $\{A_j : j \in J\}$ be a family of fuzzy α-closed sets with the finite intersection property. Supposed that $\bigwedge_{j\in J} A_j = 0_X$. Then $\bigvee_{j\in J} A^c{}_j = 1_X$. Since $\{A^c{}_j : j \in J\}$ is a collection of fuzzy α-open sets cover of X, then from the α-compactness of X it follows that there exists a finite subset $F \subseteq J$ such that $\bigvee_{j\in F} A^c{}_j = 1_X$. Then $\bigwedge_{j\in F} A_j = 0_X$ which gives a contradiction and therefore $\bigwedge_{j\in J} A_j \neq 0_X$.

$\Leftarrow$Let $\{A_j : j \in J\}$ be a collection of fuzzy α-open sets cover of X. Suppose that for every finite subset $F \subseteq J$, we have $\bigvee_{j\in F} A_j \neq 1_X$. Then $\bigwedge_{j\in F} A^c{}_j \neq 0_X$. Hence $\{A^c{}_j : j \in J\}$ satisfies the finite intersection property. Then from hypothesis we have $\bigwedge_{j\in J} A^c{}_j \neq 0_X$ which implies $\bigvee_{j\in J} A_j \neq 1_X$ and this contradicting that $\{A_j : j \in J\}$ is a fuzzy α-open cover of X. Then X is fuzzy α-compact.

Theorem 3.18 An ftsX is fuzzy α-compact if and only if every fuzzy net in X has an α-Q-adherent point.

Proof :

$\Longrightarrow$ Let X be a fuzzy α-compact space. If possible, let $\{\mathcal{S}(n): n \in D\}$,be a fuzzy net in X which has no α-Q-adherent point. For each fuzzy point x_r, there is a fuzzy α-q-nbdV_{x_r} of x_r and an $n_{V_{x_r}} \in D$, such that $x^m_{r_m} \tilde{q} V_{x_r}$ for all $m \in D$ with $m \geq n_{V_{x_r}}$. Let $\mathcal{V}$ denote the collection of all fuzzy α- q-nbd of $x_r$such V_{x_r}, where x_r runs over all fuzzy points in X. Now, we will prove that the collection $\mathcal{W}=\{1 - V_{x_r}: V_{x_r} \in \mathcal{V}\}$ is a family of fuzzy α-closed sets in X possessing finite intersection property. In fact, let $\mathcal{W}_0 = \{ 1 - V_{x^i_{r_i}}: i = 1,2,\ldots,m\}$ be a finite subfamily of $\mathcal{W}$. Then there exists $k \in D$, such that $k \geq n_{V_{x^i_{r_i}}},\ldots,n_{V_{x^m_{r_m}}}$ and so $x^p_{r_p} \tilde{q} V_{x^i_{r_i}}$ for $i = 1,2,\ldots,m$ and for all $p \geq k$ $(p \in D)$, i.e., $x^p_{r_p} \in 1 - \bigvee^m_{i=1} V_{x^i_{r_i}} = \bigwedge^m_{i=1}(1 - V_{x^i_{r_i}})$ for all $p \geq k$. Hence $\bigwedge \mathcal{W}_0 \neq 0_X$. Since Xis fuzzy α-compact, by Theorem 3.17, there exists a fuzzy point y_s in X, such that $y_s \in \bigwedge\{1 - V_{x_r}: V_{x_r} \in \mathcal{V}\} = 1 - \bigvee\{V_{x_r}: V_{x_r} \in \mathcal{V}\}$.

Thus $y_s \in 1 - V_{x_r}$, for all $V_{x_r} \in \mathcal{V}$, and hence in particular, $y_s \in 1 - V_{y_s}$ i.e., $y_s \tilde{q} V_{y_s}$. But by construction, for each fuzzy point x_r there exists a $V_{x_r} \in \mathcal{V}$, such that $x_r q V_{x_r}$, and we arrive a contradiction.

$\Longleftarrow$ Let $\mathcal{A} = \{A_j : j \in J\}$ be a family of fuzzy α-closed sets with finite intersection property. Let $D = \{\bigwedge_{j \in J_0} A_j : J_0 \in J$ and J_0 is finite$\}$. Then $\mathcal{A} \subseteq D$. For each $\lambda_j \in D$ let us choose a fuzzy point $x^{\lambda_j}_{r_{\lambda_j}}$ and consider the fuzzy net $\mathfrak{F} = \{x^{\lambda_j}_{r_{\lambda_j}} : \lambda_j \in D\}$ with the directed set $(D, \succcurlyeq)$, where for $\lambda_1, \lambda_2 \in D$, $\lambda_1 \succcurlyeq \lambda_2$ iff $\lambda_1 \leq \lambda_2$. By hypothesis, $\mathfrak{F}$ has an α-Q-adherente point x_r. Let $B \in N^{Q\alpha}_{x_r}$ and $A_j \in \mathcal{A}$. Since $A_j \in D$, there is $\lambda \in D$ with $\lambda \succcurlyeq A_j$ (that is $\lambda \leq A_j$) such that $x_\lambda q B$. As $x_\lambda \leq \lambda \leq A_j$ and hence $A_j q B$. Thus $x_r \in \overline{A_j}^{\alpha} = A_j$, for $j \in J_0$ and so $\bigwedge_{j \in J} A_j \neq 0_X$. Hence by Theorem 3.17, X is fuzzy α-compact.

Proposition 3.19 An ftsX is fuzzy α-compact if and only if each fuzzy net in X has anα-Q-convergent fuzzy subnet.

Proof :

It follows from Theorem 3.18 and Proposition 2.17.

Proposition3.20 In any ftsX, the intersection of any fuzzy α-closed set with any fuzzy α-compact set is fuzzy α-compact.

Proof :

Let A be a fuzzy α-closed set and B be a fuzzy α-compact set in X. Suppose that $\{x^n_{r_n}\}_{n \in D}$ be a fuzzy net in $A \wedge B$, then $\{x^n_{r_n}\}_{n \in D}$ in A and B. Since B is fuzzy α-compact, then by Proposition 3.19, $\{x^n_{r_n}\}_{n \in D}$ has an α-Q-convergent fuzzy subnet. Since A is a fuzzy α-closed set in X, then $x_r \in \overline{A}^{\alpha} = A$, therefore $\{x^n_{r_n}\}_{n \in D}$ be a fuzzy net in $A \wedge B$ which has an α-Q-convergent fuzzy subnet, hence $A \wedge B$ is fuzzy α-compact.

Corollary3.21A fuzzy α-closed subset of a fuzzy α- compact space is fuzzy α-compact.

Proof :

Let A be a fuzzy α-closed set in a fuzzy α-compact space X. Since $A \wedge 1_X = A$, then by Proposition 3.20, A is a fuzzy α-compact set in X.

Proposition 3.22 Let X be a fuzzy α-compact and fuzzy T_2-space and A be a fuzzy set in X. Then A is fuzzy compact if and only if A is fuzzy α-compact.

Proof :
⟹ Let A be a fuzzy compact set in X. Since X is a fuzzy T_2-space, then by Proposition 3.6, A is a fuzzy closed set in X, and then it's a fuzzy α-closed set in X. Since X is a fuzzy α-compact space, then by Corollary 3.21, A is a fuzzy α-compact set in X.
⟸ By Proposition 3.16.

Proposition 3.23 Let $f: (X, T) \longrightarrow (Y, T')$ be a mapping.If f is fuzzy α-irresolute, then an image f(A) of any fuzzy α-compact set A in X is a fuzzy α-compact subset of Y.
Proof :
Let $\Omega = \{G_j : j \in J\}$ be a family of fuzzy α-open set cover of f(A). Since f is fuzzy α-irresolute, then $\{f^{-1}(G_j) : j \in J\}$ is a fuzzy α-open cover of A. Since A is a fuzzy α-compact set in X, there is a finite subfamily $\{f^{-1}(G_j) : j = 1,2,\ldots, n\}$, such that $A \leq \vee_{j\in J}^{n} f^{-1}(G_j)$ which implies $A \leq f^{-1}(\vee_{j\in J}^{n} G_j)$ and then $f(A) \leq f(\, f^{-1} (\vee_{j\in J}^{n} G_j)\,) \leq \vee_{j\in J}^{n} G_j$. Therefore f(A) is a fuzzy α-compact set in Y.

4. Fuzzy α-Compact Mapping

This section will contain the concept of fuzzy α-compact mapping and we give new results.

Definition 4.1 Let X and Y be fts's. We say that the mapping $f: X \longrightarrow Y$ is fuzzy compact if the inverse image of each fuzzy compact set in Y, is a fuzzy compact set in X.

Definition 4.2 Let X and Y be fts's. A mapping $f: X \longrightarrow Y$ is called fuzzy α-compact if the inverse image of each fuzzy α-compact set in Y, is a fuzzy compact set in X.

Remark 4.3 Every fuzzy compact mapping is fuzzy α-compact.
Proposition 4.4 Let X and Y be fts's, such that Y is a fuzzy α-compact and fuzzy T_2-space. A mapping $f: X \longrightarrow Y$ is fuzzy compact if and only if it is fuzzy α-compact.
Proof :
⟹ By Remark 4.3.
⟸ Let A be a fuzzy compact set in Y. Since Y is a fuzzy α-compact and fuzzy T_2-space, then by Proposition 3.22, A is a fuzzy α-compact set in Y, then $f^{-1}(A)$ is a fuzzy compact set in X. Hence f is a fuzzy compact mapping.

Proposition 4.5 Let X, Y and Z be fts's, and $f: X \longrightarrow Y$, $g: Y \longrightarrow Z$ be mappings. Then:
(a) If f is fuzzy compact and gis fuzzy α-compact, then $g \circ f$ is a fuzzy α-compact mapping.
(b) If $g \circ f$ is fuzzy α-compact, f is onto and fuzzy continuous, then g is fuzzy α-compact.
(c) If $g \circ f$ is fuzzy α-compact, g is one to one and fuzzy α-irresolute, then f is fuzzy α-compact.
Proof :
(a) Let A be a fuzzy α-compact set in Z, then $g^{-1}(A)$ is a fuzzy compact set in Y, and so $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$ is a fuzzy compact set in X. Hence $g \circ f: X \longrightarrow Z$ is a fuzzyα-compact mapping.
(b) Let A be a fuzzy α-compact set in Z, then $(g \circ f)^{-1}(A)$ is a fuzzy compact set in X and then $(f(g \circ f)^{-1})(A)$ is a fuzzy compact set in Y. Now, since f is onto, then $(f(g \circ f)^{-1})(A) = g^{-1}(A)$, hence $g^{-1}(A)$ is a fuzzy compact set in Y. Therefore g is a fuzzy α-compact mapping.

(c) Let A be a fuzzy α-compact set in Y, then by Proposition 3.23, $g(A)$ is a fuzzy α-compact set in Z, thus $(g \circ f)^{-1}(g(A))$ is a fuzzy compact set in X. Since g is one to one, then$(g \circ f)^{-1}(g(A)) = f^{-1}(A)$, hence $f^{-1}(A)$ is a fuzzy compact set in X. Thus f is a fuzzy α-compact mapping.

5. Fuzzy α-Coercive Mapping

This section will contain the definition of a fuzzy α-coercive mapping and the relation between fuzzy α-compact mapping and fuzzy α-coercive mapping.

Definition 5.1 Let X and Y be fts's. A mapping $f: X \longrightarrow Y$ is called fuzzy coercive if for every fuzzy compact set B in Y, there exists a fuzzy compact set A in X such that $f(1_X \backslash A) \leq (1_Y \backslash B)$.

Definition 5.2 Let X and Y be fts's. A mapping $f: X \longrightarrow Y$ is called fuzzy α-coercive if for every fuzzy α-compact set B in Y, there exists a fuzzy compact set A in X such that$f(1_X \backslash A) \leq (1_Y \backslash B)$.

Example 5.3 If X is a fuzzy compact space, then the mapping $f: X \longrightarrow Y$ is fuzzy α-coercive. Let B be a fuzzy α-compact set in Y. Since X is fuzzy compact and$f(1_X \backslash 1_X) = f(0_X) = 0_Y \leq (1_Y \backslash B)$, then f is a fuzzy α-coercive mapping.

Proposition 5.4 Let $f: X \longrightarrow Y$ be a fuzzy coercive mapping and $g: Y \longrightarrow Z$ be a fuzzy α-coercive mapping, then $g \circ f: X \longrightarrow Z$ is a fuzzy α-coercive mapping.

Proof :

Let C be a fuzzy α-compact set in Z. Since g is a fuzzy α-coercive mapping, then there exists a fuzzy compact set B in Y, such that $g(1_Y \backslash B) \leq 1_Z \backslash C$. Since f is a fuzzy coercive mapping, then there exists a fuzzy compact set A in X, such that $f(1_X \backslash A) \leq (1_Y \backslash B)$, then $g(f(1_X \backslash A)) \leq g(1_Y \backslash B) \leq 1_Z \backslash C$, hence $(g \circ f)(1_X \backslash A) \leq 1_Z \backslash C$. Thus $g \circ f$ is a fuzzy α-coercive mapping.

Proposition 5.5 Every fuzzy coercive mapping is fuzzy α-coercive.

Proof :

Let $f: X \longrightarrow Y$ be a fuzzy coercive mapping, and B be a fuzzy α-compact set in Y, so it is a fuzzy compact set, since f is fuzzy coercive, then there exists a fuzzy compact set A in X, such that$f(1_X \backslash A) \leq 1_Y \backslash B$. Hence f is a fuzzy α-coercive mapping.

Proposition 5.6 Let X and Y be fts's, such that Y is a fuzzy α-compact and fuzzy T_2-space. Then $f: X \longrightarrow Y$ is a fuzzy coercive mapping if and only if it is fuzzy α-coercive.

Proof :

$\Longrightarrow$ By Proposition 5.5.

$\Longleftarrow$ Let B be a fuzzy compact set in Y. Since Y is a fuzzy α-compact and fuzzy T_2-space, then by Proposition 3.22, B is a fuzzy α-compact set in Y, since f is a fuzzy α-coercive mapping, then there exists a fuzzy compact set A in X, such that $f(1_X \backslash A) \leq 1_Y \backslash B$. Hence f is a fuzzy coercive mapping.

Proposition 5.7 Every fuzzy α-compact mapping is fuzzy α-coercive.

Proof :

Let $f: X \longrightarrow Y$ be a fuzzy α-compact mapping. To prove that f is a fuzzy α-coercive mapping. Let B be a fuzzy α-compact set in Y. Since f is fuzzy α-compact, then $f^{-1}(B)$ is a fuzzy compact set in X. Thus $f(1_X \backslash f^{-1}(B)) \leq 1_Y \backslash B$. Hence $f: X \longrightarrow Y$ is a fuzzy α-coercive mapping.

Proposition 5.8 Let X and Y be fts's, such that Y is a fuzzy T_2-space, and $f: X \longrightarrow Y$ be a fuzzy continuous mapping. Then f is fuzzy α-compact if and only if f is fuzzy α-coercive mapping.

Proof :

$\Longrightarrow$ By Proposition 5.7.

$\Longleftarrow$ Let B be a fuzzy α-compact set in Y, so it is fuzzy compact. To prove that $f^{-1}(B)$ is a fuzzy compact set in X. Since Y is a fuzzy T_2-space and f is a fuzzy continuous mapping, then by Proposition 3.6 and Proposition 1.6, $f^{-1}(B)$ is a fuzzy closed set in X. Since f is a fuzzy α-coercive mapping, then there exists a fuzzy compact set A in X, such that $f(1_X \backslash A) \leq 1_Y \backslash B$. Then $f(A^c) \leq B^c$, therefore $f^{-1}(B) \leq A$, then by Corollary 3.5, $f^{-1}(B)$ is a fuzzy compact set in X. Hence f is a fuzzy α-compact mapping.

6. Fuzzy α-Proper Mapping

This section will contain the definition of fuzzy α-proper mapping and addition to studying relation among fuzzy α-proper mapping, fuzzy α-compact mapping and fuzzy α-coercive mapping.

Definition 6.1 Let X and Ybe fts's. A fuzzy continuous mapping $f: X \longrightarrow Y$ iscalled fuzzy proper if:

1. f is fuzzy closed.
2. $f^{-1}(\{y_r\})$ is fuzzy compact, for each $y_r \in FP(Y)$.

Definition 6.2 Let X and Ybe fts's. A fuzzy continuous mapping $f: X \longrightarrow Y$ iscalled fuzzy α-proper if:

1. f is fuzzy α-closed.
2. $f^{-1}(\{y_r\})$ is fuzzy compact, for each $y_r \in FP(Y)$.

Remark 6.3 Every fuzzy proper mapping is fuzzy α-proper.

The converse of Remark 6.3, need not be true as the following example.

Example 6.4 Let $X = \{a, b\}$ and $Y = \{x, y, z\}$ be sets, and $T = \{0_X, 1_X, \{a_{0.3}\}, \{b_{0.5}\}, \{a_{0.3}, b_{0.5}\}, \{a_{0.4}, b_{0.6}\}\}$ and $T' = \{0_Y, 1_Y, \{x_{0.6}\}\}$ be fuzzy topologies on X and Y, respectively. Let $f: X \longrightarrow Y$ be a mapping which is defined by: $f(a) = y$, $f(b) = z$. Notice that:
$f^{-1}(0_Y) = 0_X \in T$, $f^{-1}(1_Y) = 1_X \in T$, $f^{-1}(\{x_{0.6}\}) = 0_X \in T$.
Thus f is a fuzzy continuous mapping. Now, the fuzzy closed sets in X are: $0_X, 1_X, \{a_{0.7}, b_1\}, \{a_1, b_{0.5}\}, \{a_{0.7}, b_{0.5}\}, \{a_{0.6}, b_{0.4}\}$ and since:
$f(0_X) = 0_Y$ is a fuzzy α-closed set in Y.
$f(1_X) = \{y_1, z_1\}$ is a fuzzy α-closed set in Y.
$f(\{a_{0.7}, b_1\}) = \{y_{0.7}, z_1\}$ is a fuzzy α-closed set in Y.
$f(\{a_1, b_{0.5}\}) = \{y_1, z_{0.5}\}$ is a fuzzy α-closed set in Y.
$f(\{a_{0.7}, b_{0.5}\}) = \{y_{0.7}, z_{0.5}\}$ is a fuzzy α-closed set in Y.
$f(\{a_{0.6}, b_{0.4}\}) = \{y_{0.6}, z_{0.4}\}$ is a fuzzy α-closed set in Y.
Thus f is a fuzzy α-closed mapping. Since X and Y are finite fuzzy spaces, then $f^{-1}(\{u_r\})$ is a fuzzy compact set in X, for all $u_r \in FP(Y)$. Therefore f is a fuzzy α-proper mapping. But f is not a fuzzy proper mapping, since 1_X is a fuzzy closed set in X, but $f(1_X) = \{y_1, z_1\}$ is not a fuzzy closed set in Y. Hence f is not a fuzzy closed mapping, and then f is not a fuzzy proper mapping.

Proposition 6.5 Let $f: X \longrightarrow Y$ be a fuzzy proper mapping and $g: Y \longrightarrow Z$ be a fuzzy α-proper mapping, then $g \circ f: X \longrightarrow Z$ is a fuzzy α-proper mapping.

Proof :

Since f and g are fuzzy continuous mappings, then $g \circ f$ is fuzzy continuous. Also, since f is a fuzzy proper mapping, then f is fuzzy closed. Similarly, since g is a fuzzy α-proper mapping, then g is fuzzy α-closed. Thus by Proposition 2.9, $g \circ f$ is a fuzzy α-closed mapping. Let $z_r \in FP(Z)$, since g is a fuzzy α-proper mapping, then $g^{-1}(\{z_r\})$ is a fuzzy compact set in Y, since f is a fuzzy proper mapping, then $f^{-1}\left(g^{-1}(\{z_r\})\right)$ is a fuzzy compact set in X. But $\left(g \circ f\right)^{-1}(\{z_r\})\big) = f^{-1}\left(g^{-1}(\{z_r\})\right)$, hence $(g \circ f)^{-1}(\{z_r\})$ is a fuzzy compact set in X, then $g \circ f$ is a fuzzy α-proper mapping.

Proposition 6.6 Let $f: X \longrightarrow Y$ and $g: Y \longrightarrow Z$ be fuzzy continuous mappings, such that $g \circ f: X \longrightarrow Z$ is a fuzzy α-proper mapping. If f is onto, then g is a fuzzy α-proper mapping.

Proof :

Let A be a fuzzy closed set in Y, since f is fuzzy continuous, then $f^{-1}(A)$ is a fuzzy closed set in X. Since $g \circ f$ is a fuzzy α-proper mapping, then $(g \circ f)(f^{-1}(A))$ is a fuzzy α-closed set in Z. But f is onto. Thus g is a fuzzy α-closed mapping. Let $z_r \in FP(Z)$, since $g \circ f$ is a fuzzy α-proper mapping, then $(g \circ f)^{-1}(\{z_r\}) = f^{-1}(g^{-1}(\{z_r\}))$ is a fuzzy compact set in X. Now, since f is fuzzy continuous and onto, then $f\left(f^{-1}\left(g^{-1}(\{z_r\})\right)\right) = g^{-1}(\{z_r\})$ is fuzzy compact in Y, for each $z_r \in FP(Z)$. So g is a fuzzy α-proper mapping.

Proposition 6.7 Let $f: X \longrightarrow Y$ and $g: Y \longrightarrow Z$ be mappings, such that $g \circ f: X \longrightarrow Z$ is a fuzzy α-proper mapping. If f is a fuzzy continuous mapping and g is a one to one and fuzzy α-irresolute mapping. Then f is a fuzzy α-proper mapping.

Proof :

Let A be a fuzzy closed set in X, since $g \circ f$ is a fuzzy α-proper mapping, then $(g \circ f)(A)$ is a fuzzy α-closed set in Z, since g is fuzzy α-irresolute, then by Proposition 2.11, $g^{-1}(g \circ f)(A)$ is a fuzzy α-closed set in Y. But g is one to one, then $g^{-1}(g \circ f)(A) = f(A)$. Hence $f(A)$ is a fuzzy α-closed set in Y. Thus f is a fuzzy α-closed mapping. Let $y_r \in FP(Y)$, $g(y_r) = (g(y))_r = z_r \in FP(Z)$. Now, since $g \circ f: X \longrightarrow Z$ is fuzzy α-proper and g is a one to one mapping, then the fuzzy set $(g \circ f)^{-1}(\{z_r\}) = (g \circ f)^{-1}\left(g(\{y_r\})\right) = f^{-1}(g^{-1}(g(\{y_r\}) = f^{-1}(\{y_r\})$ is fuzzy compact in X. Thus the mapping $f: X \longrightarrow Y$ is fuzzy α-proper.

Proposition 6.8 If $f: X \longrightarrow Y$ is a fuzzy α-proper mapping, then f is a fuzzy α-compact mapping.

Proof :

Let A be a fuzzy α-compact subset of Y and let $\{V_\lambda\}_{\lambda \in \Lambda}$ be a fuzzy open cover of $f^{-1}(A)$. Put $B = \{x_r \in FP(Y): A(x) = r\}$. Since f is a fuzzy α-proper mapping, then $f^{-1}(\{x_r\})$ is a fuzzy compact set, $\forall\, x_r \in B$.

But $f^{-1}(x_r) \le f^{-1}(A) \le \vee_{\lambda \in \Lambda} V_\lambda$, thus there exists $\lambda_1, \lambda_2, \ldots, \lambda_n$, such that $f^{-1}(x_r) \le \vee_{i=1}^{n} V_{\lambda_i}$, let $V_{\lambda_{x_r}} = \vee_{i=1}^{n} V_{\lambda_i}$, so for all $x_r \in B$, $f^{-1}(x_r) \le V_{\lambda_{x_r}}$. Notice that, for all $x_r \in B$, $f^{-1}(x_r)\tilde{q}(1_X - V_{\lambda_{x_r}})$, hence $x_r \tilde{q} f(1_X - V_{\lambda_{x_r}})$, therefore $x_r \in 1_Y - (f(1_X - V_{\lambda_{x_r}}))$, then $A \le \vee_{x_r \in B}(1_Y - (f(1_X - V_{\lambda_{x_r}})))$. Since $V_{\lambda_{x_r}}$ is a fuzzy open set, then $1_X - V_{\lambda_{x_r}}$ is a fuzzy closed set, thus $f(1_X - V_{\lambda_{x_r}})$ is a fuzzy α-closed set, then $1_Y - (f(1_X - V_{\lambda_{x_r}}))$ is a fuzzy α-open set. Since A is a fuzzy α-compact set in Y and $\vee_{x_r \in B}(1_Y - (f(1_X - V_{\lambda_{x_r}}))$ is a fuzzy α-open cover of A, i.e., $A \leq \vee_{x_r \in B}(1_Y - (f(1_X - V_{\lambda_{x_r}})))$, then there exists$y^1_{r_1}, y^2_{r_2}, \ldots, y^m_{r_m}$ in A, such that: $A \leq \vee_{i=1}^{m}(1_Y - (f(1_X - V_{\lambda_{y^i_{r_i}}})))$, so $f^{-1}(A) \leq \vee_{i=1}^{m} V_{\lambda_{y^i_{r_i}}}$. Therefore $f^{-1}(A)$ is a fuzzy compact set in X. Hence the mapping $f: X \longrightarrow Y$ is a fuzzy α-compact mapping.

The converse of Proposition 6.8, is not true in general as in the following example.

Example 6.9 Let X ={a, b} and Y ={x,y} be sets and $T = \{0_X, 1_X, \{a_{0.4}\}, \{a_{0.5}, b_{0.7}\}\}$ and $T' = \{0_Y, 1_Y, \{x_{0.4}\}\}$ be fuzzy topologies on X and Y, respectively. Let $f: X \to Y$ be a mapping which is defined by:$f(a) = x$, $f(b) = y$.
Since X and Y are finite fuzzy spaces, then $f^{-1}(A)$ is a fuzzy compact set in X, for each fuzzy α-compact subset A of Y. Hence $f: X \longrightarrow Y$ is a fuzzy α-compact mapping.
Notice that f is a fuzzy continuous mapping. But f is not fuzzy α-closed, since $\{a_{0.5}, b_{0.3}\}$ is a fuzzy closed set in X, but $f(\{a_{0.5}, b_{0.3}\}) = \{x_{0.5}, y_{0.3}\}$ is not a fuzzy α-closed set in Y. Hence $f: X \longrightarrow Y$ is not a fuzzy α-proper mapping.

Theorem 6.10 Let X and Y be fts's, such that Y be a fuzzy T_2-space and fuzzy sc-space. If $f: X \longrightarrow Y$ is a fuzzy continuous mapping. Then f is fuzzy α-proper if and only if f is fuzzy α- compact.
Proof :
⟹ By Proposition 6.8.
⟸Let F be a fuzzy closed subset of X. To prove that $f(F)$ is a fuzzy α-closed set in Y, let K be a fuzzy α-compact set in Y, then $f^{-1}(K)$ is a fuzzy compact set in X, thus by Proposition 3.4, $F \wedge f^{-1}(K)$ is a fuzzy compact set. Since f is fuzzy continuous , then by Proposition 3.3, $f(F \wedge f^{-1}(K))$ is a fuzzy compact set in Y. But $f(F \wedge f^{-1}(K)) = f(F) \wedge K$, then $f(F) \wedge K$ is a fuzzy compact set in Y. Therefore $f(F)$ is a fuzzy compactly closed set in Y. Since Y is a fuzzy T_2-space, then by Theorem 3.10, f(F) is a fuzzy closed set in Y and so it is fuzzy α-closed. Hence f is a fuzzy α-closed mapping. Let $y_r \in FP(Y)$, then $\{y_r\}$ is fuzzy α-compact in Y. Since f is a fuzzy α-compact mapping, then $f^{-1}(\{y_r\})$ is fuzzy compact in X. Therefore f is a fuzzy α-proper mapping.

Theorem 6.11 Let X and Y be fts's, such that Y is a fuzzy T_2-space and fuzzy sc-space. If $f: X \longrightarrow Y$ is a fuzzy continuous mapping. Then the following statements are equivalent:
(a) f is a fuzzy α-coercive mapping.
(b) f is a fuzzy α-compact mapping.
(c) f is a fuzzy α-proper mapping.
Proof :
(a)⟹(b) By Proposition 5.8.
(b)⟹(c) By Proposition 6.10.
(c)⟹(a) Since f is fuzzy α-proper mapping, then by Proposition 6.8, f is a fuzzy α-compact mapping, hence by Proposition 5.7, $f: X \longrightarrow Y$ is a fuzzy α-coercive mapping.

References:

[1] Abdullah, H. K. and Kareem, N. R., " Fuzzy Compact and Coercive Mappings ", Journal of Kerbala University, 10, 2012, 111-121.
[2] Chang, C.L., " Fuzzy Topological Spaces ", J. Math. Anal. Appl., 24, 1968, 182-190.
[3] Nouh, A. A., " On Convergence Theory in Fuzzy Topological Spaces and Its Applications ", Czechoslovak Mathematical Journal, 55(2), 2005, 295-316.
[4] Othman, H.A. and Latha, S., " New Results of Fuzzy Alpha- Open Sets Fuzzy Alpha-Continuous Mappings ", Int. J. Contemp. Math. Sciences, 4(29), 1415-1422.
[5] Pinchuck, A., " Extension Theorems on L-Topological Spaces and L-Fuzzy Vector Spaces ", M. Sci. Thesis, Rhodes University., 2001.
[6] Srivastava, R., Lal, S. N. and Srivastava, A. K., " Fuzzy Hausdorff Topological Spaces ", Banaras Hindu University, J. Math. Anal. Appl., 81, 1981, 497-506.
[7] Tang, X., " Spatial Object Modeling in Fuzzy Topological Spaces with Applications to Land Cover Change in China ", Ph. D. dissertation, University of Twente , Enscheda, The Netherlands, 2004.
[8] Warren, R. H.," Neighborhoods, Bases and Continuity in Fuzzy Topological Spaces ", Nebraska University, The Rocky Mountain Journal of Mathematics, 8(3), 1978, 459-470.
[9] Zadeh, L. A., " Fuzzy Sets ", Information and Control, 8, 1965, 338-353.