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Solving General Differential Equations of Fractional Orders Via Rohit Transform.

Rohit Gupta [1*], Rahul Gupta [2], Dinesh Verma [3]

[1*] Department of Applied Sciences (Physics), Yogananda College of Engineering and Technology, Jammu, J&K, India
[2]Department of Physics, G.D. Goenka Public School., Jammu, J&K, India.
[3] Department of Mathematics, NIILM University, Kaithal, Haryana, India.
*Corresponding author : guptarohit565@gmail.com

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Abstract

Fractional calculus is a mathematical branch inspecting the attributes of derivatives and integrals of fractional orders known as fractional derivatives and integrals. In this article, a far-out complex integral transform known as the Rohit transform (RT) is put into use for working out general homogeneous and non-homogeneous differential equations of fractional or non-integral orders of particular forms entailing the Caputo fractional derivative operator and the Riemann-Liouville fractional derivative operator. The Rohit transform (RT) of the Mittag-Leffler function, the Caputo fractional derivative operator. and the Riemann-Liouville fractional derivative operator are obtained, and then the solutions of fractional systems characterized by fractional homogeneous and fractional non-homogeneous differential equations entailing the Caputo fractional derivative operator and the Riemann-Liouville fractional derivative operator are obtained by utilizing the Rohit transform (RT). This article showcases the ability and efficacy of the Rohit transform (RT) to straighten out fractional systems characterized by fractional homogeneous and fractional non-homogeneous differential equations. While other methods, present in the literature such as the homotopy-perturbation method, Adomian decomposition method, fractional variational iteration method, Lyapunov direct method, and generalized Mittag Leffler stability, may also be capable of solving the examples presented in the paper, the Rohit transform introduced innovative concepts or methodologies that offer new insights or perspectives on the problems examined in the paper, distinguishing itself from existing transforms and potentially opening up new research directions. The simplicity and ease of implementation of the Rohit transform make it a preferable choice for practical applications, requiring fewer computational steps, less complex algorithms, and simpler parameter tuning.

1. Introduction:

Differential equations of fractional or non-integral orders characterize the mathematical models in science, economics, finance, and engineering [1]. Some notable fields where differential equations of fractional or non-integral orders find utility include: Physics (equipping of anomalous diffusion, visco-elasticity and systems with memory effects), Biology (explanation of population dynamics, biological processes, and complex physiological systems), Engineering (electrical networks, control theory of dynamical systems, materials with non-local systems), fluid mechanics, electro-chemistry of corrosion, probability and statistics, signal processing (deducing non-local signal behaviors and processing techniques in complex systems), Medicine (equipping biological systems, drug discharge kinetics, and physiological processes with memory effects), Economics and Finance (equipping of market behavior, price dynamics, and economic systems with memory effects) [2, 3, 4, 5]. Differential equations of fractional or non-integral orders provide a way to illustrate phenomena that entail memory, hereditary attributes, and non-local behaviors and offer a more precise illustration of various real-life processes and systems. Fractional calculus is a mathematical branch inspecting the attributes of derivatives and integrals of fractional orders known as fractional derivatives and integrals [6]. In the previous decades, more and more researchers have paid attention to fractional calculus since they found that fractional-order derivatives and fractional-order integrals were more suitable for the description of phenomena in the real world, such as visco-elastic systems, dielectric polarization, electromagnetic waves, heat conduction, robotics, biological systems, and so on, [7, 8, 9, 10, 11]. Owing to the great efforts of researchers, there have been rapid developments in the theory of fractional calculus and its applications, including well-posedness, stability, bifurcation, and chaos in fractional differential equations and their control. In general, this discipline entails the techniques of solving the differential equations entailing fractional derivatives of the unknown function called differential equations of fractional or non-integral orders. There are many ways, such as Laplace transform [12, 13, 14], homotopy perturbation method [15], Adomian decomposition method [16], variational iteration method [17], differential transform method [18], and others, to solve these differential equations of fractional or non-integral orders, to which Laplace transform is frequently applied. For instance, in [19, 20], the authors investigated the stability of fractional-order nonlinear dynamical systems using the Laplace transform method and the Lyapunov direct method, with the introduction of Mittag-Leffler stability and generalized Mittag-Leffler stability concepts. In [21], Deng et al. studied the stability of n-dimensional linear fractional differential equations with time delays using the Laplace transform method. However, the Rohit transform (RT) has not been sufficiently brought to bear in such differential equations of fractional or non-integral orders due to its recent appearance. The author Rohit Gupta has proffered the Rohit transform (RT) in recent years to expedite the process of solving differential equations. This transform has been successfully applied to solve many initial value problems in physical sciences and engineering [22, 23, 24, 25, 26, 27]. This article puts forth the Rohit transform (RT) for solving homogeneous and non-homogeneous differential equations of fractional or non-integral orders. The Rohit transform (RT) is defined [22, 23] for a function of exponential order by the integral equations as follows: $Rh(t) = q^3\int_0^\infty e^{-qt}h(t)dt, t\geq 0, q_1\leq q\leq q_2$. The variable q is used to factor the variable t into the argument of the function h. The Rohit transform (RT) of unidentified functions is given by

$$i. R\{t^n\} = q^3\int_0^\infty e^{-qt}t^n dt = \int_0^\infty e^{-z}\left(\frac{z}{q}\right)^n\frac{dz}{q}, z = qt$$

$$R\{t^n\} = \frac{q^2}{q^n}\int_0^\infty e^{-z}(z)^n dz = \frac{q^2}{q^n}\lceil(n+1) = \frac{q^2}{q^n}n! = \frac{n!}{q^{n-2}}$$

Hence, $R\{t^n\} = \frac{n!}{q^{n-2}}$

$$ii.\ R\{\ sinbt\} = q^3\int_0^\infty e^{-qt}sinbt\ dt = q^3\int_0^\infty e^{-qt}\left(\frac{e^{ibt}-e^{-ibt}}{2i}\right)dt$$

$$R\{\ sinbt\} = q^3\int_0^\infty\left(\frac{e^{-(q-ib)t}-e^{-(q+ib)t}}{2i}\right)dt$$

$$= -\frac{q^3}{2i(q-ib)}\left(e^{-\infty}-e^{-0}\right)+\frac{q^3}{2i(q+ib)}\left(e^{-\infty}-e^{-0}\right)$$

$$R\{\ sinbt\} = \frac{q^3}{2i(q-ib)}-\frac{q^3}{2i(q+ib)} = \frac{b\,q^3}{q^2+b^2}$$

Hence, $R\{sinbt\} = \frac{b\,q^3}{q^2+b^2}$

$$iii.\ R\{\ cosbt\} = q^3\int_0^\infty e^{-qt}cosbt\ dt = q^3\int_0^\infty e^{-qt}\left(\frac{e^{ibt}+e^{-ibt}}{2}\right)dt$$

$$R\{\ cosbt\} = q^3\int_0^\infty\left(\frac{e^{-(q-ib)t}+e^{-(q+ib)t}}{2}\right)dt$$

$$R\{\ cosbt\} = -\frac{q^3}{2(q-ib)}\left(e^{-\infty}-e^{-0}\right)$$
$$-\frac{q^3}{2(q+ib)}\left(e^{-\infty}-e^{-0}\right)$$
$$= \frac{q^3}{2(q-ib)}+\frac{q^3}{2(q+ib)} = \frac{q^4}{q^2+b^2}$$

Hence, $R\ \{cosbt\} = \frac{q^4}{q^2+b^2}$

$$iv.\ R\left\{e^{bt}\right\} = q^3\int_0^\infty e^{-qt}e^{bt}\,dt$$
$$= q^3\int_0^\infty\left(e^{-(q-b)t}\right)dt$$
$$= -\frac{q^3}{(q-b)}\left(e^{-\infty}-e^{-0}\right) = \frac{q^3}{(q-b)}$$

Hence, $R\{e^{bt}\} = \frac{q^3}{q-b}$

The Rohit transform (RT) of some derivatives [24, 25] is given by: Let g(t) be a piecewise continuous function in some interval, then the Rohit transform (RT) of $g'(t)$ is given by

$$R\ \{g'(t)\} = q^3\int_0^\infty e^{-qt}g'(t)\,dt$$

Integrating by parts and applying limits, we have

$$R\ \{g'(t)\} = q^3\left[g(0) - \int_0^\infty -qe^{-qt} g(t)\, dt\right]$$

$$= q^3\left[-g(0) + q\int_0^\infty e^{-qt} g(t)\, dt\right]$$

$$R\ \{g'(t)\} = qRg(t) - q^3 g(0)$$

Hence $R\ \{g'(t)\} = qG(q) - q^3 g(0)$

By replacing $g(t)$by $g'(t)$ and $g'(t)$ by $g''(t)$, we have

$$R\{g''(t)\} = qR\{g'(t)\} - q^3 g'(0)$$

$$= q\{qR\{g(t)\} - q^3 g(0)\} - q^3 g'(0)$$

$$R\{g''(t)\} = q^2 R\{g(t)\} - q^4 g(0) - q^3 g'(0)$$

$$= q^2 G(q) - q^4 g(0) - q^3 g'(0)$$

Hence, $R\{g''(t)\} = q^2 G(q) - q^4 g(0) - q^3 g'(0)$

Similarly, $R\{g'''(t)\} = q^3 G(q) - q^5 g(0) - q^4 g'(0) - q^3 g''(0)$.

In general, $R\{g^n(t)\} = q^n R\{g(t)\} - \sum_{k=1}^{n} q^{n-k+3} g^{k-1}(0)$

The Rohit transform (RT) of convolution,
i.e. $R\{(f*g)(t)\} = \frac{1}{q^3} F(q)G(q)$.

Proof: Since $(f*g)(t) = \int_0^t f(t-x)\, g(x)\, dx$,, therefore,

$$R\{(f*g)(t)\} = q^3 \int_0^\infty e^{-qt} (f)(t)\ dt$$

$$R\{(f*g)(t)\} = q^3 \int_0^\infty e^{-qt} \int_0^t f(t-x)\, g(x)\ dx\, dt$$

$$R\{(f*g)(t)\} = q^3 \int_0^\infty \int_0^t e^{-qt} f(t-x)\, g(x)\ dx\, dt$$

By altering the order of integration, the above equation becomes

$$R\{(f*g)(t)\} = q^3 \int_0^\infty \int_t^\infty e^{-qt} f(t-x)\, g(x)\ dt\, dx$$

$$R\{(f*g)(t)\} = q^3 \int_0^\infty e^{-qx} g(x)\, dx \int_x^\infty e^{-q(t-x)} f(t-x)\ dt$$

Let t-x=y, then the above equation becomes

$$R\{(f*g)(t)\} = q^3 \int_0^\infty e^{-qx} g(x)\, dx \int_0^\infty e^{-qy} f(y)\ dy$$

$$R\{(f*g)(t)\} = \frac{1}{q^3}\left[q^3 \int_0^\infty e^{-qx} g(x)\, dx\right]\left[q^3 \int_0^\infty e^{-qy} f(y)\, dy\right]$$

$$R\{(f*g)(t)\} = \frac{1}{q^3} G(q)F(q)$$

The article is organized as follows:

Firstly, brief information on a special function known as the Mittag-Leffler function and fractional operators such as the Caputo fractional derivative operator and the Riemann-Liouville fractional derivative operator and their attributes is provided.

Secondly, the Rohit transform (RT) of the Mittag-Leffler function, Caputo fractional derivative operator, and Riemann-Liouville fractional derivative operator are obtained.

Thirdly, the solutions of fractional systems characterized by fractional homogeneous and fractional non-homogeneous differential equations entailing the Caputo fractional derivative operator and the Riemann-Liouville fractional derivative operator are obtained by applying the Rohit transform (RT), and the graphs of some solutions are plotted.

Finally, the conclusions are presented.

2. Special Functions and Their Attributes:

The exponential function e^x can be written in the form of a series as follows:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{\Gamma(n+1)}$$

This exponential function e^x is very significant in the theory of integer-order differential equations. The modifications of this function, so-called Mittag-Leffler functions, play a significant role in the theory of differential equations [4, 5] of fractional or non-integral orders (FDEs).

The Mittag-Leffler function, with two parameters, is defined as follows:

$$E_{a,\,b}\ (t) = \sum_{n=0}^{\infty} \frac{t^n}{\Gamma(an+b)}$$

Here t belongs to the complex plane, $a > 0$, b belongs to real numbers, and Γ is the gamma function. For the particular values of the parameters a and b, we find well-known classical functions. For example,

- $E_{0,\,1}\ (t)=\sum_{n=0}^{\infty}t^{n}=\frac{1}{1-t},\ |t|<1$
- $E_{1,\,1}\ (t)\ =\sum_{n=0}^{\infty}\frac{t^{n}}{\Gamma(n+1)}=e^{t}$
- $E_{1,\,2}\ (t)\ =\sum_{n=0}^{\infty}\frac{t^{n}}{\Gamma(n+2)}=\frac{(e^{t}-1)}{t}$
- $E_{2,\,1}\ (-t^{2})\ =\sum_{n=0}^{\infty}\frac{(-t^{2})^{n}}{\Gamma(2n+1)}=cost$
- $E_{2,\,2}\ (-t^{2})\ =\sum_{n=0}^{\infty}\frac{(-t^{2})^{n}}{\Gamma(2n+2)}=\frac{sint}{t}$
- $E_{2,\,2}\ (t^{2})\ =\sum_{n=0}^{\infty}\frac{t^{2n}}{\Gamma(2n+2)}=\frac{sinht}{t}$
- $E_{1,\,1}\ (t^{2})\ =\sum_{n=0}^{\infty}\frac{t^{2n}}{\Gamma(2n+1)}=cosht$
- $E_{\frac{1}{2}\ ,\,1}\ (t)\ =\sum_{n=0}^{\infty}\frac{t^{n}}{\Gamma(\frac{1}{2}n+1)}=\ e^{t^{2}}\ erfc(-t)$

The Riemann-Liouville fractional integral [4] of order α is put into words as:

$$ {}_{\alpha 0}I_{x}^{\alpha}f(x)=\frac{1}{\Gamma(\alpha)}\int_{\alpha_{0}}^{\alpha}(x-t)^{\alpha-1}f(t)\,dt,\ \ \alpha>0 $$

Some of the attributes of the Riemann-Liouville fractional integral are given by:

- ${}_{\alpha 0}I_{x}^{0}f(x)=f(x)$
- ${}_{\alpha 0}I_{x}^{\alpha}\left({}_{\alpha 0}I_{x}^{\beta}f(x)\right)={}_{\alpha 0}I_{x}^{\alpha+\beta}f(x)$
- ${}_{0}I_{x}^{\alpha}(C)=\frac{C}{\Gamma(\alpha+1)}x^{\alpha},\ \ \alpha>0$
- ${}_{0}I_{x}^{\alpha}(x^{n})=\frac{\Gamma(n+1)}{\Gamma(n+\alpha+1)}x^{n+\alpha},\ \ \alpha,(n+1)>0$
- ${}_{-\infty}I_{x}^{\alpha}\left(e^{kx}\right)=\frac{e^{kx}}{k^{\alpha}},\ \ \alpha,\ k>0$
- ${}_{-\infty}I_{x}^{\alpha}(sinkx)=k^{\alpha}\sin\left(kx-\frac{\alpha\pi}{2}\right),\ \ \alpha>0$
- ${}_{-\infty}I_{x}^{\alpha}(coskx)=k^{\alpha}\cos\left(kx-\frac{\alpha\pi}{2}\right),\ \ \alpha,k>0$

The Riemann-Liouville fractional derivative with order α is put into words as:

$$ {}_{\alpha 0}^{RL}D_{x}^{\alpha}f(x)=\left(\frac{d}{dx}\right)^{n}{}_{\alpha 0}I_{x}^{n-\alpha}f(x) $$

$$ =\frac{1}{\Gamma(n-\alpha)}\left(\frac{d}{dx}\right)^{n}\int_{\alpha_{0}}^{\alpha}(x-t)^{n-\alpha-1}f(t)\,dt, $$

where $\alpha>0$ and $n-1<\alpha\leq n$.

Some of the attributes of the Riemann-Liouville fractional derivative are given by

- ${}_{0}^{RL}D_{x}^{\alpha}(C)=\frac{x^{-\alpha}}{\Gamma(1-\alpha)}\ ,\qquad \alpha>0$
- ${}_{0}^{RL}D_{x}^{\alpha}(x^{n})=\frac{\Gamma(n+1)}{\Gamma(n-\alpha+1)}x^{n-\alpha}\ ,\ \ \alpha,(n+1)>0$
- ${}_{-\infty}^{RL}D_{x}^{\alpha}\left(e^{kx}\right)=k^{\alpha}e^{kx},\qquad \alpha,\ k>0$
- ${}_{-\infty}^{RL}D_{x}^{\alpha}(sinkx)=k^{\alpha}\sin\left(kx+\frac{\alpha\pi}{2}\right),\ \ \alpha>0$
- ${}_{-\infty}^{RL}D_{x}^{\alpha}(coskx)=k^{\alpha}\cos\left(kx+\frac{\alpha\pi}{2}\right),\ \ \alpha>0$

The Riemann-Liouville fractional derivative [5] with order α is put into words as:

$$ {}_{\alpha 0}^{C}D_{x}^{\alpha}f(x)=\left(\frac{d}{dx}\right)^{n}{}_{\alpha 0}I_{x}^{n-\alpha}f(x) $$

$$ =\frac{1}{\Gamma(n-\alpha)}\left(\frac{d}{dx}\right)^{n}\int_{\alpha_{0}}^{\alpha}(x-t)^{n-\alpha-1}f(t)\,dt, $$

where$\alpha>0$ and $(n-1)<\alpha\leq n$.

Some of the attributes of the Caputo fractional derivative are given by:

- ${}_{0}^{C}D_{x}^{\alpha}(C)=0\ ,\ \ \alpha>0$
- ${}_{0}^{C}D_{x}^{\alpha}(x^{n})=\frac{\Gamma(n+1)}{\Gamma(n-\alpha+1)}x^{n-\alpha}\ ,\ \ \alpha,(n+1)>0$
- ${}_{0}^{C}D_{x}^{\alpha}\left(e^{kx}\right)=k^{n}x^{n-\alpha}E_{1,n-\alpha+1}\ (kx),\ \ \alpha,\ k>0$
- ${}_{0}^{C}D_{x}^{\alpha}(sinkx)=-\frac{i}{2}(ik)^{n}x^{n-\alpha}[E_{1,n-\alpha+1}\ (ikx)-(-1)^{n}$ $E_{1,n-\alpha+1}\ (-ikx)],\alpha>0$

3. Rohit Transform (RT) of Special Functions:

i.
$$ R\left\{{}_{0}I_{x}^{\alpha}f(x)\right\}=R\left\{\frac{1}{\Gamma(\alpha)}\int_{0}^{\alpha}(x-t)^{\alpha-1}f(t)\,dt\right\} $$
$$ =R\left\{\frac{1}{\Gamma(\alpha)}(x)^{\alpha-1}(x)\right\} $$
$$ =\frac{1}{\Gamma(\alpha)}\frac{1}{q^{3}}R\left\{x^{\alpha-1}\right\}R\left\{f(x)\right\} $$
$$ =\frac{1}{\Gamma(\alpha)}\frac{1}{q^{3}}\frac{\Gamma(\alpha)}{q^{\alpha-1-2}}F(cq) $$
$$ =q^{-\alpha}F(q) $$

Hence, $R_{\mathbf{0}}I_x^{\alpha} f(x) = q^{-\alpha} F(q)$

ii. $R_0^{RL}D_x^{\alpha} f(x) = R\left(\frac{d}{dx}\right)^n {}_0I_x^{n-\alpha} f(x)$

Since $R\{h^n(t)\} = q^n R\{h(t)\} - \sum_{k=1}^{n} q^{n-k+3} h^{k-1}(0)$

Or $R\{h^n(t)\} = q^n R\{h(t)\} - \sum_{k=0}^{n-1} q^{n-k+2} h^k(0)$

Or $R\{h^n(t)\} = q^n R\{h(t)\} - \sum_{k=0}^{n-1} q^{k+3} h^{n-k-1}(0)\, , therefore,$

$$R_0^{RL}D_x^{\alpha} f(x) = R\left(\frac{d}{dx}\right)^n {}_0I_x^{n-\alpha} f(x)$$

$$R_0^{RL}D_x^{\alpha} f(x) = q^n R\left\{{}_0I_x^{n-\alpha} f(x)\right\} - \sum_{k=0}^{n-1} q^{k+3} \left(\frac{d}{dx}\right)^{n-k-1} {}_0I_x^{n-\alpha} f(0)$$

$$R_0^{RL}D_x^{\alpha} f(x) = q^n q^{-n+\alpha} F(q) - \sum_{k=0}^{n-1} q^{k+3} \left(\frac{d}{dx}\right)^{n-k-1} {}_0D_x^{\alpha-n} f(0) \left[\because\ {}_0D_x^{\alpha-n} = {}_0I_x^{n-\alpha}\right]$$

$$R_0^{RL}D_x^{\alpha} f(x) = q^{\alpha} F(q) - \sum_{k=0}^{n-1} q^{k+3}\, {}_0D_x^{\alpha-k-1} f(0)$$

Hence

$$R\left\{{}_{\mathbf{0}}^{RL}D_x^{\alpha} f(x)\right\} = q^{\alpha} F(q) - \sum_{k=\mathbf{0}}^{n-\mathbf{1}} q^{k+\mathbf{3}}\, {}_{\mathbf{0}}^{RL}D_x^{\alpha-k-\mathbf{1}} f(\mathbf{0})$$

iii. $R_0^{C}D_x^{\alpha} f(x) = q^{\alpha} F(q) - \sum_{k=0}^{n-1} q^{\alpha-k+2} f^k(0)$

Let $g(x) = \left(\frac{d}{dx}\right)^n f(x)$.

Since $R_0I_x^{n-\alpha} f(x) = q^{-n+\alpha} F(q)$, therefore,

$$R_0I_x^{n-\alpha} g(x) = q^{-n+\alpha} G(q)$$

$$R_0I_x^{n-\alpha} g(x) = q^{-n+\alpha} R\left(\frac{d}{dx}\right)^n f(x)$$

$$R_0I_x^{n-\alpha} g(x) = q^{-n+\alpha}\left[q^n R\{f(t)\} - \sum_{k=0}^{n-1} q^{n-k+2} f^k(0)\right]$$

$$R_0I_x^{n-\alpha} g(x) = q^{\alpha} R\{f(t)\} - \sum_{k=0}^{n-1} q^{\alpha-k+2} f^k(0)$$

Now, $R_0^{C}D_x^{\alpha} f(x) = R\left(\frac{d}{dx}\right)^n {}_0I_x^{n-\alpha} f(x)$

$$R_0^{C}D_x^{\alpha} f(x) = R\left\{{}_0I_x^{n-\alpha} g(x)\right\}$$

$$R_0^{C}D_x^{\alpha} f(x) = q^{\alpha} F(q) - \sum_{k=0}^{n-1} q^{\alpha-k+2} f^k(0)$$

Hence, $R_{\mathbf{0}}^{C}D_x^{\alpha} f(x) = q^{\alpha} F(q) - \sum_{k=\mathbf{0}}^{n-\mathbf{1}} q^{\alpha-k+\mathbf{2}} f^k(\mathbf{0})$

iv. $R\left\{t^{b-1} E_{a,\,b}\,(\sigma t^a)\right\} = \frac{q^{a-b+3}}{q^{\alpha} - \sigma}$

Proof:
As

$$R\left\{t^{an+b-1}\right\} = q^3 \int_0^{\infty} e^{-qt} t^{an+b-1} dt$$

Put x = qt, we have

$$R\left\{t^{an+b-1}\right\} = q^2 \int_0^{\infty} e^{-x} \left(\frac{x}{q}\right)^{an+b-1} dx$$

$$R\left\{t^{an+b-1}\right\} = q^{-an-b+3} \int_0^{\infty} e^{-x} x^{an+b-1} dx$$

$$R\left\{t^{an+b-1}\right\} = q^{-an-b+3}\, \Gamma(an+b)$$

Also

$$\sum_{n=0}^{\infty} \sigma^n\, q^{-(n+1)a} = (q^{\alpha} - \sigma)^{-1}$$

Therefore,

$$R\left\{t^{b-1} E_{a,\,b}\,(\sigma t^a)\right\} = R\left\{t^{b-1} \sum_{n=0}^{\infty} \frac{(\sigma t^a)^n}{\Gamma(an+b)}\right\}$$

$$R\left\{t^{b-1} E_{a,\,b}\,(\sigma t^a)\right\} = \sum_{n=0}^{\infty} \frac{\sigma^n R\left\{t^{an+b-1}\right\}}{\Gamma(an+b)}$$

$$R\left\{t^{b-1} E_{a,\,b}\,(\sigma t^a)\right\} = \sum_{n=0}^{\infty} \frac{\sigma^n\, q^{-an-b+3}\, \Gamma(an+b)}{\Gamma(an+b)}$$

$$R\left\{t^{b-1} E_{a,\,b}\,(\sigma t^a)\right\} = \sum_{n=0}^{\infty} \sigma^n\, q^{-an-b+3}$$

$$R\left\{t^{b-1} E_{a,\,b}\,(\sigma t^a)\right\} = q^{a-b+3} \sum_{n=0}^{\infty} \sigma^n\, q^{-an-a}$$

$$R\left\{t^{b-1} E_{a,\,b}\,(\sigma t^a)\right\} = q^{a-b+3} \sum_{n=0}^{\infty} \sigma^n\, q^{-(n+1)a}$$

$$R\left\{t^{b-1} E_{a,\,b}\,(\sigma t^a)\right\} = q^{a-b+3}\, (q^a - \sigma)^{-1}$$

Hence, $\mathbf{R}\left\{t^{b-\mathbf{1}} E_{a,\,b}\,(\sigma t^a)\right\} = \frac{q^{a-b+\mathbf{3}}}{q^a - \sigma}$

4. Material and Method:

In this section, general differential equations of fractional or non-integral orders involving fractional operators such as the Caputo fractional derivative operator and the Riemann-Liouville fractional derivative operator are solved via the Rohit transform (RT).

Consider the fractional system involving the Caputo fractional derivative of the form:

$$ {}_0^CD_x^{\alpha} f(x) + \gamma f(x) = 0 \tag{1} $$

where $x > 0$ and $f^k(0) = C_k, k = 1, 2, 3, \ldots$

Solution: Taking the RT of equation (1), we get:

$$ R\left\{ {}_0^CD_x^{\alpha} f(x) \right\} + \gamma R\{f(x)\} = 0 \tag{2} $$

Since $R_0^CD_x^{\alpha} f(x) = q^{\alpha} F(q) - \sum_{k=0}^{n-1} q^{\alpha-k+2} f^k(0)$ *and* $(n-1) < \alpha < n$, therefore,

equation (2) becomes

$$ q^a F(q) - \sum_{k=0}^{n-1} q^{\alpha-k+2} C_k + \gamma F(q) = 0 $$

$$ F(q) = \frac{\sum_{k=0}^{n-1} q^{\alpha-k+2} C_k}{q^a + \gamma} \tag{3} $$

As $R^{-1}\left\{ \frac{q^{a-b+3}}{q^a - \sigma} \right\} = x^{b-1} E_{a,\,b}(\sigma x^a)$, applying inverse RT to equation (3), we get

$f(x) = C_k\, x^{k+1-1} E_{\alpha,\,k+1}(-\gamma x^{\alpha})$, where b = k+1

$$ f(x) = \sum_{k=0}^{n-1} C_k x^k\, E_{\alpha,\,k+1}(-\gamma x^{\alpha}) \tag{4} $$

The equation (4) illustrates the solution to equation (1).

Consider the fractional system involving the Caputo fractional derivative of the form:

$$ {}_0^CD_x^{\alpha} f(x) + \gamma f(x) = 0 \tag{5} $$

where x¿0, $f(0) = C$, and $0 < \alpha < 1$.

Solution:
Taking the RT of equation (5), we get

$$ R\left\{ {}_0^CD_x^{\alpha} f(x) \right\} + \gamma R\{f(x)\} = 0 \tag{6} $$

Since $R_0^CD_x^{\alpha} f(x) = q^{\alpha} F(q) - \sum_{k=0}^{n-1} q^{\alpha-k+2} f^k(0)$ and $(n-1) < \alpha < n$, therefore, equation (6) becomes

$$ q^a F(q) - \sum_{k=0}^{n-1} q^{\alpha-k+2} f^k(0) + \gamma F(q) = 0 $$

For n = 1,

$$ q^{\alpha} F(q) - \sum_{k=0}^{0} q^{\alpha-k+2} f^k(0) + \gamma F(q) = 0 $$

$$ q^{\alpha} F(q) - q^{\alpha+2} C + \gamma F(q) = 0 $$

$$ F(q) = \frac{C q^{\alpha+2}}{q^a + \gamma} \tag{7} $$

As $R^{-1}\left\{ \frac{q^{a-b+3}}{q^a - \sigma} \right\} = x^{b-1} E_{a,\,b}(\sigma x^a)$, applying inverse RT to equation (7), we get

$f(x) = C\, x^{1-1} E_{\alpha,\,1}(-\gamma x^{\alpha})$, where b =1

$$ f(x) = C\, E_{\alpha,\,1}(-\gamma x^{\alpha}) \tag{8} $$

The equation (8) illustrates the solution to equation (5).

For $\alpha = \frac{1}{2}, \gamma = 1$, and C=1, equation (8) becomes
$f(x) = E_{\frac{1}{2},\,1}\left(-x^{\frac{1}{2}}\right) = e^x\, erfc\left(x^{\frac{1}{2}}\right)$

$$ f(x) = e^x[\, erf\left(-x^{\frac{1}{2}}\right) + 1] \tag{9} $$

The graphs of equation (9) for different ranges of x are shown in Figures 1a to 1f.
Consider the fractional system involving the Caputo fractional derivative of the form:

$$ {}_0^CD_x^{\frac{1}{2}} f(x) - \gamma f(x) = 0 \tag{10} $$

where x¿0 and $f(0) = C$.

Solution:
Taking the RT of equation (10), we get

$$ R\left\{ {}_0^CD_x^{\frac{1}{2}} f(x) \right\} - \gamma R\{f(x)\} = 0 \tag{11} $$

Since $R_0^CD_x^{\alpha} f(x) = q^{\alpha} F(q) - \sum_{k=0}^{n-1} q^{\alpha-k+2} f^k(0)$ and $(n-1) < \alpha < n$, therefore, equation (11) becomes

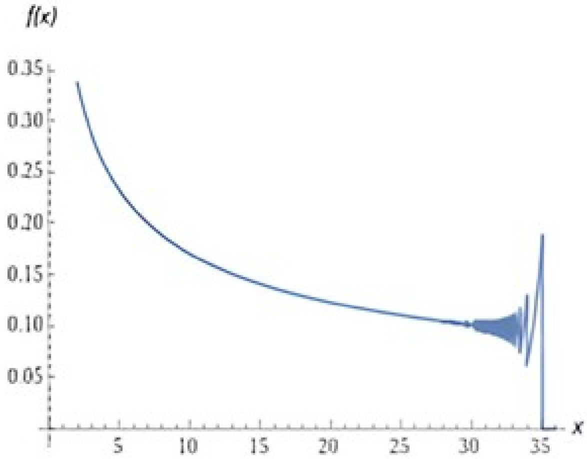

Figure 1a. The numerical solution of equation (5) for x ranges from 0.00 to 37.00.

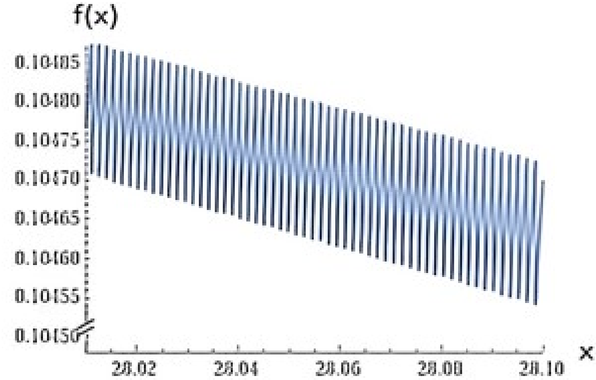

Figure 1d. The numerical solution of equation (5) for x ranges from 28.01 to 28.10.

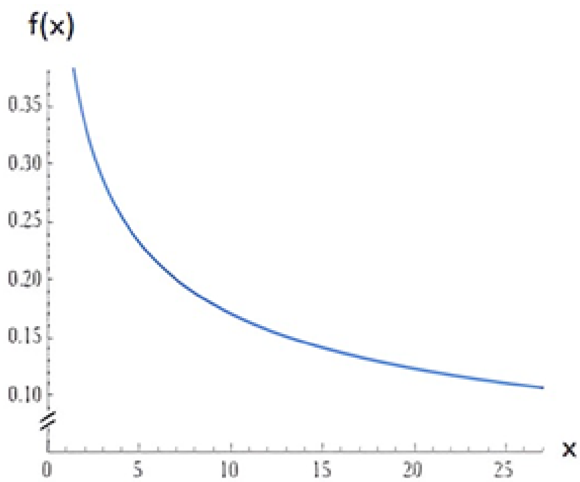

Figure 1b. The numerical solution of equation (5) for x ranges from 0.00 to 27.00.

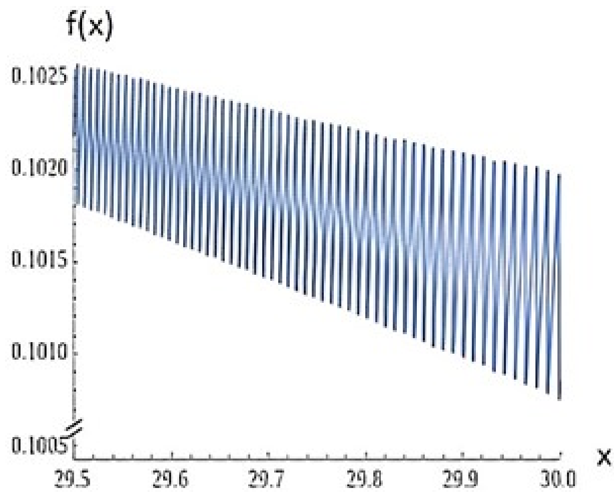

Figure 1e. The numerical solution of equation (5) for x ranges from 29.50 to 30.00.

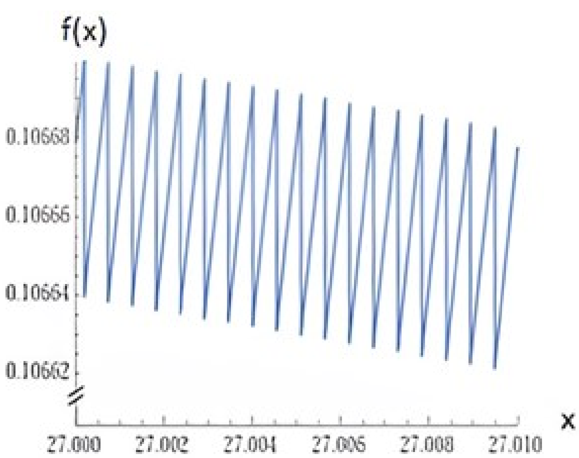

Figure 1c. The numerical solution of equation (5) for x ranges from 27.00 to 27.01.

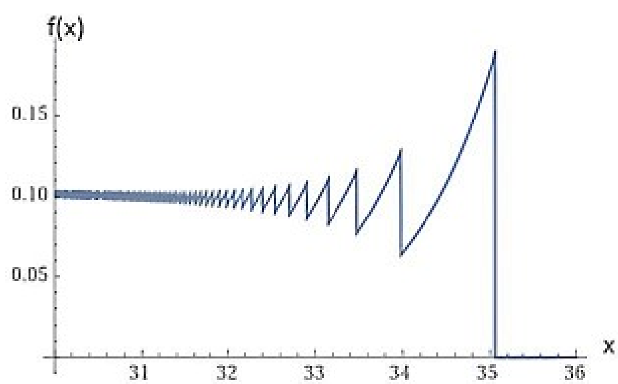

Figure 1f. The numerical solution of equation (5) for x ranges from 30.00 to 36.00.

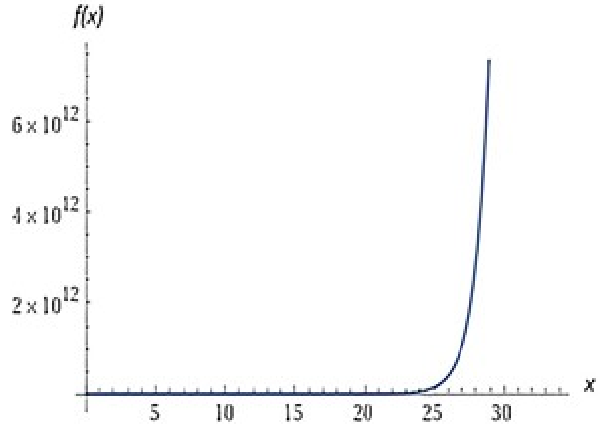

Figure 2. The numerical solution of equation (10).

$q^{\frac{1}{2}}F(q)-\sum_{k=0}^{n-1}q^{\frac{1}{2}-k+2}f^{k}(0)-\gamma F(q)=0$

$q^{\frac{1}{2}}F(q)-\sum_{k=0}^{0}q^{\frac{1}{2}-k+2}f^{k}(0)-\gamma F(q)=0,$

where n = 1

$q^{\frac{1}{2}}F(q)-q^{\frac{5}{2}}C-\gamma F(q)=0$

$$F(q)=\frac{Cq^{\frac{5}{2}}}{q^{\frac{1}{2}}-\gamma} \tag{12}$$

As $R^{-1}\left\{\frac{q^{a-b+3}}{q^{a}-\sigma}\right\}=x^{b-1}E_{a,\,b}\,(\sigma x^{a})$, applying inverse RT to equation (12), we get

$f(x)=C\,x^{1-1}E_{\frac{1}{2},\,1}\left(\gamma x^{\frac{1}{2}}\right)$, where $a=\frac{1}{2}$ and b=1

$$f(x)=C\,E_{\frac{1}{2},\,1}\left(\gamma x^{\frac{1}{2}}\right) \tag{13}$$

The equation (13) illustrates the solution to equation (10). The graph of equation (13) is shown in Figure 2.
Consider the fractional system involving the Caputo fractional derivative of the form:

$${}_{0}^{C}D_{x}^{\alpha}f(x)-\gamma f(x)=h(x) \tag{14}$$

where x¿0 and $f^{k}(0)=C_{k},\ k=1,\ 2,\ 3,\ \ldots$

Solution:
Taking the RT of equation (14), we get

$$R\left\{{}_{0}^{C}D_{x}^{\alpha}f(x)\right\}-\gamma R\{f(x)\}=h(x) \tag{15}$$

Since $R_{0}^{C}D_{x}^{\alpha}f(x)=q^{\alpha}F(q)-\sum_{k=0}^{n-1}q^{\alpha-k+2}f^{k}(0)$ and $(n-1)<\ \alpha<n$, therefore, equation (15) becomes $q^{a}F(q)-\sum_{k=0}^{n-1}q^{\alpha-k+2}C_{k}-\gamma F(q)=H(q)$

$$F(q)=\frac{\sum_{k=0}^{n-1}q^{\alpha-k+2}C_{k}}{q^{a}-\gamma}+\frac{H(q)}{q^{a}-\gamma} \tag{16}$$

As $R^{-1}\left\{\frac{q^{a-b+3}}{q^{n}-\sigma}\right\}=x^{b-1}E_{a,\,b}\,(\sigma x^{a})$, applying inverse RT to equation (16), we get

$$f(x)=R^{-1}\left\{\frac{\sum_{k=0}^{n-1}q^{\alpha-k+2}C_{k}}{q^{a}-\gamma}\right\}+R^{-1}\left\{\frac{H(q)}{q^{a}-\gamma}\right\}$$

$$f(x)=R^{-1}\left\{\frac{\sum_{k=0}^{n-1}q^{\alpha-k+2}C_{k}}{q^{a}-\gamma}\right\}+R^{-1}\left\{\frac{q^{3}H(q)}{q^{3}(q^{a}-\gamma)}\right\}$$

$$f(x)=R^{-1}\left\{\frac{\sum_{k=0}^{n-1}q^{\alpha-k+2}C_{k}}{q^{a}-\gamma}\right\}+R^{-1}\left\{\frac{1}{q^{3}}H(q).G(s)\right\} \tag{17}$$

where $G(s)=\frac{q^{3}}{q^{a}-\gamma}$

As $R^{-1}\left\{\frac{q^{a-b+3}}{q^{a}-\sigma}\right\}=x^{b-1}E_{a,\,b}\,(\sigma x^{a})$, therefore,

$R^{-1}\{G(s)\}=g(x)=\left\{\frac{q^{3}}{q^{a}-\gamma}\right\}=x^{a-1}E_{a,\,a}\,(\gamma x^{a})=x^{a-1}E_{a,\,a}\,(\gamma x^{a})$, where b = a

And $R^{-1}\left\{\frac{\sum_{k=0}^{n-1}q^{\alpha-k+2}C_{k}}{q^{a}-\gamma}\right\}=\sum_{k=0}^{n-1}C_{k}\,x^{k+1-1}E_{\alpha,\,k+1}\,(\gamma x^{\alpha})=\sum_{k=0}^{n-1}C_{k}\,x^{k}E_{\alpha,\,k+1}\,(\gamma x^{\alpha})$, where b = k + 1

Hence equation (17) becomes

$$f(x)=\sum_{k=0}^{n-1}C_{k}\,x^{k}E_{\alpha,\,k+1}\,(\gamma x^{\alpha})+R^{-1}\left\{\frac{H(q)\ G(s)}{q^{3}}\right\}$$

$$f(x)=\sum_{k=0}^{n-1}C_{k}x^{k}\,E_{\alpha,\,k+1}\,(\gamma x^{\alpha})+h(x)*g(x) \tag{18}$$

Since $g(x)*h(x)=\int_{0}^{x}g(x-t)^{h}(t)\,dt=\int_{0}^{x}(x-t)^{\alpha-1}E_{a,\,a}\,(\gamma(x-t)^{a})\,h(t)\,dt$, therefore, equation (18) becomes

$$f(x)=\sum_{k=0}^{n-1}C_{k}x^{k}\,E_{\alpha,\,k+1}\,(\gamma x^{\alpha})+\int_{0}^{x}(x-t)^{\alpha-1}E_{a,\,a}\,(\gamma(x-t)^{a})\,h(t)\,dt \tag{19}$$

The equation (19) illustrates the solution to equation (14).

Consider the fractional system involving the Caputo fractional derivative of the form:

$$ {}_0^C D_x^{\frac{1}{2}} f(x) - \gamma f(x) = x^2, \ \ x > 0 \tag{20} $$

Solution:
Taking the RT of equation (20), we get

$$ R\left\{ {}_0^C D_x^{\frac{1}{2}} f(x) \right\} - \gamma R\{f(x)\} = Rx^2 \tag{21} $$

Since $R_0^C D_x^{\alpha} f(x) = q^{\alpha} F(q) - \sum_{k=0}^{n-1} q^{\alpha-k+2} f^k(0)$ and$(n-1) < \alpha < n$, therefore, equation (21) becomes

$q^{\frac{1}{2}} F(q) - \sum_{k=0}^{n-1} q^{\frac{1}{2}-k+2} f^k(0) - \gamma F(q) = 2$
$q^{\frac{1}{2}} F(q) - \sum_{k=0}^{0} q^{\frac{1}{2}-k+2} f^k(0) - \gamma F(q) = 2$
where $\alpha = 1/2$ and n = 1
$q^{\frac{1}{2}} F(q) - q^{\frac{5}{2}} C - \gamma F(q) = 2$

$$ F(q) = \frac{Cq^{\frac{5}{2}}}{q^{\frac{1}{2}} - \gamma} + \frac{2q^0}{q^{\frac{1}{2}} - \gamma} \tag{22} $$

As $R^{-1}\left\{ \frac{q^{a-b+3}}{q^a - \sigma} \right\} = x^{b-1} E_{a,\,b}\,(\sigma x^a)$, applying inverse RT to equation (22), we get

$$ f(x) = C\,x^{1-1} E_{\frac{1}{2},\ 1}\left(\gamma x^{\frac{1}{2}}\right) + 2x^{\frac{1}{2}+3-1} E_{\frac{1}{2},\ \frac{1}{2}+3}\left(\gamma x^{\frac{1}{2}}\right) $$

$$ f(x) = C\,E_{\frac{1}{2},\ 1}\left(\gamma x^{\frac{1}{2}}\right) + 2x^{\frac{5}{2}} E_{\frac{1}{2},\ \frac{7}{2}}\left(\gamma x^{\frac{1}{2}}\right) \tag{23} $$

The equation (23) illustrates the solution to equation (20). The graph of equation (23) is shown in Figure 3.

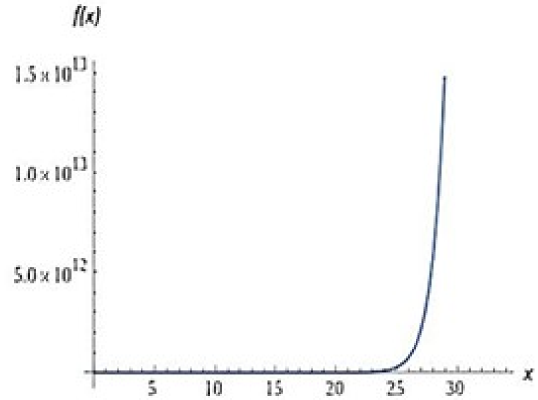

Figure 3. The numerical solution of equation (20).

$$ {}_0^C D_x^{\frac{1}{2}} f(x) + \gamma f(x) = x^2, \ \ x > 0 \tag{24} $$

Solution:
Taking the RT of equation (24), we get

$$ R\left\{ {}_0^C D_x^{\frac{1}{2}} f(x) \right\} + \gamma R\{f(x)\} = Rx^2 \tag{25} $$

Since $R_0^C D_x^{\alpha} f(x) = q^{\alpha} F(q) - \sum_{k=0}^{n-1} q^{\alpha-k+2} f^k(0)$ and $(n-1) < \ \alpha < n$, therefore, equation (25) becomes
$q^{\frac{1}{2}} F(q) - \sum_{k=0}^{n-1} q^{\frac{1}{2}-k+2} f^k(0) + \gamma F(q) = 2$
$q^{\frac{1}{2}} F(q) - \sum_{k=0}^{0} q^{\frac{1}{2}-k+2} f^k(0) + \gamma F(q) = 2$
where $\alpha = 1/2$and n = 1
$q^{\frac{1}{2}} F(q) - q^{\frac{5}{2}} C + \gamma F(q) = 2$

Consider the fractional system involving the Caputo fractional derivative of the form:

$$ F(q) = \frac{Cq^{\frac{5}{2}}}{q^{\frac{1}{2}} + \gamma} + \frac{2q^0}{q^{\frac{1}{2}} + \gamma} \tag{26} $$

As $R^{-1}\left\{ \frac{q^{a-b+3}}{q^a - \sigma} \right\} = x^{b-1} E_{a,\,b}\,(\sigma x^a)$, applying inverse RT to equation (26), we get

$$ f(x) = C\,x^{1-1} E_{\frac{1}{2},\ 1}\left(-\gamma x^{\frac{1}{2}}\right) + 2x^{\frac{1}{2}+3-1} E_{\frac{1}{2},\ \frac{1}{2}+3}\left(-\gamma x^{\frac{1}{2}}\right) $$

$$ f(x) = C\,E_{\frac{1}{2},\ 1}\left(-\gamma x^{\frac{1}{2}}\right) + 2x^{\frac{5}{2}} E_{\frac{1}{2},\ \frac{7}{2}}\left(-\gamma x^{\frac{1}{2}}\right) \tag{27} $$

The equation (27) illustrates the solution to equation (24). For C = 1 and$\gamma = 1$, the graphs of equation (27) for different ranges of x are shown in Figures 4a to 4c.

$$ f''(x) + d\,{}_0^C D_x^{\alpha} f(x) + b f(x) = 0 \tag{28} $$

where $1 < \alpha < 2, \ \ x > 0, \ \ f(0) = C_1$ and$'(0) = C_2$

Solution:
Taking the RT of equation (28), we have
$q^2 F(q) - \ q^4 f(0) - \ q^3 f'(0) + d\left[q^{\alpha} F(q) - \ sum_{k=0}^{n-1} q^{\alpha-k+2} f^k(0) \right] + bF(q) = 0$
$\left(q^2 + dq^{\alpha} + b\right) F(q) - \ q^4 C_1 - {}^3C_2 - d\sum_{k=0}^{1} q^{\alpha-k+2} f^k(0) = 0$
$\left(q^2 + dq^{3+\alpha} + b\right) F(q) - \ q^4 C_1 - {}^3C_2 - dq^{\alpha+2} C_1 - dq^{\alpha+1} C_2 = 0$

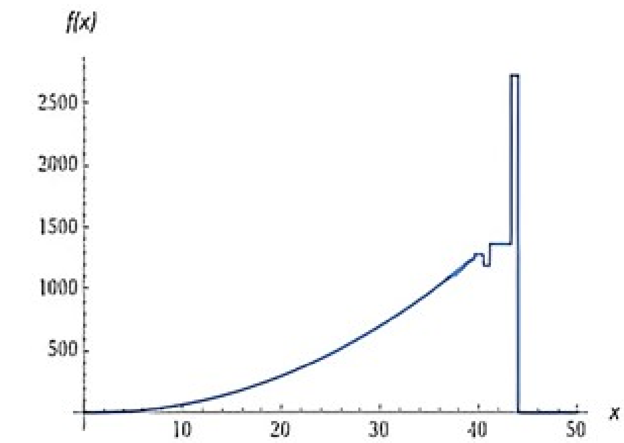

Figure 4a. The numerical solution of equation (24) for x ranges from 0.00 to 50.00.

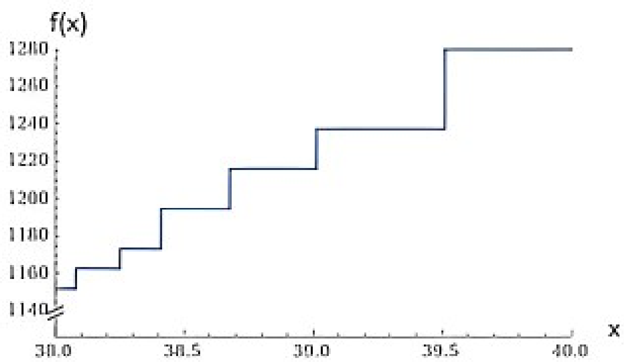

Figure 4b. The numerical solution of equation (1) for x ranges from 38.00 to 40.00.

Consider the fractional system involving the Caputo fractional derivative of the form:

$$F\left(q\right)=\frac{q^4C_1+\ q^3C_2+dq^{\alpha+2}C_1+dq^{\alpha+1}C_2}{(q^2+dq^{\alpha}+b)} \tag{29}$$

Now, let us simplify the term: $\frac{1}{(q^2+dq^{3+\alpha}+b)}$ as follows:

$$\frac{1}{(q^2+dq^{3+\alpha}+b)}=\frac{q^{-\alpha}}{(q^{2-\alpha}+d+bq^{-\alpha})}$$

$$\frac{1}{(q^2+dq^{\alpha}+b)}=\frac{q^{-\alpha}}{(q^{2-\alpha}+d)\left(1+\frac{bq^{-\alpha}}{q^{2-\alpha}+d}\right)}$$

$$\frac{1}{(q^2+dq^{\alpha}+b)}=\frac{q^{-\alpha}}{(q^{2-\alpha}+d)}\frac{1}{\left(1-\frac{-bq^{-\alpha}}{q^{2-\alpha}+d}\right)}$$

$$\frac{1}{(q^2+dq^{\alpha}+b)}=\frac{q^{-\alpha}}{(q^{2-\alpha}+d)}\sum_{k=0}^{\infty}\left(\frac{-bq^{-\alpha}}{q^{2-\alpha}+d}\right)^k$$

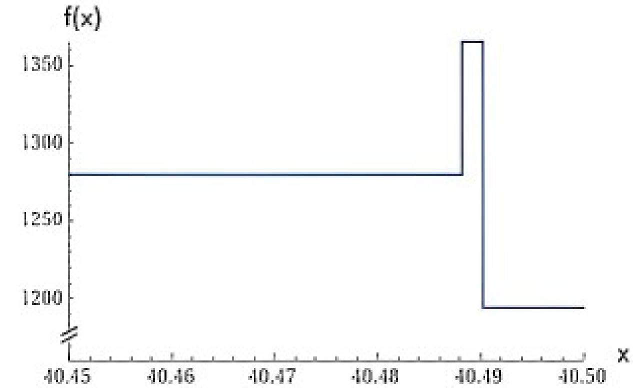

Figure 4c. The numerical solution of equation (1) for x ranges from 40.45 to 40.50.

$$\frac{1}{(q^2+dq^{\alpha}+b)}=\sum_{k=0}^{\infty}(-b)^k\frac{q^{-\alpha k-\alpha}}{\left(q^{2-\alpha}+d\right)^{k+1}}$$

$$\frac{1}{(q^2+dq^{\alpha}+b)}=\sum_{k=0}^{\infty}(-b)^k\frac{q^{-\alpha k-\alpha}}{\left(q^{2-\alpha}\right)^{k+1}\left(1+dq^{\alpha-2}\right)^{k+1}}$$

$$\frac{1}{(q^2+dq^{\alpha}+b)}=\sum_{k=0}^{\infty}(-b)^k\frac{q^{-\alpha k-\alpha}\left(q^{2-\alpha}\right)^{-k-1}}{\left(1+dq^{\alpha-2}\right)^{k+1}}$$

$$\frac{1}{(q^2+dq^{\alpha}+b)}=\sum_{k=0}^{\infty}(-b)^k\frac{q^{-\alpha k-\alpha}q^{-2k+\alpha k-2+\alpha}}{\left(1+dq^{\alpha-2}\right)^{k+1}}$$

$$\frac{1}{(q^2+dq^{\alpha}+b)}=\sum_{k=0}^{\infty}(-b)^k\frac{q^{-2k-2}}{\left(1+dq^{\alpha-2}\right)^{k+1}}$$

$$\frac{1}{(q^2+dq^{\alpha}+b)}=\sum_{k=0}^{\infty}(-b)^k q^{-2k-2}\sum_{r=0}^{\infty}\binom{k+1+r-1}{r}\left(-dq^{\alpha-2}\right)^r$$

$$\frac{1}{(q^2+dq^{\alpha}+b)}=\sum_{k=0}^{\infty}(-b)^k\sum_{r=0}^{\infty}\binom{k+r}{r}(-d)^r(q^{\alpha r-2r-2k-2}) \tag{30}$$

Using equation (30) in (29), we get

$$F\left(q\right)=\sum_{k=0}^{\infty}(-b)^k\sum_{r=0}^{\infty}\binom{k+r}{r}(-d)^r (q^{\alpha r-2r-2k-2})(\ q^4C_1+q^3C_2+dq^{\alpha+2}C_1+dq^{\alpha+1}C_2)$$

$$F\left(q\right)=\sum_{k=0}^{\infty}(-b)^k\sum_{r=0}^{\infty}\binom{k+r}{r}(-d)^r)(q^{\alpha r-2r-2k+2})C_1 +(q^{\alpha r-2r-2k+1})C_2+d(q^{\alpha r+\alpha-2r-2k})C_1 +d(q^{\alpha r+\alpha-2r-2k-1})C_2 \tag{31}$$

As $R^{-1}\left(\frac{1}{q^{n-2}}\right) = R^{-1}\left(q^{2-n}\right) = \frac{x^n}{\Gamma(n+1)}$, or $R^{-1}\left(q^{z}\right) = \frac{x^{2-z}}{\Gamma(3-z)}$, applying inverse RT to equation (**??**), we have

$$f(x) = \sum_{k=0}^{\infty}(-b)^k \sum_{r=0}^{\infty}\binom{k+r}{r}(-d)^r\Big(\frac{x^{2r-\alpha r+2k}}{\Gamma(2r-\alpha r+2k+1)}C_1 + \frac{x^{2r-\alpha r+2k+1}}{\Gamma(2r-\alpha r+2k+2)}C_2 + \frac{dx^{2r-\alpha r-\alpha+2k+2}}{\Gamma(2r-\alpha r-\alpha+2k+3)}C_1 + \frac{dx^{2r-\alpha r-\alpha+2k+3}}{\Gamma(2r-\alpha r-\alpha+2k+4)}\ C_2\Big)$$

$$f(x) = \sum_{k=0}^{\infty}(-b)^k \sum_{r=0}^{\infty}\frac{(k+r)!}{r!k!}(-d)^r\Big(\frac{x^{2r-\alpha r+2k}}{\Gamma(2r-\alpha r+2k+1)}C_1 + \frac{dx^{2r-\alpha r-\alpha+2k+2}}{\Gamma(2r-\alpha r-\alpha+2k+3)}C_1 + \frac{x^{2r-\alpha r+2k+1}}{\Gamma(2r-\alpha r+2k+2)}C_2 + \frac{dx^{2r-\alpha r-\alpha+2k+3}}{\Gamma(2r-\alpha r-\alpha+2k+4)}\ C_2\Big) \tag{32}$$

The equation (32) illustrates the solution to equation (28).

Consider the fractional system involving the Caputo fractional derivative of the form:

$${}_{0}^{C}D_{x}^{\alpha} f(x) + {}_{0}^{C}D_{x}^{\beta} f(x) = h(x) \tag{33}$$

where $x > 0,\ 0 < \alpha < \beta < 1$ and $f(0) = C$.

Solution:
Taking the RT of ${}_{0}^{C}D_{x}^{\alpha} f(x) + {}_{0}^{C}D_{x}^{\beta} f(x) = h(x)$, we get

$$R\left\{{}_{0}^{C}D_{x}^{\alpha} f(x)\right\} + R\left\{{}_{0}^{C}D_{x}^{\beta} f(x)\right\} = Rh(x) \tag{34}$$

Since $R{}_{0}^{C}D_{x}^{\alpha} f(x) = q^{\alpha}F(q) - \sum_{k=0}^{n-1} q^{\alpha-k+2} f^{k}(0)$
and $(n-1) <\ \alpha < n$, therefore, equation (34) becomes

$q^{a}F(q) - \sum_{k=0}^{n-1} q^{\alpha-k+2} f^{k}(0) + q^{\beta}F(q) - \sum_{k=0}^{n-1} q^{\beta-k+2} f^{k}(0) = H(q)$

$q^{a}F(q) - \sum_{k=0}^{0} q^{\alpha-k+2} f^{k}(0) + q^{\beta}F(q) - \sum_{k=0}^{0} q^{\beta-k+2} f^{k}(0) = H(q)$

$q^{a}F(q) - q^{\alpha+2}C + q^{\beta}F(q) - q^{\beta+2}C = H(q)$

$$F(q) = \frac{(q^{\alpha+2}+q^{\beta+2})C}{q^{a}+q^{\beta}} + \frac{H(q)}{q^{a}+q^{\beta}}$$

$$F(q) = q^{2}C + \frac{H(q)}{q^{a}+q^{\beta}} \tag{35}$$

As $R^{-1}\left\{\frac{q^{a-b+3}}{q^{a}-\sigma}\right\} = x^{b-1}E_{a,\,b}\ (\sigma x^{a})$, applying inverse RT to equation (35), we get

$$f(x) =\ R^{-1}\left\{q^{2}C\right\} +\ R^{-1}\left\{\frac{H(q)}{q^{a}+q^{\beta}}\right\}$$

$$f(x) = C +\ R^{-1}\left\{\frac{1}{q^{3}}H(q).G(s)\right\} \tag{36}$$

where $G(s) = \frac{q^{3}}{q^{a}+q^{\beta}} = \frac{q^{3-a}}{q^{\beta-a}+1}$

Since:
$R^{-1}\{G(s)\} = g(x) = \left\{\frac{q^{3-\beta}}{q^{a-\beta}-1}\right\} = x^{\beta-1}E_{\beta-a,\beta}\ \left(-x^{\beta-a}\right)$

and $g(x) * h(x) = \int_0^x g(x-t)^{h}(t)\,dt = \int_0^x (x-t)^{\beta-1}E_{\beta-a},$
$\beta\ \left(-(x-t)^{\beta-a}\right)h(t)\,dt,$
therefore, equation (**??**) becomes

$$f(x) = C + \int_0^x (x-t)^{\beta-1}E_{\beta-a,\quad\beta}\ \left(-(x-t)^{\beta-a}\right)h(t)\,dt \tag{37}$$

The equation (37) illustrates the solution to equation (33).

Consider the general fractional system involving the Riemann-Liouville fractional derivative of the form:

$${}_{0}^{RL}D_{x}^{\alpha} f(x) - \gamma f(x) = 0 \tag{38}$$

where $x > 0$ and ${}_{0}^{RL}D_{x}^{\alpha-k-1} f(0) = C_k,\ \ k = 1,\ 2,\ 3, \ldots.$.
Solution:
Taking the RT of equation (38), we get

$$R\left\{{}_{0}^{RL}D_{x}^{\alpha} f(x)\right\} - \gamma R\{f(x)\} = 0 \tag{39}$$

Since $R{}_{0}^{RL}D_{x}^{\alpha} f(x) = q^{\alpha}F(q) - \sum_{k=0}^{n-1} q^{k+3}{}_{0}^{RL}D_{x}^{\alpha-k-1} f(0)$

and $(n-1) <\ \alpha < n$, therefore, equation (39) becomes

$q^{a}F(q) - \sum_{k=0}^{n-1} q^{k+3}C_k - \gamma\,F(q) = 0$

$$F(q) = \frac{\sum_{k=0}^{n-1} q^{k+3}C_k}{q^{a}-\gamma} \tag{40}$$

As $R^{-1}\left\{\frac{q^{a-b+3}}{q^{a}-\sigma}\right\} = x^{b-1}E_{a,\,b}\ (\sigma x^{a})$, applying inverse RT to equation (40), we get

$$f(x) = \sum_{k=0}^{n-1} C_k x^{a-k-1} E_{\alpha,\,a-k}\ (\gamma t^{\alpha}) \tag{41}$$

where b = a-k

The equation (41) illustrates the solution to equation (38).

Consider the general fractional system involving the Riemann-Liouville fractional derivative of the form:

$${}_{0}^{RL}D_{x}^{\alpha} f(x) - \gamma f(x) = 0 \tag{42}$$

where $x > 0$, α belongs to (0, 1) and ${}_{0}^{RL}D_{x}^{\alpha-1} f(0) = C$.

Solution:
Taking the RT of equation (42), we get

$$R\left\{{}_{0}^{RL}D_{x}^{\alpha} f(x)\right\} - \gamma R\{f(x)\} = 0 \tag{43}$$

Since $R_{0}^{RL}D_{x}^{\alpha} f(x) = q^{\alpha}F(q) - \sum_{k=0}^{n-1} q^{k+3}{}_{0}^{RL}D_{x}^{\alpha-k-1} f(0)$ and $(n-1) < \ \alpha < n$, therefore, equation (43) becomes

$$q^{a}F(q) - \sum_{k=0}^{n-1} q^{k+3}{}_{0}^{RL}D_{x}^{\alpha-k-1} f(0) - \gamma F(q) = 0$$

Since n = 1, therefore,

$$q^{3}F(q) - \sum_{k=0}^{0} q^{k+3}{}_{0}^{RL}D_{x}^{\alpha-k-1} f(0) - \gamma F(q) = 0$$

$q^{a}F(q) - q^{3}\ {}_{0}^{RL}D_{x}^{\alpha-1} f(0) - \gamma\ F(q) = 0$ $q^{a}F(q) - q^{3}\ C - \gamma F(q) = 0$

$$F(q) = \frac{q^{3}\ C}{q^{a} - \gamma} \tag{44}$$

As $R^{-1}\left\{\frac{q^{a-b+3}}{q^{a}-\sigma}\right\} = x^{b-1}E_{a,\,b}\ (\sigma x^{a})$, applying inverse RT to equation (44), we get

$$f(x) = Cx^{a-1}E_{\alpha,\,a}\ (\gamma t^{\alpha}) \tag{45}$$

where b = a

The equation (45) illustrates the solution to equation (42).

Consider the fractional system involving the Riemann-Liouville fractional derivative of the form:

$${}_{0}^{RL}D_{x}^{\frac{1}{2}} f(x) - \gamma f(x) = 0 \tag{46}$$

where $x > 0$ and ${}_{0}^{RL}D_{x}^{-\frac{1}{2}} f(0) = C$.

Solution:
Taking the RT of equation (46), we get

$$R\left\{{}_{0}^{RL}D_{x}^{\frac{1}{2}} f(x)\right\} - \gamma R\{f(x)\} = 0 \tag{47}$$

Since $R_{0}^{RL}D_{x}^{\alpha} f(x) = q^{\alpha}F(q) - \sum_{k=0}^{n-1} q^{k+3}{}_{0}^{RL}D_{x}^{\alpha-k-1} f(0)$

and $(n-1) < \ \alpha < n$, therefore, to equation (47), we get

$$q^{\frac{1}{2}}F(q) - q^{3}{}_{0}^{RL}D_{x}^{-\frac{1}{2}} f(0) - \gamma F(q) = 0$$

$$q^{\frac{1}{2}}F(q) - q^{3}C - \gamma F(q) = 0$$

$$F(q) = \frac{q^{3}C}{q^{\frac{1}{2}} - \gamma} \tag{48}$$

As $R^{-1}\left\{\frac{q^{a-b+3}}{q^{a}-\sigma}\right\} = t^{b-1}E_{a,\,b}\ (\sigma t^{a})$, applying inverse RT to equation (48), we get

$$f(x) = C\,t^{-\frac{1}{2}}E_{\frac{1}{2},\ \ \frac{1}{2}}\ \left(\gamma t^{\frac{1}{2}}\right) \tag{49}$$

The equation (49) illustrates the solution to equation (46).

Consider the general fractional system involving the Riemann-Liouville fractional derivative of the form:

$${}_{0}^{RL}D_{x}^{\alpha} f(x) - \gamma f(x) = h(x) \tag{50}$$

where $x > 0$ and ${}_{0}^{RL}D_{x}^{\alpha-k-1} f(0) = C_k,\ \ k = 1,\ 2,\ 3, \ldots$

Solution:
Taking the RT of equation (50), we get

$$R\left\{{}_{0}^{RL}D_{x}^{\alpha} f(x)\right\} - \gamma R\{f(x)\} = Rh(x) \tag{51}$$

Since $R_{0}^{RL}D_{x}^{\alpha} f(x) = q^{\alpha}F(q) - \sum_{k=0}^{n-1} q^{k+3}{}_{0}^{RL}D_{x}^{\alpha-k-1} f(0)$

and $(n-1) < \ \alpha < n$, therefore, equation (**??**) becomes

$$q^{a}F(q) - \sum_{k=0}^{n-1} q^{k+3}{}_{0}^{RL}D_{x}^{\alpha-k-1} f(0) - \gamma F(q) = H(q)$$

$$q^{a}F(q) - \sum_{k=0}^{n-1} q^{k+3}C_k - \gamma F(q) = H(q)$$

$$F(q) = \frac{\sum_{k=0}^{n-1} q^{k+3}C_k}{q^{a}-\gamma} + \frac{H(q)}{q^{a}-\gamma}$$

$$F(q) = \frac{\sum_{k=0}^{n-1} q^{k+3}C_k}{q^a-\gamma} + \frac{q^3 H(q)}{q^3(q^a-\gamma)}$$

$$F(q) = \frac{\sum_{k=0}^{n-1} q^{k+3}C_k}{q^a-\gamma} + \frac{G(q)\,H(q)}{q^3} \tag{52}$$

Taking inverse RT of equation (52), we get

$$f(x) = R^{-1}\left\{\frac{\sum_{k=0}^{n-1} q^{k+3}C_k}{q^a-\gamma}\right\} + R^{-1}\left\{\frac{G(q)\,H(q)}{q^3}\right\} \tag{53}$$

Where$H(q) = \frac{q^3}{q^a-\gamma} = \frac{q^3}{q^a-\gamma}$

Since $R^{-1}\left\{\frac{q^{a-b+3}}{q^a-\sigma}\right\} = x^{b-1}E_{a,\,b}\,(\sigma x^a)$, therefore,

$R^{-1}\{H(q)\} = g(x) = x^{a-1}E_{a,\ \ a}\,(\gamma x^a) = x^{a-1}E_{a,\ a}\,(\gamma x^a)$

And $R^{-1}\left\{\frac{\sum_{k=0}^{n-1} q^{k+3}C_k}{q^a-\gamma}\right\} = \sum_{k=0}^{n-1} C_k x^{a-k-1}E_{\alpha,\,a-k}\,(\gamma t^{\alpha})$.

Hence, equation (53) becomes

$$f(x) = \sum_{k=0}^{n-1} C_k x^{a-k-1}E_{\alpha,\,a-k}\,(\gamma t^{\alpha}) + g(x)*h(x) \tag{54}$$

Since $g(x)*h(x) = \int_0^x g(x-t)^h(t)\,dt = \int_0^x (x-t)^{\alpha-1}E_a,$

$_a\left(\gamma(x-t)^a\right)h(t)\,dt$, therefore, equation (54) becomes

$$f(x) = \sum_{k=0}^{n-1} C_k x^{a-k-1}E_{\alpha,\,a-k}\,(\gamma t^{\alpha}) + \int_0^x (x-t)^{\alpha-1}E_a,$$

$$_a\,(\gamma(x-t)^a)\,h(t)\,dt \tag{55}$$

The equation (55) illustrates the solution to equation (50).

Consider the fractional system involving the Riemann-Liouville fractional derivative of the form:

$${}_0^{RL}D_x^{\frac{1}{2}}f(x) - \gamma f(x) = x^2 \tag{56}$$

where $x > 0$ and ${}_0^{RL}D_x^{-\frac{1}{2}}f(0) = C$.

Solution:
Taking the RT of equation (56), we get

$$R\left\{{}_0^{RL}D_x^{\frac{1}{2}}f(x)\right\} - \gamma R\{f(x)\} = Rx^2 \tag{57}$$

Since $R_0^{RL}D_x^{\alpha}f(x) = q^{\alpha}F(q) - \sum_{k=0}^{n-1} q^{k+3}{}_0^{RL}D_x^{\alpha-k-1}f(0)$ and $(n-1) < \ \alpha < n$, therefore, equation (57) becomes

$$q^{\frac{1}{2}}F(q) - q^3{}_0^{RL}D_x^{-\frac{1}{2}}f(0) - \gamma F(q) = 2,$$

where $\alpha = \frac{1}{2}$ and $n = 1q^{\frac{1}{2}}F(q) - q^3C - \gamma F(q) = 2q^0$

$$F(q) = \frac{q^3C}{q^{\frac{1}{2}}-\gamma} + \frac{2q^0}{q^{\frac{1}{2}}-\gamma} \tag{58}$$

As $R^{-1}\left\{\frac{q^{a-b+3}}{q^a-\sigma}\right\} = x^{b-1}E_{a,\,b}\,(\sigma x^a)$, applying inverse RT to equation (58), we get

$$f(x) = C\,x^{\frac{1}{2}\,-1}E_{\frac{1}{2}\,,\ \ \frac{1}{2}}\left(\gamma x^{\frac{1}{2}}\right) + 2\,x^{\frac{1}{2}+3-1}E_{\frac{1}{2}\,,\ \ \frac{1}{2}+3}\left(\gamma x^{\frac{1}{2}}\right)$$

$$f(x) = C\,x^{-\frac{1}{2}}E_{\frac{1}{2}\,,\ \ \frac{1}{2}}\left(\gamma x^{\frac{1}{2}}\right) + 2\,x^{\frac{5}{2}}E_{\frac{1}{2}\,,\ \ \frac{7}{2}}\left(\gamma x^{\frac{1}{2}}\right) \tag{59}$$

The equation (59) illustrates the solution to equation (56).

5. Conclusions:

In this article, the operation of the Rohit transform (RT) technique has been extended to obtain the solutions of fractional systems described by general homogeneous and non-homogeneous differential equations of fractional or non-integral orders. The solutions of fractional systems are derived in the form of the Mittag-Leffler function.

This article has validated the ability and efficacy of the Rohit transform method for solving the differential equations of fractional or non-integral orders within the framework of the Caputo fractional derivative operator, and the Riemann-Liouville fractional derivative operator. Consequently, the Rohit transform is proved to be valid for solving the differential equations of fractional or non-integral orders under general conditions. So, the validity of the Rohit transform (RT) for solving the differential equations of fractional or non-integral orders is justified. There is no doubt that the models submitted are simplified examples rather than models of real physical settings.

However, it is hoped that the examples submitted help to elucidate the general ideas that trussed possible physical administrations of fractional time derivatives. While other methods, present in the literature may also be capable of solving the examples presented in the paper, the Rohit transform introduced innovative concepts or methodologies that offer new insights or perspectives on the problems examined in the paper, distinguishing itself from existing methods and potentially opening up new research directions.

The simplicity and ease of implementation of the Rohit transform make it a preferable choice for practical applications, requiring fewer computational steps, less complex algorithms, and simpler parameter tuning. The uniqueness of the Rohit transform lies in its ability to solve fractional systems more facilely and effectively compared to other methods present in the literature, such as the homotopy-perturbation method [15], the Adomian decomposition method [16], the fractional variational iteration method [17], the Lyapunov direct method, and generalized Mittag Leffler stability [19, 20].

Future Scope of Rohit Transform: The future scope of the Rohit transform (RT) holds promise across various domains, including signal processing, image processing, data compression, and cryptography. Here are some potential avenues for its development and application:Further Research and Refinement: Continuous research efforts can refine and enhance the Rohit transform, exploring its mathematical properties, optimizing its algorithms, and extending its capabilities to address a broader range of problems.

Integration into Existing Systems: The Rohit transform can be integrated into existing signal and image processing systems to improve their performance, efficiency, and accuracy. Its simplicity and computational advantages make it a viable option for real-time applications and resource-constrained environments.

Multimedia Compression: The Rohit transform's ability to efficiently represent and process signals and images can be leveraged for multimedia compression techniques, enabling higher compression ratios with minimal loss of information.

Security and Cryptography: The Rohit transform's cryptographic properties can be explored for developing secure communication protocols, encryption algorithms, and steganographic techniques, ensuring data confidentiality and integrity in sensitive applications.

Overall, the future of the Rohit transform is bright, with ample opportunities for innovation, collaboration, and practical application across various disciplines. Continued exploration and development of its capabilities are likely to yield valuable contributions to the advancement of science and technology.

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تحويل روهيت لحل المعادلات التفاضلية العامة من الرتب الكسرية

روهيت غوبتا [1*] ، راهول غوبتا [2] ، دينيش فيرما [3]

[1*] قسم العلوم التطبيقية (الفيزياء)، كلية يوغاناندا للهندسة والتكنولوجيا، جامو، جامو وكشمير، الهند.

[2] قسم الفيزياء، مدرسة جي دي جوينكا العامة، جامو، جامو وكشمير، الهند.

[3] قسم الرياضيات، جامعة نيلم ،كيثال، هاريانا، الهند.

* الباحث المسؤول: *guptarohit565@gmail.com*

الخلاصة

التفاضل والتكامل الكسري فرع من فروع الرياضيات يدرس فيه المشتقات والتكاملات من الرتب الكسرية. في هذه الورقة البحثية تم استخدام تحويل *Rohit* للتكاملات المعقدة لحل المعادلات التفاضلية المتجانسة من الرتب الكسرية او غير التكاملية ذات صيغ محددة المسمى بمؤثر *Caputo* التفاضلي الكسري. حيث تم إيجاد تحويل *Rohit* لدالة $Mittag-leffler$ ومؤثر $Riemann-Liouville$ التفاضلي. وباستخدام نفس التحويل (*Rohit*) تم صياغة حلول النظم الكسرية المتجانسة وغير المتجانسة لمؤثري *Caputo* و $Riemann-Liouville$ التفاضلي الكسري ومن خلال الحلول الموصوفة أعلاه تم تبيان كفاءة تحويل *Rohit* عن الطرق الأخرى (مثل طريقة $homotopy-perturbation$ ، طريقة تحليل *Adomian* ، طريقة $fractionalvariationaliteration$ ، طريقة *Lyapunovdirect* ، طريقة $generalizedMittagLefflerstability$) والتي يمكن استخدامها لحل الحالات أعلاه، ولكن تحويل *Rohit* تتميز عنها باستخدام آليات وطرق مبتكرة تهيئة لمفاهيم وأفكار جديدة على النماذج التي تم دراستها في هذا البحث مما يهيئ لأبحاث جديدة. اضف الى ذلك سهولة تطبيق تحويل *Rohit* جعلها الاختيار المناسب للتطبيق العملي لقلة العمليات الحسابية وبساطة الخوارزميات ذات العلاقة وسهولة تنعيم المعاملات.

الكلمات الدالة : تحويل روهيت *Rohit* ، الدوال الخاصة، المعادلات التفاضلية للأوامر الكسرية أو غير التكاملية

التمويل: لايوجد.

بيان توفر البيانات: جميع البيانات الداعمة لنتائج الدراسة المقدمة يمكن طلبها من المؤلف المسؤول.

اقرارات:

تضارب المصالح: يقر المؤلفون أنه ليس لديهم تضارب في المصالح.

الموافقة الأخلاقية: لم يتم نشر المخطوطة أو تقديمها لمجلة أخرى، كما أنها ليست قيد المراجعة.