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## Comparison of Some Wavelet Transformations to Estimate Nonparametric Regression Function

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### Abstract:

The purpose of this article is to improve and minimize noise from the signal by studying wavelet transforms and showing how to use the most effective ones for processing and analysis. As both the Discrete Wavelet Transformation method was used, we will outline some transformation techniques along with the methodology for applying them to remove noise from the signal. Proceeds based on the threshold value and the threshold functions Lifting Transformation, Wavelet Transformation, and Packet Discrete Wavelet Transformation. Using AMSE, A comparison was made between them , and the best was selected. When the aforementioned techniques were applied to actual data that was represented by each of the prices, it became evident that the lifting transformation method (LIFTINGW) and the discrete transformation method with a soft threshold function and the Sure threshold value (SURESDW) were the best. Consumer prices will be the dependent variable for the period of 2015–2020, and Iraqi oil (Average price of a barrel of Iraqi oil) will serve as the explanatory variable. The methods described above have proven effective in estimating the nonparametric regression function for the study model.

**Paper type:** Research paper.

**Keywords:** Discrete Wavelet Transformation, non-parametric regression, Lifting Transformation, Continues Wavelet Transformation, Packet Discrete Wavelet Transformation

## 1.Introduction :

Both the Fourier transform (FT) and the Short Fourier transform (FFT) extract information from time while ignoring frequency, or vice versa, they do not yield accurate results or efficient analysis of the signal. Wave transforms are an example of another transform that was needed because they are more accurate and produce results quickly, which has the capacity to conduct local signal analysis (Andrew, 1996), given that many applications require simultaneous knowledge of frequency and time information.

Wavelet analysis reveals unique signal characteristics, such as tendencies, breaking points, stopping points, etc., that are not visible using any other technique or analysis. Additionally, this transformation This transformation can also perform multi-discrimination analysis, which other transformations fail to achieve.

In wavelet analysis, the convergence between the tested signal and the wavelet function over time intervals is calculated independently, and the correlation between the signal to be tested and a function known as the wavelet function is calculated (Donoho, 1993).

A small, finite-time signal function that has specific properties such as analysis and windowing is called a wavelet function  $\psi(t)$ . The wavelet functions differed, with each one depending on the characteristics that help us during conversion processes.

Wavelet transformations have been developed in accordance with certain characteristics unique to each transformation, such as the time needed to extract the information from the signal or the maximum amount of information that can be extracted; or they are tailored to a particular type of data and not others, indicating greater application flexibility.

### 1.1 Literature review:

Wavelet transforms, their properties, their accuracy, and their efficiency in estimating have all been covered in a number of studies; some related research will be reviewed in this study.

Burus et al (1998) talked about how wavelet transform techniques, such as discrete wavelet transform methods, which had high estimation accuracy, were developed to estimate wavelets and their properties.

In 2001, Ahmad and Kumal demonstrated The efficiency of the sine wave transform in estimation and its adaptability to a variety of disturbed data types were demonstrated by the study, which also included a comparison of the sine wave transform with Fourier transform techniques

Swelden and Claypoole (2003) demonstrated the effectiveness of estimation techniques utilizing the lifting method through examination of the simulation and the applied aspect, the researchers focused on wavelet transforms, particularly the lifting transform method and adaptive lifting.

Raimondo and Kulik (2009) talked about the study of nonparametric regression in stochastic models that include error correlation and heterogeneity issues. The study demonstrated that wavelet transforms, particularly discrete ones, can estimate and adapt to the aforementioned issues.

Abdullah (2012) demonstrated the efficiency of the sine wave transform demonstrated by the simulation study, and discussed its properties and show that it can adapt to a variety of data types.

Abdulrahman (2013) proved through simulation studies that it is possible to create frames in  $H_2(\mathbb{R})$  by stretching and shifting them to the Schwartz class. The researcher also discussed the sine wave transform while studying the  $L_2(\mathbb{R})$  and  $L_2(\mathbb{R})$  tight wave packet frames.

Marina and Nason (2016) through this approach marked the start of additional research on long-term memory in environmental and climate science applications, they based on long-term memory in time series.

Khalil (2018) showed the effectiveness of the sinusoidal wavelet transform in estimating the nonparametric regression function, and the researcher discussed wavelet transform methods in statistical applications.

Hamza and Hmood (2021) used the discrete wavelet transform was applied with two filtering parameters (db5, Haar) to lower the error, they used discrete wavelet transform methods for estimating the Hurst parameter for fractional Brownian motion of a multivariate regression function.

Damian (2021) studied The topic of selecting the best transformation technique for surface texture-related Problems solved. Since the results were extrapolated to three-dimensional data, he studied the process also talked about the process of selecting the mother function and how it affected the estimation's accuracy.

Ali and Hamza (2022) concluded that the discrete waveform transform was more effective in estimation depending on the mother functions after discussing techniques for selecting the mother function in discrete and continuous waveform transformation and demonstrating the extent of the impact of selecting the appropriate mother function on the accuracy of the estimation.

Hamza and Mahdi (2022) discussed how to choose the threshold value in wavelet transforms and how much the ideal threshold value affects the estimation accuracy when using the discrete wavelet transform.

The research problem is that, despite the efficiency of wavelet estimates in estimating the nonparametric regression function, they are greatly affected by the nature of the data and the lack of clarity of the most appropriate transformation method to estimate the nonparametric regression function for the problem under study.

The Objective of the study is to find the best wavelet transform method to estimate the wavelet regression function for the problem under study.

## 2. Material and Methods :

### 2.1 Continues Wavelet Transformation:

The signal's continuous wavelet transform yields the relationship below.

$$X_{WT}(t, s) = \frac{1}{\sqrt{|s|}} \int_{-\infty}^{+\infty} x(t) \psi^* \left( \frac{t - \tau}{s} \right) d(t) \quad (1)$$

The outcome is a signal that depends  $\tau$  on the medium  $s$  (also known as the scale factor) and the variable (also known as the displacement coefficient). In the case of nodal wavelets, the upper index \* indicates the use of the nodal conjugate, and the mother wavelet is indicated by  $\psi$  (Daubechies, 1992).

By dividing the wavelet coefficients on  $\frac{1}{\sqrt{|s|}}$ , we are able to determine the signal energy for each

measurement. This demonstrates that the wavelets' energy is constant across all measures.

The mother wavelet is compressed and expanded with changes in the scaling factor  $s$ . However, changing this parameter not only affects the frequency and center  $fe$ , but also affects the length of the analysis window. Therefore, scale (scale factor)  $s$  is used instead of frequency to represent the resulting signal in wavelet analysis  $\tau$ . The shift factor determines the temporal location of the wavelet by changing the shift of the wavelet along the signal. The components of the signal  $X_{WT}(t, s)$  are called wavelet coefficients, as each wavelet coefficient is associated with a measurement (frequency) and a specific time point within the time domain (Donoho, 1993).

The CWT has an inverse transform called (ICWT) and it is according to the following formula

$$X(t) = \frac{1}{C_{\psi}^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} X_{WT}(t, s) \frac{1}{s^2} \psi\left(\frac{t-\tau}{s}\right) d\tau ds \quad (2)$$

Every scale in the wavelet function has a central frequency (fc), and the relationship between the frequency and the scale is reversed. A low frequency that provides general information about the signal is correlated with a large scale. High frequencies that provide detailed signal information are correlated with small scales. The Heisenberg principle, which states that the bandwidth in time remains constant and finite  $s$ , is followed by CWT. Decreasing, or putting it another way, A smaller window will result in less frequency discrimination and greater time discrimination. This suggests that, for frequency  $f$ , frequency discrimination is appropriate (Burrus, 1998). The discrete values of the displacement and scale factors are used to recalculate the continuous wavelet transform, and the resultant wavelet coefficients  $s$  are referred to as (wavelet series). The following relationship can be used to calculate the wavelet series.

$$XWT(m, n) = \int_{-\infty}^{+\infty} x(t) \psi_{m,n}(t) dt \quad (3)$$

The original signal can be reconstructed from the following equation (Avekamp, 2003) since integers  $(m, n)$  determine the amount of expansion and displacement of the orthogonal wavelet  $S_0 = 0, \tau_0 = 1$  of the dyadic grid.

$$\psi_{m,n} = S_0^{-m/2} \psi(S_0^{-m} t - n \tau_0) \quad (4)$$

## 2.2 Discrete Wavelet Transformation (DWT):

Discrete Wavelet Transform (DWT) uses special wavelet filters to analyze and reconstruct the signal, and so-called filter banks to achieve distinct multiple analysis (frequency-time), whereas the continuous wavelet transform (CWT) performs multiple analysis by expanding and compressing the wavelet function (Abramovich, 2000).

A subset of the measurements and displacements required for the processing process can be chosen, as opposed to performing the wavelet transformation on all measurements and displacements by executing time interruptions in the signal, which is the fundamental distinction between continuous and discrete wavelet transformation. This conversion produces enough information to maintain the signal's essential information while reducing the computation time.

The relationship in the discrete wave transform that expresses the mother wave function is where the CWT and DWT equations diverge mathematically (Donoho and Johnstone, 1995).

$$S = S_0^j, \tau = k \tau_0 S_0^j \quad (5)$$

$$\psi_{j,k}(t) = \frac{1}{\sqrt{S_0^j}} \psi\left(\frac{t - k \tau_0 S_0^j}{S_0^j}\right) \quad (6)$$

Where  $\tau$  the displacement coefficient and  $S$  Standardization factor.

The measurement step is denoted by the integers  $j$  and the displacement step on the time axis is represented by the integer  $k$ .

The discrete wavelet transform is expressed mathematically as the following relationship.

$$W_x(j, k) = \sum_t x(t) \psi_{(j,k)}^*(t) \quad (7)$$

where  $W_x(j, k)$  the DWT coefficients are represented.

The following formula represents the inverse discontinuous wave transform (IDWT):

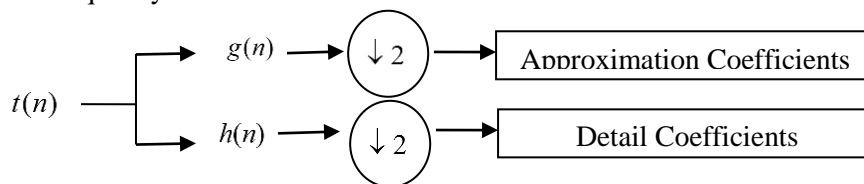
$$\psi_{j,k}(t) x(t) = \sum_{j=-\infty}^{+\infty} \sum_{k=-\infty}^{+\infty} W_x(j,k) \quad (8)$$

Typically, dyadic sampling is achieved by selecting values for the measurements and displacements in such a way  $S_0 = 2, t_0 = 1$  that the relationship expressing the mother wave function looks like this (Daubechies, 1990):

$$\psi_{j,k}(t) = 2^{-\frac{j}{2}} \psi(2^{-j}t - k) \quad (9)$$

Using filters, Mallat (1988) suggested an efficient method of applying this transformation. The signal  $x(n)$  is split into a low-pass filter  $g(n)$  to obtain the approximate coefficients, also called scaling coefficients, which represent the signal's high-frequency and low-frequency components. Additionally, to acquire the precise coefficients—also referred to as wavelet coefficients—that represent the signal's low-frequency and high-frequency components, apply a high-pass filter  $h(n)$  (Mahdi and Hamza, 2022).

This separation makes it possible to analyze the signal with varying degrees of accuracy within various frequency bands.



**Figure 1:** The process for obtaining detail and approximate coefficients using the Mallat algorithm

Starting from the last analysis process with the same number of analysis stages, where the original signal is obtained, all the resulting coefficients—detailed and approximate—are sequentially collected to yield the original signal, or the composition of the signal (Younis and Hmood 2020 ; Mahdi and Hamza 2022) .

### 2.3 Lifting Transformation (LT):

This transformation technique has just recently emerged because it was designed in a way that allows multi-discrimination analysis to be applied to a wider range of data—that is, data that is irregularly spaced.

The three primary steps of this technique—division, prediction (double raising), and updating (prime preparation)—are outlined below and were first presented by (Sweldens 1996) .

- **Division:** The following determines how points  $y_i$  are divided into odd and even equally.

$$Split(c_j) = (c_j^{k\tau}, c_j^{M\tau})$$

- **Prediction:** The following task is prediction. Based on the information found in the evenly indexed values, we predict the  $y_i$  values for the oddly indexed values. This gives us a prediction for the  $y_i$  function with an objective and an agency smoothing degree.

$$dj - 1 = c_j^{M\tau} - P(c_j^{M\tau})$$

Because  $P$  is selected during the prediction process, which establishes a connection between  $c_j^{k\tau}$  and  $c_j^{M\tau}$  .

- **Update:** The last step involves using a linear combination of the previous even-indexed  $y_i$  values and the detail vector to update the  $y_i$  values on the even locations.

Maintaining the amount of signal in the initial updated values (the average value of the signal) throughout the process's subsequent iterations is the aim of the updating phase (Mathewi and Nunes, 2005). The initial data  $y_i$  is replaced with the remaining updated data in the subsampling (which produces coarse scale features of the signal) and the detailed coefficients accumulate during the process. This is done after repeating the update prediction evaluation procedure on the updated values. This work bears similarities to the discrete wavelet transform method (Piella and Heijmans, 2002), where  $y_i$  is substituted with a set of father functions and mother wavelet coefficients.

#### 2.4 Wavelet Packet Transformation (WPT):

According to Robert and Ruqiang (2010), the discrete wavelet transform exhibits flexibility in both the frequency and time domains; however, its sensitivity to high frequencies makes it challenging to differentiate between high-frequency transient information.

Wave packet transform (WPT), in contrast, overcomes this restriction by increasing the fragmentation (deconstruction) of the signal's detailed information in the high frequency region.

The wavelet band transformation process is depicted in the following figure. It yields four levels, or a total of sixteen sub-bands, with each sub-band covering one to sixteen of the signal's frequency spectrum. This means that the analysis's ability to enhance the signal and, consequently, its performance, makes it appealing for the detection and distinction of transient elements with high frequency characteristics. The definition of the wavelet packet transform is as follows (Kercel and Klein, 2001):

$$u_{2n}^{(j)}(t) = \sqrt{2} \sum_k h(k) u_n^{(j)}(2t - k), n = 0, 1, 2, \dots \quad (10)$$

Given that  $u_0^{(0)}(t)$  is the fundamental wavelet function  $\phi(t)$  and where is  $u_0^{(0)}(t) = b(t)$ ,  $u_1^{(0)}(t)$  the measurement function  $\psi(t)$ , it follows  $u_1^{(0)}(t) = \psi(t)$  (Wickerhauser 1991)

$$u_{2n+1}^{(j)}(t) = \sqrt{2} \sum_k g(k) u_n^{(j)}(2t - k), k = 0, 1, 2, \dots, m \quad (11)$$

Since the  $j$ th level of the wavelet packet rules is denoted by (J) in Equation (5), binary wavelet packet rules will exist at that level (Salman, 2009).

The Mallat algorithm in 1999 is used to obtain the wave packet once its basis has been established using (5). An iterative algorithm can then be designed to implement (WPT) in order to decompose the signal as a result of the division process.

$$d_{j+1,2n} = \sum_m h(m - 2k) d_{j,n} \quad (12)$$

$$d_{j+1,2n+1} = \sum_m g(m - 2k) d_{j,n} \quad (13)$$

In this case  $d_{j,n}$ ,  $m$  stands for the number of wavelet coefficients, and denotes the wavelet coefficients at the  $j$ th level,  $2n$ th, and  $(j+1)$ th bands, as well as the  $n, 2n$  and  $d_{j+1,2n}$ ,  $d_{j+1,2n+1}$  sub-band (Stephen and Marvin, 2001).

L-level decomposition can be applied in a variety of ways to analyze the signal theoretically. This allows for increased efficiency and improvement of the signal segmentation process. To aid in the optimization process, various criteria can be applied, such as entropy and logarithm as a cost function. According to this transformation, the most commonly used criterion for the best representation of the signal is the Shannon entropy (Coifman & Wickerhauser 1992) of the wavelet coefficients in the  $n$ th sub-frequency band inside the  $j$ th level  $d_{j,n} = \{d_{j,n} : n = 1, 2, \dots, 2^j\}$ . The definition of the Shannon entropy is as follows:

$$Entropy(d_{j,n}) = -\sum p_i \text{Log}(p_i) \tag{14}$$

The probability distribution function is defined as follows: where  $p_i$  is the probability distribution of the energy contained in the wavelet coefficients in the  $n$ th sub-frequency band within the  $j$ th level.

$$p_i = |d_{j,n}(i)| / \|d_{j,n}\|^2 \tag{15}$$

$$\sum_1^m p_i = 1, p_i \text{Log}(p_i) = 0 \text{ if } p_i = 0. \tag{16}$$

The number of wavelet coefficients at the  $n$ th sub-frequency bands in the  $j$ th level is indicated by the upper limit,  $m$ .

If the energy content is distributed among the wavelet coefficients formed within the sub-frequency band, the Shannon entropy has a large value. Conversely, if the energy is focused on a small number of dominant components, it takes on a small value. The goal of signal decomposition is to contain the minimum Shannon entropy in the wavelet coefficients, which is necessary to concentrate the signal information into the fewest number of coefficients (Trappe and Liu, 2000).

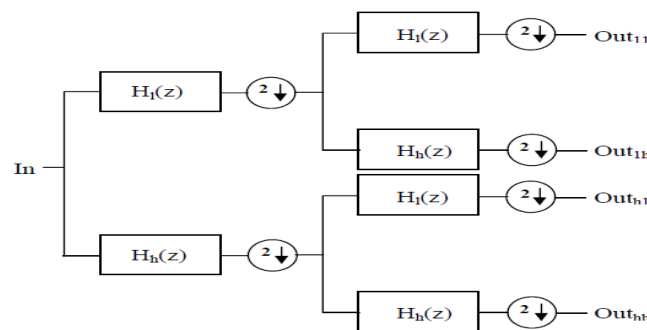


Figure 2: Mechanism for implementing the packet wavelet transform

### 2.5 Thresholding Rules:

As we saw in the previous section, shrinking the wavelet coefficients is necessary for the second step of estimation using the shrinkage wavelet. The reduction is accomplished mathematically by applying the law of hard threshold cut or soft threshold cut, depending on which of their formulas is used (Younis and Hmood, 2020).

$$\mathcal{D}_\lambda^H(\hat{\omega}_{jk}) = \begin{cases} 0 & \text{if } |\hat{\omega}_{jk}| \leq \lambda \\ \hat{\omega}_{jk} & \text{if } |\hat{\omega}_{jk}| > \lambda \end{cases} \tag{17}$$

$$\mathcal{D}_\lambda^S(\hat{\omega}_{jk}) = \begin{cases} 0 & \text{if } |\hat{\omega}_{jk}| \leq \lambda \\ \hat{\omega}_{jk} - \lambda & \text{if } \hat{\omega}_{jk} > \lambda \\ \hat{\omega}_{jk} + \lambda & \text{if } \hat{\omega}_{jk} < -\lambda \end{cases} \tag{18}$$

Where is  $\lambda$  the threshold's value (Value Thresholding) located? "A threshold cut allows the data itself to decide which wavelet coefficients are important," according to Antoniadis.

The smooth threshold cut (continuous function) is the law of "reduction" or "termination," whereas this one is the law of "keep" or "termination" (Nason, 2002)

## 2.6 Threshold Value:

Since selecting the right threshold value will directly affect the noise in the signal, the threshold value is a crucial parameter in the wavelet reduction algorithm to reduce signal noise. Threshold values come in various varieties (Nason a, 1996).

## 2.7 VisuShrink Thresholding:

Donoho and Jonstone introduced the global threshold method, which is expressed using the following formula:

$$\lambda_{\text{VisuShrink}} = \sigma \sqrt{2 \log(n)} \quad (19)$$

Because of:  $n$  The signal's duration ;  $\sigma$  The noise level's standard deviation.

Selecting the threshold would almost certainly result  $\sqrt{2 \log}$  in an increase in noise (Younis and Hmood, 2020).

## 2.8 Sure Thresholding:

Donoho and Jonstone created the crucial "Sure Shrink" method for determining the threshold value. The Unbiased Risk Estimation (SURE) technique, which is an acronym for the term "Stein Unbiased Risk Estimation" for each wavelet level, is the foundation of this methodology. It was developed by Stein in 1981)

Stein explained that this estimate is not biased towards risk because the wavelet transform is orthogonal, which means that the transform for noise is also orthogonal, meaning that the coefficients  $d_{j,k}$  are also orthogonal. Additionally, because the noise is distributed (Gaussian), it follows that it is  $d^*$  distributed (Gaussian).

Therefore, the threshold value for (SURE) can be found from the following formula (Donoho and Johnstone, 1995a)

$$SURE(\lambda_j, d_{jk}) = N - 2 \sum_{k=1}^N I(|d_{jk}| \leq \lambda_j) + \sum_{k=1}^N \min(|d_{jk}|, \lambda)^2 \quad (20)$$

$$\lambda_{j,SURE} = \arg \min_{0 \leq \lambda \leq \sqrt{2 \log N}} SURE(\lambda_j, d_{jk}) \quad (21)$$

## 3. Discussion of Results:

### 3.1 Simulation:

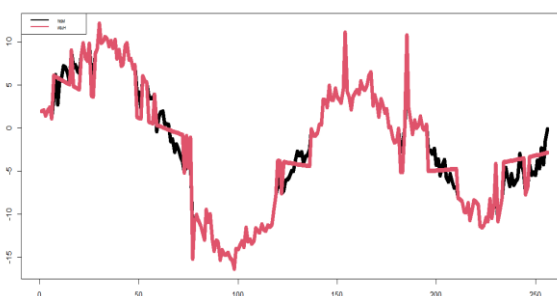
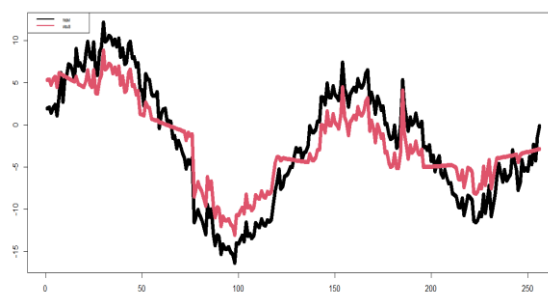
Errors are generated as an uncorrelated errors for the nonparametric regression model within the interval[0,1], and the nonparametric regression function will be estimated. The dependent variable, on the other hand, is produced by adding noise to the test functions. To conduct the simulation experiments, make use of the following elements:

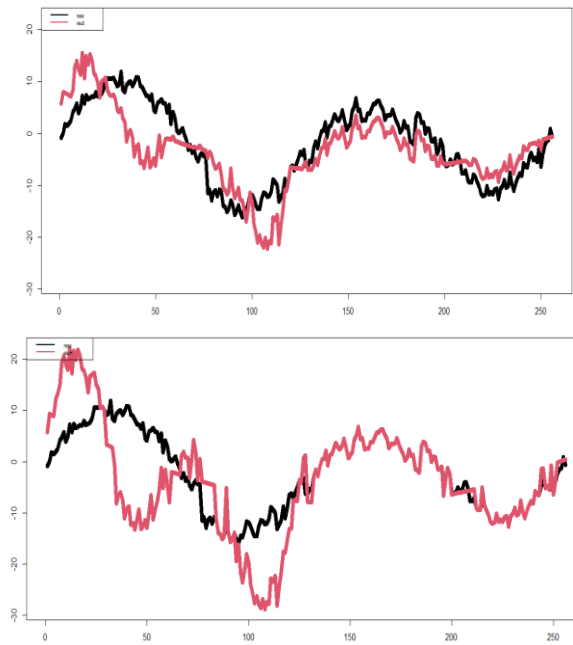
- i. Sample numbers  $n$ . Two different sample sizes were employed  $2^6 = 64$  ,  $2^7 = 128$  , since wavelet transforms necessitate a dyadic sample size  $n = 2^j$  that is, one for each positive integer.
- ii. signal to Noise ratios (SNR): There were two noise ratios used: (SNR=5), which represent to a low noise ratio, and (SNR=10), which represent to a high noise ratio (He and Xing, 2015).
- iii. The Heavesin function  $g(t_j)$  , which is characterized by being discontinuous rather than continuous because it is similar to the state of the data in the applied aspect after it is plotted, was chosen as one of the test functions to represent various cases of real-world scenarios.
- iv. The number of vanishing moments served  $N = 10$  as the mother function for a Daubechies wavelet



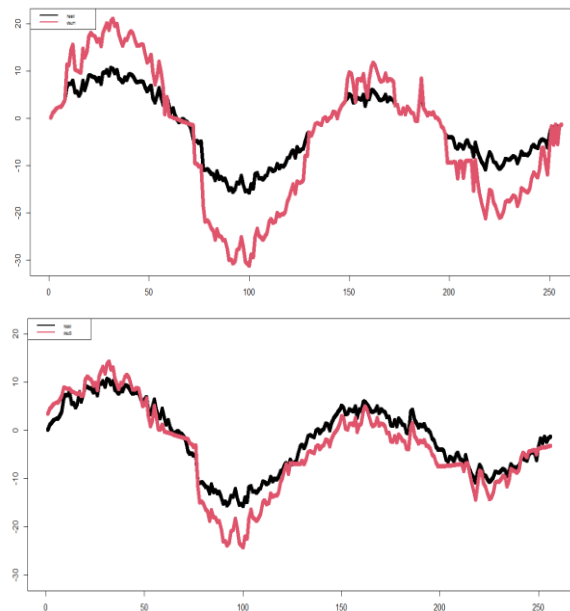
**Table 1:** The Heavisin distortion function estimates for sample sizes  $n = 256$ ,  $n = 512$ ,  $n = 128$  and  $n = 64$ , and a signal-to-noise ratio (SNR) of 5 are compared using the MASE criterion,

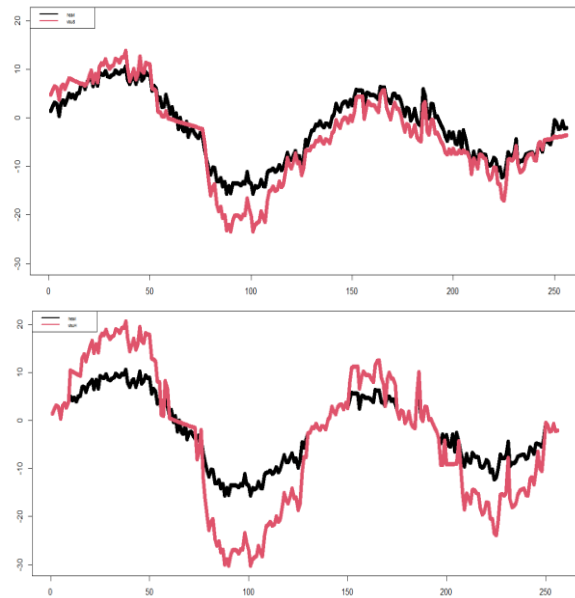
SNR=5					
n	sureSDW	sureHDW	visuSDW	visuHDW	DPW
64	0.05275395	0.05312389	0.01036879	0.05134537	0.05906202
128	0.006100051	0.006116155	0.003978631	0.006082060	0.02553661
SNR=5					
n	sureSCW	sureHCW	visuSCW	visuHCW	LIFTINGW
64	0.0004281427	0.0014516428	0.0083625874	0.0015371940	0.04955564
128	0.004369257	0.004354152	0.003724166	0.004521735	0.02282738
SNR=10					
n	sureSDW	sureHDW	visuSDW	visuHDW	DPW
64	0.05091246	0.05391514	0.01053423	0.05195679	0.05916158
128	0.006116024	0.009890102	0.006113160	0.006115993	0.05449302
SNR=10					
n	sureSCW	sureHCW	visuSCW	visuHCW	LIFTINGW
64	0.0003629877	0.00007511986	0.008970747	0.001171968	0.003642707
128	0.003517455	0.003520740	0.002980504	0.003642707	0.002980504





**Figure 3:** The true values and the estimated values of the dependent variable Y using a noise ratio (5)





**Figure 4:** The true values and the estimated values of the dependent variable Y using a noise ratio (10)

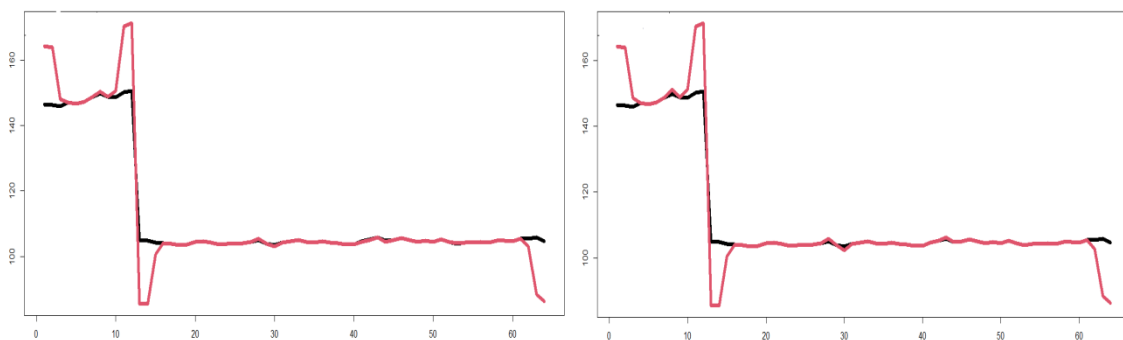
### 3.2 Real Data:

The average cost per barrel of Iraqi oil (Basra Crude) is one of the study variables. It is considered as an explanatory variable ( $x_i$ ) from the Organization of Arab Petroleum Exporting Countries (OAPEC) website for the period from 1/31/2015 till 12/31/ 2020. Regarding the pricing information, Consumption (CPI) ( $y_i$ ), a dependent variable, was acquired from the Central Bank of Iraq (CBI) data website for the same time period. As we discussed in the theoretical section, the study is crucial for determining which transformation techniques will yield the best wavelet regression function estimate, which will yield the most information from the signal under investigation. The study is also economically significant because the stated objective of policy is to determine whether or not oil prices are stable in international markets. By creating the best wave regression, cash in Iraq and analyzing how changes in global oil prices reflect on overall price levels in light of shifting financial and economic conditions assist the relevant authorities in managing the basic determinants of inflation and interpreting the degree to which it contributes to the consumer price index over the short- and long-terms.

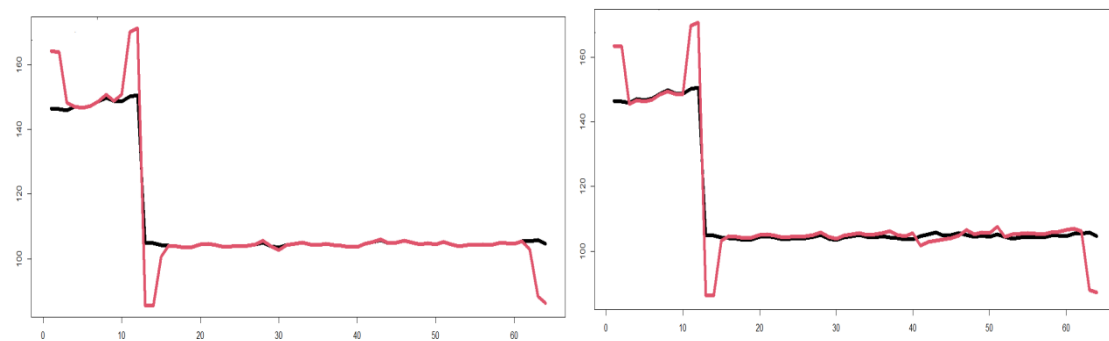
SPSS version (26) was used to test the data. As we note the failure of the parametric model in explaining the relationship in a more detailed and precise manner, it turned out that the relationship between the explanatory and dependent variables is non-linear and that the model is useless in explaining the relationship between both (Average cost per Barrel) as an explanatory variable and the consumer price index as a dependent variable (Aviral and Juncel, 2019).

**Table 2:** The MSE value for estimation techniques utilizing various transformation techniques, threshold laws, and threshold values is displayed

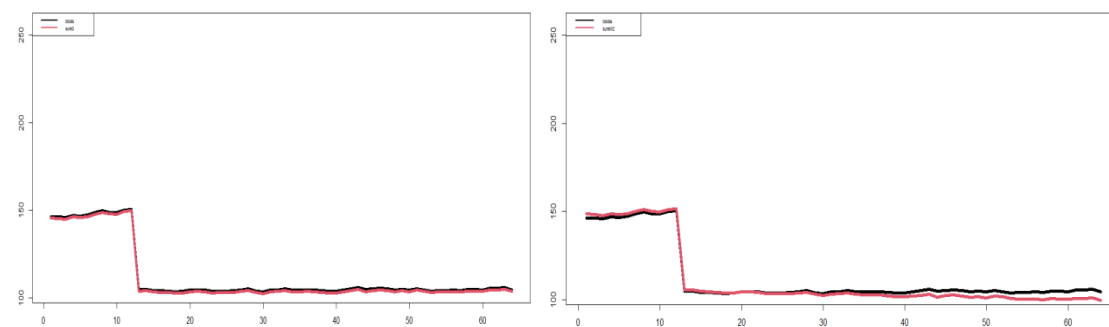
sureSDW	sureHDW	visu SDW	visu HDW	DPW
0.000140194	0.0101164	0.0107933	0.0117519	0.129730
sureSCW	sureHCW	visu SCW	visu HCW	LIFTINGW
0.04066264	0.04514032	0.03089632	0.04497149	0.0002288469



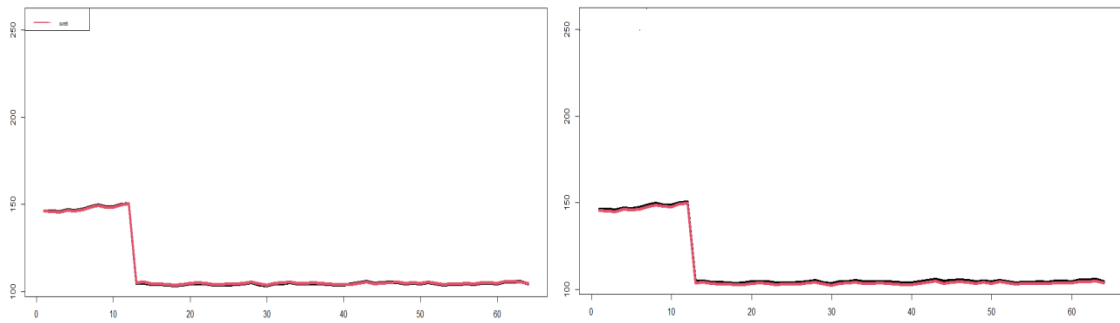
**Figure 5:** Curves of true values and estimated values of response variable Y for the real data using sureHCW and DPW methods



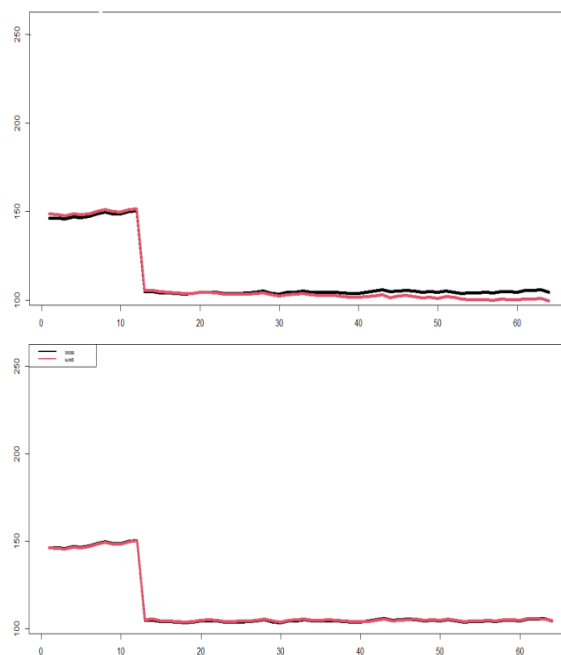
**Figure 6:** Curves of true values and estimated values of response variable Y for the real data using sureCW and visuHCW methods



**Figure 7:** Curves of true values and estimated values of response variable Y for the real data using visuSCW and visuHDW methods



**Figure 8:** Curves of true values and estimated values of response variable Y for the real data using sureHDW and visuSDW methods



**Figure 9:** Curves of true values and estimated values of response variable Y for the real data using sureSDW and LIFTINGW methods

The estimated function is shown in red in Figures above, while the real function is shown in black. The accuracy of the estimation made with the previously mentioned transformation methods is displayed in the above figure

### 3.3 Findings :

1-Plotting the estimated function against the real function makes it clear that the discrete wavelet transformation method, which used a smooth threshold function and a certain threshold value, was the best wavelet transformation method for these kinds of data because It was lower MSE. The lift transformation method is In the next line, followed by the transformation method. There is a certain threshold value and a strong threshold function for the discontinuous function.

2. The continuous transform (CW) approach with various threshold values and functions performed better than the discrete packet transform (DPW) method, which had the worst performance.

3. Compared to the parametric performance model, the performance of the nonparametric model was more accurate, which is explained by the drawing that shows the accuracy of estimating the nonparametric regression function, while the parametric regression failed to explain that relationship efficiently, as the coefficient of determination was 1.5%, which is a very weak value for interpreting the model.

4. It is evident from paragraph (3) that the primary driver of consumer prices in Iraq is the price of oil. This is because oil prices have a significant impact on the national budget, which in turn affects salaries and the overall state of the economy. As the primary driver of the economy and the primary source of rent, oil prices have a fundamental impact on prices.

5- The decline in oil prices in the last year (2020) clearly cast a negative impact on consumer prices, and the reason for this is that Iraq mainly depends for its economic revenues on oil as a primary source in the budget.

#### **4. Conclusion:**

It turned out that the discrete transformation method with a smooth threshold function and the Sure threshold value (SURESDW) was the best, followed after that by the lifting transformation method (LIFTINGW), as the aforementioned methods were applied to real data represented by both Iraqi oil prices (Basra crude oil) as an explanatory variable and consumer prices.

#### **Authors Declaration:**

Conflicts of Interest: None

-We Hereby Confirm That All The Figures and Tables In The Manuscript Are Mine and Ours. Besides, The Figures and Images, Which are Not Mine, Have Been Permitted Republication and Attached to The Manuscript.

- Ethical Clearance: The Research Was Approved By The Local Ethical Committee in The University.

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## مقارنة تحويلات الموجات لتقدير دالة الانحدار اللامعلمي

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هذا العمل مرخص تحت اتفاقية المشاع الإبداعي تُسبب المُصنَّف - غير تجاري - الترخيص العمومي الدولي 4.0

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### مستخلص البحث

يتم في هذا البحث دراسة التحويلات الموجية وبيان امكانية توظيف اكثرها كفاءة في معالجة وتحليل الاشارة وذلك بهدف تحسينها وازالة الضجيج منها حيث سنطرح بعض طرائق التحويل والية تطبيقها من اجل التخلص من الضجيج في الاشارة اذ تم اخذ كلاً من طريقة التحويل الموجي المتقطع (Discrete Wavelet Transformation) باختلاف دوال العتبة وقيمة العتبة ، والتحويل الموجي المستمر (Continues Wavelet Transformation) والتحويل الجيبي المتقطع ( Packet Discrete Wavelet Transformation) وتحويل الرفع (Lifting Transformation) ، اذ تم المقارنة فيما بينها عن طريق AMSE واختيار الافضل وقد اتضح ان طريقة التحويل المتقطع مع دالة عتبة ناعمة وقيمة عتبة Sure (SURESDW) كانت الافضل تلاها بعد ذلك طريقة تحويل الرفع (LIFTINGW) اذ تم تطبيق الطرائق المذكورة على بيانات حقيقية تمثلت بكل من اسعار نفط العراق (نفط خام البصرة) كمتغير توضيحي واسعار المستهلك كمتغير معتمد للفترة من (2015-2020) اذ اثبتت الطرائق المذكورة كفاءتها في تفسير علاقة التأثير للمتغيرات المذكورة .

نوع البحث: ورقة بحثية.

المصطلحات الرئيسية للبحث: التحويل الموجي المتقطع ، الانحدار اللامعلمي ، تحويل الرفع ، التحويل الموجي المستمر ، التحويل الجيبي المتقطع.