



# Cayley Neutrosophic Graphs on the Neutrosophic Groups

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## الرسوم البيانية النيوتروسوفية لكايلى على المجموعات النيوتروسوفية

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### ABSTRACT

**Background:** Although graph theory plays a significant role in representing the causes of a problem and the relationship between them to facilitate its solution, there are many problems in our real lives that cannot be represented accurately due to the inaccuracy and ambiguity of the available data. In this article, Cayley (SVNG) will define and look into some of its characteristics. In terms of algebraic structures, we demonstrate a few intriguing characteristics of SVNGs. Additionally, planarity in Cayley (SVNG)s will be discussed.

**Materials and Methods:** The single-valued Neutrosophic sets (SVNS) were used, which depend on three functions from the universal set, say  $X$ , to the standard range  $[0, 1]$  and emanating from the non-standard range  $]0, 1^+[$  which the Neutrosophic sets rely on. The Cayley table of algebraic groups was used.

**Result:** This article introduced the idea of neutrosophic Cayley graphs (NCG) as a combination of graph theory and algebraic structure, and some of its properties were studied in different environments where the algebraic Cayley structure exists. For example, it was proven that every reflexive and transitive Cayley graph is regular and so on.

**Conclusion:** It can be concluded that a relationship has been built to draw a weighted directed graph for SVNS that contains three components (truth, indeterminacy, and false). and many interesting features of the neutrosophic graph (NG) were displayed, such as transitivity and regularity. The planarity of Cayley's neutrosophic graph has also been demonstrated, and observations and theorems have been formulated in relation to it.

**Keywords:** Neutrosophic set, single valued neutrosophic set, algebraic group ,neutrosophic subgroups, Cayley neutrosophic graphs.



## INTRODUCTION

A recently developed mathematical framework called a fuzzy set (FS) is used to illustrate how uncertainty manifests itself in trials in everyday life. It was first presented in 1965 by Zadeh, and the ideas were developed by numerous separate studies [1].

Kaufmann's original idea of a fuzzy graph (FG) was built on Zadeh's fuzzy relations [1,2]. Rosenfeld [2] was the first to present the fuzzy equivalent of some basic concepts in graph theory. The notion of (FG) complement was defined by Mordeson and Peng [3], who also looked at several fuzzy graph operations.

Krassimir T. Atanassov introduced the Intuitionistic fuzzy graph (IFG) for the first time in 1994 [4]. In 1983, Atanassov [5], [6] suggested intuitionistic fuzzy sets (IFS) as a popularization of the idea of fuzz sets. With the single requirement that the sum both grades do not exceed 1. Atanassov expands on the idea of a fuzzy set (FS) by introducing new element that describes the non-membership grade  $\nu$  in addition of the membership grade  $\mu$ . IFS offers  $\mu$  and  $\nu$  in varying degrees that are more or less distinct from one another. In contrast, [7]fuzzy sets only reveal an element's in a particular set. Even though (IFS)s provides both and that are varying degrees of independence from one another, the sole prerequisite is that the sum of two degrees is not greater than 1.

Later, interval-valued fuzzy sets [6] were added to the IF sets notion (Atanassov & Gargov, 1989). Generalizing from (FS) and (IFS)[8], [9](Smarandache, 1998, 1999, 2002, 2005, 2006, 2010) created "neutrosophic sets" (NS). These sets are especially helpful for handling the partial, ambiguous, and unreliable data that exists in real life. The three features of (NS) are truth (T), indeterminacy (I), and falsehood (F) membership maps. This idea is crucial in many implementation disciplines due to the explicit quantification of indeterminacy and the independence of T, I, F functions. A single-valued neutrosophic set (SVNS) was first proposed by Smarandache, and the name was first used by Wang et al. in 2010 [10]. Single-valued neutrosophic relations based on SVNS were suggested by Yang et al

## 2.MATERIAL AND METHODS

This section goes through some fundamental ideas that must be understood in order to completely appreciate this work.

**Definition 2.1:[11]** A directed graph (digraph) is a couple  $D = (V, A)$ , such that  $A$  is the subset of the collection of ordered pairs of various elements of  $V$ . In digraph, a vertex set is nonempty set that contains all other elements. The arcs of  $D$  refer to the elements of  $A$ .

In discrete mathematics, the vertex-transitive graph study has a long and famous history. Vertex-transitive graphs, or Cayley graphs (CG), are well-known examples that have important theoretical and practical implications. Cayley graphs, for instance, are a great model for interconnection networks.

**Definition 2.2:[10]** Assume  $M$  is the minimum generating set of the finite group  $N$ . The vertices of a Cayley graph  $(N, M)$  belong to  $N$ , and the edge set is provided by  $\{(n, nm): n \in N, m \in M\}$ , where  $m \in M$ . Two vertices are adjacent if  $n_2 = n_1m$

Note: If  $M$  generates  $H$  but no appropriate subset of  $M$  does, then  $M$  is a minimally generated set.



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**Definition 2.3:** Consider that  $(V, *)$  is a group,  $A \subset V$ . Then  $G = (V, A)$  is (CG) induced by  $(V, *, A)$ , where  $A = \{(x, y) : x^{-1}y \in A\}$ .

**Definition 2.4[12]:** Let a set  $H \neq \emptyset$  be given, then a set  $A = \{ \langle h, \mu_A(h) \rangle : h \in H \}$  that is drawn from H is said to be fuzzy set (FS), with  $\mu_A: H \rightarrow [0,1]$ .

**Definition 2.5:** A fuzzy graph (FG) is order triple with the formula  $G = (V, \sigma, \mu)$ , where V is the vertex set,  $\sigma$  is a (FS) on V,  $\mu$  is value on  $\sigma$  and  $\sigma(u) \wedge \sigma(v) \geq \mu(u, v)$  for every  $u, v \in V$ .

**Definition 2.6 [13]:** Assume that X is a universal set.  $A = \{(x, \mu_A(x), \nu_A(x)) | x \in X\}$  is an (IFS) on X, such that  $\nu_A, \mu_A: X \rightarrow [0, 1]$  are mappings of  $\mu_A$  as a membership and  $\nu_A$  as a non-membership with  $\mu_A(x) + \nu_A(x) \leq 1, \forall x \in X$ .

**Definition 2.7 [14]:** Assume that x is an unspecific element within a set of points X.  $A = \{x: T_A(x), I_A(x), F_A(x)\}$ , and  $T_A, I_A, F_A: X \rightarrow [0,1]$  represent a (SVNS) in X. Such that they are known as (truth, indeterminacy, and falsity) functions where  $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3, \forall x \in X$ .

**Definition 2.8:** Assume X be SVNS. With respect any subset A, and for  $\alpha \in [0, 1]$ ,

- 1)  $A_\alpha = \{x | T_A(x) \geq \alpha, I_A(x) \leq \alpha, \text{ and } F_A(x) \leq \alpha\}$  is known as  $\alpha$ - cut of A.
- 2)  $A_\alpha^+ = \{x | T_A(x) > \alpha, I_A(x) < \alpha, \text{ and } F_A(x) < \alpha\}$  is known as strong  $\alpha$ - cut of A.
- 3)  $A_0^+ = \{x \in X | T_A(x) \geq 0, I_A(x) \geq 0, \text{ and } F_A(x) \geq 0\}$  is called support of A

**Definition 2.9:** Neutrosophic relationship  
 $R = \{ \langle (x, y) : T_R(x, y), I_R(x, y), F_R(x, y) \rangle | (x, y) \in X \times X \}$   
 is a neutrosophic set (NS) in  $X \times X$ , where  $T_R, I_R, F_R : X \times X \rightarrow [0, 1]$ , and  $0 \leq T_R(x, y) + I_R(x, y) + F_R(x, y) \leq 3$  is satisfied  $\forall x, y \in X$ .

**Definition 2.10:** let R be used to symbolize a neutrosophic relation R on the universe set. Therefore

- a) R is neutrosophic reflexive if  $R(x, x) = (1,0,0), \forall x \in X$
- b) R is neutrosophic symmetric if  $R(x, y) = R(y, x), \forall x, y \in X$ .
- c) R is neutrosophic anti-symmetric if  $R(x, y) \neq R(y, x), \forall x, y \in X$ .
- d) R is neutrosophic transitive if  $R(x, z) \geq \wedge (R(x, y) \vee R(y, z))$ .
- e) If the neutrosophic relation R meets conditions a, b, and d is said to be equivalence (EQ) on X, and it is said to be a neutrosophic partial order (NPO) if the conditions a, c, and d are satisfied.

**Definition 2.11:** Assume that R represents a neutrosophic relationship on universe X. If the following criteria are met, R is pointing to a neutrosophic linear order relation (NLO) on X:

- (a) R is a neutrosophic partial relation.
- (b)  $R(x, y) \vee R^{-1}(x, y) > 0, \forall x, y \in X$ .



### 3. CAYLEY SINGLE VALUED NEUTROSOPHIC GRAPHS

This section introduces Cayley neutrosophic graphs and demonstrates a few properties of it.

**Definition 3.1:** A (SVN)-digraph, where the underlying digraph  $G^* = (V, E)$  is represented by  $G = (A, B)$ , where

- 1) The (truth  $T_A$ , indeterminacy  $I_A$ , and false  $F_A$ )-memberships degree of the elements of  $V$  are shown, respectively, by the maps  $T_A, I_A, F_A: V \rightarrow [0, 1]$ , and
- 2)  $0 \leq T_A(x_i) + I_A(x_i) + F_A(x_i) \leq 3$  for each  $x_i \in V$  and  $i = 1, 2, 3, \dots, n$ .
- 3) The functions  $T_B, I_B, F_B: V \times V \rightarrow [0, 1]$  are known by comparative equations

$$T_B(x_i x_j) \leq T_A(x_i) \wedge T_A(x_j)$$

$$I_B(x_i x_j) \leq I_A(x_i) \wedge I_A(x_j), \text{ and}$$

$$F_B(x_i x_j) \geq F_A(x_i) \vee F_A(x_j)$$

indicates the  $(T_B, I_B, F_B)$ -membership values of the edge  $(u_i, u_j) \in E$ , when  $0 \leq T_B(x_i x_j) + I_B(x_i x_j) + F_B(x_i x_j) \leq 3, \forall (x_i x_j) \in E, i, j = 1, 2, 3, \dots, n$ .

We call  $A, B$  as the (SVN)-set of vertex  $V$  and edges  $E$  respectively. We can write

$$B(x, y) = (T_B(x, y), I_B(x, y), F_B(x, y))$$

**Remark 3.2 :** Assume a SVNG  $G = (A, B)$  with underlined graph  $G^* = (V, E)$ . For any pair of vertices  $x, y \in V$  are known as adjacent if and only if the following comparative of the edge and vertices neutrosophic values satisfy the three comparative equations.

**Definition 3.3:** Take  $G$  to be SVN-digraph.

An indegree of  $x \in V(G)$  is known as

$$ind(x) = (indT(x), indI(x), indF(x)), \text{ where}$$

$$indT(x) = \sum_{x \neq y} T_B(xy), \text{ } indI(x) = \sum_{x \neq y} I_B(xy) \text{ and } indF(x) = \sum_{x \neq y} F_B(xy)$$

In similarly way, out-degree of  $x \in V(G)$  will be

$$outd(x) = (outdT(x), outdI(x), outdF(x)), \text{ where}$$

$$outdT(x) = \sum_{x \neq y} T_B(yx), \text{ } outdI(x) = \sum_{x \neq y} I_B(yx) \text{ and } outdF(x) = \sum_{x \neq y} F_B(yx)$$

**Remark 3.4:** A SVN- digraph is said to be

- 1) Out-regular if  $outd(x) = outd(y), \forall x, y \in V(G)$
- 2) In-regular if  $ind(x) = ind(y), \forall x, y \in V(G)$

**Example 3.5:** Let the SVN-digraph  $G$  of  $G^* = (V, E)$  be given, where  $V = \{D, C, B, A\}$ ,  $E = \{\overrightarrow{AB}, \overrightarrow{AD}, \overrightarrow{BC}, \overrightarrow{DC}, \overrightarrow{CA}\}$ .

It is easy to note that from the following SVN - digraph it is in-regular digraph but not out-regular digraph.

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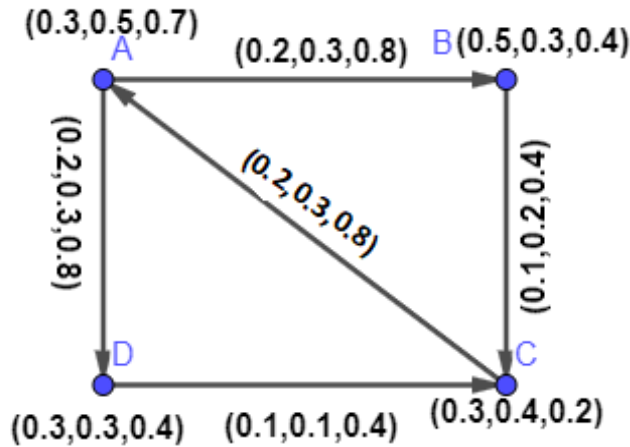


Figure: 1 in-regular neutrosophic graph but not out-regular

**Definition 3.6:** Consider the group  $(H, *)$  and  $S \subset H, S \neq \emptyset$ . The Cayley SVNG  $G = (V, R)$  is a neutrosophic graph with  $V = H$  and if  $A = (T_A, I_A, F_A)$  be SVN-subset of  $V$ ,  $R(x, y)$  be SVN- relation on  $V$  is specified as  $R(x, y) = A(x^{-1}y) | x, y \in H \text{ and } x^{-1}y \in N$ .

**Example 3.7:** Assume the group  $(H, *)$  was given ,where  $H = \{1, 3, 5, 7\}$ , and  $*: H \rightarrow H$  defined by  $*(a, b) = (a \times b) \text{ mod } 8$ , and let  $S = \{3, 5\} \subseteq H$  then  $G = (H, R)$  is Cayley graph with vertices  $H$ ,  $A = (T_A, I_A, F_A)$  be single valued neutrosophic subset of  $H$ , and  $\vec{ab} \in R$  where  $b = sa \forall a, b \in H \text{ and } s = a^{-1}b \in S$ . Then

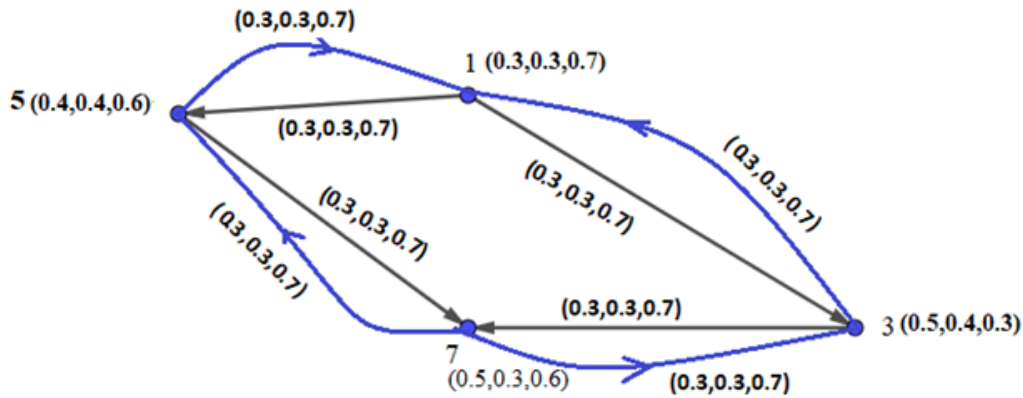
$$(T_A, I_A, F_A)(1) = (0.3, 0.3, 0.7), (T_A, I_A, F_A)(3) = (0.5, 0.4, 0.3),$$

$$(T_A, I_A, F_A)(5) = (0.4, 0.4, 0.6), (T_A, I_A, F_A)(7) = (0.5, 0.3, 0.6)$$

x	1	1	3	3	5	5	7	7
y	3	5	1	7	1	7	3	5
$S = x^{-1}y$	3	5	3	5	5	3	5	3

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Figur 2: Cayley regular graph

Fig. 2 makes it clear that  $G = (H,R)$  is a regular Cayley neutrosophic digraph. Additionally, each directed edge's strength is described by the relation “R” in the aforementioned description.

**Definition 3.8 [6]:** A semigroup is an algebraic structure  $(S, \circ)$  that meets the criteria listed below:

- 1)  $a \circ b \in S$
- 2)  $\forall a, b, c \in S$

**Definition 3.9 [6]:** Consider a semigroup  $(S, *)$  and  $A' = (T_{A'}, I_{A'}, F_{A'})$  is a neutrosophic subset of  $S$ . Thereafter  $A'$  known as neutrosophic sub-semigroup of  $S$  if the neutrosophic comparison conditions for edges and vertices are met.

**Theorem 3.10:** Every (CNG)  $G$  is vertex-transitive.

**Proof.** Suppose that  $a, b \in V$ , and  $\theta: V \rightarrow V$  defined by  $\varphi(x) = ba^{-1}x, \forall x \in V$ . Obviously  $\varphi$  is a bijective function.  $\forall x, y \in V$ ,

$$R(\theta(x), \theta(y)) = (R_T(\theta(x), \theta(y)), R_I(\theta(x), \theta(y)), R_F(\theta(x), \theta(y))).$$

$$\begin{aligned} \text{Now } R_T(\theta(x), \theta(y)) &= R_T(ba^{-1}x, ba^{-1}y) \\ &= T_A((ba^{-1}x)^{-1}(ba^{-1}y)) \\ &= T_A(x^{-1}y) = R_T(x, y). \end{aligned}$$

$$\begin{aligned} R_I(\theta(x), \theta(y)) &= R_I(ba^{-1}x, ba^{-1}y) \\ &= I_A((ba^{-1}x)^{-1}(ba^{-1}y)) \\ &= I_A(x^{-1}y) = R_I(x, y), \text{ and} \end{aligned}$$

$$\begin{aligned} R_F(\theta(x), \theta(y)) &= R_F(ba^{-1}x, ba^{-1}y) \\ &= F_A((ba^{-1}x)^{-1}(ba^{-1}y)) \\ &= F_A(x^{-1}y) = R_F(x, y). \end{aligned}$$

$R(\theta(x), \theta(y))$  hence equals  $R(x, y)$ . Consequently,  $\theta$  is an automorphism on  $G$ .

As well  $\theta(a) = b$ , therefore  $G$  is transitive. ■

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**Theorem 3.11:** Every vertex-transitive graph with constant neutrosophic edge set values is regular.

**Proof.** Assume that any vertex-transitive neutrosophic graph  $G = (V, R)$  exists. If you take, then  $f$  is automorphism function on  $G$  with  $f(u) = v$ .

Recall that

$$\begin{aligned}
 \text{ind}(u) &= \sum_{x \in V} R(x, u) = \sum_{x \in V} (R_T(x, u), R_I(x, u), R_F(x, u)) \\
 &= \sum_{x \in V} (R_T(f(x), f(u)), R_I(f(x), f(u)), R_F(f(x), f(u))) \\
 &= \text{ind}(v), \text{ where } f(x) = y. = \sum_{x \in V} (R_T(y, v), R_I(y, v), R_F(y, v)) \\
 \text{Outd}(u) &= \sum_{x \in V} R(x, u) = \sum_{x \in V} (R_T(u, x), R_I(u, x), R_F(u, x)) \\
 &, R_F(f(u), f(x)) = \sum_{x \in V} (R_T(f(u), f(x)), R_I(f(u), f(x))) \\
 &= \text{outd}(v), \text{ where } f(x) = y. = \sum_{x \in V} (R_T(v, y), R_I(v, y), R_F(v, y))
 \end{aligned}$$

Hence,  $G$  is regular. ■

**Corollary 3.12:** Every Cayley neutrosophic graph with constant neutrosophic edge set values is regular.

**Proof:** Theorems 3.8 and 3.9 directly contribute to the proof. ■

**Theorem 3.13:** Consider a (NG)  $G = (V, R)$ , and  $R$  is a neutrosophic relation on  $G$ . Then  $R$  is a reflexive, and  $R(1, 1) = (1, 0, 0)$ , if and only if  $T_A(1) = 1, I_A(1) = 0$ , and  $F_A(1) = 0$ .

**Proof.** Assume  $R$  to reflexive with  $R(x, x) = (1, 0, 0), \forall x \in V$ , Then

$$\begin{aligned}
 R(x, x) &= (T_A(x^{-1}x), I_A(x^{-1}x), F_A(x^{-1}x)) \\
 &= (T_A(1), I_A(1), F_A(1)) = R(1,1) \text{ for all } x \in V. \\
 \text{Since } R(1,1) &= (1,0,0) \text{ then } T_A(1) = 1, I_A(1) = 0 \text{ and } F_A(x) = 0. \\
 \text{Conversely suppose that } &T_A(1) = 1, I_A(1) = 0, \text{ and } F_A(1) = 0
 \end{aligned}$$

Then  $(1, 0, 0) = (T_A(1), I_A(1), F_A(1)) = R(1,1)$

Also,

$$(T_A(1), I_A(1), F_A(1)) = (T_A(x^{-1}x), I_A(x^{-1}x), F_A(x^{-1}x)) = R(x, x) \text{ then } R \text{ is reflexive. } \blacksquare$$

**Theorem 3.14:** If  $G = (V, R)$  is NG and  $R$  is a neutrosophic relation on  $G$ , then  $R$  is a symmetric relation if and only if  $A(x^{-1}) = A(x), \forall x \in V$

**Proof.** Assume that  $R$  is symmetric. Then  $\forall y, x \in V, R(y, x) = R(x, y)$

Now take  $y = x^2$  then

$$\begin{aligned}
 A(x) &= A(x^{-1}x^2) = R(x, x^2) = R(x, y) \\
 \text{since } R \text{ is symmetric then } &R(x, x^2) = R(x, y) = R(y, x) = R(x^2, x) \\
 &= (T_A((x^2)^{-1}x), I_A((x^2)^{-1}x), F_A((x^2)^{-1}x)) \\
 &= (T_A(x^{-2}x), I_A(x^{-2}x), F_A(x^{-2}x)) \\
 &= (A(x^{-1}))
 \end{aligned}$$

Conversely, suppose that  $A(x) = A(x^{-1}), \forall x \in V$ .

Then  $R(x, y) = A(x^{-1}y), \forall x, y \in V$ ,

$$\begin{aligned}
 R(x, y) &= A(x^{-1}y) \\
 &= A(x^{-1}x^2) = A(x) = A(x^{-1})
 \end{aligned}$$



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$$= (A(x^{-2}x) = A(y^{-1}x) = R(y, x)$$

Hence R is symmetric. ■

**Theorem 3.15:** Neutrosophic Relation R be antisymmetric if and only if

$$\{x: A(x) = A(x^{-1})\} = \{(1,1,1)\}$$

**Proof:** let  $x \in V$  and take R as anti-symmetric.

Then  $(T_R(x), I_R(x), F_R(x)) = (T_A(x^{-1}), I_A(x^{-1}), F_A(x^{-1}))$ , that implies  $R(1, x) = R(x, 1)$

As a result,  $x = 1$  [since R is antisymmetric].

Conversely, imagine  $\{x: R(x) = A(x^{-1})\} = \{(1, 0,0)\}$ .

So,  $\forall x, y \in V, R(x, y) = R(y, x) \Leftrightarrow A(x^{-1}y) = A(y^{-1}x)$ .

This implies that  $A(y^{-1}x) = A(x^{-1}y)^{-1}$ . So that  $x^{-1}y = 1$ .

In other words,  $x = y$ . R is hence antisymmetric ■

**Theorem 3.16:** Let R be SVN- relation. Then R is transitive iff  $(T_A, I_A, F_A)$  is neutrosophic sub-semigroup of  $(G, *)$ .

**Proof.** Let  $x, y \in V$  and R be transitive, then  $R \geq R^2$ .

Now  $\forall x \in V$ , we have  $R(1, x) = A(x)$ . This implies that

$$\vee \{R(1, x) \wedge R(z, xy) | z \in V\} = R^2(1, xy) \leq R(1, xy)$$

That is  $\vee \{(T_R(z) \wedge (T_R(z^{-1}xy) | z \in V) \leq (T_R(xy) \wedge \{I_R(z) \vee I_R(z^{-1}xy) | z \in V\} \geq I_R(xy)$  and  $\wedge \{F_R(z) \vee F_R(z^{-1}xy) | z \in V\} \geq F_R(xy)$ .

Then  $T_A(xy) \geq (T_A(x) \wedge T_A(y))$ ,

$$I_A(xy) \geq (I_A(x) \wedge I_A(y)), \text{ and } F_A(xy) \geq (F_A(x) \wedge F_A(y))$$

Hence  $(T_A, I_A, F_A)$  is neutrosophic sub-semigroup of  $(S, *)$ .

Conversely, suppose that  $A = (T_A, I_A, F_A)$  is neutrosophic sub-semigroup of  $(G, *)$ .

That is,  $\forall x, y \in V$ , comparative of the edge and vertices neutrosophic values conditions meet next for any  $x, y \in V$ ,

$$R^2(x, y) = (R_T^2(x, y), R_I^2(x, y), R_F^2(x, y))$$

$$R_T^2(x, y) = \vee \{R_T(x, z) \wedge R_T(z, y) | z \in V\}$$

$$= \vee \{T_A(x^{-1}z) \wedge T_A(z^{-1}, y) | z \in V\} \leq T_A(x^{-1}, y) = R_T(x, y)$$

$$R_I^2(x, y) = \wedge \{R_I(x, z) \vee R_I(z, y) | z \in V\}$$

$$= \wedge \{I_A(x^{-1}z) \vee I_A(z^{-1}, y) | z \in V\} \leq I_A(x^{-1}, y) = R_I(x, y)$$

$$R_F^2(x, y) = \wedge \{R_F(x, z) \vee R_F(z, y) | z \in V\}$$

$$= \wedge \{F_A(x^{-1}z) \vee F_A(z^{-1}, y) | z \in V\} \leq F_A(x^{-1}, y) = R_F(x, y)$$

Hence  $R_T^2(x, y) \leq R_T(x, y)$ ,  $R_I^2(x, y) \leq R_I(x, y)$ , and  $R_F^2(x, y) \leq R_F(x, y)$

Therefore, R be transitive. ■

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**Theorem 3.17:** If  $R$  be a neutrosophic relation then  $R$  is a partial order (PO) if and only if  $A = (T_B, I_B, F_B)$  is neutrosophic sub-semigroup of  $(V, *)$  satisfy the following

- (i)  $T_A(1) = 1, I_A(1) = 0, F_A(1) = 0,$
- (ii)  $\{x: (T_A(x), I_A(x), F_A(x)) = (T_A(x^{-1}), I_A(x^{-1}), F_A(x^{-1}))\} = \{(1,0,0)\}$

Proof: since  $R$  is (PO) then it is

- 1)  $R$  is transitive it means that (i) is satisfied by theorem 3.13
- 2)  $R$  is anti-symmetric which leads (ii) is met by theorem 3.15

Conversely let (i) , (ii) satisfied and  $A$  is neutrosophic sub-semigroup then from (i) and (ii) it can be proven that  $R$  is reflexive and anti-symmetric by theorems(3.13 and 3.15) respectively.

Additionally , since  $A$  is sub-semigroup then  $R$  is transitive by theorem 3.16 ■

**Theorem 3.18:**

Neutrosophic relation ( $R$ ) is linear order(LO) if and only if  $(T_B, I_B, F_B)$  is a neutrosophic sub-semigroup of  $(V, *)$ , In addition to conditions of theorem 3.17, the following conditions are satisfying:

- (iii)  $R^2 \leq R$ , that is,  
 $\{x, y | T_R(x, y) \geq T_{R \circ R}(x, y), I_R(x, y) \leq I_{R \circ R}(x, y), \text{ and } F_R(x, y) \leq F_{R \circ R} | x, y \in V \},$
- (iv)  $\{x | T_A(x) \vee T_A(x^{-1}) > 0, I_A(x) \wedge I_A(x^{-1}) > 0, \text{ and } F_A(x) \wedge F_A(x^{-1}) > 0\}.$

**Proof.** Suppose that  $R$  to be (LO). Therefore, the points (i), (ii) and (iii) are satisfied by Theorem 3.17.  $\forall x \in V, (R \vee R^{-1})(1, x) > 0$ , it follows from this  $R(1, x) \vee R(x, 1) > 0$  Therefore  $\{x | T_A(x) \vee T_A(x^{-1}) > 0, I_A(x) \wedge I_A(x^{-1}) > 0, \text{ and } F_A(x) \wedge F_A(x^{-1}) > 0\}.$

Conversely, assume that (i), (ii), and (iii) are held. According to the theorem 3.15,  $R$  is (PO). Now,  $\forall x, y \in V$  , we will have  $(x^{-1}y), (y^{-1}x) \in V$  .

Then by condition (iv),  
 $\{x | T_A(x) \vee T_A(x^{-1}) > 0, I_A(x) \wedge I_A(x^{-1}) > 0, \text{ and } F_A(x) \wedge F_A(x^{-1}) > 0\}.$

That is  $R(x, 1) \vee R(1, x) > 0$ . So,  $(R \vee R^{-1})(x, y) > 0$  ,and therefore,  $R$  is linear order. ■

**Theorem 3.19:** Suppose  $R$  be a neutrosophic relation. Thus, the relation  $R$  is (EQ) if and only if  $(T_A, I_A, F_A)$  be a neutrosophic sub-semigroup of  $(G, *)$  satisfy the following:

- (i)  $A(1) = (1,0,0).$
- (ii)  $A(x^{-1}) = A(x)$  for all  $x \in V$  .

**Proof:** Theorems 3.17 and 3.18 directly lead to the proof. ■

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#### 4. CAYLEY SINGLE VALUED NEUTROSOPHIC GRAPHS (CSVNG) INDUCED BY CAYLEY (SVN) GRAPHS

**Definition 4.1:** Assume that  $L$  is a semigroup and that  $A(x) \subset L$ . Then, all of the neutrosophic sub-semigroups of  $L$  that contain  $A$  are met in the sub-semigroup created by  $A$ , it has been shown by  $\langle A \rangle$ .

**Example 4.2:** Suppose that  $L = Z_3$ , and  $A = (T_A, I_A, F_A)$  as in example 3.5.

Then  $\langle A \rangle$  is provided by  $\langle A(0) \rangle = (1, 0, 0)$ , and  $\langle A(y) \rangle = (0.5, 0.5, 0.5)$ , when  $y = 1, 2$ .

**Theorem 4.3:** Suppose that  $(L, *)$  is a semigroup, and  $A = (T_A, I_A, F_A)$  is neutrosophic subset of  $L$ . So, neutrosophic subset  $\langle A \rangle$  is exactly stated by

$$\langle T_A(x) \rangle = \vee \{ (T_A(x_1) \wedge T_A(x_2) \wedge \dots \wedge T_A(x_n) : x = x_1, x_2, \dots, x_n \text{ with } T_A(x_i) > 0 \text{ for } i = 1, 2, \dots, n),$$

$$\langle I_A(x) \rangle = \vee \{ (I_A(x_1) \wedge I_A(x_2) \wedge \dots \wedge I_A(x_n) : x = x_1, x_2, \dots, x_n \text{ with } I_A(x_i) > 0 \text{ for } i = 1, 2, \dots, n) \text{ for any } x \in L.$$

$$\text{And } \langle F(x) \rangle = \wedge \{ F_A(x_1) \vee F_A(x_2) \vee \dots \vee F_A(x_n) : x = x_1, x_2, \dots, x_n \text{ with } F_A(x_i) > 0, \text{ for } i = 1, 2, \dots, n \} \forall x \in L.$$

**Proof.** Let  $A' = (T'_A, I'_A, F'_A)$ , be neutrosophic subset of  $L$  defined by

$$T'_A(x) = \vee \{ T_A(x_1) \wedge T_A(x_2) \wedge \dots \wedge T_A(x_n) : (T_A(x_n)) : x = x_1, x_2, \dots, x_n \text{ with } T_A(x_i) > 0 \text{ for } i = 1, 2, \dots, n),$$

$$I'_A(x) = \wedge \{ I_A(x_1) \vee I_A(x_2) \vee \dots \vee I_A(x_n) : x = x_1, x_2, \dots, x_n \text{ with } I_A(x_i) > 0 \text{ for } i = 1, 2, \dots, n),$$

$$F'_A(x) = \wedge \{ F_A(x_1) \vee F_A(x_2) \vee \dots \vee F_A(x_n) : x = x_1, x_2, \dots, x_n \text{ with } F_A(x_i) > 0 \text{ for } i = 1, 2, \dots, n), \text{ for any } x \in L.$$

Let  $x, y \in L$ . If  $T_A(x) = 0$  or  $T_A(y) = 0$ , then  $T_A(x) \wedge T_A(y) = 0$

and  $I_A(x) = 0$  or  $I_A(y) = 0$ , then  $I_A(x) \wedge I_A(y) = 0$ .

Therefore,  $T'_A(xy) \geq T_A(x) \wedge T_A(y)$ , and  $I'_A(xy) \geq I_A(x) \wedge I_A(y)$ .

Again, if  $T_A(x) \neq 0, I_A(x) \neq 0$ , and  $F_A(x) \neq 0$

then by definition of  $T'_A(x), I'_A(x)$ , and  $F'_A(x)$  we have  $T'_A(xy) \geq T_A(x) \wedge T_A(y)$  and  $I'_A(xy) \geq I_A(x) \vee I_A(y)$  and  $F'_A(xy) \leq F_A(x) \vee F_A(y)$ .

Hence  $(T'_A, I'_A, F'_A)$  is neutrosophic sub-semigroup of  $L$  containing  $(T_A, I_A, F_A)$ .

Now, let  $L'$  represents any neutrosophic sub-semigroup of  $L'$  that contains  $(T_A, I_A, F_A)$ .

Then,  $\forall x \in L$  with  $x = x_1, x_2, \dots, x_n$  with  $T_A(x_i) > 0, I_A(x_i) > 0$ , and  $F_A(x_i) > 0$ ,

for  $i = 1, 2, \dots, n$ , we have  $T_{L'}(x_i) \geq T_{L'}(x_1) \wedge T_{L'}(x_2) \wedge \dots \wedge T_{L'}(x_n) \geq$

$T_A(x_1) \wedge T_A(x_2) \wedge \dots \wedge T_A(x_n), I_{L'}(x_i) \leq I_{L'}(x_1) \wedge I_{L'}(x_2) \wedge \dots \wedge I_{L'}(x_n) \geq$

$I_A(x_1) \wedge I_A(x_2) \wedge \dots \wedge I_A(x_n)$  and  $F_{L'}(x_i) \leq F_{L'}(x_1) \wedge F_{L'}(x_2) \wedge \dots \wedge F_{L'}(x_n) \geq$

$F_A(x_1) \wedge F_A(x_2) \wedge \dots \wedge F_A(x_n) . T_{L'}(x_i) \geq T_A(x_1) \wedge T_A(x_2) \wedge \dots \wedge T_A(x_n)$

$x = x_1, x_2, \dots, x_n$  with  $T_A(x_i) > 0$  for  $i = 1, 2, \dots, n$ ,  $I_{L'}(x_i) \leq I_A(x_1) \wedge I_A(x_2) \wedge \dots \wedge$

$I_A(x_n) : x = x_1, x_2, \dots, x_n$  with  $I_A(x_i) > 0$  for  $i = 1, 2, \dots, n$ , and  $F_{L'}(x_i) \leq F_A(x_1) \wedge$

$F_A(x_2) \wedge \dots \wedge F_A(x_n) : x = x_1, x_2, \dots, x_n$  with  $F_A(x_i) > 0$ , for  $i = 1, 2, \dots, n$ .

$\forall x \in L, T'_A(x) \leq T_{L'}(x), I'_A(x) \geq I_{L'}(x)$ , and  $F'_A(x) \leq F_{L'}(x)$ , for all  $x \in L$ .

Thus  $A' = (T'_A, I'_A, F'_A)$  is the meet of all neutrosophic sub-semigroup containing  $(T_A, I_A, F_A)$ . ■



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## 5.PLANARITY IN CAYLEY (SVN) GRAPHS

**Definition 5.1:** A graph having the formula  $G=(V,E)$  with set of vertices  $V$  and edges set  $E$  is called planer graph if for any two edges do not overlap, except potentially at their end-vertice.

**Definition 5.2:** Let  $G$  be a group with generator  $X$  then both  $G$  and its generating set  $X$  are said to be planar if the Cayley graph  $C(G,X)$  is planar.

**Theorem 5.3:** For any subgroup of planar group is planar.

Proof :let  $G$  be planar group then there exists a set  $X \subset G$  in which  $C(G,X)$  be Cayley planar graph.

Take  $H$  as a subgroup of  $G$ . Since any subgraph of planar graph is planar then any subgraph of  $C(G,X)$  induced by vertices points of  $H$  is planar, then  $H$  is planar. ■

In reality, suppose that  $H \subset G$  and that is a Cayley graph for group  $G$  determined by

- 1) select a spanning tree  $T$  for the quotient  $H \backslash \Gamma$  of the  $H$ -action on  $\Gamma$ ,
- 2) Constrict each pre-image of  $T$  in  $\Gamma$  to a point.

**Definition 5.4:** With  $x$  denoting  $X$ 's general components. A SVN dual-set  $A$  taken from  $X$  described by the three functions counts( truth  $T_A$ , indeterminacy  $I_A$ , and falsity  $F_A$ )-memberships, from  $X$  into the real interval  $[0, 1]$ . Next, we define a SVN dual-set  $A$  as

$$A = \{ \langle x, (T_A^1(x), T_A^2(x), I_A^1(x), I_A^2(x), F_A^1(x), F_A^2(x)) \rangle \}$$

where the truth-membership sequence  $T_A^1(x), T_A^2(x)$ , the indeterminacy-membership sequence  $I_A^1(x), I_A^2(x)$ , and the falsity-membership sequence  $F_A^1(x), F_A^2(x)$  either a descending or ascent order, and sum of  $T_A^i(x), I_A^i(x), F_A^i(x) \in [0, 1]$ , fulfills the requirement

$$0 \leq \sup T_A^i(x) + \sup I_A^i(x) + \sup F_A^i(x) \leq 3$$

for  $x \in X$ . It is convenient to represent a (SVN) dual-set  $A$  as follows:

$$A = \{ \langle x, T_A(x)_i, I_A(x)_i, F_A(x)_i \mid x \in X, i = 1, 2 \rangle \}.$$

**Note:** Using the idea of (SVN) dual-sets, we first propose the SVN dual-graph notion.

**Definition 5.5:** Let  $A = (T_A, I_A, F_A)$  be SVNS on  $X \neq \emptyset$  and

$$B = \{ \langle xy: T_B(xy)_i, I_B(xy)_i, F_B(xy)_i \mid x, y \in X \times X, i = 1, 2 \rangle \}$$

be a (SVN) dual-set of  $X \times X$  such that neutrosophic comparative equations are satisfied, then  $G = (A, B)$  is said to be SVN dual-graph.

Keep in mind that there can be several edges connecting the vertices  $x$  and  $y$ . the edge  $xy$ 's three types of membership values: truth, indeterminacy, and falsehood reflect, respectively.  $B$  is referred to as a SVN dual-edge set in the SVN dual-graph  $G$ .

**Example 5.6:** Example in figure 2 is Cayley SVN-Dual graph.

**Theorem 5.7:** Every neutrosophic Cayley graph is planar if and only if its underline graph is planar

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**Definition 5.8:** Let  $B(xy)_i, i = 1, 2 | xy \in X \times X$  be a (SVN) dual-edge set in SVN dual-graph  $G$ . Both degree type of a vertex  $x \in V$  is defined by

$$in\text{-deg}(x) = (\sum_{i=1}^m T_B(\overline{xy})_i, \sum_{i=1}^m I_B(\overline{xy})_i, \sum_{i=1}^m F_B(\overline{xy})_i) .Or \text{ by}$$

$$out\text{-deg}(x) = (\sum_{i=1}^m T_B(\overline{xy})_i, \sum_{i=1}^m I_B(\overline{xy})_i, \sum_{i=1}^m F_B(\overline{xy})_i), \text{ for all } y \in X$$

**Example 5.9:** In example 2

$In\text{-deg}(1) = In\text{-deg}(3) = In\text{-deg}(5) = In\text{-deg}(7) = (0.6, 0.6, 1.4)$ . also  
 $Out\text{-deg}(1) = Out\text{-deg}(3) = Out\text{-deg}(5) = Out\text{-deg}(7) = (0.6, 0.6, 1.4)$ .  
 Then the graph is in-regular and out-regular i.e., it is regular.

**Definition 5.10:** Let  $B = \{(uv, T_B(uv)_i, I_B(uv)_i, F_B(uv)_i), i = 1, 2 | uv \in V \times V\}$  be (SVN) dual-edge set in SVN dual-graph  $G$ . A dual-edge  $xy$  of  $G$  is strong if the equalities hold

$$\begin{aligned} \text{Min}\{T_A(x), T_A(y)\} &= T_B(x, y) \\ \text{Min}\{I_A(x), I_A(y)\} &= I_B(x, y) \\ \text{Max}\{T_A(x), T_A(y)\} &= T_B(x, y), \text{ for all } i = 1, 2. \end{aligned}$$

**Definition 5.11:** Let  $B = \{(xy, T_B(xy)_i, I_B(xy)_i, F_B(xy)_i), | xy \in V \times V, i = 1, 2\}$  be a SVN dual-edge set in SVN dual-graph  $G$ . SVN dual-graph  $G$  is complete if only the equalities in the neutrosophic comparative equations hold,  $\forall i = 1, 2$ , and  $\forall x, y \in V$ .

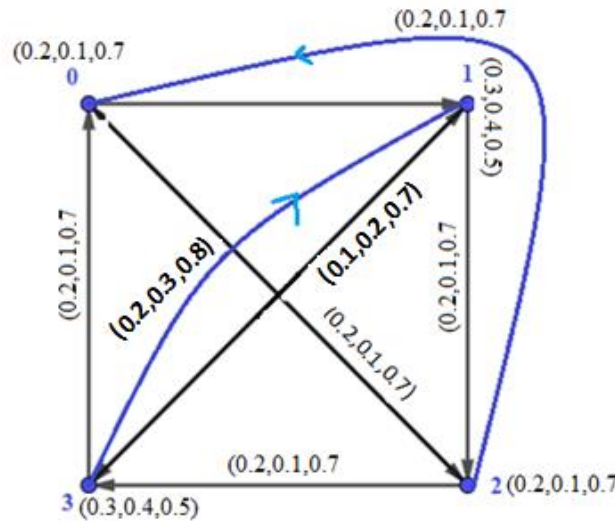


Figure 3 :Cayley graph  $C(Z_4, \{1,2\})$

**Remark 5.12:** A SVN-graph may be in-complete or out-complete

**Definition 5.13:** The strength of the SVN edge  $ab$  determined by the value

$$S_{ab} = ((ST)_{ab}, (SI)_{ab}, (SF)_{ab})$$

$$= (\min(\frac{T_B(ab)_i}{\text{Min}\{T_A(a), T_A(b)\}}), \min(\frac{I_B(ab)_i}{\text{Min}\{I_A(a), I_A(b)\}}), \max(\frac{\text{Max}\{F_A(a), F_A(b)\}}{F_B(ab)_i}))$$



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**Definition 5.14:** Make  $G$  an SVN dual graph. An edge  $ab$  is referred to a SVN strong edge if  $(ST)_{ab} \geq 0.5$ ,  $(SI)_{ab} \geq 0.5$ ,  $(SF)_{ab} \geq 0.5$ , unlike that, it is referred to as a weak-edge.

**Definition 5.15:** Consider the SVN dual-graph  $G = (A, B)$  with two edges  $(ab, T_B(ab)_i, I_B(ab)_i, F_B(ab)_i)$  and  $(cd, T_B(cd)_j, I_B(cd)_j, F_B(cd)_j)$  in  $B$ , which met at a point  $P$ . The intersection value at  $P$  is given by  $S_P = ((ST)_P, (SI)_P, (SF)_P) = (\frac{(ST)_{ab} + (ST)_{cd}}{2}, \frac{(SI)_{ab} + (SI)_{cd}}{2}, \frac{(SF)_{ab} + (SF)_{cd}}{2})$

A SVN dual-graph's planarity decreases as the number of intersection points rises.  $S_P$  is thus inversely proportional to the planarity for SVN dual-graph. Now, the idea of an SVN planar graph is introduced.

**Definition 5.16:**  $G$  is known SVN planar graph if  $P_1, P_2, P_3, \dots, P_Z$  are the points where the edges of a particular geometric representation intersect and  $G$  is a SVN dual-graph.  $f = (f_T, f_I, f_F)$ , where

$$f = (f_T, f_I, f_F) = (\frac{1}{1+(ST)_{P_1}+(ST)_{P_2}+\dots+(ST)_{P_i}}, \frac{1}{1+(SI)_{P_1}+(SI)_{P_2}+\dots+(SI)_{P_i}}, \frac{1}{1+(SF)_{P_1}+(SF)_{P_2}+\dots+(SF)_{P_i}})$$

Clearly,  $f = (f_T, f_I, f_F)$  is bounded and  $0 < f_T \leq 1$ ,  $0 < f_I \leq 1$ ,  $0 < f_F \leq 1$ .

The SVN- planarity value of a particular geometric representation of an SVN- planar network is  $(1, 1, 1)$  if there is no point of intersection.

In this situation, the crisp planar graph is the underlying crisp graph of the SVNG. The degree of planarity varies, and as a result, there are more and fewer places where the edges cross. In the event that  $f_F$  rises while  $f_T$  and  $f_I$  fall. We come to the conclusion that every SVNG is a planar and has a specific value for the single-valued neutrosophic planarity.

**Example 5.17:** in figure 3

$$\begin{aligned}
 \text{value } S_{31} &= ((ST)_{31}, (SI)_{31}, (SF)_{31}) \\
 &= (\min(\frac{T_B(31)}{\min\{T_A(3), T_A(1)\}}, \frac{T_B(13)}{\min\{T_A(3), T_A(1)\}}), \min(\frac{I_B(31)}{\min\{T_A(3), T_A(1)\}}, \frac{I_B(13)}{\min\{T_A(3), T_A(1)\}}), \max(\frac{\max\{F_A(1), F_A(3)\}}{F_B(31)}, \frac{\max\{F_A(1), F_A(3)\}}{F_B(13)})) \\
 &= ((\frac{0.2}{0.3} \wedge \frac{0.1}{0.3}), (\frac{0.3}{0.4} \wedge \frac{0.2}{0.4}), (\frac{0.5}{0.8} \vee \frac{0.5}{0.7})) = (\frac{0.1}{0.3}, \frac{0.1}{0.2}, \frac{0.5}{0.7}) = (\frac{1}{3}, \frac{1}{2}, \frac{5}{7})
 \end{aligned}$$

$$\begin{aligned}
 \text{value } S_{02} &= ((ST)_{02}, (SI)_{02}, (SF)_{02}) \\
 &= (\min(\frac{T_B(02)}{\min\{T_A(0), T_A(2)\}}, \min(\frac{I_B(02)}{\min\{I_A(0), I_A(2)\}}, \max(\frac{\max\{F_A(0), F_A(2)\}}{F_B(02)}))) = (\frac{0.2}{0.2}, \frac{0.2}{0.2}, \frac{0.2}{0.2}) = (1, 1, 1)
 \end{aligned}$$

$$\begin{aligned}
 S_{P_1} &= ((ST)_{P_1}, (SI)_{P_1}, (SF)_{P_1}) = (\frac{(ST)_{31} + (ST)_{02}}{2}, \frac{(SI)_{31} + (SI)_{02}}{2}, \frac{(SF)_{31} + (SF)_{02}}{2}) \\
 &= (\frac{\frac{1}{3} + 1}{2}, \frac{\frac{1}{2} + 1}{2}, \frac{\frac{5}{7} + 1}{2}) = (\frac{2}{3}, \frac{3}{4}, \frac{3}{7})
 \end{aligned}$$



## CONCLUSION

In this article, the concept of Neutrosophic Cayley graphs (NCG), which combines graph theory with algebraic structure, is presented. Then its features were studied in various contexts where the Cayley algebraic structure exists. A weighted directed graph of neutrosophic groups was designed that is expressed by three components (truth, indefiniteness, and falsity)—memberships as they were used and built into a relation. This is an improvement over FSs and IFSs. Numerous intriguing properties of the neutrosophic graph (NG), including transitivity and regularity, are visible. Additionally, Cayley's neutrosphere diagrams' planarity was established, and observations and ideas were developed in response.

## Conflict of interests.

There are non-conflicts of interest.

## References.

- [1] F. Karaaslan, "Gaussian single-valued neutrosophic numbers and its application in multi-attribute decision making," *Neutrosophic sets and Systems*, vol. 22, no. 1, pp. 101-117, 2018.
- [2] A. U. Rahman, M. Saeed, and A. Dhital, "Decision making application based on neutrosophic parameterized hypersoft set theory". *Neutrosophic Sets and Systems*, Vol. 41, PP2-14, 2021
- [3] B. C. Cuong and V. Kreinovich, "Picture fuzzy sets-a new concept for computational intelligence problems," in *2013 third world congress on information and communication technologies (WICT 2013)*, IEEE, pp. 1-6.2013.
- [4] M. Karunambigai, R. Parvathi, and O. Kalaivani, "A study on atanassov's intuitionistic fuzzy graphs," in *2011 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE 2011)*, 2011: IEEE, pp. 649-655.
- [5] M. Akram and W. A. Dudek, "Intuitionistic fuzzy hypergraphs with applications," *Information Sciences*, vol. 218, pp. 182-193, 2013.
- [6] Y. Pan, L. Zhang, Z. Li, and L. Ding, "Improved fuzzy Bayesian network-based risk analysis with interval-valued fuzzy sets and D-S evidence theory," *IEEE Transactions on Fuzzy Systems*, vol. 28, no. 9, pp. 2063-2077, 2019.
- [7] Z. Xu and X. Gou, "An overview of interval-valued intuitionistic fuzzy information aggregations and applications," *Granular Computing*, vol. 2, pp. 13-39, 2017.
- [8] A. Majeed and N. Arif, "Closed neutrosophic dominating set in neutrosophic graphs," *Neutrosophic Sets and Systems*, vol. 55, no. 1, p. 31, 2023.
- [9] G. Shahzadi, M. Akram, and A. B. Saeid, "An application of single-valued neutrosophic sets in medical diagnosis," *Neutrosophic sets and systems*, vol. 18, pp. 80-88, 2017.
- [10] S. A. Neamah and A. Erfanian, "Some Results on the Generalized Cayley Graph of Complete Graphs," *Iraqi Journal of Science*, pp. 3424-3436, 2023.
- [11] H. Nagesh and M. Kumar, "Block digraph of a directed graph," *Open Journal of Mathematical Sciences*, vol. 2, no. 1, pp. 202-208, 2018.
- [12] T. Asir and T. Tamizh Chelvam, "On the genus of generalized unit and unitary Cayley graphs of a commutative ring," *Acta Mathematica Hungarica*, vol. 142, pp. 444-458, 2014.
- [13] P. Mukherjee, R. Mukherjee, and S. K. Sardar, "Action graph of a semigroup act & its functorial connection," *Categories and General Algebraic Structures with Applications*, vol. 18, no. 1, pp. 43-80, 2023.
- [14] S. Broumi, M. Talea, A. Bakali, and F. Smarandache, "Single valued neutrosophic graphs," *Journal of New theory*, no. 10, pp. 86-101, 2016.

## الخلاصة

**الخلفية :** على الرغم من أن نظرية الرسم البياني تلعب دوراً هاماً في تمثيل أسباب المشكلة والعلاقة بين الأسباب لتسهيل حلها، إلا أن هناك العديد من المشاكل في حياتنا الواقعية لا يمكن تمثيلها بدقة بسبب عدم دقة البيانات المتاحة وغموضها .

**المواد والطرق:** تم استخدام المجموعات النيوتروسوفكية ذات القيمة الواحدة والتي تعتمد على ثلاث دوال من المجموعة العالمية مثلاً  $X$  إلى المدى القياسي  $[0,1]$  والمنبثقة من المدى غير القياسي  $[0, 1^+]$  التي تعتمد عليها المجموعات النيوتروسوفية. تم استخدام جدول كايلي للمجموعات الجبرية

**النتيجة:** قدم هذا المقال فكرة رسوم كايلي النيوتروسوفكية (NCG) كمزيج من نظرية الرسوم البيانية والبنية الجبرية، وتمت دراسة بعض خصائصها في بيئات مختلفة حيث توجد بنية كايلي الجبرية. على سبيل المثال، ثبت أن كل رسم بياني كايلي الانعكاسي والتعدي هو منتظم وكذلك تم إيجاد درجة الاستواء وهكذا.

**الاستنتاج:** يمكن استنتاج أنه تم بناء علاقة لرسم رسم بياني موجه مرجح لـ SVNS يحتوي على ثلاثة مكونات (الحقيقة، وعدم التحديد، والخطأ). وتم عرض العديد من الميزات المثيرة للاهتمام للرسم البياني النيوتروسوفكي (NG)، مثل العبور والانتظام. كما تم إثبات استواء الرسم البياني النيوتروسوفكي لكايلي، وتمت صياغة ملاحظات ونظريات فيما يتعلق به.

**الكلمات المفتاحية:** المجموعة النيوتروسوفكية، المجموعة النيوتروسوفكية ذات القيمة الواحدة، المجموعة الجبرية، المجموعات الجزئية النيوتروسوفكية، رسوم كايلي النيوتروسوفكية.