



The β^m – Homeomorphism in Double Fuzzy Topological Spaces

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ABSTRACT

The purpose of this study is to introduce the concept of homeomorphism via β^m – closed set and study its behavior and properties in double fuzzy topological spaces. This objective is achieved through the definitions of $df - \beta^m$ continuous functions and $df - \beta^m$ closed functions. The results of this study represent important relationships and proofs, in addition to providing some necessary examples.

التشاكل - β^m في الفضاءات التوبولوجية المضطربة المزدوجةسناء ابراهيم عبدالله¹، طه حميد جاسم²، علي عبد المجيد شهاب³^{2,1} قسم الرياضيات، كلية علوم الحاسوب والرياضيات، جامعة تكريت، تكريت، العراق³ قسم الرياضيات، كلية التربية للعلوم الصرفة، جامعة تكريت، تكريت، العراق

الملخص

في هذا البحث قدمنا مفهوم التشاكل التوبولوجي df-hom وكذلك مفهوم التشاكل التوبولوجي من النمط df- β^m hom. من خلال تقديمنا لتعريف الدوال المغلقة والدوال المستمرة على المجموعات المغلقة في الفضاءات التوبولوجية المضطربة المزدوجة. β^m closed set. $(\check{r}, \check{s}) - f$ وتم دراسة العلاقات بينهم من خلال مبرهنات والفرضيات والملاحظات والعديد من الامثلة الضرورية.

1. Introduction

Zadeh [1] presented his study on the concept of fuzzy set in 1965, which is a generalization of the classical sets. After that, in 1968 [2], Chang introduced his study on the fuzzy topological space. In 1993 [3], Atanassov introduced a new concept of sets, namely the intuitionistic fuzzy sets. On the basis of these sets, in 1997 [4], Coker introduced the fuzzy intuitionistic topological spaces. This study adopts the concept of Garcia and Rodabaugh [5] that the term double fuzzy topological spaces in mathematics is more appropriate than the intuitionistic fuzzy topological spaces. It studies df - β^m continuous functions, df - homeomorphism and df - β^m homeomorphism, as well as their properties and relationships.

2. Preliminaries

This section introduces some basic and essential concepts in this work. In this concern, a non-empty set is denoted by the symbol G and the closed unit interval $[0, 1]$ by I , also $I_{\check{r}}$ is denoted by $(0, 1]$ and $I_{\check{s}}$ is denoted by $[0, 1)$. The family of all fuzzy sets is denoted by I^G . Hence, $\check{0}$ and $\check{1}$ represent the smallest and greatest fuzzy sets, respectively. For the fuzzy set $\check{\alpha} \in I^G$, $\check{1} - \check{\alpha}$ denotes the complement of $\check{\alpha}$. The symbols \leq , \wedge and \vee represent the less or equal value, intersection and union, respectively. These symbols are used for fuzzy sets.

2.1. Definition: [1]

Let G be a non-empty set. A fuzzy set in G is characterized by its membership function $\delta_{\check{\alpha}}: G \rightarrow [0, 1]$ where $\delta_{\check{\alpha}}(g)$ is interpreted as the degree of membership of element g in fuzzy set $\check{\alpha}$, for each $g \in G$. It is clear that $\check{\alpha}$ is completely determined by the set of tuples $\check{\alpha} = \{(g, \delta_{\check{\alpha}}(g)): g \in G\}$.

2.2. Definition: [5]

A double fuzzy topology $(\mathcal{J}, \mathcal{J}^*)$ on a non-empty set G is a pair of functions $\mathcal{J}, \mathcal{J}^*: I^G \rightarrow I$, which satisfies the following properties:

- (i) $\mathcal{J}(\check{\alpha}) \leq \check{1} - \mathcal{J}^*(\check{\alpha})$ for each $\check{\alpha} \in I^G$.
- (ii) $\mathcal{J}(\check{\alpha}_1 \wedge \check{\alpha}_2) \geq \mathcal{J}(\check{\alpha}_1) \wedge \mathcal{J}(\check{\alpha}_2)$ and $\mathcal{J}^*(\check{\alpha}_1 \wedge \check{\alpha}_2) \leq \mathcal{J}^*(\check{\alpha}_1) \vee \mathcal{J}^*(\check{\alpha}_2)$ for each $\check{\alpha}_1, \check{\alpha}_2 \in I^G$.
- (iii) $\mathcal{J}(\bigvee_{k \in r} \check{\alpha}_k) \geq \bigwedge_{k \in r} \mathcal{J}(\check{\alpha}_k)$ and $\mathcal{J}^*(\bigvee_{k \in r} \check{\alpha}_k) \leq \bigvee_{k \in r} \mathcal{J}^*(\check{\alpha}_k)$ for each $\check{\alpha}_k \in I^G, k \in r$.

The triple $(G, \mathcal{J}, \mathcal{J}^*)$ is called double fuzzy topological spaces (dftss, for short).

2.3. Definition: [7]

Let $(G, \mathcal{J}, \mathcal{J}^*)$ be a dftss, then for each $\check{r} \in I_{\check{r}}$ and $\check{s} \in I_{\check{s}}$ and $\check{\alpha}, \delta \in I^G$, the double fuzzy closure $(C_{\mathcal{J}, \mathcal{J}^*})$ and interior $(I_{\mathcal{J}, \mathcal{J}^*})$ operator $C_{\mathcal{J}, \mathcal{J}^*}, I_{\mathcal{J}, \mathcal{J}^*}: I^G \times I_{\check{r}} \times I_{\check{s}} \rightarrow I^G$ are defined as follows:

$$C_{\mathcal{J}, \mathcal{J}^*}(\check{\alpha}, \check{r}, \check{s}) = \bigwedge \{ \delta \in I^G : \check{\alpha} \leq \delta, \mathcal{J}(\check{1} - \delta) \geq \check{r}, \mathcal{J}^*(\check{1} - \delta) \leq \check{s} \}.$$

$$I_{\mathcal{J}, \mathcal{J}^*}(\check{\alpha}, \check{r}, \check{s}) = \bigvee \{ \delta \in I^G : \delta \leq \check{\alpha}, \mathcal{J}(\delta) \geq \check{r}, \mathcal{J}^*(\delta) \leq \check{s} \}.$$

2.4. Definition:

Let $(G, \mathcal{J}, \mathcal{J}^*)$ be a dfts. For each $\tilde{\alpha} \in I^G, \check{r} \in I_{\check{r}}$ and $\check{s} \in I_{\check{s}}$.

(1) $\tilde{\alpha}$ is an (\check{r}, \check{s}) -fuzzy open set $((\check{r}, \check{s})$ -f open) if $\mathcal{J}(\tilde{\alpha}) \geq \check{r}$ and $\mathcal{J}^*(\tilde{\alpha}) \leq \check{s}$ and $\tilde{\alpha}$ is an (\check{r}, \check{s}) -fuzzy closed set $((\check{r}, \check{s})$ -f closed) if $\mathcal{J}(1 - \tilde{\alpha}) \geq \check{r}$ and $\mathcal{J}^*(1 - \tilde{\alpha}) \leq \check{s}$ [9].

(2) $\tilde{\alpha}$ is an (\check{r}, \check{s}) -fuzzy semi open set $((\check{r}, \check{s})$ -fs open) if $\tilde{\alpha} \leq C_{\mathcal{J}, \mathcal{J}^*}(I_{\mathcal{J}, \mathcal{J}^*}(\tilde{\alpha}, \check{r}, \check{s}), \check{r}, \check{s})$ and an $((\check{r}, \check{s})$ -fuzzy semi closed set $((\check{r}, \check{s})$ -fs-closed), if $I_{\mathcal{J}, \mathcal{J}^*}(C_{\mathcal{J}, \mathcal{J}^*}(\tilde{\alpha}, \check{r}, \check{s}), \check{r}, \check{s}) \leq \tilde{\alpha}$ [9].

(3) $\tilde{\alpha}$ is an (\check{r}, \check{s}) -fuzzy preopen set $((\check{r}, \check{s})$ -fp open) if $\tilde{\alpha} \leq I_{\mathcal{J}, \mathcal{J}^*}(C_{\mathcal{J}, \mathcal{J}^*}(\tilde{\alpha}, \check{r}, \check{s}), \check{r}, \check{s})$ and an $((\check{r}, \check{s})$ -fuzzy semi closed set $((\check{r}, \check{s})$ -fp closed), if $C_{\mathcal{J}, \mathcal{J}^*}(I_{\mathcal{J}, \mathcal{J}^*}(\tilde{\alpha}, \check{r}, \check{s}), \check{r}, \check{s}) \leq \tilde{\alpha}$ [10].

(4) $\tilde{\alpha}$ is an (\check{r}, \check{s}) -fuzzy α open set $((\check{r}, \check{s})$ -f α open) if $\tilde{\alpha} \leq I_{\mathcal{J}, \mathcal{J}^*}(C_{\mathcal{J}, \mathcal{J}^*}(I_{\mathcal{J}, \mathcal{J}^*}(\tilde{\alpha}, \check{r}, \check{s}), \check{r}, \check{s}), \check{r}, \check{s})$ and an $((\check{r}, \check{s})$ -fuzzy semi closed set $((\check{r}, \check{s})$ -f α closed), if $C_{\mathcal{J}, \mathcal{J}^*}(I_{\mathcal{J}, \mathcal{J}^*}(C_{\mathcal{J}, \mathcal{J}^*}(\tilde{\alpha}, \check{r}, \check{s}), \check{r}, \check{s}), \check{r}, \check{s}) \leq \tilde{\alpha}$ [8].

(5) $\tilde{\alpha}$ is an (\check{r}, \check{s}) -fuzzy β open set $((\check{r}, \check{s})$ -f β open) if $\tilde{\alpha} \leq C_{\mathcal{J}, \mathcal{J}^*}(I_{\mathcal{J}, \mathcal{J}^*}(C_{\mathcal{J}, \mathcal{J}^*}(\tilde{\alpha}, \check{r}, \check{s}), \check{r}, \check{s}), \check{r}, \check{s})$ and an $((\check{r}, \check{s})$ -fuzzy β closed set $((\check{r}, \check{s})$ -f β closed), if $I_{\mathcal{J}, \mathcal{J}^*}(C_{\mathcal{J}, \mathcal{J}^*}(I_{\mathcal{J}, \mathcal{J}^*}(\tilde{\alpha}, \check{r}, \check{s}), \check{r}, \check{s}), \check{r}, \check{s}) \leq \tilde{\alpha}$ [12].

(6) $\tilde{\alpha}$ is called (\check{r}, \check{s}) -generalized fuzzy closed $((\check{r}, \check{s})$ -gf closed) set if $C_{\mathcal{J}, \mathcal{J}^*}(\tilde{\alpha}, \check{r}, \check{s}) \leq \delta$, whenever, $\tilde{\alpha} \leq \delta$ and $\mathcal{J}(\delta) \geq \check{r}, \mathcal{J}^*(\delta) \leq \check{s}$. Complement of (\check{r}, \check{s}) -gf closed set is an (\check{r}, \check{s}) -gf open set [8].

(7) $\tilde{\alpha}$ is called (\check{r}, \check{s}) -fuzzy α^m closed $((\check{r}, \check{s})$ -f α^m closed) set if $I_{\mathcal{J}, \mathcal{J}^*}(C_{\mathcal{J}, \mathcal{J}^*}(\tilde{\alpha}, \check{r}, \check{s}), \check{r}, \check{s}) \leq \delta$, whenever, $\tilde{\alpha} \leq \delta$ and δ is an (\check{r}, \check{s}) -f α open. $\tilde{\alpha}$ is called an (\check{r}, \check{s}) -fuzzy α^m open $((\check{r}, \check{s})$ -f α^m open) if $1 - \tilde{\alpha}$ is an (\check{r}, \check{s}) -f α^m closed set [11].

2.5. Definition: [12]

Let $(G, \mathcal{J}, \mathcal{J}^*)$ be a dfts, for each $\tilde{\alpha}, \delta \in I^G, \check{r} \in I_{\check{r}}$ and $\check{s} \in I_{\check{s}}$. $\tilde{\alpha}$ is called an (\check{r}, \check{s}) -fuzzy β^m closed $((\check{r}, \check{s})$ -f β^m closed) set if $I_{\mathcal{J}, \mathcal{J}^*}(C_{\mathcal{J}, \mathcal{J}^*}(\tilde{\alpha}, \check{r}, \check{s}), \check{r}, \check{s}) \leq \delta$, whenever $\tilde{\alpha} \leq \delta$ and δ is an (\check{r}, \check{s}) -f β open. $\tilde{\alpha}$ is called an (\check{r}, \check{s}) -fuzzy β^m open $((\check{r}, \check{s})$ -f β^m open) if $1 - \tilde{\alpha}$ is an (\check{r}, \check{s}) -f β^m closed set.

2.6. Definition

Let $f: (G, \mathcal{J}_1, \mathcal{J}_1^*) \rightarrow (H, \mathcal{J}_2, \mathcal{J}_2^*)$, then f is said to be:

- (1) double fuzzy – closed (df – closed) function if image for every (\check{r}, \check{s}) -f closed set is an (\check{r}, \check{s}) -f closed set in H whenever $\tilde{\alpha} \in I^G, \check{r} \in I_{\check{r}}$ and $\check{s} \in I_{\check{s}}$ [9].
- (2) double fuzzy – continuous (df – con) function if inverse image for every (\check{r}, \check{s}) -f closed set is an (\check{r}, \check{s}) -f closed set in G whenever $\tilde{\alpha} \in I^G, \check{r} \in I_{\check{r}}$ and $\check{s} \in I_{\check{s}}$ [9].
- (3) double fuzzy – semi continuous (df – s con) function if inverse image for every (\check{r}, \check{s}) -f closed set is an (\check{r}, \check{s}) -f s closed set in G whenever $\tilde{\alpha} \in I^G, \check{r} \in I_{\check{r}}$ and $\check{s} \in I_{\check{s}}$ [6].
- (4) double fuzzy – generalized continuous (df – g con) function if inverse image for every (\check{r}, \check{s}) -f closed set is an (\check{r}, \check{s}) -gf closed set in G whenever $\tilde{\alpha} \in I^G, \check{r} \in I_{\check{r}}$ and $\check{s} \in I_{\check{s}}$ [6].
- (5) double fuzzy – α^m continuous (df – α^m con) function if inverse image for every (\check{r}, \check{s}) -f closed set is an (\check{r}, \check{s}) -f α^m closed set in G whenever $\tilde{\alpha} \in I^G, \check{r} \in I_{\check{r}}$ and $\check{s} \in I_{\check{s}}$ [11].

In this part, the double fuzzy β^m -homeomorphism is introduced.

3.1. Definition

Let $f: (G, \mathcal{J}_1, \mathcal{J}_1^*) \rightarrow (H, \mathcal{J}_2, \mathcal{J}_2^*)$, then f is said to be double fuzzy – β^m closed (df – β^m closed) function if image for every (\check{r}, \check{s}) -f closed set is an (\check{r}, \check{s}) -f β^m closed set in H whenever $\tilde{\alpha} \in I^G, \check{r} \in I_{\check{r}}$ and $\check{s} \in I_{\check{s}}$.

3.2. Example

Let $G = \{c, d\}, H = \{k, v\}$ and $(\mathcal{J}_1(\varphi), \mathcal{J}_1^*(\varphi))$ is defined on G by:

$$\mathcal{J}_1(\varphi) = \begin{cases} 1, & \text{if } \varphi \in \{0, 1\}, \\ \frac{1}{2}, & \text{if } \varphi(g) = \varphi_1, \\ 0, & \text{otherwise.} \end{cases}, \quad \mathcal{J}_1^*(\varphi) = \begin{cases} 0, & \text{if } \varphi \in \{0, 1\}, \\ \frac{1}{2}, & \text{if } \varphi(g) = \varphi_1 \\ 1, & \text{otherwise.} \end{cases}$$

Such that,

$$\varphi_1(c) = 0.4, \quad \varphi_1(d) = 0.6,$$

Also, $(\mathcal{J}_2(Y), \mathcal{J}_2^*(Y))$ is defined on H by:

$$\mathcal{J}_2(Y) = \begin{cases} 1, & \text{if } Y \in \{0, 1\}, \\ \frac{1}{2}, & \text{if } Y(h) = Y_1 \\ \frac{1}{4}, & \text{if } Y(h) = Y_2 \\ 0, & \text{otherwise.} \end{cases}, \quad \mathcal{J}_2^*(Y) = \begin{cases} 0, & \text{if } Y \in \{0, 1\}, \\ \frac{1}{2}, & \text{if } Y(h) = Y_1 \\ \frac{3}{4}, & \text{if } Y(h) = Y_2 \\ 1, & \text{otherwis.} \end{cases}$$

Such that,

$$Y_1(k) = 0.6, \quad Y_1(v) = 0.3.$$

And,
$$Y_2(k) = 0.4, \quad Y_2(v) = 0.7.$$

When, the function f between two dfts $(\mathcal{J}_1(\varphi), \mathcal{J}_1^*(\varphi))$ and $(\mathcal{J}_2(Y), \mathcal{J}_2^*(Y))$ is defined by: $f: (G, \mathcal{J}_1, \mathcal{J}_1^*) \rightarrow (H, \mathcal{J}_2, \mathcal{J}_2^*)$, as $f(c) = k$ and $f(d) = v$.

So, $\mathcal{J}_1(\varphi_1) \geq \frac{1}{2}, \mathcal{J}_1^*(\varphi_1) \leq \frac{1}{2}$, then $\mathcal{J}_1(1 - \varphi_1) \geq \frac{1}{2}, \mathcal{J}_1^*(1 - \varphi_1) \leq \frac{1}{2}$, such that $f(1 - \varphi_1) = f(\varphi_1^c) = (c_{0.6}, d_{0.4}) \leq Y_1$, where Y_1 is an $(\frac{1}{2}, \frac{1}{2}) - f\beta$ open set. So, $f(\varphi_1^c)$ is an $(\frac{1}{2}, \frac{1}{2}) - f\beta^m$ closed set. Hence, f is $df - \beta^m$ closed function.

3.3. Remark

Every $df -$ closed function is $df - \beta^m$ closed function, but need not conversely.

3.4. Example

Refer to example 3.2., a fuzzy set φ_1 is taken for dfts $(\mathcal{J}_1(\varphi), \mathcal{J}_1^*(\varphi))$ and the fuzzy sets Y_1 and Y_2 for dfts $(\mathcal{J}_2(Y), \mathcal{J}_2^*(Y))$, such that

$$\varphi_1(c) = 0.5, \quad \varphi_1(d) = 0.6$$

And,
$$Y_1(k) = 0.6, \quad Y_1(v) = 0.3$$

$$Y_2(k) = 0.4, \quad Y_2(v) = 0.6.$$

So, $\mathcal{J}_1(\varphi_1) \geq \frac{1}{2}, \mathcal{J}_1^*(\varphi_1) \leq \frac{1}{2}$, then $\mathcal{J}_1(1 - \varphi_1) \geq \frac{1}{2}, \mathcal{J}_1^*(1 - \varphi_1) \leq \frac{1}{2}$. Now, $f(1 - \varphi_1) = f(\varphi_1^c) = (c_{0.5}, d_{0.4}) \leq Y_1$, where Y_1 is an $(\frac{1}{2}, \frac{1}{2}) - f\beta$ open set. So, $f(\varphi_1^c)$ is an $(\frac{1}{2}, \frac{1}{2}) - f\beta^m$ closed set. Hence, f is $df - \beta^m$ closed function. But f is not $df -$ closed function, since $f(\varphi_1^c)$ is not an $(\frac{1}{2}, \frac{1}{2}) - f$ closed set.

3.5. Definition

Let $f: (G, \mathcal{J}_1, \mathcal{J}_1^*) \rightarrow (H, \mathcal{J}_2, \mathcal{J}_2^*)$ be a function between these dftss $(G, \mathcal{J}_1, \mathcal{J}_1^*)$ and $(H, \mathcal{J}_2, \mathcal{J}_2^*)$. Then, f is said to be a double fuzzy - β^m continuous ($df - \beta^m$ con) function if inverse image for every $\mathcal{J}_2(1 - \tilde{\alpha}) \geq \check{r}$, and $\mathcal{J}_2^*(1 - \tilde{\alpha}) \leq \check{s}$ is an $(\check{r}, \check{s}) - f\beta^m$ closed set in M, whenever $\tilde{\alpha} \in I^G, \check{r} \in I_f$ and $\check{s} \in I_s$.

3.6. Example

Let $G = \{c, d\}$, $H = \{k, v\}$ and $(\mathcal{J}_1(\varphi), \mathcal{J}_1^*(\varphi))$ is defined on G by:

$$\mathcal{J}_1(\varphi) = \begin{cases} 1, & \text{if } \varphi \in \{0, 1\}, \\ \frac{1}{2}, & \text{if } \varphi(g) = \varphi_1 \\ \frac{1}{4}, & \text{if } \varphi(g) = \varphi_2 \\ 0, & \text{otherwise.} \end{cases}, \quad \mathcal{J}_1^*(\varphi) = \begin{cases} 0, & \text{if } \varphi \in \{0, 1\}, \\ \frac{1}{2}, & \text{if } \varphi(g) = \varphi_1 \\ \frac{3}{4}, & \text{if } \varphi(g) = \varphi_2 \\ 1, & \text{otherwis.} \end{cases}$$

Such that, $\varphi_1(c) = 0.5, \varphi_1(d) = 0.4,$

And, $\varphi_2(c) = 0.4, \varphi_2(d) = 0.7.$

Also, $(\mathcal{J}_2(Y), \mathcal{J}_2^*(Y))$ is defined on H by:

$$\mathcal{J}_2(Y) = \begin{cases} 1, & \text{if } Y \in \{0, 1\}, \\ \frac{1}{2}, & \text{if } Y(h) = Y_1 \\ 0, & \text{otherwise.} \end{cases}, \quad \mathcal{J}_2^*(Y) = \begin{cases} 0 & \text{if } Y \in \{0, 1\}, \\ \frac{1}{2}, & \text{if } Y(h) = Y_1 \\ 1 & \text{otherwise.} \end{cases}$$

Such that, $Y_1(k) = 0.6$ and $Y_1(v) = 0.4,$

When the function f between the two dftss $(\mathcal{J}_1(\varphi), \mathcal{J}_1^*(\varphi))$ and $(\mathcal{J}_2(Y), \mathcal{J}_2^*(Y))$ is defined by: $f: (G, \mathcal{J}_1, \mathcal{J}_1^*) \rightarrow (H, \mathcal{J}_2, \mathcal{J}_2^*)$, as $f(c) = k$ and $f(d) = v$. So, $\mathcal{J}_2(Y_1) \geq \frac{1}{2}, \mathcal{J}_2^*(Y_1) \leq \frac{1}{2}, f^{-1}(Y_1^c) = (k_{0.4}, v_{0.6}) \leq \varphi_1$. Then, $f^{-1}(Y_1^c)$ is an $(\frac{1}{2}, \frac{1}{2}) - f \beta^m$ closed set. This implies that f is $df - \beta^m$ con function.

3.7. Definition

Let $(G, \mathcal{J}_1, \mathcal{J}_1^*)$ and $(H, \mathcal{J}_2, \mathcal{J}_2^*)$ be dftss. A bijection function $f: (G, \mathcal{J}_1, \mathcal{J}_1^*) \rightarrow (H, \mathcal{J}_2, \mathcal{J}_2^*)$ is said to be a double fuzzy – homeomorphism (df – hom) if f and f^{-1} are df - con functions.

3.8. Example

Let $G = \{c, d\}$, $H = \{k, v\}$ and $(\mathcal{J}_1(\varphi), \mathcal{J}_1^*(\varphi))$ is defined on G by:

$$\mathcal{J}_1(\varphi) = \begin{cases} 1, & \text{if } \varphi \in \{0, 1\}, \\ \frac{1}{2}, & \text{if } \varphi(g) = \varphi_1 \\ 0, & \text{otherwise.} \end{cases}, \quad \mathcal{J}_1^*(\varphi) = \begin{cases} 0 & \text{if } \varphi \in \{0, 1\}, \\ \frac{1}{2}, & \text{if } \varphi(g) = \varphi_1 \\ 1 & \text{otherwise.} \end{cases}$$

Such that,

$$\varphi_1(c) = 0.4, \quad \varphi_1(d) = 0.6,$$

Also, $(\mathcal{J}_2(Y), \mathcal{J}_2^*(Y))$ is defined on H by:

$$\mathcal{J}_2(Y) = \begin{cases} 1, & \text{if } Y \in \{0, 1\}, \\ \frac{1}{2}, & \text{if } Y(v) = Y_1 \\ 0, & \text{otherwise.} \end{cases}, \quad \mathcal{J}_2^*(Y) = \begin{cases} 0 & \text{if } Y \in \{0, 1\}, \\ \frac{1}{2}, & \text{if } Y(v) = Y_1 \\ 1 & \text{otherwise.} \end{cases}$$

Such that,

$$Y_1(k) = 0.4, \quad Y_1(v) = 0.6,$$

Doi:

When the bijection function f between the two dftss $(\mathcal{J}_1(\varphi), \mathcal{J}_1^*(\varphi))$ and $(\mathcal{J}_2(Y), \mathcal{J}_2^*(Y))$ is defined by: $f: (G, \mathcal{J}_1, \mathcal{J}_1^*) \rightarrow (H, \mathcal{J}_2, \mathcal{J}_2^*)$, as $f(c) = k$ and $f(d) = v$.

So, $\mathcal{J}_2(Y_1) \geq \frac{1}{2}, \mathcal{J}_2^*(Y_1) \leq \frac{1}{2}$, so $\mathcal{J}_2(1 - Y_1) \geq \frac{1}{2}, \mathcal{J}_2^*(1 - Y_1) \leq \frac{1}{2}$. Then, f is df - con function. And

$\mathcal{J}_1(\varphi_1) \geq \frac{1}{2}, \mathcal{J}_1^*(\varphi_1) \leq \frac{1}{2}$, so $\mathcal{J}_1(1 - \varphi_1) \geq \frac{1}{2}, \mathcal{J}_1^*(1 - \varphi_1) \leq \frac{1}{2}$.

Then, f^{-1} is df - con function. That is f is df - hom.

3.9. Remark

Every df - hom is df - closed (df - β^m closed) function, but need not conversely.

3.10. Example

Refer to example 3.2., a fuzzy set φ_1 is taken for dfts $(\mathcal{J}_1(\varphi), \mathcal{J}_1^*(\varphi))$ and the fuzzy sets Y_1 and Y_2 for dfts $(\mathcal{J}_2(Y), \mathcal{J}_2^*(Y))$, such that:

$$\varphi_1(c) = 0.5, \quad \varphi_1(d) = 0.5$$

And,

$$Y_1(k) = 0.5, \quad Y_1(v) = 0.5$$

$$Y_2(k) = 0.4, \quad Y_2(v) = 0.6.$$

So, $\mathcal{J}_1(\varphi_1) \geq \frac{1}{2}, \mathcal{J}_1^*(\varphi_1) \leq \frac{1}{2}$, then $\mathcal{J}_1(1 - \varphi_1) \geq \frac{1}{2}, \mathcal{J}_1^*(1 - \varphi_1) \leq \frac{1}{2}$. Then, f is df - closed function, and since every df - closed function is df - β^m closed function, so f is df - β^m closed function. But, f is not df - hom, since f is not df - con function.

3.11. Remark

Every df - hom is df - con (df - β^m con, df - α^m con, df - semi con, df - g con) function, but need not conversely.

3.12. Example

Refer to example 3.6., a fuzzy set φ_1 is taken for dfts $(\mathcal{J}_1(\varphi), \mathcal{J}_1^*(\varphi))$ and the fuzzy sets Y_1 and Y_2 for dfts $(\mathcal{J}_2(Y), \mathcal{J}_2^*(Y))$, such that: $\varphi_1(c) = 0.5, \varphi_1(d) = 0.5$,

$$\varphi_2(c) = 0.4, \quad \varphi_2(d) = 0.7.$$

And

$$Y_1(k) = 0.5, \quad Y_1(v) = 0.5$$

f is df - con (df - β^m con, df - α^m con, df - semi con, df - g con) function, but f is not df - hom, since f^{-1} is not df - con function.

3.13. Definition

A bijection function $f: (G, \mathcal{J}_1, \mathcal{J}_1^*) \rightarrow (H, \mathcal{J}_2, \mathcal{J}_2^*)$ is said to be a double fuzzy - β^m homeomorphism (df - β^m hom) if f and f^{-1} are df - β^m con functions.

3.14. Example

Let $G = \{c, d\}, H = \{k, v\}$ and $(\mathcal{J}_1(\varphi), \mathcal{J}_1^*(\varphi))$ is defined on G by:

$$\mathcal{I}_1(\varphi) = \begin{cases} 1, & \text{if } \varphi \in \{0, 1\}, \\ \frac{1}{2}, & \text{if } \varphi(g) = \varphi_1 \\ \frac{1}{4}, & \text{if } \varphi(g) = \lambda_2 \\ 0, & \text{otherwise.} \end{cases}, \quad \mathcal{I}_1^*(\varphi) = \begin{cases} 0, & \text{if } \varphi \in \{0, 1\}, \\ \frac{1}{2}, & \text{if } \varphi(g) = \varphi_1 \\ \frac{3}{4}, & \text{if } \varphi(g) = \varphi_2 \\ 1, & \text{otherwis.} \end{cases}$$

Such that, $\varphi_1(c) = 0.5, \varphi_1(d) = 0.4,$

And, $\varphi_2(c) = 0.4, \varphi_2(d) = 0.7.$

Also, $(\mathcal{I}_2(Y), \mathcal{I}_2^*(Y))$ is defined on H by:

$$\mathcal{I}_2(Y) = \begin{cases} 1, & \text{if } Y \in \{0, 1\}, \\ \frac{1}{2}, & \text{if } Y(h) = Y_1 \\ \frac{1}{4}, & \text{if } Y(h) = Y_2 \\ 0, & \text{otherwise.} \end{cases}, \quad \mathcal{I}_2^*(Y) = \begin{cases} 0, & \text{if } Y \in \{0, 1\}, \\ \frac{1}{2}, & \text{if } Y(gh) = Y_1 \\ \frac{3}{4}, & \text{if } Y(h) = Y\varphi_2 \\ 1, & \text{otherwis.} \end{cases}$$

Such that, $Y_1(k) = 0.6, Y_1(v) = 0.4,$

And, $Y_2(k) = 0.5, Y_2(v) = 0.6.$

Now, f is df- β^m con function, since $f^{-1}(1 - Y_1)$ is an $(\frac{1}{2}, \frac{1}{2}) - f \beta^m$ closed set. And, f^{-1} is df- β^m con function, since $(f^{-1})^{-1}(1 - \varphi_1)$ is an $(\frac{1}{2}, \frac{1}{2}) - f \beta^m$ closed set. That is, f is df- β^m hom.

3.15. Theorem

Every df – hom is df – β^m hom, but need not conversely.

Proof

Let a function $f: (G, \mathcal{I}_1, \mathcal{I}_1^*) \rightarrow (H, \mathcal{I}_2, \mathcal{I}_2^*)$ be a df – hom.

Then, f and f^{-1} are df– con function. This implies that f and f^{-1} are df– β^m con function. That is a function f is a df – β^m hom.

The following example demonstrates that the converse of this theorem is not true.

3.16. Example

Refer to example 3.14., f is df– β^m hom. But, f is not df– hom, since $f^{-1}(1 - Y_1)$ is not $(\frac{1}{2}, \frac{1}{2}) - f$ closed set.

3.17. Theorem

Let $f: (G, \mathcal{I}_1, \mathcal{I}_1^*) \rightarrow (H, \mathcal{I}_2, \mathcal{I}_2^*)$ be a function, if f is df – bijective function, then the following statements are equivalent:

- i) $f^{-1}: (H, \mathcal{I}_2, \mathcal{I}_2^*) \rightarrow (G, \mathcal{I}_1, \mathcal{I}_1^*)$ is df – β^m con function.
- ii) f is df – β^m open function.
- iii) f is df – β^m closed function.

Proof

Doi:

i → ii: Let $\mathcal{J}_1(\tilde{\alpha}) \geq \check{r}$ and $\mathcal{J}_1^*(\tilde{\alpha}) \leq \check{s}$, then $\mathcal{J}_1(\check{1} - \tilde{\alpha}) \geq \check{r}$ and $\mathcal{J}_1^*(\check{1} - \tilde{\alpha}) \leq \check{s}$, where $\check{r} \in I_{\check{r}}$ and $\check{s} \in I_{\check{s}}$. Since, f^{-1} is $df - \beta^m$ con function. Then, $(f^{-1})^{-1}(\check{1} - \tilde{\alpha})$ is an $(\check{r}, \check{s}) - f\beta^m$ closed set in H.

This implies that $f(\check{1} - \tilde{\alpha})$ is an $(\check{r}, \check{s}) - f\beta^m$ closed set in H. That is $\check{1} - f(\tilde{\alpha})$ is an $(\check{r}, \check{s}) - f\beta^m$ closed set in H.

So, $f(\tilde{\alpha})$ is an $(\check{r}, \check{s}) - f\beta^m$ open set in H. This implies that f is $df - \beta^m$ open function.

ii → iii: Let $\mathcal{J}_1(\check{1} - \delta) \geq \check{r}$ and $\mathcal{J}_1^*(\check{1} - \delta) \leq \check{s}$, then $\mathcal{J}_1(\check{1} - (\check{1} - \delta)) \geq \check{r}$ and $\mathcal{J}_1^*(\check{1} - (\check{1} - \delta)) \leq \check{s}$, where $\check{r} \in I_{\check{r}}$ and $\check{s} \in I_{\check{s}}$. Since f is $df - \beta^m$ open function, then $f(\check{1} - (\check{1} - \delta))$ is an $(\check{r}, \check{s}) - f\beta^m$ open set in H. This implies that $\check{1} - f(\check{1} - \delta)$ is an $(\check{r}, \check{s}) - f\beta^m$ open set in H. That is $f(\check{1} - \delta)$ is an $(\check{r}, \check{s}) - f\beta^m$ closed set in H.

This implies that f is $df - \beta^m$ closed function.

iii → i: Let $\mathcal{J}_1(\check{1} - \gamma) \geq \check{r}$ and $\mathcal{J}_1^*(\check{1} - \gamma) \leq \check{s}$, where $\check{r} \in I_{\check{r}}$ and $\check{s} \in I_{\check{s}}$.

Since f is $df - \beta^m$ closed function, then $f(\check{1} - \gamma)$ is an $(\check{r}, \check{s}) - f\beta^m$ closed set in H. That is $(f^{-1})^{-1}(\check{1} - \gamma)$ is an $(\check{r}, \check{s}) - f\beta^m$ closed set in H. That is f^{-1} is $df - \beta^m$ con function.

3.18. Theorem

Let a function $f: (G, \mathcal{J}_1, \mathcal{J}_1^*) \rightarrow (H, \mathcal{J}_2, \mathcal{J}_2^*)$ be $df - \beta^m$ hom. Then, a function f is $df - \text{hom}$ if G and H represent $df - (\mathcal{J}, \mathcal{J}^*)_{T\beta^m}$ space.

Proof

Let $\mathcal{J}_2(\check{1} - \tilde{\alpha}) \geq \check{r}$ and $\mathcal{J}_2^*(\check{1} - \tilde{\alpha}) \leq \check{s}$, where $\check{r} \in I_{\check{r}}$ and $\check{s} \in I_{\check{s}}$. Since, f is $df - \beta^m$ hom, then f is $df - \beta^m$ con function. That is, $f^{-1}(\check{1} - \tilde{\alpha})$ is an $(\check{r}, \check{s}) - f\beta^m$ closed set in G. Now,

since G is $df - (\mathcal{J}, \mathcal{J}^*)_{T\beta^m}$ space, this implies that $f^{-1}(\check{1} - \tilde{\alpha})$ is an $(\check{r}, \check{s}) - f$ closed set in G.

Hence, f is $df - \text{con}$ function, by hypothesis, $f^{-1}: (H, \mathcal{J}_2, \mathcal{J}_2^*) \rightarrow (G, \mathcal{J}_1, \mathcal{J}_1^*)$ is $df - \beta^m$ con function.

Let $\mathcal{J}_1(\check{1} - \gamma) \geq \check{r}$ and $\mathcal{J}_1^*(\check{1} - \gamma) \leq \check{s}$, where $\check{r} \in I_{\check{r}}$ and $\check{s} \in I_{\check{s}}$.

Then, $(f^{-1})^{-1}(\check{1} - \gamma) = f(\check{1} - \gamma)$ is an $(\check{r}, \check{s}) - f\beta^m$ closed set in H, since H is $df - (\mathcal{J}, \mathcal{J}^*)_{T\beta^m}$ space. That

is $f(\check{1} - \gamma)$ is an $(\check{r}, \check{s}) - f$ closed set in H. This implies that f^{-1} is $df - \text{con}$ function. Therefore, a function f is $df - \text{hom}$.

3.19. Proposition

The composition of two $df - \beta^m$ hom does not need $df - \beta^m$ hom, as clarified in the following example.

3.20. Example

Let $G = \{c, d\}$, $H = \{k, h\}$ and $F = \{q, w\}$, $(\mathfrak{I}_1(\varphi), \mathfrak{I}_1^*(\varphi))$ is defined on M by:

$$\mathfrak{I}_1(\varphi) = \begin{cases} \check{1}, & \text{if } \varphi \in \{\check{0}, \check{1}\}, \\ \frac{1}{2}, & \text{if } \varphi(g) = \varphi_1 \\ \frac{1}{4}, & \text{if } \varphi(g) = \varphi_2 \\ \check{0}, & \text{otherwise.} \end{cases}, \quad \mathfrak{I}_1^*(\varphi) = \begin{cases} \check{0}, & \text{if } \varphi \in \{\check{0}, \check{1}\}, \\ \frac{1}{2}, & \text{if } \varphi(g) = \varphi_1 \\ \frac{3}{4}, & \text{if } \varphi(g) = \varphi_2 \\ \check{1}, & \text{otherwis.} \end{cases}$$

Such that,

$$\varphi_1(c) = 0.2, \quad \varphi_1(d) = 0.4,$$

And, $\varphi_2(c) = 0.8, \quad \varphi_2(d) = 0.6.$

Also, $(\mathcal{J}_2(Y), \mathcal{J}_2^*(Y))$ is defined on H by:

$$\mathcal{I}_2(\Upsilon) = \begin{cases} 1, & \text{if } \Upsilon \in \{0, 1\}, \\ \frac{1}{2}, & \text{if } \Upsilon(h) = \Upsilon_1 \\ \frac{1}{4}, & \text{if } \Upsilon(h) = \Upsilon_2 \\ 0, & \text{otherwise.} \end{cases}, \quad \mathcal{I}_2^*(\Upsilon) = \begin{cases} 0, & \text{if } \Upsilon \in \{0, 1\}, \\ \frac{1}{2}, & \text{if } \Upsilon(h) = \Upsilon_1 \\ \frac{3}{4}, & \text{if } \Upsilon(h) = \Upsilon_2 \\ 1, & \text{otherwis.} \end{cases}$$

Such that,

$$\Upsilon_1(k) = 0.4, \quad \Upsilon_1(v) = 0.9.$$

And, $\Upsilon_2(k) = 0.4, \quad \Upsilon_2(v) = 0.3.$

Also, $(\mathcal{I}_3(\vartheta), \mathcal{I}_3^*(\vartheta))$ is defined on F by:

$$\mathfrak{I}_3(\vartheta) = \begin{cases} 1, & \text{if } \vartheta \in \{0, 1\}, \\ \frac{1}{2}, & \text{if } \vartheta(f) = \vartheta_1 \\ \frac{1}{4}, & \text{if } \vartheta(f) = \vartheta_2 \\ 0, & \text{otherwise.} \end{cases}, \quad \mathfrak{I}_3^*(\vartheta) = \begin{cases} 0, & \text{if } \vartheta \in \{0, 1\}, \\ \frac{1}{2}, & \text{if } \vartheta(f) = \vartheta_1 \\ \frac{3}{4}, & \text{if } \vartheta(f) = \vartheta_2 \\ 1, & \text{otherwis.} \end{cases}$$

Such that,

$$\vartheta_1(q) = 0.6, \quad \vartheta_1(w) = 0.6.$$

And, $\vartheta_2(q) = 0.4, \quad \vartheta_2(w) = 0.8.$

When the bijection function \mathcal{F} between the two dftss $(\mathfrak{I}_1(\varphi), \mathfrak{I}_1^*(\varphi))$ and $(\mathfrak{I}_2(\Upsilon), \mathfrak{I}_2^*(\Upsilon))$ is defined by:

$$f: (G, \mathcal{I}_1, \mathcal{I}_1^*) \rightarrow (H, \mathcal{I}_2, \mathcal{I}_2^*) \text{ as, } f(c) = k \text{ and } f(d) = v.$$

Now, $\mathcal{I}_2(\Upsilon_1) \geq \frac{1}{2}, \mathcal{I}_2^*(\Upsilon_1) \leq \frac{1}{2}$, so $\mathcal{I}_2(1 - \Upsilon_1) \geq \frac{1}{2}, \mathcal{I}_2^*(1 - \Upsilon_1) \leq \frac{1}{2}$,

$f^{-1}(1 - \Upsilon_2) \leq \varphi_2$, φ_2 is an $(\frac{1}{2}, \frac{1}{2})$ - $f\beta$ open set. So,

$f^{-1}(\Upsilon_2^c)$ is an $(\frac{1}{2}, \frac{1}{2})$ - $f\beta^m$ closed set. Hence, f is $df - \beta^m$ closed function. And f^{-1} is $df - \beta^m$ con function, since $\mathfrak{I}_1(\varphi) \geq \frac{1}{2}, \mathfrak{I}_1^*(\varphi_2) \leq \frac{1}{2}$, so $\mathfrak{I}_1(1 - \varphi_2) \geq \frac{1}{2}, \mathfrak{I}_1^*(1 - \varphi_2) \leq \frac{1}{2}, (f^{-1})^{-1}(1 - \varphi_2) \leq \Upsilon_2, \Upsilon_2$ is an $(\frac{1}{2}, \frac{1}{2})$ - $f\beta$ open set. So, $(f^{-1})^{-1}(\varphi_2^c)$ is an $(\frac{1}{2}, \frac{1}{2})$ - $f\beta^m$ closed set. Hence, f^{-1} is $df - \beta^m$ con function. That is f is $df - \beta^m$ hom. And, the

bijection function g between the two dftss $(\mathfrak{I}_2(\Upsilon), \mathfrak{I}_2^*(\Upsilon))$ and $(\mathfrak{I}_3(\vartheta), \mathfrak{I}_3^*(\vartheta))$ is defined by:

$$\mathcal{I}_2^* \rightarrow (F, \mathcal{I}_3, \mathcal{I}_3^*) \text{ as, } g(k) = q \text{ and } g(v) = w.$$

$\mathcal{I}_3^*(\vartheta_1) \leq \frac{1}{2}$, so $\mathcal{I}_3(1 - \vartheta_1) \geq \frac{1}{2}, \mathcal{I}_3^*(1 - \vartheta_1) \leq \frac{1}{2}, g^{-1}(1 - \vartheta_1) \leq \Upsilon_2, \Upsilon_2$ is an $(\frac{1}{2}, \frac{1}{2})$ - $f\beta$ open set. $(g^{-1})^{-1}(1 - \Upsilon_2) \leq \vartheta_1, \vartheta_1$ is an $(\frac{1}{2}, \frac{1}{2})$ - $f\beta$ open set. So, $(g^{-1})^{-1}(\Upsilon_2^c)$ is an $(\frac{1}{2}, \frac{1}{2})$ - $f\beta^m$ closed set. Hence, g is $df - \beta^m$ closed function.

is $df - \beta^m$ con function, since $\mathcal{I}_2(\Upsilon_2) \geq \frac{1}{2}, \mathcal{I}_2^*(\Upsilon_2) \leq \frac{1}{2}$, so $\mathcal{I}_2(1 - \Upsilon_2) \geq \frac{1}{2}, \mathcal{I}_2^*(1 - \Upsilon_2) \leq \frac{1}{2}$,

$(g^{-1})^{-1}(1 - \Upsilon_2) \leq \vartheta_1, \vartheta_1$ is an $(\frac{1}{2}, \frac{1}{2})$ - $f\beta$ open set. So, $(g^{-1})^{-1}(\Upsilon_2^c)$ is an $(\frac{1}{2}, \frac{1}{2})$ - $f\beta^m$ closed set.

Hence, g^{-1} is $df - \beta^m$ con function. That is g^{-1} is $df - \beta^m$ hom.

Now, the bijection function f between the two dftss $(\mathcal{I}_1(\varphi), \mathcal{I}_1^*(\varphi))$ and $(\mathcal{I}_3(\vartheta), \mathcal{I}_3^*(\vartheta))$ is defined by: $g \circ f: (G, \mathcal{I}_1, \mathcal{I}_1^*) \rightarrow (F, \mathcal{I}_3, \mathcal{I}_3^*)$ as, $g \circ f(c) = q$ and $g \circ f(d) = w$.

$\mathcal{I}_3^*(\vartheta_1) \leq \frac{1}{2}$, so $\mathcal{I}_3(1 - \vartheta_1) \geq \frac{1}{2}, \mathcal{I}_3^*(1 - \vartheta_1) \leq \frac{1}{2}$,

$(g \circ f)^{-1}(1 - \vartheta_1) \leq 1, 1$ is an $(\frac{1}{2}, \frac{1}{2})$ - $f\beta$ open set. So, $(g \circ f)^{-1}(\vartheta_1^c)$ is an $(\frac{1}{2}, \frac{1}{2})$ - $f\beta^m$ closed set.

$(g \circ f)$ is $df - \beta^m$ con function. But $(g \circ f)^{-1}$ is not $df - \beta^m$ con function.

$\mathcal{I}_1^*(\varphi_2) \leq \frac{1}{2}$, so $\mathcal{I}_1(1 - \varphi_2) \geq \frac{1}{2}, \mathcal{I}_1^*(1 - \varphi_2) \leq \frac{1}{2}$,

$((g \circ f)^{-1})(1 - \varphi_2) \leq \varphi_1, \varphi_1$ is an $(\frac{1}{2}, \frac{1}{2})$ - $f\beta$ open set. So,

$(g \circ f)^{-1}(\varphi_2^c)$ is not $(\frac{1}{2}, \frac{1}{2})$ - $f\beta^m$ closed set. Hence, $(g \circ f)$ is not $df - \beta^m$ hom.

3.21. Proposition

The composition of two $df - \beta^m$ hom needs $df - \beta^m$ hom if H is $df - (\mathcal{J}, \mathcal{J}^*)_{T\beta^m}$ space.

Proof

Let a function $f: (G, \mathcal{J}_1, \mathcal{J}_1^*) \rightarrow (H, \mathcal{J}_2, \mathcal{J}_2^*)$ and $g: (H, \mathcal{J}_2, \mathcal{J}_2^*) \rightarrow (F, \mathcal{J}_3, \mathcal{J}_3^*)$ be $df - \beta^m$ hom. And let $\mathcal{J}_3(\bar{1} - \bar{\alpha}) \geq \check{r}$ and $\mathcal{J}_3^*(\bar{1} - \bar{\alpha}) \leq \check{s}$, since g is $df - \beta^m$ hom. So, $g^{-1}(\bar{1} - \bar{\alpha})$ is an $(\check{r}, \check{s}) - f\beta^m$ closed set in H . Since a space H is $df - (\mathcal{J}, \mathcal{J}^*)_{T\beta^m}$ space. Then, $g^{-1}(\bar{1} - \bar{\alpha})$ is an $(\check{r}, \check{s}) - f$ closed set in H , since f is $df - \beta^m$ hom. This implies that $f^{-1}(g^{-1}(\bar{1} - \bar{\alpha}))$ is an $(\check{r}, \check{s}) - f\beta^m$ closed set in G . Then, $f^{-1}(g^{-1}(\bar{1} - \bar{\alpha})) = (g \circ f)^{-1}(\bar{1} - \bar{\alpha})$ is an $(\check{r}, \check{s}) - f\beta^m$ closed set in G . Therefore, $g \circ f$ is $df - \beta^m$ hom.

3. Conclusion

In this study, the ideas and definitions of symmetry on functions in the fuzzy double topological space are presented and studied. This definition is generalized to a definition of a new concept, which is $df - \beta^m$ homomorphism. In addition, the study develops the relationship between these new types of symmetry of functions in double fuzzy topological spaces with some other types of functions that were previously studied through theorems and a number of examples. Finally, the results are as follows:

- (1) Every $df - \beta^m$ hom is $df - \beta^m$ con ($df - \beta^m$ con, $df - \alpha^m$ con, $df - \text{semi con}$, $df - g$ con) function.
- (2) Every $df - \beta^m$ hom is $df - \beta^m$ hom.

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