

Extended Cantor's Algorithm for Non-Singular Hyperelliptic
curves to Singular Hyperelliptic curves

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Abstract

There are many applications for elliptic curves in cryptography. Cantor's algorithm relies on the Mumford representation of the points in jacobians. This compact representation of points in jacobians and Cantor's algorithm make non-singular hyperelliptic curves suitable for many applications in cryptography. This paper show the extension of cantor's representation for singular hyperelliptic curves.

الخلاصة:

يوجد استخدام واسع لموضوع المنحنيات البيضاوية في التشفير. خوارزمية كانتور تعتمد على تمثيل ممفورد للنقاط في الجاكوبيان. هذا الترابط الشديد للنقاط يعمل على جعل المنحنيات البيضاوية غير الأحادية مناسبة الاستخدام في كثير من التطبيقات لموضوع التشفير. في هذا البحث سوف نستعرض خوارزمية جديدة لتوسيع هذا الاستخدام ليشمل المنحنيات البيضاوية الأحادية.

1- Introduction

Cantor's Algorithm gives an efficient way for computing in the Jacobian of a non-singular hyperelliptic curve[3]. This algorithm relies on the Mumford Representation of points in Jacobian and Cantor's Algorithm make non-singular hyperelliptic curves suitable for many applications in cryptography. This paper show the extension of the Cantor's Algorithm for singular hyperelliptic curves. The algorithm that we present in section 3 mainly depends on this extension.

The use of non-singular hyperelliptic curves, especially lower genus ones, mainly depends on the hardness of the Discrete Logarithm Problem (DLP) on their Jacobians in a finite field [4].

The DLP in the Jacobian of a singular hyperelliptic curve is at most hard as the DLP in the multiplicative group of the finite field.

Key words: Hyperelliptic curves, Jacobian, cantor's algorithm, Mumford representation

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2-Background

2-1 Hyperelliptic curves:

Definition: A curve over a field k is a reduced Noetherian connected scheme of dimension, a complete irreducible curve x is called hyperelliptic if there is a morphism $h: x \rightarrow P_1$ of degree 2 [5].

2-2 Jacobian

Definition: $Jac(x)$, of a hyperelliptic curve x is isomorphic to the identity component, the group $Pic(x)$ which is the free abelian group of divisor classes of x modulo principle divisor [1].

2.3 Cantor's algorithm[2]

This algorithm takes two divisor classes $D_1 = [u_1(x), v_1(x)]$ and $D_2 = [u_2(x), v_2(x)]$ on X and outputs the unique reduced divisor D such that $D = D_1 + D_2$. The algorithm can be clarified as shown below :-

1. $D = gcd(u_1, u_2, v_1 + v_2)$ With *polynomials* h_1, h_2, h_3 such that $d = h_1 u_1 + h_2 u_2 + h_3 (v_1 + v_2)$
2. $u = \frac{u_1 u_2}{d^2}$ and $v \equiv \frac{h_1 u_1 v_2 + h_2 u_2 v_1 + h_3 (v_1 v_2 + f)}{d} \pmod{u}$
3. Repeat:
4. $\tilde{u} = \frac{v^2 - f}{u}$ and $\tilde{v} \equiv v \pmod{\tilde{u}}$
5. $u = \tilde{u}$ and $v = \tilde{v}$

Until $deg(u) \leq g$

6. Multiply u by a constant to make u manic.

2-4 Mumford representation[5]

Let $P_i = (x_i, y_i)$ be a point on X . the map $w : X \rightarrow X, w(x_i, y_i) = (x_i, -y_i)$ is called the hyperelliptic involution. Let D be a divisor class in $Jac(X)$. Then D can be written as $\sum_i n_i (P_i - \infty)$.

Let $D = \sum_i n_i (P_i - \infty)$ with $P_i = (x_i, y_i)$ be a divisor class in $Jac(X)$. D is called a reduced divisor if it satisfies the following:

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1. $n_i \geq 0$ for all i
2. If $y_i=0$ then $n_i=0$ or 1
3. If P_i with $y_i \neq 0$ Occurs in the sum, then $[w(P_i)]$ does not occurs.
4. $\sum_i n_i \leq g$.

There is a unique reduced divisor D for each divisor class in $\text{Jac}(X)$. The representation $\sum_i n_i(P_i - \infty)$ of a divisor class is not concrete enough to perform group operations efficiently in $\text{Jac}(X)$.

3-The Developed Algorithm

We show in this section that the above methods for non-singular hyperelliptic curves can be extended to singular hyperelliptic curves. The curve $X : y^2 = f(x)$ is singular if $f(x)$ has a multiple root (**deg $f(x)$ is still $2g + 1$**). The curve \hat{X} is normalization of X . The singular points of X are of the form $(a, 0)$ where (a) is a root of $f(x)$ with multiplicity greater than 1. Any divisor class D in $\text{Jac}(X)$ has a unique representative $(u(x), v(x))$ satisfying the following:

1. $u(x)$ is a monic polynomial in $k[x]$
2. $\deg(v(x)) < \deg(u(x)) \leq g$
3. $v(x)^2 - f(x)$ is divisible by $u(x)$.
4. If $u(x)$ and $v(x)$ are multiples of $(x - a)$ for a singular point $(a, 0)$ then $\frac{f(x) - v(x)^2}{u(x)}$ is not a multiple of $(x - a)$.

4-Conclusion:

This paper show that Cantor's Algorithm works for singular Hyperelliptic curves as the same way in the non-singular case.

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