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Carrier Heating Effects in Quantum Dot Semiconductor Optical Amplifiers

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Abstract

In this paper, we are introduce a new model to simulate the influence of carrier heating in quantum dot optical amplifiers. depending on density matrix theory, rate equations for two-level and the analytical solution of pulse propagation in quantum dot (QD) the nonlinear gain coefficient due carrier heating has been derive. The effects of energy splitting, detuning, carrier density and time of spectral hole burning versus nonlinear gain coefficient have been investigated.Also, the time recovery of ground state (GS) with carrier heating effects hasbeen calculated, the resultsshows a good agreement with research.

Keyword: Quantum dot, semiconductor optical amplifier, nonlinear gain coefficients.

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تأثيرات الحاملات الحرارية في المضخمات البصرية الشبة موصلة الكمية النقطية

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الخلاصة

في هذا البحث نقدم نموذج جيد لمحاكاة تأثير حرارة الحاملات في المضخم البصري شبة الموصل النانوي.بالاعتماد على نظرية كثافة المصفوفة و معادلات التغير الزمني لنظام مستويين. و أن نموذج الحل التحليلي لسريان النبضات في النقط الكمية. ان معاملات الربح اللاخطية بسبب حرارة الحاملات مشتق.

كذلك زمن الاستعادة بوجود تأثير حرارة الحاملات تم حسابه و النتائج توضح توافق جيد مع البحوث العالمية .

الكلمات المفتاحية: النقط الكمية ، المضخم البصري لأشباه الموصلات ،معاملات الربح اللاخطية

1. Introduction

The effects of carrier heating (CH) in semiconductor optical amplifier (SOA) is no less important from spectral hole brining, it is intraband process and affect strongly the gain dynamic of bulk and quantum well (QW) with subpicosecond time scale [1-3],makea strong contribution to the high-speed performance of thedevices [4,5] . The main sources of CH are injected carrier from barrier to dot structure[6] , carrier recombination , where the “cold carriers” which are close to the bandedge are removed [7] , free carrier absorption which includes the photonabsorption by the interaction of free carrier within the same band [8] . In all of these process the temperature of carriers will be higher than the lattice temperature. To reach thermal equilibrium, the carriers will transfer the access energy to the lattice through the interaction with phonons [9].

Carrier heating effect has been studied by nonlinear gain coefficients, which affects strongly the maximum modulation bandwidth and wavelength conversion [10-13]. Carrier heating influence on the performance of lasers and amplifiers in bulk and QW has been reported by a number of work [13-15], and used the CH nonlinear gain coefficient in QW to model the conversion efficiency of four-wave mixing (FWM) in QD[12]. The theoretical study of Auger capture induced by CH in QD has been introduce by [4].

In this work, a model to simulate CH in QD structure depending density matrix theory and rate equations is present, and the nonlinear gain coefficient due CH in the FWM stated here which not done earlier .

2. The rate equations

The rate equations describe dynamic of carriers for QD 2-level system including Fermi function at relaxation and CH contribution are given as [7,12]

$$\frac{dN_w}{dt} = D\frac{\rho_c}{\tau_e} - \frac{N_w(1-\rho_c)}{\tau_c} - \frac{N_w}{\tau_s} + \frac{I}{eV} \quad (1)$$

$$\frac{d\rho_c}{dt} = -\frac{(\rho_c - f_c)}{\tau_e} + \frac{1}{D}\frac{N_w(1-(\rho_c - f_c))}{\tau_c} - \frac{(\rho_c - f_c)}{\tau_s} - \frac{(\rho_c - f_c^{\,L})}{\tau_{CH}} + 2a(f\ )E(t) \quad (2)$$

ρ_c is the occupation probability of ground state, N_w is the carriers density, τ_e and τ_c are the carrierescape and carrier capture times respectively, D is the total number of states, τ_s is the spontaneous time, τ_{CH} is the CH lifetime , I is the injected current, V is the volume, e is the electron charge, $E(t)$ is the electric field of the interacting light, f_c is Fermi function which is a function of temperature.

$$f_c(t) = F(T(t)) \tag{3}$$

f_c^L is Fermi function at lattice temperature, the steady state and small signal of Fermi function can be written as [7]

$$f_{Fc} = \bar{f}_{c,k} + \frac{\partial f_{c,k}}{\partial T_c}(\tilde{T}_c \exp(-i\,\delta t)) \tag{4}$$

where $a_n(f_c)$ is the absorption renormalized for the occupation probability. According to density matrix theory $a_n(f)$ is given as [12]

$$a(f) = -\frac{i}{2DV\hbar}\sum_i \mu_{cv,i}\left(\rho_{cdvd,i} - \rho_{vdcd,i}\right) \tag{5}$$

$\rho_{cdvd,i}$ is the coherence term of the density-matrix equations, μ_{cv} is the transition dipole moment[12]

3. Theory of Carrier Heating in QD SOA

Nonlinearities in the semiconductor material leads to the generation of fields at the combination frequencies $\omega_\ell = \omega_0 + \ell.\delta,\ \ell = \pm1, \pm2, ...$, where δ is the detuning frequency. The pump signal at ω_0 is much stronger than all other fields, so that only the pump saturates the medium.This allows us to neglect higher harmonics after $\omega_2 = \omega_0 - \delta$, while the signal generated at ω_2 denoted the conjugatesignal.[12] Thus, consider a total electric field propagatingin the amplifier of the form

$$\vec{E}(z,t) = \vec{E}_0(z)e^{-i\omega_0 t} + \vec{E}_1(z)e^{-i\omega_1 t} + \vec{E}_2(z)e^{-i\omega_2 t} + c.c \tag{6}$$

$\vec{E}_0$ is the slowly varying amplitude of the pump, $\vec{E}_1$ is that of the probe and $\vec{E}_2$ is the conjugate formed through nonlinear mixing . The field $\vec{E}(z,t)$ induces a polarization $\vec{P}(z,t)$ in theactive medium of the amplifier [12]

$$\vec{P}(z,t) = \vec{P}_0(z)e^{-i\omega_0 t} + \vec{P}_1(z)e^{-i\omega_1 t} + \vec{P}_2(z)e^{-i\omega_2 t} + c.c \tag{7}$$

where $\vec{P}_0(z)$, $\vec{P}_1(z)$ and $\vec{P}_2(z)$ are the components of polarization, the orders of polarization are given by the following equations [7].

$$\vec{P}_0 = \frac{1}{V}\sum_j \frac{\left|\mu_{cv,j}\right|^2}{\hbar}\chi_j(\omega_o)(\bar{\rho}_{c,j} + \bar{\rho}_{v,j} - 1)E_0 \tag{8}$$

$$\vec{P}_1 = \frac{1}{V}\sum_j \frac{|\mu_{cv,j}|^2}{\hbar}\chi_j(\omega_1)\left[(\bar{\rho}_{c,j}+\bar{\rho}_{v,j}-1)E_1 + (\tilde{\rho}_{c,j}+\tilde{\rho}_{v,j})E_0\right] \quad (9)$$

$$\vec{P}_2 = \frac{1}{V}\sum_j \frac{|\mu_{cv,j}|^2}{\hbar}\chi_j(\omega_2)\left[(\bar{\rho}_{c,j}+\bar{\rho}_{v,j}-1)E_2 + (\tilde{\rho}^*_{c,j}+\tilde{\rho}^*_{v,j})E_0\right] \quad (10)$$

($(\bar{\rho}_{c,j}+\bar{\rho}_{v,j}-1)$ and $(\tilde{\rho}_{c,j}+\tilde{\rho}_{v,j})$) are consider Fermi function at steady state and small-signal analysis. The small-signal analysis for carrier density and occupation probability is [7,12]

$$\rho_c = \bar{\rho}_c + \tilde{\rho}_c\, e^{-i\delta t} + \tilde{\rho}_c\, e^{i\delta t} \quad (11)$$

$$N_w = \bar{N}_w + \tilde{N}_w\, e^{-i\delta t} + \tilde{N}_w\, e^{i\delta t} \quad (12)$$

where all the variables on the RHS of the Eqs.(11) and (12) are time independent. For the steady-state solution, the combination of Eq. (2) for conduction and valance band, one obtain [12].

$$(\bar{\rho}_c + \bar{\rho}_v - 1) = \frac{\left(\frac{2}{D}\frac{\bar{N}_w}{\tau_c}\tau_{in} - 1\right)}{1 + \frac{2i\,\tau_{in}}{\hbar^2}|\mu_{cv}|^2\left[\chi_j(\omega_0) - \chi^*_j(\omega_0)\right]} \quad (13)$$

where

$$\tau_{in} = \left(\frac{1}{D}\frac{\bar{N}_w}{\tau_c} + \frac{1}{\tau_e} + \frac{1}{\tau_{CH}}\right)^{-1} \quad (14)$$

and $\chi_j(\omega)$ is the Lorentzian lineshape determined by the decoherencetime and is responsible for homogeneous broadening and defined as

$$\chi_j(\omega) = \frac{1}{\left((\omega - \omega_j) + i\,\gamma_2\right)} \quad (15)$$

When the pump is turned off it is expected that the dot occupation probabilities should be the same as the occupationprobability under thermal equilibrium, F, such that $(\bar{\rho}_{c,j} + \bar{\rho}_{v,j} - 1) = (F_c + F_v - 1)$ [12]. By taking $E_0 = 0$ in Eq. (13), one obtains

$$(F_c + F_v - 1) = \left(\frac{2}{D}\frac{\bar{N}_w}{\tau_c}\tau_{in} - 1\right) \quad (16)$$

The derivation of eq. (16) gives

$$\left(\frac{d\bar{\rho}_{c,j}}{dN_w} + \frac{d\bar{\rho}_{v,j}}{dN_w}\right) = \left(\frac{2}{D}\frac{\tau_{in}}{\tau_c}\right)\left(1 - \frac{1}{D}\frac{\tau_{in}}{\tau_c}\bar{N}_w\right) \quad (17)$$

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In QD structure, the time of spectral-hole burning time constant is equivalent $\left(\frac{\bar{N}_w}{D\tau_c}+\frac{1}{\tau_e}\right)^{-1}$ which represents the rate at which the quantum-dot ensemblewill relax to thermal equilibrium via these captureand escape dynamics and it limits by carrier densities in wetting layer, so that τ_{in} can be considered as a total intraband time constant.

Similar to the steady-state analysis, the small-signal analysis of Eq. (2)in conduction and valence bands respected to $\left(e^{-i\delta t}\right)$ is derive with substituting Eqs.(11), (12) and (4) in Eq. (2) and combine for conduction and valence bands, one obtain

$$\left(-i\delta+\frac{1}{\tau_e}+\frac{N_w}{D}\frac{1}{\tau_c}+\frac{1}{\tau_{CH}}\right)\left(\tilde{\rho}_{c,k}+\tilde{\rho}_{v,k}\right)=-\left(\frac{N_w}{D}\frac{1}{\tau_c}\right)\left(\bar{\rho}_{c,k}+\bar{\rho}_{v,k}\right)+\left(\frac{1}{\tau_e}+\frac{N_w}{D}\frac{1}{\tau_c}+\frac{1}{\tau_{CH}}\right)$$

$$\left(\bar{f}_{c,k}+\bar{f}_{v,k}\right)+\frac{N_w}{D\,\tau_c}+\left(\frac{1}{\tau_e}+\frac{N_w}{D}\frac{1}{\tau_c}+\frac{1}{\tau_{CH}}\right)\left(\frac{\partial f_{c,k}}{\partial T_c}\tilde{T}_c+\frac{\partial f_{v,k}}{\partial T_v}\tilde{T}_v\right)-\frac{i\,|\mu_k|^2}{\hbar^2}\left(\bar{\rho}_{c,k}+\bar{\rho}_{v,k}-1\right)\times$$

$$\left[\hat{\chi}(\omega_1)-\hat{\chi}^*(\omega_0)\right]E_1E_0^*+\left[\hat{\chi}(\omega_0)-\hat{\chi}^*(\omega_2)\right]E_0E_2^*)-\frac{i\,|\mu_{cv}|^2}{\hbar^2}\left(\tilde{\rho}_{c,k}+\tilde{\rho}_{v,k}\right)\left[\hat{\chi}(\omega_1)-\hat{\chi}^*(\omega_2)\right]|E_0|^2 \qquad (18)$$

$$\left(\tilde{\rho}_{c,k}+\tilde{\rho}_{v,k}\right)=\frac{1}{(1-i\,\tau_{in}\delta)}\{\tilde{N}_w\,[(-\frac{N_w}{D}\frac{1}{\tau_c})\left(\bar{\rho}_{c,k}+\bar{\rho}_{v,k}-1\right)+(\frac{N_w}{D}\frac{1}{\tau_c})]+\left(\frac{\partial f_{c,k}}{\partial T_c}\tilde{T}_c+\frac{\partial f_{v,k}}{\partial T_v}\tilde{T}_v\right)$$

$$-\frac{2i\,\tau_{in}\,|\mu_k|^2}{\hbar^2}\left(\bar{\rho}_{c,k}+\bar{\rho}_{v,k}-1\right)\times\left[\hat{\chi}(\omega_1)-\hat{\chi}^*(\omega_0)\right]E_1E_0^*+\left[\hat{\chi}(\omega_0)-\hat{\chi}^*(\omega_2)\right]E_0E_2^*) \qquad (19)$$

The small signal of Eq. (1) and (19) and using Eqs.(11), (12) and (4), the result

$$\tilde{N}_w=\frac{-i}{D}\frac{1}{V}X\sum_i\frac{|\mu_i|^2}{\hbar^2}\left(\frac{2\bar{N}_w\tau_{in}}{D\,\tau_c}-1\right)\times\left[\hat{\chi}(\omega_1)-\hat{\chi}^*(\omega_0)\right]E_1E_0^*+\left[\hat{\chi}(\omega_0)-\hat{\chi}^*(\omega_2)\right]E_0E_2^*)*$$

$$\frac{\{1-\frac{2i\,\tau_{in}\,|\mu_k|^2}{\hbar^2}|E_0|^2\left[\hat{\chi}_i(\omega_1)-\hat{\chi}_i^*(\omega_2)\right]\}}{[(WY-XZ)+\frac{i}{D}\frac{1}{V}X\sum_i\frac{|\mu_i|^2}{\hbar^2}|E_0|^2\left[\hat{\chi}_i(\omega_1)-\hat{\chi}_i^*(\omega_2)\right]\left(\frac{2\tau_{in}}{D\,\tau_c}\right)[1-\frac{2\bar{N}_w\tau_{in}}{D\,\tau_c}]}+$$

$$\frac{\frac{-i}{D}\frac{1}{V}X\sum_i\frac{|\mu_i|^2}{\hbar^2}|E_0|^2\left[\hat{\chi}_i(\omega_1)-\hat{\chi}_i^*(\omega_2)\right]\left(\frac{\partial f_{c,k}}{\partial T_c}\tilde{T}_c+\frac{\partial f_{v,k}}{\partial T_v}\tilde{T}_v\right)}{[(WY-XZ)+\frac{i}{D}\frac{1}{V}X\sum_i\frac{|\mu_i|^2}{\hbar^2}|E_0|^2\left[\hat{\chi}_i(\omega_1)-\hat{\chi}_i^*(\omega_2)\right]\left(\frac{2\tau_{in}}{D\,\tau_c}\right)[1-\frac{2\bar{N}_w\tau_{in}}{D\,\tau_c}]} \qquad (20)$$

where

$$Y=\left(\frac{1}{\tau_e}+\frac{1}{D}\frac{\bar{N}_w}{\tau_c}+\frac{1}{\tau_s}+\frac{1}{\tau_{CH}}-i\,\delta\right) \tag{21}$$

$$Z=\frac{1}{D}\left(\frac{1-\bar{\rho}_c}{\tau_c}\right) \tag{22}$$

$$W=\left(\frac{1}{\tau_e}+\frac{1}{\tau_s}-\frac{\bar{\rho}_c}{\tau_c}+\frac{1}{\tau_{CH}}-i\,\delta\right) \tag{23}$$

Taking equations (9-10) and combining them with the earlier expressions for the polarization densities, it found that the pump polarization density and linear susceptibility, $X^{(l)}$, to be by using equation (13) written as

$$P_0(t)=\frac{1}{V}\sum_{j=i,k}\frac{|\mu_j|^2}{\hbar}\hat{\chi}_j(\omega_0)\left(\frac{\left(\frac{2\bar{N}_w\tau_{in}}{D\,\tau_c}-1\right)}{\{1+\frac{2i\,\tau_{in}|\mu_k|^2}{\hbar^2}|E_0|^2\left[\hat{\chi}_i(\omega_0)-\hat{\chi}_i^*(\omega_0)\right]\}}\right)E_0 \tag{24}$$

The linear susceptibility is simply extracted by comparison with $P_0=\varepsilon_0\chi_1 E$

$$X^{(l)}=\frac{1}{\varepsilon_0}\frac{1}{V}\sum_{j=i,k}\frac{|\mu_i|^2}{\hbar}\hat{\chi}_j(\omega_0)\left(\frac{\left(\frac{2\bar{N}_w\tau_{in}}{D\,\tau_c}-1\right)}{\{1+\frac{2i\,\tau_{in}|\mu_k|^2}{\hbar^2}|E_0|^2\left[\hat{\chi}_i(\omega_0)-\hat{\chi}_i^*(\omega_0)\right]\}}\right) \tag{25}$$

The second order polarization can be obtained by substituting Eq.(13) and (19) in Eq.(9), we get

$$P_1(t)=\varepsilon_0\chi^{(l)}(\omega_1)E_1+\frac{1}{V}\sum_{j=i,k}\frac{|\mu_i|^2}{\hbar}\hat{\chi}_j(\omega_1)\frac{1}{(1-i\,\tau_{in}\delta)}\{\tilde{N}_w[(-\frac{1}{D}\frac{\tau_{in}}{\tau_c})(\bar{\rho}_{c,k}+\bar{\rho}_{v,k}-1)$$
$$+(\frac{1}{D}\frac{\tau_{in}}{\tau_c})]\}+\frac{1}{V}\sum_{j=i,k}\frac{|\mu_i|^2}{\hbar}\hat{\chi}_j(\omega_1)\frac{1}{(1-i\,\tau_{in}\delta)}\{(\frac{\partial f_{c,k}}{\partial T_c}\tilde{T}_c+\frac{\partial f_{v,k}}{\partial T_v}\tilde{T}_v)\}-\frac{1}{V}\sum_{j=i,k}\frac{|\mu_i|^2}{\hbar}\hat{\chi}_j(\omega_1)$$
$$\frac{1}{(1-i\,\tau_{in}\delta)}\frac{2i\,\tau_{in}|\mu_k|^2}{\hbar^2}(\bar{\rho}_{c,k}+\bar{\rho}_{v,k}-1)\times(\left[\hat{\chi}(\omega_1)-\hat{\chi}^*(\omega_0)\right]E_1E_0^*+\left[\hat{\chi}(\omega_0)-\hat{\chi}^*(\omega_2)\right]E_0E_2^*)E_0$$
(26)

The induced polarization density is split into threeterms. The First is linear polarization density associatedwith gain or absorption in the optical amplifier, the second is the interaction between the pump and probe due to carrier density pulsation, spectral hole burning and finally the nonlinear interaction due to CH.

The susceptibility due carrier density pulsation (CDP) and spectral hole burning (SHB) can be simplified using the definition of gain and differential gain which are expressed by

$$g(\omega)=\frac{i\,\omega}{2c\,n\,\varepsilon_0}\frac{1}{V}\sum_j\frac{|\mu_j|^2}{\hbar}\left(\frac{2}{D}\frac{\bar{N}_w}{\tau_c}\tau_{in}-1\right)\left(\chi_j(\omega)-\chi_j^*(\omega)\right) \qquad (27)$$

$$\frac{dg}{dN}(\alpha+i)=\frac{-\omega}{c\,n\,\varepsilon_0}\frac{1}{V}\sum_j\frac{|\mu_j|^2}{\hbar}\left(\frac{2}{D}\frac{\tau_{in}}{\tau_c}\right)\left(1-\frac{1}{D}\frac{\tau_{in}}{\tau_c}\bar{N}_w\right)\chi_j(\omega) \qquad (28)$$

Eqs.(27) and (28) havebeen derived by substituting the relation of $\left(\bar{\rho}_{c,j}+\bar{\rho}_{v,j}-1\right)$ and $\left(\frac{d\bar{\rho}_{c,j}}{dN_w}+\frac{d\bar{\rho}_{v,j}}{dN_w}\right)$ in QD system identified into Eqs. (16), (17).These identities also introduce an important parameterwhich can be compared to experiment including linewidth enhancement factor, α_N , the refractive index, n , and the material gain, $g(\omega)$, which is calculated from the susceptibility defined in Eq (25). Substitute Eqs. (20), (27), (28) into Eq. (26), we can determine generalized susceptibilities due to CDP and SHB as

$$X^{CDP}(\omega_q;\omega_m;\omega_n)=\frac{\varepsilon_0\dfrac{2(cn)^2}{\hbar\omega_0\omega_q}\dfrac{dg(\omega_q)}{dN}\tau_s\,g(\omega_0)|E_0|^2}{(1-i\,\delta_{nm}\,\tau_{in})}\frac{\left(\alpha(\omega_q)+i\right)}{\left(\dfrac{D\tau_s}{X}(WY-XZ)+\dfrac{|E_0|^2}{\left(E_{sat}^{2l}\right)^2}\right)} \qquad (29)$$

$$X^{SHB}(\omega_q;\omega_n;\omega_m)=\frac{-2i\,\tau_{in}}{V\,\varepsilon_0}\frac{|E_0|^2}{(1-i\,\delta_{mn}\tau_{in})}\times\sum\frac{|\mu|^4}{\hbar^3}\chi_j(\omega_q)\left(\frac{2}{D}\frac{\bar{N}_w}{\tau_c}\tau_{in}-1\right)\left(\chi_j(\omega_m)-\chi_j^*(\omega_n)\right) \qquad (30)$$

E_{sat}^{2l} is the saturated field for two-level QDs system which can be defined as [12]

$$E_{sat}^{2l}=\sqrt{\frac{\hbar\omega}{2c\,n\,\varepsilon_0\tau_s\dfrac{dg_{2l}}{dN_w}}} \qquad (31)$$

The susceptibility due CH is derive as

$$X^{CH}(\omega_q,\omega_m,\omega_n)=\frac{1}{V}\sum_{j=i,k}\frac{|\mu_i|^2}{\hbar}\hat{\chi}_j(\omega_q)\frac{1}{(1-i\,\tau_{in}\delta_{mn})}\{\frac{\partial f_{c,k}}{\partial T_c}\tilde{T}_c+\frac{\partial f_{v,k}}{\partial T_v}\tilde{T}_v\} \tag{32}$$

To calculate the temperature at small signal, we must use the definition of energy density (U) which is given by the following equation [7]

$$U_x(t)=\sum_j \varepsilon_{x,j}\rho_{x,j}(t) \tag{33}$$

Multiplying (2) by $\varepsilon_{x,j}$, and summing over j, one obtain

$$\dot{U}=-\frac{U}{\tau_e}+\frac{N_w(\langle E_x\rangle-U)}{D\,\tau_c}-\frac{U}{\tau_s}-\frac{U}{\tau_{CH}}+K_x\langle|E(z,t)|^2\rangle-\frac{1}{V}\frac{i}{\hbar}\sum_i\sum E_x(\mu_{vc,i}\rho_{cdvd,i}-\mu_{cv,i}\rho_{cdvd,i})E(t) \tag{34}$$

The term $(K_x\langle|E(z,t)|^2\rangle)$ is phenomenologically added to represent the contribution of CH induced by FCA, K_x is a coefficient that can be express bythe
cross section for free carrier absorption (FCA) in the conduction and valence bands which is given by [12]

$$K_x=\varepsilon_0\, n\, n_g\, \upsilon_g\sigma_x\, N_w \tag{35}$$

Where σ_x is the cross-section, ε_0 is the permittivity of free space, n is the refractive index, n_g is the group refractive index and υ_g is the group velocity. x refers to conduction (c) or valance band (v). To determine the expression of temperature at small-signal, we use the expansions [7].

$$U_x=\bar{U}_x+h_x\left(\tilde{T}e^{-i\delta t}+c.c\right) \tag{36}$$

Substituting Eq. (36) and Eq.(35)in Eq.(34) , to obtain

$$\begin{aligned}-i\delta h_x\left(\tilde{T}_x\exp(-i\delta t)\right)=&-\frac{h_x\left(\tilde{T}_x\exp(-i\delta t)\right)}{\tau_e}-\frac{1}{D}\frac{N_w}{\tau_c}h_x\left(\tilde{T}_x\exp(-i\delta t)\right)-\\ &\frac{h_x\left(\tilde{T}_x\exp(-i\delta t)\right)}{\tau_s}-\frac{h_x\left(\tilde{T}_x\exp(-i\delta t)\right)}{\tau_{CH}}+2K_x(E_1E_0^*+E_0E_2^*)\exp(-i\delta t)-\frac{1}{V}\frac{i\,|\mu_{cv}|^2}{\hbar^2}\\ &\sum E_x(\bar{\rho}_{c,k}+\bar{\rho}_{v,k}-1).\{\left[\hat{\chi}(\omega_1)-\hat{\chi}^*(\omega_0)\right]E_1E_0^*+\left[\hat{\chi}(\omega_0)-\hat{\chi}^*(\omega_2)\right]E_0E_2^*\}\exp(-i\delta t)-\\ &\frac{1}{V}\frac{i\,|\mu_{cv}|^2}{\hbar^2}\sum E_x(\tilde{\rho}_{c,k}+\tilde{\rho}_{v,k})\left[\hat{\chi}(\omega_1)-\hat{\chi}^*(\omega_2)\right]|E_0|^2\exp(-i\delta t)\end{aligned} \tag{37}$$

For simplicity, we neglect the last term in Eq.(37) and used Eq.(5), then from definition of the material gain (Eq.(27)) and free carrier absorption factor, Eq.(35) can simplified as

$$\tilde{T}_x = \frac{-\tau_{in} h_x^{-1}}{(1-i\,\tau_{in}\delta)}\{E_x \frac{2c\,\eta\varepsilon_0}{\omega_0\hbar}\,\mathrm{g}(\omega_0) - 2\varepsilon_0\,\eta\,\eta_g v_g \sigma_X N\}(E_1 E_0^* + E_0 E_2^*) \tag{38}$$

Substituting Eq.(38) in Eq.(32) and use the identity

$$\frac{1}{V}\sum_k \frac{|\mu_{cv}|^2}{\hbar}\,\hat{\chi}(\omega_0)(\frac{\partial f_{x,k}}{\partial T_x}) = \varepsilon_0\,\frac{\partial\bar{\chi}(\omega)}{\partial T_x} = -\varepsilon_0\,\frac{c\,\eta}{\omega}\frac{\partial\bar{g}(\omega)}{\partial T_x}(\alpha_{T_x}(\omega)+i) \tag{39}$$

The susceptibility due to CH will become

$$\tilde{\chi}^{CH} = -\varepsilon_0\,\frac{c\,\eta}{\omega}\frac{\partial\bar{g}(\omega)}{\partial T_x}\,\frac{(\alpha_{T_x}(\omega)+i)}{(1-i\,\tau_{in}\delta)}\,\frac{-\tau_{in}h_x^{-1}}{(1-i\,\tau_{in}\delta)}\,v_g\,\frac{2\varepsilon_0\,\eta\eta_g}{\omega\hbar}\,\bar{\mathrm{g}}(\omega_0)$$
$$[E_x - \frac{\sigma_X N_w \omega\hbar}{\bar{\mathrm{g}}(\omega_0)}](E_1 E_0^* + E_0 E_2^*) \tag{40}$$

4. Nonlinear gain coefficient

The nonlinear gain coefficient depends on the analytical solution of pulse propagation inside QD SOA [3]. FWM has three different physical mechanisms contributing toward its conversion. The first mechanism is called carrier density pulsation, the beating between the pump and probeallowing wave mixing by producing a temporal grating in the device[12].Four-wave mixing also has contributions due to spectral-hole burning and carrier heating, which are governed by the scattering process such as carrier-carrier and carrier phonon-scattering[12]. In general, nonlinear gain coefficient due CDP is assumed to be equal to unity [9]. Other nonlinear gain coefficients are deduced from the normalized nonlinear susceptibility.As we do with SHB nonlinear coefficient [3], the nonlinear gain coefficient is derived as

$$\kappa_{CH}(1-i\,\alpha_{CH}) = \frac{X^{CH}}{\left|X^{CDP}\right|_{Normalize}} \tag{41}$$

$$\kappa_{CH,x}(1-i\,\alpha_{CH}) = \frac{\bar{N}_w E_{0,x}}{K_\beta T^2}\,\frac{\tau_{in}h_x^{-1}}{(1-i\,\delta\tau_{in})\tau_s}\left(\Delta E - \frac{\sigma_x \bar{N}_w \hbar\omega_0}{g(\omega_0)}\right) \tag{42}$$

5. Results and discussion

The QD structure chosen in this study is InAs grown on $In_{0.53}Ga_{0.47}As$ wetting layer (WL), while the substrate is GaAs. Many researches uses this structure to study different QD properties [16-19]. The calculations of Energy levels is achieve by using Quantum Disk Model where the Schrödinger equation is solve using cylindrical coordinates. It is assumed here that the disk have a radius of (13 nm) and a high of (2nm) . The parameters used in the calculations energy levels are given by Tab.1.

Table (1): Band structure parameters at 300 K [20, 21]

	InAs	$In_{0.53}Ga_{0.47}As$	**GaAs**
Energy gab (eV)	0.354	0.75	1.42
Electron effective mass $(m_e^*)/m_e$	0.023	0.041	0.067
Heavy hole effective mass $(m_{hh}^*)/m_e$	0.4	0.465	0.50

Carrier heating effect in dynamic of carrier is very important parameter and its influence is obviously show in carrier density and carrier occupation probability in QD. Figure (1) illustrated the reduction of carrier density due to carrier heating in semiconductor, Figure (2) shown the occupation probability of ground state (GS) with CH and
Without it. this behave is agree with global research [4], One can see the time recovery of SOA is effect by CH phenomena.

In this work, we are introduce new form to represent CH nonlinear gain coefficient for two-level system, depending the density matrix theory and the analytic model introduced by[22]. Figure (3) shows the 3-dimensional (3-D) plot of $K_{CH} - N_{WL} - \delta$. The WL carrier density dependence shown, here, is different from that one using in the previous (QW-like) model. Here, a complex dependence comes from both WL and τ_{in} as in Eq. (13). Figure (4) shows the $K_{CH} - N_w - \Delta E$ plot, the nonlinear coefficient increase with energy difference between WL and GS level. We also check the contribution of time constant τ_{in} through Figure (5). The increasing of τ_{in} leads to rise the nonlinear gain coefficients.

6. Conclusions

We conclude that, the nonlinear gain coefficient is considered a good technique to steadying the nonlinear behavior in semiconductor material, and the heating in QD structure that generated by carrier can be represented by CH gain coefficient. This phenomena effects on the dynamic of carriers, it is restricttransition between WL and dot level , so the carrier occupation and carrier density will be reduce.

7. References

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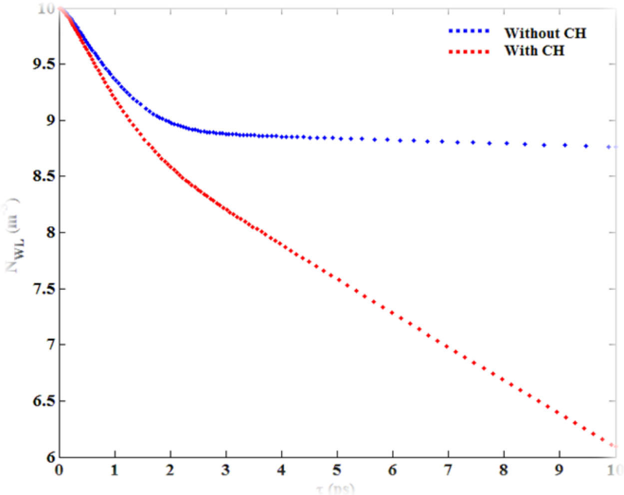

Figure (1): The carrier density versus time.

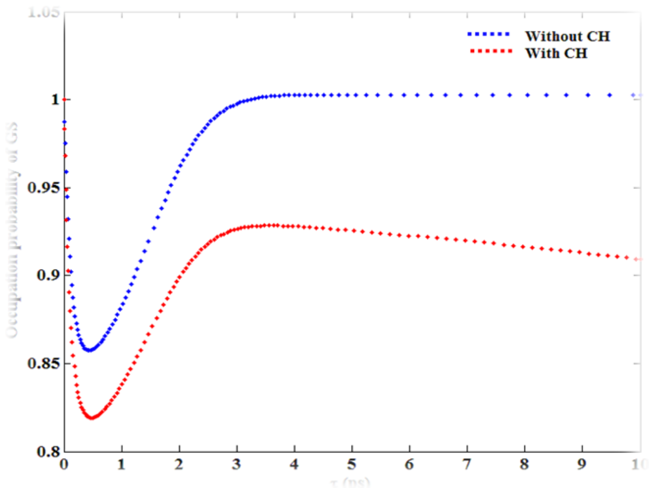

Figure (2): The occupation probability of GS versus time.

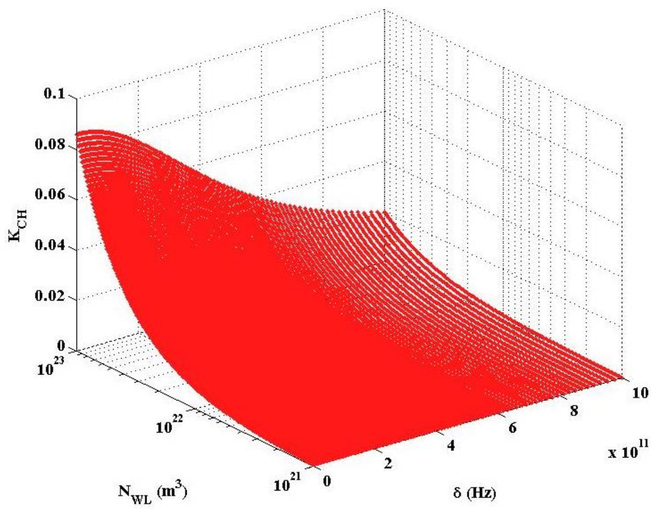

Figure (3): The 3-dimensional (3-D) plot of $K_{CH} - N_{WL} - \delta$.

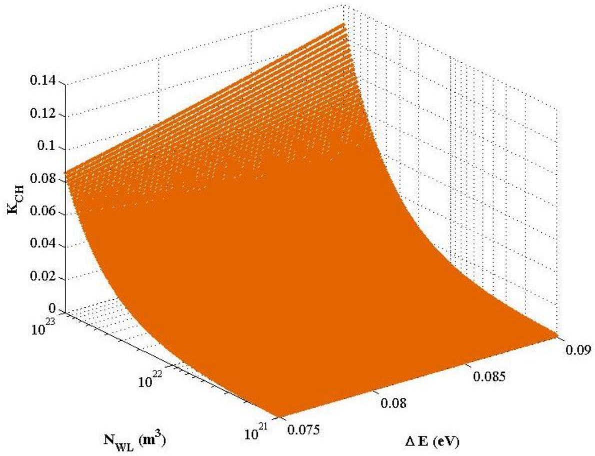

Figure (4): The $K_{CH} - N_w - \Delta E$ plot**.**

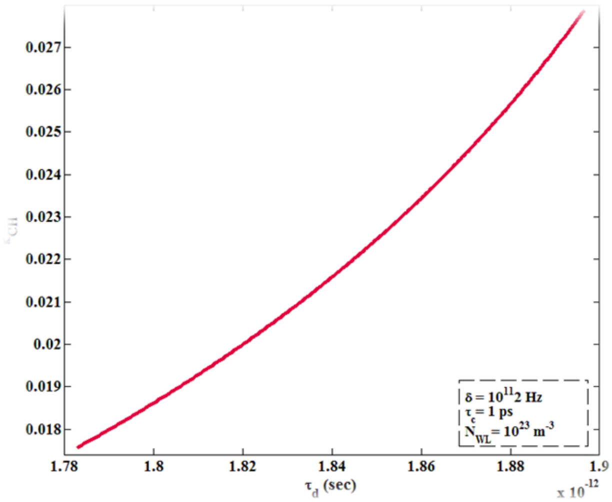

Figure (5): The Nonlinear gain coefficients versus the time τ_{in}