MODIFICATION OF PORTAL METHOD FOR ANALYZING THE FRAMES

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Abstract: The portal method is one of the common approximate methods in the analysis of statically indeterminate structures. This method is used to analyze the frames which subjected to lateral loadings such as wind, earthquake, and blast loadings. The portal method is still used in the planning phase of projects, preliminary designs, and quick checking for analysis. In this paper an improvement is presented to make the portal method analysis more closer to the accurate analysis for one bay frames. In this work, the analysis by using the finite element method is carried out for a twenty seven building frames with various numbers of bays and stories. The outputs indicate that some improvements in the portal method will be useful to make this method more accurate. The improvements have been written in a new "modified portal method". In order to compare the results of the analysis after improvement, a typical five frames have been analyzed by using three methods: namely, portal method, modified portal method (presented in this paper), and the finite element method via SAP2000 V14. The analysis by using the modified portal method gave more accurate results than the basic portal method for the multi-story frames with one bay, but didn't improve the results for the frames with more than one bay.

Keywords: *Portal method, Modified portal method, Approximate structural analysis, Frames.*

تعديل طريقة البوابة لتحليل الهياكل ذات الفضاء الواحد

الخلاصة: تعتبر طريقة البوابة من الطرق التقريبية الشائعة في تحليل المنشآت الغير محددة سكونيا. تستخدم هذه الطريقة لتحليل الهياكل المعرضة للأحمال الجانبية مثل أحمال الرياح, والهزات الأرضية, والانفجارات. إن طريقة البوابة لا تزال تستخدم في مرحلة التخطيط للمشروع, مرحلة التصاميم الأولية , وفي التدقيق السريع للتحليل الانشائي. في هذا البحث, تم تقديم تحسين لجعل التحليل بطريقة البوابةللهياكل ذات الفضاء الواحد أكثر اقترابا الى التحليل الدقيق. في هذا العمل تم اجراء التحليل باستخدام طريقة العناصر المحددة لسبع وعشرين بناية هيكلية بمختلف أعداد الفضاءات وأعداد الطوابق. مخرجات التحليل بينت بأن بعض التحسينات في طريقة البوابة ستكون مفيدة لجعل هذه الطريقة اكثر دقة. التحسينات تم كتابتها باسم "الطريقة المعدلة للبوابة". ولغرض مقارنة نتائج الطريقة بعد التحسين, تم تحليل خمسة هياكل باستخدام ثلاث طرق وهي : طريقة البوابة, الطريقة المعدلة للبوابة (المقدمة في هذا البحث), وطريقة العناصر المحددة باستخدام برنامج "ساب 2000 الاصدار 14". التحليل باستخدام الطريقة المعدلة للبوابة أعطى نتائج أكثر دقة من نتائج التحليل باستخدام طريقة البوابة للهياكل المتعددة الطوابق ذات الفضاء الواحد, ولكن التحليل لم يتحسن بالنسبة للهياكل ذات الفضاءات التي تزيد عن فضاء واحد.

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1. Introduction

include the force methods and the displacement methods, and secondly the approximate methods. The approximate methods are useful in the planning phase of projects to compare the alternative models, also it is easier for the engineers to use them in the preliminary designs and quick checking since they are not require the section properties for the analysis and also to avoid the difficulties and the time consuming by using the "exact methods". The approximate methods can be used for various structures such as trusses, beams, and frames. For the frames, there are an approximate methods for the frames under vertical loadings, and another methods for those subjected to lateral loadings. For frames subjected to lateral loadings such as wind loads and earthquake loads, there are two main approximate methods: namely, the portal and the cantilever methods.

The portal method is still used for the analysis of frames that subjected to lateral loads. Therefore it is useful to improve the accuracy of this method.

The aim of this paper is to improve the portal method to give more accurate and more realistic results for the approximate analysis of statically indeterminate frames subjected to lateral loadings.

Presenting a modified portal method started with careful investigation for the basic assumptions of portal method by analyzing various frames, which led to more appropriate assumptions that improve the accuracy of the analysis.
The improvement is based on analyzing various frames under various lateral loadings by using three methods of analysis: namely, portal method, theproposed modified portal method, and finite element method (by using the software SAP2000 V14). Outputs for analysis have been selected to be both the member end moments and external reactions of the frame because of these two outputs give an excellent indication about the accuracy of each method. Then the selected outputs for both portal method and modified portal method have been compared with the results obtained by finite element method to specify the error percentages.Based on error percentages obtained, conclusions and a new modification for portal method have been presented in this paper.

2. The Portal Method

The portal frames are generally used in the entrance of bridges, and also used as parts or stiffeners in various buildings to resist the lateral loadings due to wind, earthquakes, and other lateral effects [5]. The portal method is an approximate method for analyzing the statically indeterminate portal frames with fixed supports and subjected to lateral loads. The portal method was initially presented by A. Smith in 1915, and it is mostly appropriate for the low building frames [4].

To make the analysis easier, this method is based on two assumptions as follows:
1. The inflection point (the zero moment point on the moment diagram) is located at the middle of each member (beam and column) of the frame.

2. For each story of the frame, the interior columns carry twice as much shear as exterior columns.

The first assumption is based on the general behavior of the single story portal frame with fixed supports that subjected to lateral force, while the second assumption is based on an approximation based on replacing the frame by two adjacent frames (the interior column of the original frame becomes double columns) in which the interior column represents the effect of two columns and carries twice the shear force in the exterior column [4].

These two assumptions make the frame analyzable by using the three equations of equilibrium for the frame and its parts (i.e. the additional compatibility equations are not needed in the analysis). The above two assumptions make the analysis easier, but in the same time make it approximated compared with the exact methods.

In 1983, Wang [1] presented an "improved portal method". The improved portal method replaced the second assumption for the basic portal method by another one, so that the relation between the shear forces in exterior and interior columns can be changed. The new assumption based on the manner of distributing the shear forces in columns, in which for a given story, the shear forces are distributed to the columns in proportion to the tributary height of the columns.

In 2011, Selvam and Bindhu [2] published a new improvement for the portal method which named as "Split Frame Method". This method is also replaced the second assumption for the basic portal method by another assumption based on developing a relation between the area of the column with its height by splitting each bay of the frame to be independent portal frame. The area of each column in the original frame is considered to be proportional to its tributary height.

In this paper a new modification has been presented to improve the basic portal method for frames with one bay.

3. The Finite Element Models

Doubtless, the analysis by using an accurate method for frame models and then investigation of the results will lead to the improvement. In this regard, twenty seven frame models with fixed supports, various numbers of bays and stories, and subjected to two lateral loading cases have been selected. Each frame model has a span length of 6 m (c/c), and a story height of 3 m (c/c).

The twenty seven models are named with first capital letter "F" followed by three numbers; the first number represents the number of bays, the second number represents the number of stories, and the third number represents the number of concentrated forces acting on the frame. For example, the model F321 is a frame with three bays, two stories, and subjected to one concentrated force. A typical models (F321 and F322 frames) are shown in Fig.1.

Also each frame having more than one story has been analyzed twice: firstly under a lateral concentrated 10 kN-force at the tip (upper joint) of the frame (loading case 1), and secondly under a series of concentrated loads each of 10 kN-force at

each joint in one side of the frame (loading case 2). Fig. 2 shows the loading types for typical frames.

All models have been analyzed by the common accurate finite element method by using the SAP2000 V14.
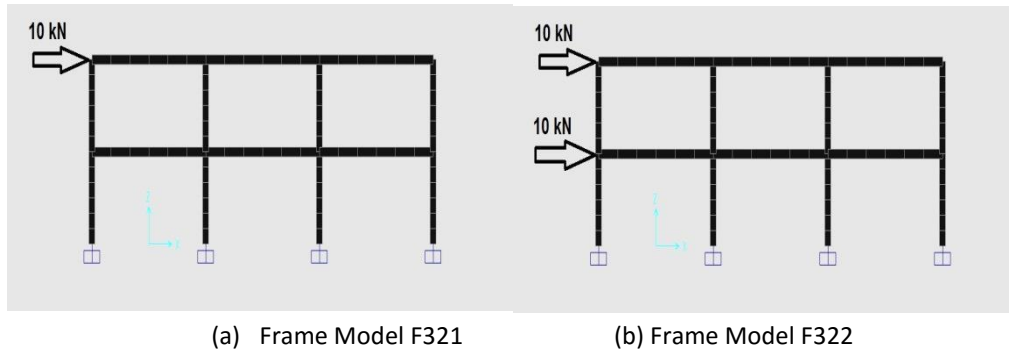

(a)   Frame Model F321   (b) Frame Model F322

Figure 1. Typical Frame Models (Not to scale)

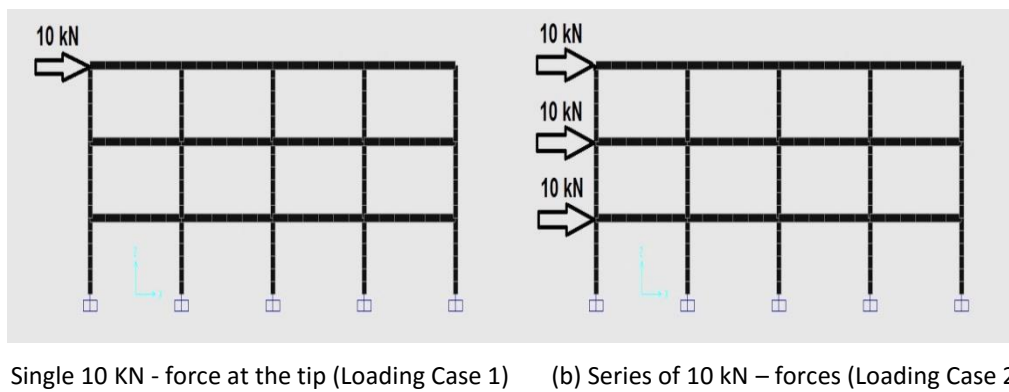(a) Single 10 KN - force at the tip (Loading Case 1)   (b) Series of 10 kN – forces (Loading Case 2)

Figure 2. Loading Cases for Typical Frames

4. The Outputs of Finite Element Analysis

Since the two assumptions of the portal method mentioned in section 2 are based on the locations of the inflection points and the relations between the shear values in exterior and interior columns, the results for the analyses (by using the finite element method) especially the moment diagrams for the beams and the columns, and the shear diagrams for the columns have been investigated to obtain a more realistic and more accurate assumptions that can improve the portal method.

The moment diagrams for the beams and columns indicate the location of the inflection points (the points of zero moments in the moment diagrams), while the shear diagrams indicate the relations between the shear values for the exterior columns with the interior columns.

The moment diagrams and the shear diagrams for typical frames are shown in Fig.3 and Fig.4 respectively.
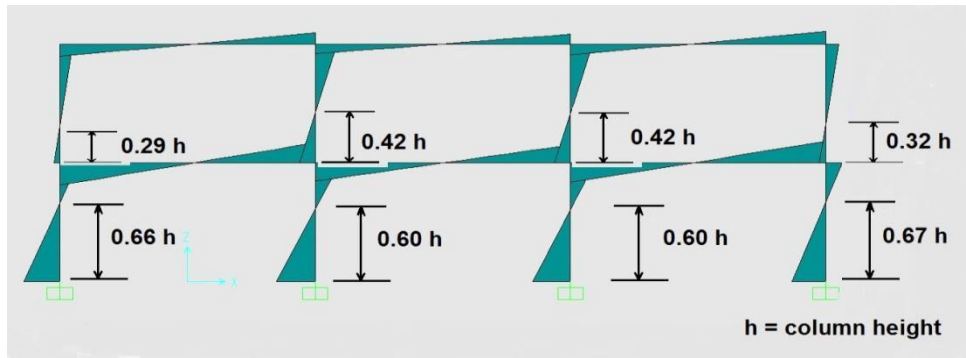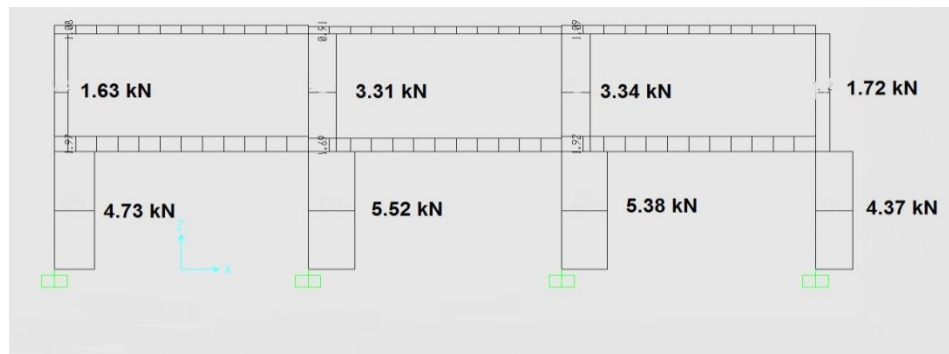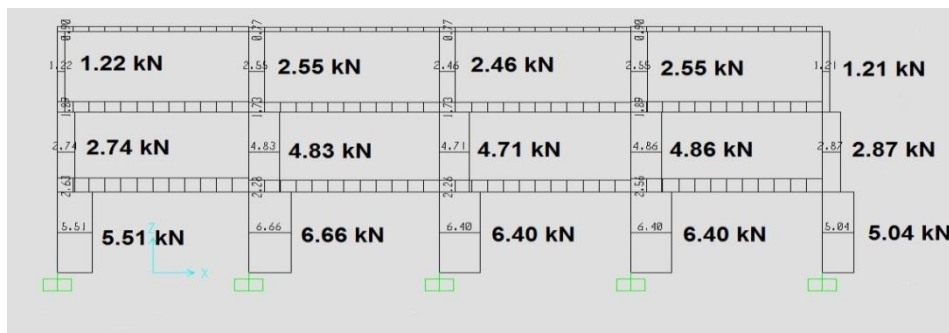
(a) The frame F322
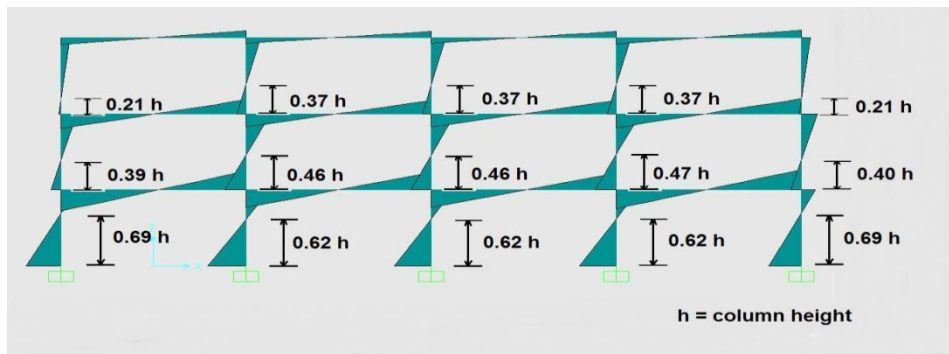
(b) The frame F433

Figure 3. Typical Moment Diagrams for the Finite Element Models

(a) The frame F322

(b) The frame F433

Figure 4. Typical Shear Diagrams for the Finite Element Models

The location of the inflection point for any column in any story is represented by a distance measured from the base of the column (the base of the column is considered to be the point of intersection between the column and the beam centerlines). The values of the ratios of the distances of the inflection points to the heights of the columns, and the values of the shear forces multiplied by 0.1 (to give a clear vision for the results) for the columns are summarized in TablesA-1 throughA-4 (see Appendix A). In these tables, the first column in the left side of a given story is numbered as a first column in the tables.

The locations of the inflection points for the beams in all the twenty seven models are all closed to the middle of each beam. Thus it is not important to present the values of the inflection points distances for the beams.

After the analysis of the finite element models are completed, a careful investigation for the results have been undertaken and led to the following two main notes:

1) The inflection points for the columns in the first story are located approximately at a distance of two thirds of the columns heights measured from the base of the column, and the inflection points for the columns in the highest story are located approximately at a distance of one third of the columns heights measured from the base of the column. The inflection points for all the beams and columns in intermediate stories (other than the first and the highest stories) are located approximately at the middle of the column.

2) For the first story and the highest story, the interior columns carry approximately 1.2 as much shear as exterior columns.

These two notes are initially considered as new assumptions to test the validity to improve the analysis by using the portal method.

It can be noticed from Tables A-1 and A-2 that the locations of inflection points for external columns differ from those for internal columns, but by making a careful investigation, and for simplicity sake in applying the approximate methods, all values of the locations for inflection points are rounded to one appropriate value for each story (i.e. 1/3, 2/3, or 1/2).

An example has been solved based on these proposed two new assumptions. The analysis results indicate that the second new proposed assumption for the relation between the interior and the exterior columns (i.e. the interior column carries 1.2 as much shear as the exterior column) does not applicable since the results for the analysis gave unbalance structure (the moment equation of equilibrium did not satisfied in the parts and in the entire structure). Thus this assumption has been cancelled.

The first proposed assumption can be considered as an applicable assumption. In the next section, the details for the analysis depending on this new assumption are illustrated in five examples.

5. Numerical Examples

Five different frames have been selected to check the suitability of using the new assumption mentioned in previous section. For each frame, three analyses have been performed namely: Basic portal method, the modified portal method (based on the new first assumption, in addition to the second assumption for the shear in basic portal method), and finally by using the finite element method which can be considered as an exact method.

5.1. Example 1

The following frame is presented in Ref.[4]. The frame has one story with two bays and subjected to 60-kN lateral force at the left upper joint. The details of this frame are shown in Fig.5 below.

Figure 5. Portal frame / Example 1. Ref.[4]

5.1.1. Method I

The frame has been analyzed by using the portal method [4]. In this method, an internal hinge inserted at the middle of each beam and column, and the interior column (column BE) carried twice as much shear as the exterior column (column AD or CF) as shown in Fig.6.

(a)Simplified frame   (b) Sectioned frame
(S=shear force in exterior column)
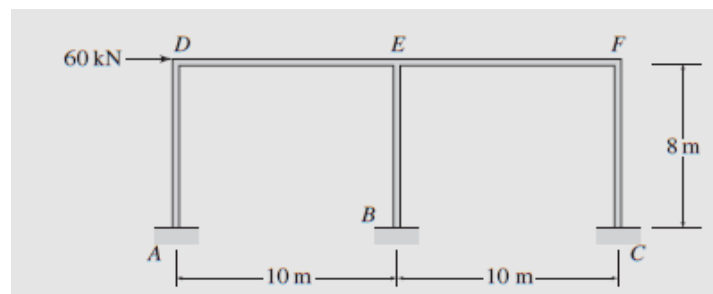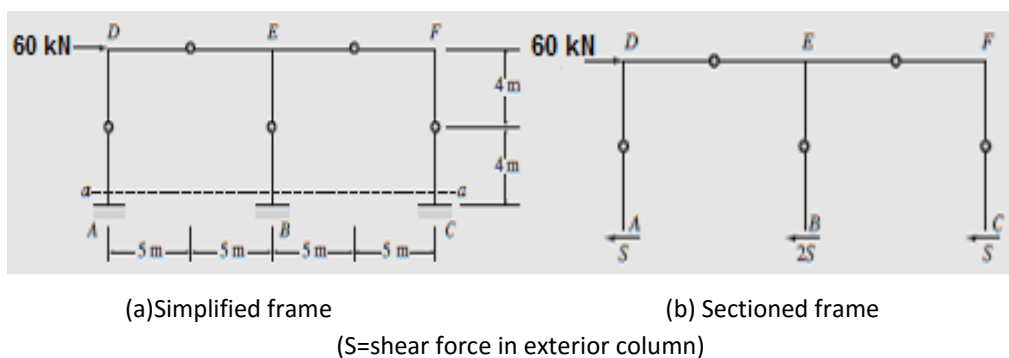
Figure 6. Application of the two assumptions for the basic portal method- Example 1. Ref.[4]

5.1.2. Method II

The frame has been analyzed by using the modified portal method. In this method, an internal hinge inserted at the middle of each beam, and at a distance of two thirds measured from the base of each column (8*2/3=5.33m from the base), since all columns are in first story. Also, each interior column carried twice as much shear as the exterior column. The difference between the simplified frame in this method and that in method 1Fig.6 (a) is the locations of the internal hinges in the columns. In this method, for each column the location of the internal hinge is 5.33m measured from the base (or 2.67m measured from the top) as shown in Fig.7.
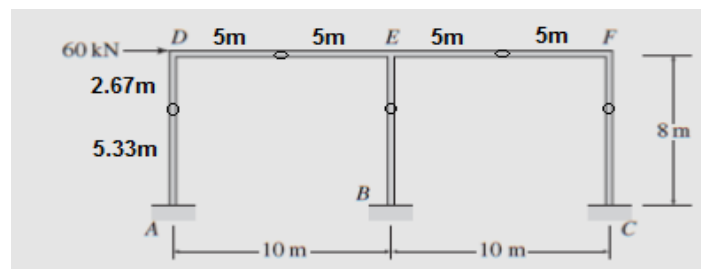
Figure 7. Application of the two assumptions for the modified portal method. The figure is modified from Figure 5.

5.1.3. Method III

The frame has been analyzed by using the finite element method via SAP2000 V14. The values of the member end moments, and the error percentages for the portal method and the modified portal method compared with the finite element method are summarized in Table 1.

Table 1. The member end moments and error percentages / Example 1

Member end moments	Values of member end moments (kN.m)			Error (%) compared with the finite element method	
	Portal method	Modified portal method	Finite element method	Portal method	Modified portal method
M_{AD}	-60	-80	-87.21	31.2	8.3
M_{DA}	-60	-40	-57.57	4.2	30.5
M_{DE}	60	40	57.57	4.2	30.5
M_{ED}	60	40	44.59	34.6	10.3
M_{BE}	-120	-160	-102.44	17.1	56.2
M_{EB}	-120	-80	-89.00	34.8	10.1
M_{EF}	60	40	44.41	35.1	11.0
M_{FE}	60	40	57.19	4.9	30.0
M_{CF}	-60	-80	-86.6	30.1	7.6
M_{FC}	-60	-40	-57.19	4.9	30

The values of the base reaction forces at the points A, B, and C in vertical and horizontal directions, and the error percentages for the portal method and the modified portal method compared with the finite element method are summarized in Table2.

Table 2. The reactions and error percentages / Example 1

Reactions	Values of reaction forces (kN)			Error (%) compared with the finite element method	
	Portal method	Modified portal method	Finite element method	Portal method	Modified portal method
A_V	-12	-8	-10.22	17.4	21.7
A_H	-15	-15	-18.10	17.1	17.1
B_V	0	0	0	0	0
B_H	-30	-30	-23.93	25.4	25.4
C_V	12	8	10.12	18.6	20.9
C_H	-15	-15	-17.97	16.5	16.5

5.2. Example 2

The frame in this example is presented in Ref.[5]. The frame has two stories with two bays. The details of this frame are shown in Fig.8 below.

Figure 8. Portal frame / Example 2. Ref.[5]

In the analysis by using the "modified portal method", an internal hinge inserted at the middle of each beam, at a distance of two thirds measured from the base (4 m) of each column in first story, and at a distance of one third measured from the base (5/3= 1.67 m) of each column in second story (highest story) . Also, each interior column carried twice as much shear as the exterior column.

The values for a selective member end moments, and the error percentages for the portal method and the modified portal method compared with the finite element method are summarized in Table 3.

Table 3. The member end moments and error percentages / Example 2

Member end moments	Values of member end moments (kN.m)			Error (%) compared with the finite element method	
	Portal method	Modified portal method	Finite element method	Portal method	Modified portal method
M_{AD}	-37.5	-50	-57.46	34.7	13.0
M_{DA}	-37.5	-25	-36.59	2.5	31.7
M_{DG}	-12.5	-8.36	-4.23	195.5	97.6
M_{GD}	-12.5	-16.64	-18.1	30.9	8.1
M_{DE}	50	33.36	40.95	22.1	18.5
M_{GH}	12.5	16.64	18.1	30.9	8.1
M_{BE}	-75	-100	-63.38	18.3	57.8
M_{CF}	-37.5	-50	-56.93	34.1	12.2

The values of the base reaction forces at the points A, B, and C in vertical and horizontal directions, and the error percentages for the portal method and the modified portal method compared with the finite element method are summarized in Table 4.

Table 4. The reactions and error percentages / Example 2.

Reactions	Values of reaction forces (kN)			Error (%) compared with the finite element method	
	Portal method	Modified portal method	Finite element method	Portal method	Modified portal method
A_V	-15.63	-12.5	-13.89	12.5	10.0
A_H	-12.5	-12.5	-15.70	20.4	20.4
B_V	0	0	0	0	0
B_H	-25	-25	-18.77	33.2	33.2
C_V	15.63	12.5	13.89	12.5	10.0
C_H	-12.5	-12.5	-15.54	19.6	19.6

5.3 Example 3

The frame in this example has one story with one bay. The height is 6m and the width is 8m, and subjected to 10 kN - force as shown in Fig.9 below.
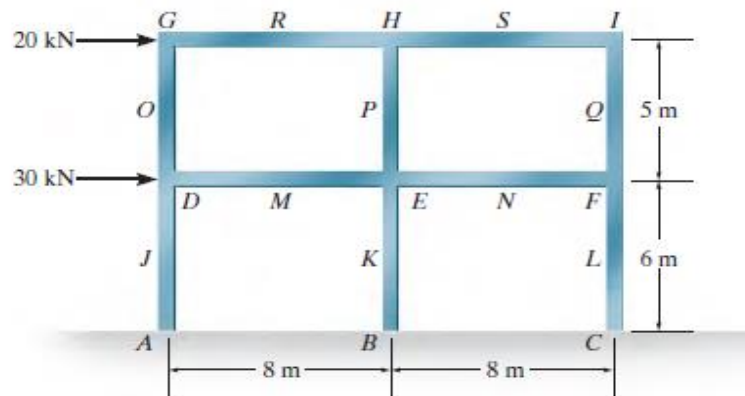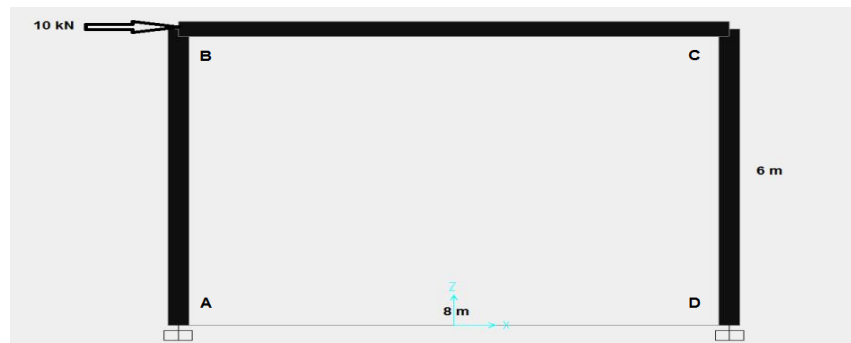

Figure 9. Portal frame / Example 3

In the analysis by using the "modified portal method", an internal hinges inserted at the middle of the beam, and at a distance of two thirds measured from the base (4 m) of each column. Also, each column carries equal shear forces (5 kN for each column).

The values of the member end moments, and the error percentages for the portal method and the modified portal method compared with the finite element method are summarized in Table 5 .

Table 5. The member end moments and error percentages / Example 3

Member end moments	Values of member end moments (kN.m)			Error (%) compared with the finite element method	
	Portal method	Modified portal method	Finite element method	Portal method	Modified portal method
M_{AB}	-15	-20	-17.81	15.8	12.3
M_{BA}	-15	-10	-12.24	22.5	18.3
M_{BC}	15	10	12.24	22.5	18.3
M_{CB}	15	10	-12.21	22.9	18.1
M_{DC}	-15	-20	-17.74	15.4	12.7
M_{CD}	-15	-10	-12.21	22.9	18.1

The values of the base reaction forces at the points A and D in vertical and horizontal directions, and the error percentages for the portal method and the modified portal method compared with the finite element method are summarized in Table 6.

146

Table 6. The reactions and error percentages / Example 3

Reactions	Values of reaction forces (kN)			Error (%) compared with the finite element method	
	Portal method	Modified portal method	Finite element method	Portal method	Modified portal method
A_V	-3.75	-2.5	-3.06	22.5	18.3
A_H	-5	-5	-5.01	0	0
D_V	3.75	2.5	3.06	22.5	18.3
D_H	-5	-5	-4.99	0	0

5.4 Example 4

The frame in this example has two stories with one bay. The details of this frame is shown in Fig.10 below.

Figure10: Portal frame / Example 4

In the analysis by using the "modified portal method", an internal hinge inserted at the middle of each beam, at a distance of two thirds measured from the base (4 m) of each column in first story, and at a distance of one third measured from the base (5/3= 1.67 m) of each column in second story (highest story) . Also, for a given story, each column carries equal shear forces (7.5 kN for each column in the first story and 5 kN for each column in the second story). The values of the member end moments, and the error percentages for the portal method and the modified portal method compared with the finite element method are summarized in Table 7.

Table 7. The member end moments and error percentages / Example 4.

Member end moments	Values of member end moments (kN.m)			Error (%) compared with the finite element method	
	Portal method	Modified portal method	Finite element method	Portal method	Modified portal method
M_{AB}	-22.5	-30	-29.00	22.4	3.4
M_{BA}	-22.5	-15	-15.95	41.1	6.0
M_{BC}	-15	-10	-12.48	20.2	19.9
M_{CB}	-15	-20	-17.48	14.2	14.4
M_{CD}	15	20	17.48	14.2	14.4
M_{DC}	15	20	17.48	14.2	14.4
M_{DE}	-15	-20	-17.48	14.2	14.4
M_{ED}	-15	-10	-12.52	19.8	20.1
M_{BE}	37.5	25	28.54	31.4	12.4
M_{EB}	37.5	25	28.52	31.5	12.3
M_{FE}	-22.5	-30	-28.97	22.3	3.6
M_{EF}	-22.5	-15	-16.00	40.6	6.3

The values of the base reaction forces at the points A,B, and C in vertical and horizontal directions, and the error percentages for the portal method and the modified portal method compared with the finite element method are summarized in Table 8 .

Table 8. The reactions and error percentages / Example 4.

Reactions	Values of reaction forces (kN)			Error (%) compared with the finite element	
	Portal method	Modified portal method	Finite element method	Portal method	Modified portal method
A_V	-13.13	-11.25	-11.5	14.2	2.2
A_H	-7.5	-7.5	-7.5	0	0
F_V	13.13	11.25	11.5	14.2	2.2
F_H	-7.5	-7.5	-7.5	0	0

5.5 Example 5

The frame in this example is has three stories with one bay. The details of this frame are shown in Fig.11 below.
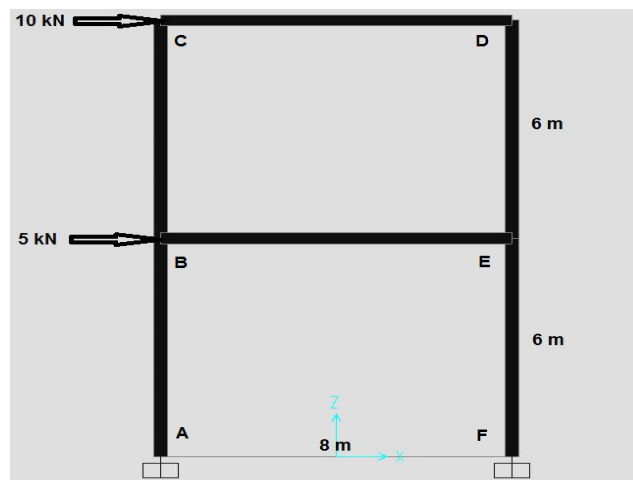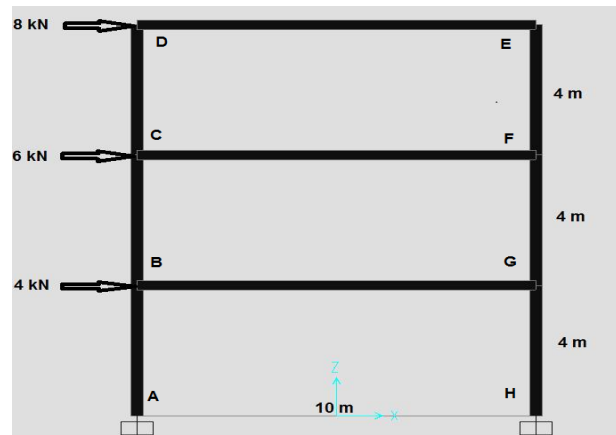

Figure 11: Portal frame / Example 5

In the analysis by using the "modified portal method", an internal hinges inserted at the middle of each beam, at a distance of two thirds measured from the base (2.67 m) of each column in first story, at the middle of each column in the second story (2m), and at a distance of one third measured from the base (1.33 m) of each column in the third story. Also, for a given story, each column carries equal shear forces (9 kN for each column in the first story, 7 kN for each column in the second story and 4 kN for each column in the third story).

The values for a selective member end moments, and the error percentages for the portal method and the modified portal method compared with the finite element method are summarized in Table 9.

Table 9. The member end moments and error percentages / Example 5

Member end moments	Values of member end moments (kN.m)			Error (%) compared with the finite element method	
	Portal method	Modified portal method	Finite element method	Portal method	Modified portal method
M_{AB}	-18	-24.03	-27.26	34.0	11.8
M_{BA}	-18	-11.97	-8.80	104.5	36.0
M_{BC}	-14	-14.00	-13.04	7.4	7.4
M_{CB}	-14	-14.00	-14.94	6.3	6.3
M_{CD}	-8	-5.32	-4.57	75.1	16.4
M_{DC}	-8	-10.68	-11.40	29.8	6.3

M_{DE}	8	10.68	11.40	29.8	6.3
M_{CF}	22	19.32	19.54	12.6	1.1
M_{BG}	32	25.97	21.84	46.5	18.9

The values of the base reaction forces at the points A and H in vertical and horizontal directions, and the error percentages for the portal method and the modified portal method compared with the finite element method are summarized in Table 10.

Table 10. The reactions and error percentages / Example 5

Reactions	Values of reaction forces (kN)			Error (%) compared with the finite element method	
	Portal method	Modified portal method	Finite element method	Portal method	Modified portal method
A_V	-12.40	-11.90	-10.56	17.4	6.0
A_H	-9.00	-9.00	-9.01	0	0
H_V	12.40	11.9	10.56	17.4	6.0
H_H	-9.00	-9.00	-8.99	0	0

6. Discussion of the Results

Based on the error percentages for the values of member end moments and the base reactions in the five frames presented in examples 1 through 5 which listed in Tables 1 through 10, two main observations have been noticed:

1. By inspection, the portal method gives slightly more accurate results than the modified portal method for the frames in examples 1 and 2 (Multi bays frames). The range of error percentages for the member end moments by using the portal method in examples 1 and 2was[2.5% , 35%], except one extreme value of 195.5%, while the range by using the modified portal method was [7.6% , 57.8%], except one extreme value of 97.6%. For the error percentages in reactions, same ranges were obtained by using both the portal method and the modified portal method [0%, 33.2%].

2. The modified portal method clearly gives more accurate results than the portal method in frames in examples 3, 4, and 5 (Frames with one bay). Most of resulted error percentages of modified portal method are less than those resulted by portal method. Also the range of error percentages for the member end moments by using the modified portal method in examples 3, 4, and 5 was [1.1% , 20.1%], with one extreme value of 36%, while the range by using the portal method was [6.3% , 46.5%], with two extreme values of 75.1% and 104.5%. For the error percentages in

reactions, the range by using the modified portal method was [0%, 18.3%], while the range by using the portal method was [0%, 22.5%].

Thus, it can be noticed that the modified portal method gives a more accurate analysis for the frames having only one bay like the frames in examples 3, 4, and 5.

For the frames having more than one bay the portal method can be considered as the more appropriate method than the modified portal method, based on the analysis of examples 1 and 2.

7. Conclusions

The following conclusions can be obtained from this work:

1. This work presented a "modified portal method" to be used in structural analysis. The modified portal method based on two main assumptions as follows:

a. The inflection points for the columns in the first story are located at a distance of two thirds of the columns heights measured from the base of the column, and the inflection point for the columns in the highest story are located at a distance of one third of the columns heights measured from the base of the column. For the columns in intermediate stories (other than the first and the highest stories), the inflection point is located at the middle of each column. For all beams in the frame, the inflection point is located at the middle of each beam.

b. For each story of the frame, the interior columns carry twice as much shear as exterior columns.

2. The modified portal method gives a more accurate analysis than the portal method for the one bay frames with one or more stories.

3. The modified portal method can be used for the analysis of frames with more than one bay but with somewhat less accurate analysis compared with the portal method. It is preferred to keep using the portal method for the frames with more than one bay.

8. References

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Appendix –A

Table A-1.The ratios of the distances of the inflection points (measured from the base of the columns) to the heights of the columns, for the frames subjected to series of lateral forces.

Model	Story	Column 1	2	3	4	5	6
F222	1st	0.67	0.60	0.67	-	-	-
	2nd	0.32	0.42	0.33	-	-	-
F322	1st	0.66	0.60	0.60	0.67	-	-
	2nd	0.29	0.42	0.42	0.32	-	-
F333	1st	0.69	0.62	0.62	0.70	-	-
	2nd	0.40	0.47	0.47	0.41	-	-
	3rd	0.22	0.37	0.37	0.22	-	-
F422	1st	0.66	0.61	0.60	0.61	0.67	-
	2nd	0.27	0.42	0.41	0.42	0.33	-
F433	1st	0.69	0.62	0.62	0.62	0.69	-
	2nd	0.39	0.46	0.46	0.47	0.40	-
	3rd	0.21	0.37	0.37	0.37	0.21	-
F444	1st	0.70	0.62	0.62	0.62	0.71	-
	2nd	0.43	0.48	0.48	0.48	0.45	-
	3rd	0.34	0.43	0.43	0.43	0.33	-
	4th	0.18	0.37	0.37	0.37	0.17	-
F455	1st	0.71	0.63	0.64	0.63	0.72	-
	2nd	0.46	0.49	0.49	0.49	0.46	-
	3rd	0.40	0.45	0.45	0.45	0.40	-
	4th	0.32	0.42	0.42	0.42	0.32	-
	5th	0.17	0.36	0.36	0.36	0.17	-
F522	1st	0.65	0.61	0.61	0.61	0.61	0.67
	2nd	0.25	0.42	0.42	0.42	0.42	0.33
F533	1st	0.69	0.61	0.62	0.62	0.62	0.70
	2nd	0.38	0.46	0.46	0.46	0.46	0.41
	3rd	0.21	0.37	0.37	0.37	0.37	0.20
F544	1st	0.70	0.62	0.62	0.62	0.62	0.70
	2nd	0.42	0.48	0.48	0.48	0.48	0.45
	3rd	0.33	0.42	0.42	0.42	0.42	0.33
	4th	0.18	0.36	0.36	0.36	0.36	0.18
F555	1st	0.71	0.63	0.64	0.64	0.64	0.72
	2nd	0.46	0.49	0.49	0.49	0.49	0.49
	3rd	0.39	0.45	0.45	0.45	0.45	0.39
	4th	0.32	0.42	0.42	0.42	0.42	0.32
	5th	0.17	0.36	0.36	0.36	0.36	0.17

Table A-2.The ratios of the distances of the inflection points (measured from the base of the columns) to the heights of the columns, for the frames subjected to a single lateral force.

Model	Story	Column 1	2	3	4	5	6
F111	1st	0.63	0.63	-	-	-	-
F211	1st	0.64	0.56	0.64	-	-	-
F221	1st	0.81	0.64	0.74	-	-	-
	2nd	0.46	0.46	0.45	-	-	-
F311	1st	0.64	0.57	0.57	0.64	-	-
F321	1st	0.75	0.65	0.64	0.73	-	-
	2nd	0.45	0.46	0.46	0.44	-	-
F331	1st	0.76	0.65	0.65	0.76	-	-
	2nd	0.53	0.52	0.52	0.52	-	-
	3rd	0.42	0.44	0.44	0.44	-	-
F411	1st	0.64	0.65	0.58	0.56	0.64	-
F421	1st	0.75	0.64	0.65	0.64	0.72	-
	2nd	0.45	0.46	0.46	0.45	0.43	-
F431	1st	0.75	0.65	0.65	0.65	0.75	-
	2nd	0.54	0.52	0.52	0.52	0.52	-
	3rd	0.43	0.44	0.44	0.44	0.39	-
F441	1st	0.76	0.65	0.66	0.65	0.76	-
	2nd	0.54	0.52	0.52	0.52	0.54	-
	3rd	0.50	0.50	0.49	0.49	0.48	-
	4th	0.41	0.44	0.44	0.44	0.38	-
F451	1st	0.76	0.65	0.65	0.65	0.76	-
	2nd	0.54	0.52	0.52	0.52	0.55	-
	3rd	0.50	0.50	0.50	0.50	0.51	-
	4th	0.50	0.50	0.49	0.49	0.47	-
	5th	0.41	0.44	0.44	0.43	0.38	-
F511	1st	0.64	0.56	0.58	0.57	0.56	0.64
F521	1st	0.76	0.64	0.64	0.64	0.64	0.72
	2nd	0.46	0.47	0.46	0.46	0.46	0.42
F531	1st	0.75	0.65	0.65	0.65	0.65	0.75
	2nd	0.55	0.52	0.52	0.52	0.52	0.52
	3rd	0.43	0.45	0.44	0.42	0.44	0.38
F541	1st	0.75	0.65	0.66	0.66	0.65	0.75
	2nd	0.54	0.52	0.52	0.52	0.52	0.54
	3rd	0.51	0.50	0.50	0.50	0.50	0.47
	4th	0.43	0.44	0.44	0.43	0.43	0.38
F551	1st	0.75	0.65	0.65	0.65	0.65	0.75
	2nd	0.54	0.52	0.52	0.52	0.52	0.53
	3rd	0.50	0.50	0.50	0.50	0.50	0.50
	4th	0.51	0.50	0.50	0.50	0.50	0.47
	5th	0.42	0.44	0.44	0.44	0.44	0.37

Table A-3.Values (multiplied by 0.1) of the shear forces in the columns (kN) for the frames subjected to series of lateral forces.

Model	Story	Column 1	2	3	4	5	6
F222	1st	0.63	0.76	0.61	-	-	-
	2nd	0.25	0.50	0.25	-	-	-
F322	1st	0.47	0.55	0.54	0.44	-	-
	2nd	0.17	0.33	0.33	0.17	-	-
F333	1st	0.69	0.84	0.82	0.65	-	-
	2nd	0.36	0.63	0.63	0.38	-	-
	3rd	0.16	0.34	0.34	0.16	-	-
F422	1st	0.38	0.44	0.42	0.42	0.34	-
	2nd	0.12	0.25	0.24	0.26	0.13	-
F433	1st	0.55	0.67	0.64	0.64	0.50	-
	2nd	0.27	0.48	0.47	0.49	0.29	-
	3rd	0.12	0.26	0.25	0.26	0.12	-
F444	1st	0.72	0.89	0.86	0.86	0.67	-
	2nd	0.41	0.73	0.71	0.73	0.42	-
	3rd	0.28	0.48	0.48	0.48	0.28	-
	4th	0.12	0.26	0.25	0.26	0.11	-
F455	1st	0.89	1.11	1.08	1.08	0.84	-
	2nd	0.56	0.97	0.94	0.97	0.56	-
	3rd	0.41	0.73	0.72	0.73	0.41	-
	4th	0.27	0.49	0.48	0.49	0.27	-
	5th	0.11	0.26	0.26	0.26	0.11	-
F522	1st	0.33	0.37	0.35	0.34	0.34	0.27
	2nd	0.09	0.20	0.20	0.20	0.20	0.22
F533	1st	0.47	0.55	0.53	0.52	0.52	0.41
	2nd	0.23	0.39	0.38	0.38	0.39	0.23
	3rd	0.10	0.20	0.20	0.20	0.20	0.10
F544	1st	0.60	0.74	0.71	0.70	0.70	0.55
	2nd	0.34	0.59	0.57	0.57	0.59	0.34
	3rd	0.22	0.39	0.39	0.39	0.39	0.22
	4th	0.09	0.21	0.20	0.20	0.21	0.09
F555	1st	0.74	0.92	0.89	0.88	0.88	0.69
	2nd	0.44	0.79	0.77	0.77	0.79	0.46
	3rd	0.33	0.59	0.58	0.58	0.59	0.33
	4th	0.22	0.39	0.39	0.39	0.39	0.22
	5th	0.09	0.21	0.20	0.20	0.21	0.09

Table A-4.Values (multiplied by 0.1) of the shear forces in the columns (kN) for the frames subjected to a single lateral force.

Model	Story	Column 1	2	3	4	5	6
F111	1st	0.50	0.50	-	-	-	-
F211	1st	0.31	0.40	0.29	-	-	-
F221	1st	0.30	0.40	0.30	-	-	-
	2nd	0.27	0.47	0.26	-	-	-
F311	1st	0.23	0.29	0.27	0.21	-	-
F321	1st	0.22	0.28	0.28	0.22	-	-
	2nd	0.20	0.32	0.30	0.18	-	-
F331	1st	0.22	0.28	0.28	0.22	-	-
	2nd	0.18	0.31	0.32	0.19	-	-
	3rd	0.19	0.32	0.31	0.18	-	-
F411	1st	0.19	0.23	0.21	0.21	0.16	-
F421	1st	0.16	0.25	0.23	0.23	0.13	-
	2nd	0.16	0.22	0.22	0.22	0.18	-
F431	1st	0.17	0.22	0.22	0.22	0.17	-
	2nd	0.14	0.24	0.24	0.24	0.14	-
	3rd	0.15	0.25	0.23	0.24	0.13	-
F441	1st	0.17	0.22	0.22	0.22	0.17	-
	2nd	0.14	0.24	0.24	0.24	0.14	-
	3rd	0.14	0.24	0.24	0.24	0.14	-
	4th	0.15	0.25	0.23	0.24	0.13	-
F451	1st	0.17	0.22	0.22	0.22	0.17	-
	2nd	0.14	0.24	0.24	0.24	0.14	-
	3rd	0.14	0.24	0.24	0.24	0.14	-
	4th	0.14	0.24	0.24	0.24	0.14	-
	5th	0.15	0.25	0.23	0.24	0.13	-
F511	1st	0.17	0.20	0.18	0.17	0.16	0.12
F521	1st	0.13	0.18	0.18	0.18	0.18	0.15
	2nd	0.13	0.21	0.19	0.18	0.18	0.11
F531	1st	0.14	0.18	0.18	0.18	0.18	0.14
	2nd	0.11	0.19	0.19	0.19	0.20	0.12
	3rd	0.13	0.21	0.19	0.18	0.18	0.11
F541	1st	0.14	0.18	0.18	0.18	0.18	0.14
	2nd	0.11	0.20	0.19	0.19	0.20	0.11
	3rd	0.11	0.19	0.19	0.19	0.20	0.12
	4th	0.13	0.21	0.19	0.18	0.18	0.11
F551	1st	0.14	0.18	0.18	0.18	0.18	0.14
	2nd	0.11	0.20	0.19	0.19	0.20	0.11
	3rd	0.11	0.20	0.19	0.19	0.20	0.11
	4th	0.11	0.19	0.19	0.19	0.20	0.12
	5th	0.13	0.21	0.19	0.18	0.18	0.11