

Critical Points of Scattering Electrons by Light Atoms at Low Energies

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Abstract

The electron energy where a minimum in the differential cross section attains its smallest value is known as the critical energy, and the corresponding scattering angle is the, critical angle. The method which requires phaseshifts only at three energies is used for obtaining critical positions for the scattering of electrons by atoms. The method of partial waves along with an optical potential is employed. To explore the possibility of the existence of critical positions for the atoms, the differential cross sections for the elastic scattering of electrons from the light atoms having atomic number 4 to 10 are obtained in the 0-100 eV region. The values of the critical positions obtained by the proposed method are in excellent agreement with the computed values and other investigated values.

Introduction

An investigation of the results of the differential cross section (DCS) that was obtained theoretically and experimentally for the elastic scattering of electrons from complex atoms reveals very marked structures in the cross section for these atoms [1-6]. Further, at some incident energy and scattering angle, depending upon the atom, the DCS becomes a minimum with respect to both the incident electron energy E and the scattering angle θ . The combination of the impact energy and the angle are known as critical points (positions) and are represented by critical energy (E_c) and critical angle (θ_c) [7] and a small change in either causes an increase of DCS [8]. The critical points (E_c, θ_c) have been the subject of a number of experimental and theoretical investigations [9-20]. It may be noted that it is not always possible to observe critical points in a normal scattering experiment or to predict them theoretically with the help of the calculations carried out at arbitrary electron energies because of the narrow energy range over which the deep minima occur [4]. However, the understanding of this process is important for a number of phenomena including the newly developed field of spin polarization phenomena in low-energy electron diffraction (LEED) from surfaces [8]. A further reason for studying critical positions was given by Buhring [7], namely that they should give a more sensitive test of the atomic field than measurements of absolute differential cross section at arbitrary electron energies.

The differential cross section I is, in the relativistic case, given by [4]

$$f = |f(\theta)|^2 + |g(\theta)|^2 \quad \dots\dots\dots(1)$$

where f is the direct and g is the spin-flip scattering amplitude. These amplitudes are evaluated from the spin-up and spin-down phase shifts δ_1^+ and δ_1^- respectively for each partial wave (l) from the expressions [4]:

$$f(\theta) = \frac{1}{2ik} \sum_{l=0}^{\infty} \left\{ (\ell + 1) [e^{2i\delta_l^+} - 1] + [e^{2i\delta_l^-} - 1] \right\} P_{\ell}(\cos \theta) \quad \dots\dots\dots(2)$$

$$g(\theta) = \frac{1}{2ik} \sum_{l=1}^{\infty} \left\{ e^{2i\delta_l^+} - e^{2i\delta_l^-} \right\} P_{\ell}(\cos \theta) \quad \dots\dots\dots(3)$$

where k is the momentum of the incident electron in atomic units and $P_l(\cos\theta)$ are $P_l(\cos\theta)$ are respectively the legendre and associated legendre polynomials. I is expressed in units of a_0^2 , where a is the Bohr atomic radius.

When unpolarised electrons are scattered by atoms, the polarisation S of the scattered electrons is given by [4]:

$$S = i (fg^* - f^*g) / I \quad \dots\dots\dots(4)$$

Hence, in general, scattered electrons are partially polarised. Since f is normally the dominant amplitude, large values of S are obtained only when f is small, which from equation(1) is seen to occur at a cross section minimum. In particular, at the critical position, f usually vanishes, and so at electron energies and scattering angle close to the critical position, the elastically scattered electrons may be fully polarised, i.e. $S = \pm 1$. Hence one reason for the establishment of the critical position is that it is the first stage in the search for electrons which have been fully polarised as a result of elastic scattering. A more detailed analysis of the behaviour of the observed polarisation features following elastic scattering has been made by Buhring [11]. Several theoretical estimates have been made of the critical positions and points of total polarisation in a number of atoms. These have been referred to by Lucas et al. [16]. Buhring [11] using a simple analytical potential which he expected to be unreliable for light atoms. The first successful measurements of some of the critical positions in argon, krypton and xenon were reported by Lucas and Liedtke [13]. More extensive results were presented by Kessler et al. [14]. Some preliminary measurements in neon have been reported by Lucas et al. [16]. The simplest theory is the static potential calculation of Khare and Kumar [2] they solved the Schrodinger equation for a simple atomic potential. They also extended their calculations by making an approximate allowance for exchange between the incident and atomic electrons, and for distortion of the atomic electrons by the polarizing effect of the incident electron.

An extension of relatively simple models to include absorption has been made by McCarthy et al. [17]; the results have also been discussed by Lucas and McCarthy. This model makes allowance for those electrons which are scattered inelastically. The remaining Schrodinger equation calculations from which critical positions have been derived are those of Thompson [21-22] who not only made a full exchange calculation but also allowed for distortion. A similar degree of sophistication has been achieved by D.W. Walker (private communication) using the relativistic Dirac equation solved for a relativistic Hartree-Fock atomic potential. For comparison, critical positions from his earlier result [12] including exchange but no distortion have also been derived. Unfortunately, no approximations such as close coupling or R-matrix, which enable more exact allowance for absorption, appear to have been applied to elastic electron scattering in argon. Electron correlation has been included in calculations by Pindzola and Kelly [23] and by Tancic [24] but neither unfortunately meets the criteria stated above for calculating critical positions. Other calculations of elastic scattering in argon which do not satisfy these criteria are the analytic potential calculations of Bery et al. [25], the optical-model calculations of Joachain et al. [26-27], the relativistic non-exchange calculations of Fink and Yates [28], the non-relativistic calculations of Ritey and Truhlar [29] which cover six different types of exchange approximations, and the polarized orbital calculations of Yau et al. [30]. The phase shifts have been derived from recent experimental results for absolute differential cross sections by Gibson and Rees [31]. They analysed the measurements of Gupta and Ress [32-33], DuBois and Rudd [34], Williams and Willis [35] and Vuskovic and Kurepa [36] at electron energies where structure was present. In fact Gibson and Rees used the corrected values for argon later reported by DuBois and Rudd [37]. In addition Westin [38] made a phase shifts analysis of his own measurements in argon, using an analogue computer he built for this purpose. Lucas [4] has suggested a method by which critical energies and critical angles can be obtained. This method uses theoretically obtained or experimentally determined phase shifts as the input. For a reasonable accuracy phase shifts corresponding to more than six incident energies should be available and the critical energy should lie within the energy range represented by these impact energies. Khare and Raj [8] proposed an alternative method of obtaining the critical points for a theoretical central potential (the static field of the atom). In this method phase shifts at only three energies are required, the energies being reasonably close to the critical energy. Unlike Lucas's method, this method does not require any iteration

and hence saves computer time .To test the accuracy of this method , critical points are also theoretically computed for light atoms ($6 \leq Z \leq 10$) in the static -field approximation and the computed values are compared with the values obtained by the proposed method with great success. It may be noted that so far no theoretical or experimental values E_c and θ_c are available for light atoms . Only Yates [10]has remarked that for $Z < 7$ the value of E_c should be less than 30 eV. Also, the linear relationship between E_c and Z proposed by Yates does not hold over a wide range of Z [12].In the present investigation we have employed the method proposed by Khare and Raj [8] to obtain the critical points for light atoms , to test the accuracy of this method , critical points (E_c , θ_c) are also theoretically computed for light atoms ($4 \leq Z \leq 10$) in the optical approach and by using the partial wave method . Also we compare our results with the other investigators results [3,5,39] our results are in good agreement with the other results . Also we have obtained results explain the relationship between the differential cross section and the incident electron energy at the constant critical angle θ_c for all atoms in this investigation.

Theory and Calculations

As already remarked , to obtain the critical points we require DCS for the electrons elastically scattered by the atom .In the present investigation we have employed the optical approach in which the many-body problem is reduced to a one-body problem. According to this approach, the wave function $\psi(r)$ of the scattered electron is a solution of the following one-body scattering equation [39]:

$$(\nabla^2 + K^2 - 2V_{opt}(r))\psi(r) = 0 \quad \dots\dots\dots (5)$$

where $V_{opt}(r)$ known as optical potential is, in general, a local, and energy- dependent potential, $1/2(K^2)$ is the energy of the incident electron and ∇^2 is the kinetic energy operator (we employ atomic units unless specified otherwise). The exact solution of the above equation is as difficult as the solution of original many -body Schrodinger equation .Hence , we solve eq.(5) using the static field exchange correlation polarization (SFCEP) approximation in which $V_{pot}(r)$ is written as [40]:

$$V_{opt}(r) = V_s(r) + V_{ex}^{HFEG}(r) + V_{cor}^{SR}(r) + V_{pol}^{LR}(r) \quad \dots\dots\dots(6)$$

the static potential $V_s(r)$ is given as [39]:

$$V_s(r) = \langle \Phi | V(r_1, r_2, \dots, r_z, r) | \Phi_0 \rangle \quad \dots\dots\dots(7)$$

where Φ_0 is the ground -state wave function of the target atom having Z electrons and $V(r_1, r_2, \dots, r_z, r)$ is the interaction potential due to the target and the incident electron interaction.

$V_{ex}^{HFEG}(r)$ is the exchange potential of Hara [41] for free electron gas as given by [42]:

$$V_{ex}^{HFEG}(r) = -\frac{2}{\pi} K_F(r) F(\xi) \quad \dots\dots\dots (8)$$

$$K_F(r) = [3\pi^2 \rho(r)]^{1/3} \quad \dots\dots\dots ..(9)$$

$$F(\xi) = (1/2) + \frac{1 - \xi^2}{4\xi} \ln \left| \frac{1 + \xi}{1 - \xi} \right| \quad \dots\dots\dots ..(10)$$

$$K^2(r) = K_F^2(r) + 2I + k^2 \quad \dots\dots\dots ..(11)$$

where $K_F(r)$ is the local fermi momentum, which relate to the electronic charge density $\rho(r)$ as in eq.(9), and $\xi = K(r) / K_F(r)$.

Hara [41] suggested that the scattered electron momentum must be equivalent with the bound electrons momentum in gas, therefore the variation r with $K(r)$ resulting from $K_F(r)$ as an eq.(11) , where in this equation , I represent the ionization potential for target atom and K is the momentum of the incident electron , therefore $V_{ex}^{HFEG}(r)$ as in eq.(8) depend on the incident electron energy through eq.(11) [42] .This potential gives a good results in the energy calculations for atomic valence states [43] and photoionization [44-45] .

In order to calculate the static potential $V_s(r)$ and exchange potential for any atom we have employed the Roothaan-Hartree-Fock wave function for the ground state of the atom, given as Clementi and Roetti [46] with:

$$\Phi_{nl}(r) = R_{nl}(r) Y_{lm}(r) \text{ or } \Phi_{nl}(r) = \Phi_{i\lambda\alpha}(r) = \sum X_{p\lambda\alpha}(r) C_{i\lambda p},$$

where $X_{p\lambda\alpha}(r, \theta, \Phi) = R_{\lambda p}(r) Y_{\lambda\alpha}(\theta, \phi) Y_{\lambda\alpha}(\theta, \phi)$ are the normalized spherical harmonics and the radial function $R_{\lambda p}(r)$ is given by:

$$R_{\lambda p}(r) = N_{\lambda p} r^{n-1} \exp(-\epsilon_{\lambda p} r),$$

$N_{\lambda p}$ are the normalization constants and are given as:

$$N_{\lambda p} = [(2n_{\lambda p})!]^{-1/2} (2\epsilon_{\lambda p})^{n_{\lambda p} + 1/2}$$

α and β are the spin-up and spin-down wave functions, respectively. In the case of Be atom ($Z=4$), the values of λ and α are zero. Thus we can write

$$\Phi_{1s}(r) = \sum_{i=1}^6 A_i X_i(r) \text{ and } \Phi_{2s}(r) = \sum_{i=1}^6 B_i X_i(r),$$

and $X_i = N_i \exp(-C_i r) Y_{00}(\theta, \phi)$ where $i=1$ to 6. The constants A_i , B_i and C_i are given by Clementi and Roetti [46].

Now, employing the wave function Φ_0 for the atom in eq.(7), $V_s(r)$ is calculated. The exchange potential, in principle, is a local potential. For the short range correlation potential, we have employed the describe model by Perdew and Wang [47], where they give an simplest analytical representation of the correlation energy for the uniform electron gas as a function of the charge density parameter r_s and relative spin polarisation $\xi(r)$. The dependent parameters on $\xi(r)$ are calculated in the low and high density limits, therefore the correlation potential $V_{cor}^{SR}(rs, \xi)$ for the case $\xi(r)=0$ (unpolarised), small r_s - density, is :-

$$V_{cor}^{SR}(rs, 0) = 0.031091 \ln r_s - 0.0570076 + 0.0044266 r_s \ln r_s - 0.0091666 r_s \quad \text{-----(12)}$$

and for large r_s -low density, the form is :-

$$V_{cor}^{SR}(rs, 0) = -0.578 r_s^{-1} + 2.1612 r_s^{-3/2} \quad \text{-----(13)}$$

while for $\xi(r) = 1$ (polarized), for small r_s is :-

$$V_{cor}^{SR}(rs, 1) = 0.015545 \ln r_s - 0.0307806 + 0.0021266 r_s \ln r_s - 0.0036233 r_s \quad \text{-----(14)}$$

and for large r_s is:

$$V_{cor}^{SR}(rs, 1) = -0.4382666 r_s^{-1} + 2.65455 r_s^{-3/2} \quad \text{-----(15)}$$

$$\text{where } r_s = \left[\frac{3}{4\pi\rho(r)} \right]^{1/3} \quad \text{-----(16)}$$

$$\xi(r) = \frac{n_{\uparrow} - n_{\downarrow}}{n_{\uparrow} + n_{\downarrow}} \quad \text{-----(17)}$$

where n_{\uparrow} and n_{\downarrow} represents spin densities of electrons.

For the polarization potential $V_{pol}^{LR}(r)$ we have employed an empirical energy dependent and spherically symmetric potential proposed by Ali [48] given as :

$$V_{pol}^{LR}(r) = -\frac{1}{2} \sum \frac{\alpha_{2L}(0) r^{2L}}{(r^2 + d^2 n)^{2L+1}} \quad \text{-----(18)}$$

where $2L$ represent the order of multipole, $\alpha_{2L}(0)$ static multipole

polarizabilities of the target atom, and d_n cut -off parameter that explained by Ali[48]. Following O'Connell and Lane [49] we have connected $V_{cor}^{SR}(r, \xi)$ with $V_{pol}^{LR}(r)$ at the point r_0 a given by:

$$V_{pol}(r) = \begin{cases} V_{cor}^{SR}(r) & ; \quad r \leq r_0 \\ V_{pol}^{LR}(r) & ; \quad r > r_0 \end{cases} \quad \text{..... (19)}$$

where r_0 represent the crossing point between $V_{cor}^{SR}(r)$ and $V_{pol}^{LR}(r)$, this point have theoretical value as given by [48]:

$$r_0 = -\left[\frac{V_{cor}^{SR}(r)}{\frac{d}{dr}V_{cor}^{SR}(r)}\right] \quad \text{-----(20)}$$

the parameters r_0 and d_n one depend on other, therefore we must compute them very well to obtain a good results.

Thus, after having obtained the value of $V_{opt}(r)$ which is spherically symmetric in the (SFECP) approximation the equation [50]:

$$\left[\frac{d^2}{dr^2} + \frac{2}{r} \frac{d}{dr} - \frac{\ell(\ell+1)}{r^2} - 2V_{opt}(r) + K^2\right]R_\ell(K, r) = 0 \quad \text{-----(21)}$$

is numerically solved to obtain the phaseshifts where $\delta_\ell(k)$ is the phaseshifts for the ℓ th partial wave due to the presence of the potential $V_{opt}(r)$. The scattering amplitude $f(k, \theta)$ is then obtained from the relation [50]:

$$f(k, \theta) = \frac{1}{k} \sum_{\ell} (2\ell + 1) e^{i\delta_\ell} \sin s_\ell p_\ell(\cos \theta) \quad \text{-----(22)}$$

it is noticed that eq.(22) is the same as eq.(2). Therefore after having obtained the scattering amplitude $f(k, \theta)$ from eq.(22), the differential cross section $I(\theta)$ is obtained by taking the square modulus of $f(\theta)$. The differential cross section thus obtained are utilised to calculate the critical points (E_c, θ_c). Let us consider the elastic scattering of a beam of unpolarised electrons by an atomic target. If we replace the target by a central potential but include the spin-orbit interaction due to the spin and orbital angular of the incident electron, the differential cross section for the incident energy E is given by eq.(1) which reduces to zero in the absence of a spin-dependent potential. Furthermore, for light atoms the spin-dependent potential is weak and the neglect of this potential does not appreciably effect the positions of the critical points. Hence, in the present investigation we neglect $g(\theta)$ and take $I(E, \theta)$ as [8]:

$$I(E, \theta) = f_R^2 + f_I^2 \quad \text{-----(23)}$$

where f_R and f_I are the real and imaginary parts of the direct scattering amplitude and are obtained from eq.(22). The values of $I(E, \theta)$ are calculated by the numerical solution of eq.(21) and with the help of eqs. (22) and (23) and thereby the values of E and θ which yield minimum values of $I(E, \theta)$ are obtained. Thus the critical points (E_c, θ_c) are computed for light atoms. In this investigation we have employed the method proposed by Khare and Raj[8] to obtain the critical points which has yielded satisfactory results for atoms having $4 \leq Z \leq 10$ as small as the other method that determine by computation. The partial waves method is used to calculate the phase shifts, (S_ℓ) and the scattering amplitude $f(k, \theta)$ with the help of eq.(6) and by utilizing the (CAVLEED) international program [51] and some auxiliary programs, also we have employed the Roothaan-Hartree-Fock wave functions for free atoms given by Clementi-Roetti[46] in the calculations. The method proposed by Khare and Raj[8] requires phase shifts only at three incident electron energies reasonably close to the critical energy. This method is based on the observation that near the critical points the value of DCS is very small. Hence, one may take:

$$I(E_c, \theta_c) = f_R^2(E_c, \theta_c) + f_I^2(E_c, \theta_c) = 0 \quad \text{-----(24)}$$

without any significant error. Therefore at the critical points one may take real as small as imaginary parts of the scattering amplitude represented by f_R and f_I , respectively, separately equal to zero. Khare and Raj[8] noted that θ_i (where $f_R = f_I = 0$) can be expressed as a polynomial in f_i , and terms up to second power of f_i , are sufficient. Hence, we take

$$\theta = af_i + bf_i^2 + c \quad \text{-----(25)}$$

the computed values of f_i , at three energies are used to evaluate a, b and c. The value of c is equal to the critical angle θ_c . Similarly, the critical energy is obtained by the following expression:

$$E_i = ef_i + gf_i^2 + h \quad \text{-----(26)}$$

where the constant e, g and h are calculated with three f_i and corresponding E_i . The value of h yields the critical energy E_c . Since eqs.(25) and (26) involve three constants, separately, hence f_i or the phase shifts are required at three energies to evaluate E_c and θ_c . After we computed the critical points, we have two sets of results, one determined by computation and other by the method of Khare-Raj[8], are in excellent agreement. Also our results compared with the results of other investigators [3,5,39], are in good agreement. Furthermore, with the help of critical angle θ_c , the relation between the values of the differential cross section and incident electron energies in the range (0-100)eV, at

constant θ_c for Be (Z=4) , C (Z=6) , N(Z=7) , O(Z=8), F(Z=9) and Ne(Z=10) atoms are evaluated.

Results and Discussion

To determine critical points , phase shift obtained by the numerical solution of the differential equation(21) are employed to generate $f_R(E, \theta)$ and $f_I(E, \theta)$ which are then plotted as a function of θ for different values of E and the curve of intersections of f_R and f_I is obtained .The intersection of this curve with the θ axis yields the critical angle θ_c and the corresponding energy at which the intersection with the θ axis has happened represent the critical energy E_c [8]. If the values of the chosen energies are reasonably close to each other and to critical energy, the obtained results are more accuracy [8].

In this investigation, the method of partial waves is employed, also we have employed the method proposed by Khare and Raj[8] to obtaine the critical points for light atoms, to test the accuracy of this method, the obtained results by this method compared with the results obtained by computation and with results of other investigator [3,5,39] where we have obtained two sets of values , one determined by computation and other by Khare –Raj[8] method, are in good agreement with each other and with other results [3,5,39]. Also we have obtained results explain the relation between the differential cross section and incident electron energy at constant critical angle θ_c for all atoms in this investigation.

Khare -Raj [8] explained that the simple proposed method is successful even if the difference between the energies chosen to obtain f_i and θ_i is as large as 20eV .The critical points vary with the uses of scattering potential for any atom because of many effects such as the static ,exchange, correlation and polarisation effects , however , for any atom θ_c is much less sensitive to the potential [8].

The dipole $\alpha_2(0)$ and quadrupole $\alpha_4(0)$ polarisabilities are employed to calculate the critical points for atoms , as shown in table (1) , for the Be-atom we have used $\alpha_2(0) = 37.0$ [52] , $\alpha_4(0) = 302.0$ [53] , for C-atom $\alpha_2(0)=1.88$ [54] , while for N-atom $\alpha_2(0) = 7.62$ [55] for O-atom $\alpha_2(0)=5.0$ [55], $\alpha_4(0)=18.746$ [56] for F-atom we have used $\alpha_2(0)=4.5904$ [48] and for Ne-atom we have utilized $\alpha_2(0) = 2.669$ and $\alpha_4(0)=5.541$ [48].

The critical points result from atoms are shown in table (2) with other investigators results , where the points values E_c and θ_c for Be-atom are 6.73 eV , 101.62° respectively , as shown in figure (1) , for C-atom are 18.40 eV , 102.90° respectively, as shown in figure(2), for N-atom are respectively, 30.05 eV , 97.44° as shown in figure (3) , for O-atom the value of E_c and θ_c are 45.50 eV, 98.64° respectively as shown in figure (4) , while for F-atom the results are 59.29 eV , 98.76° as shown in figure (5) and for Ne-atom, the critical points are shown in figure (6) where the values are 70.95 eV and 99.48° respectively. Furthermore, we have obtained results explain the relation between the differential cross section and incident electron energies in the (0-100) eV energy range, at constant critical angle θ_c for all atoms in this investigation, as in figure (7) , where we note that the differential cross section minimum for atom attains its smallest value at the critical energy E_c , for constant critical angle θ_c .

We conclude from the present study that the differential cross section minimum attains its smallest value at the critical points (E_c, θ_c) .Also we note from table (2) that the critical energy E_c increases with Z, therefore this result is in accordance with the prediction of Buhning [11].

Table (1) : The parameters (r_0 , r_s , d_1 or d_2), the correlation -polarization potential with the dipole $\alpha_2(0)$, quadrupole $\alpha_4(0)$ polarizabilities and ionization potential I(eV) for atoms (in atomic units a.u : $\alpha_0 = e = h = m = 1.0$).

Atom	Atomic number Z	r_0 (a. u.)	$r_s(r_0)$ (a. u.)	d_1 (P. W.)	d_2 (P. W.)	$\alpha_2(0)$ (a_0^3)	$\alpha_4(0)$ (a_0^5)	I (eV)
Be	4	4.050	5.9628	---	1.6149	37.0	302.5	9.32
C	6	2.806	3.5633	0.7140	---	11.88	---	11.26
N	7	2.540	3.2575	2.558	---	7.62	---	14.54
O	8	3.060	5.2987	---	2.1750	5.00	18.746	13.61
F	9	2.806	5.0381	---	1.8950	3.60	4.5904	17.42
Ne	10	2.038	2.5966	---	0.5755	2.669	5.5410	21.56

P. W. represent the present work.

The ionization potentials I(eV) in the above table are chosed from ref. [58].

Table (2) : Critical energy (E_c) and angle (θ_c) for light atoms obtained by using Khare –Raj[8] method and by computation, with other investigators results.

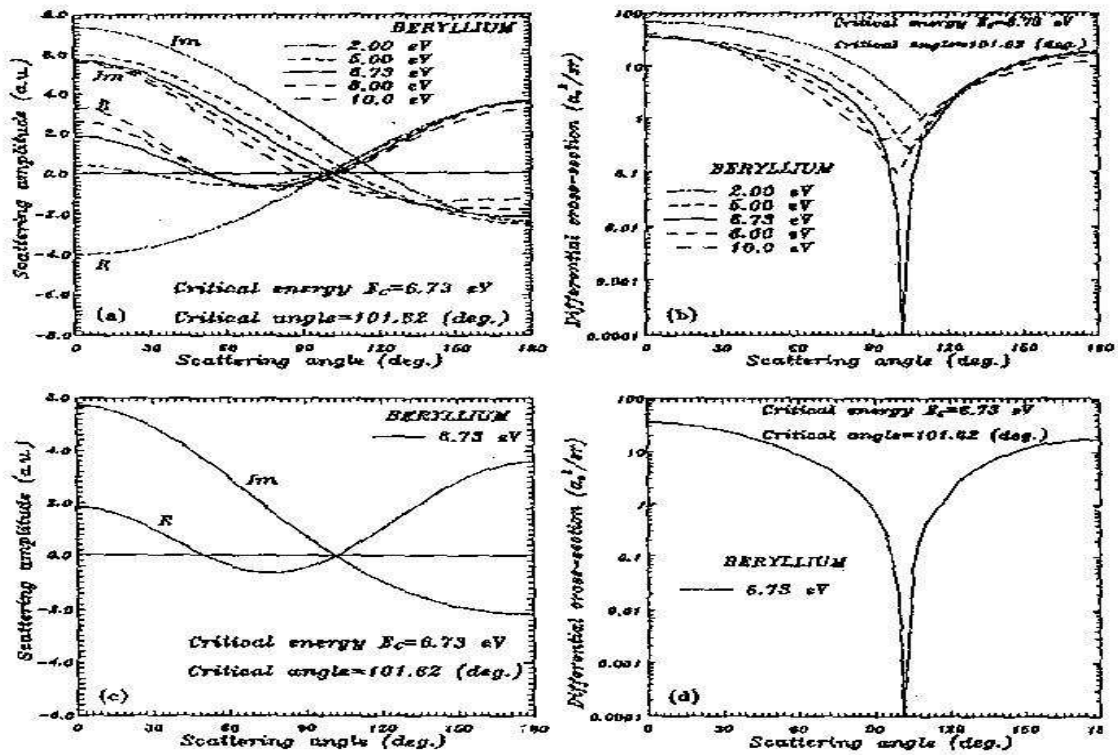
Atom	Atomic number Z	E_c (eV)	O_c (deg.)		Energies(cV) Employed in eq.(25)and(26)	E_c (cV)	θ_c (deg.)	Ref.
	From eq..(26)	Computed (P. W.)	From eq..(25)	Computed (P. W.)				
Be	6.73	6.73	101.62	101.62	5,8,18	5.80	94.80	a
C	18.40	18.40	102.90	102.90	15,20,25	17.49	97.63	b
N	35.05	35.05	97.44	97.44	30,40,45	26.80	97.50	b
O	45.50	45.50	98.64	98.64	40,50,55	35.59	98.65	b
F	59.29	59.29	98.76	98.76	55,65,70	45.20	99.73	b
Ne	70.95	70.95	99.48	99.48	50,60,80	73.7 ± 1	10 ± 0.5	c

P.W. represent the present work.

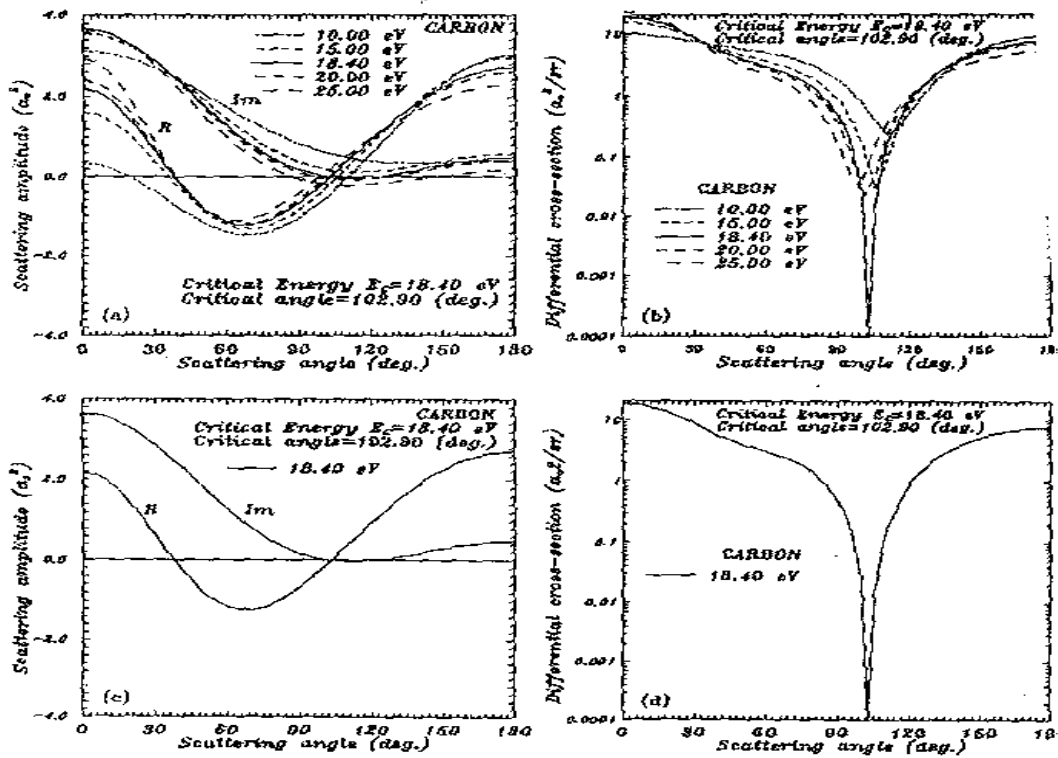
ref.(a) : Kaushik et al. (1983) , Theo.[39]

(b) : Raj(1981), Theo.[5]

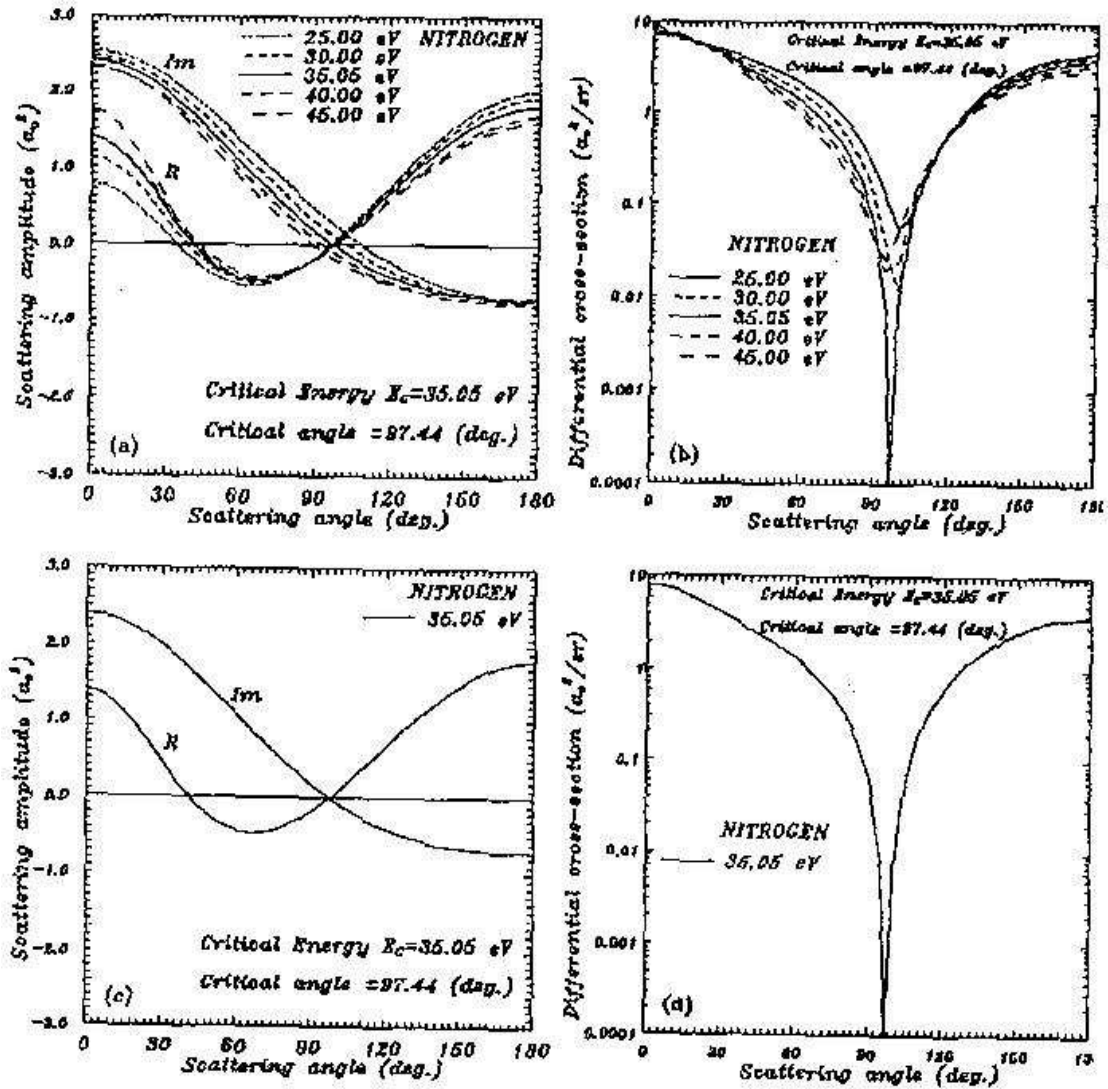
(c) : Kollath-Lucas (1979), Exp.[3]



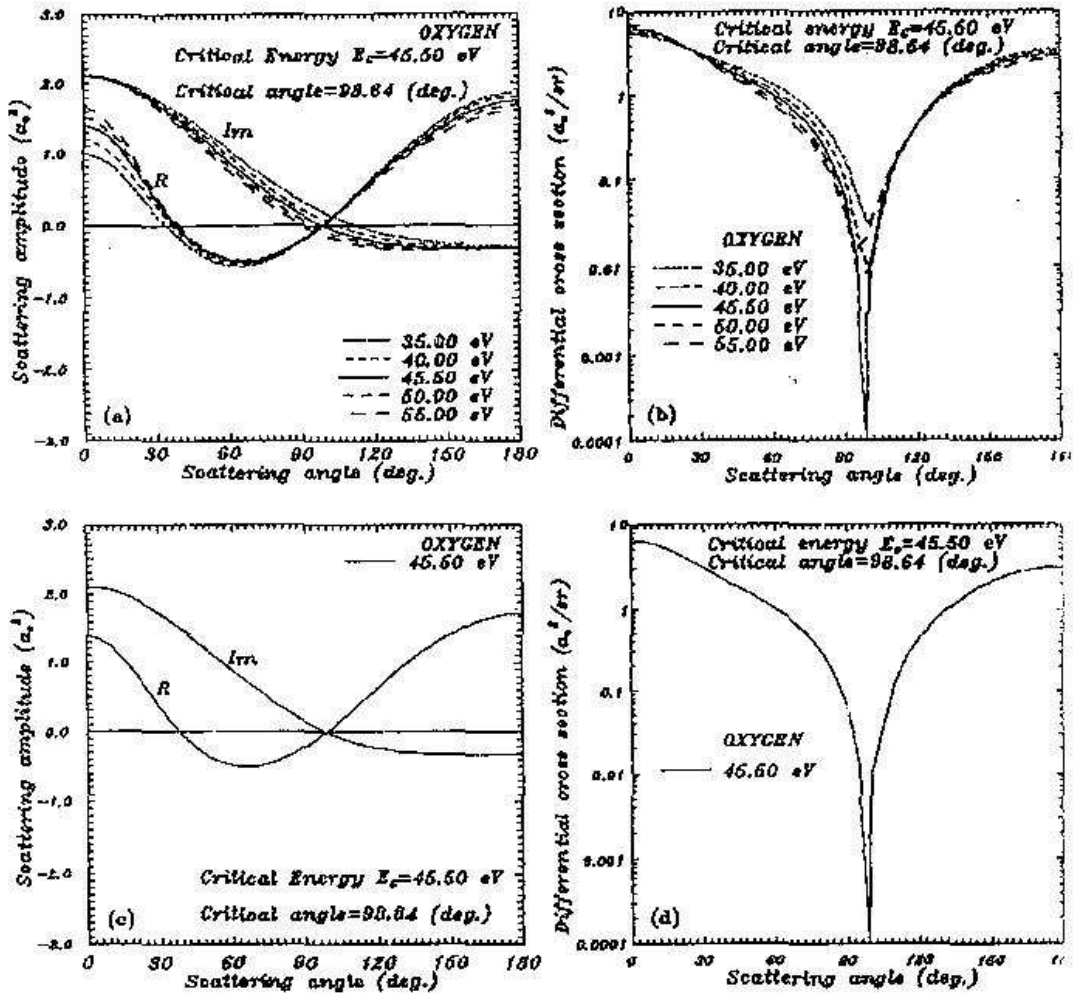
Figure(1) : Critical points results for Beryllium atom.



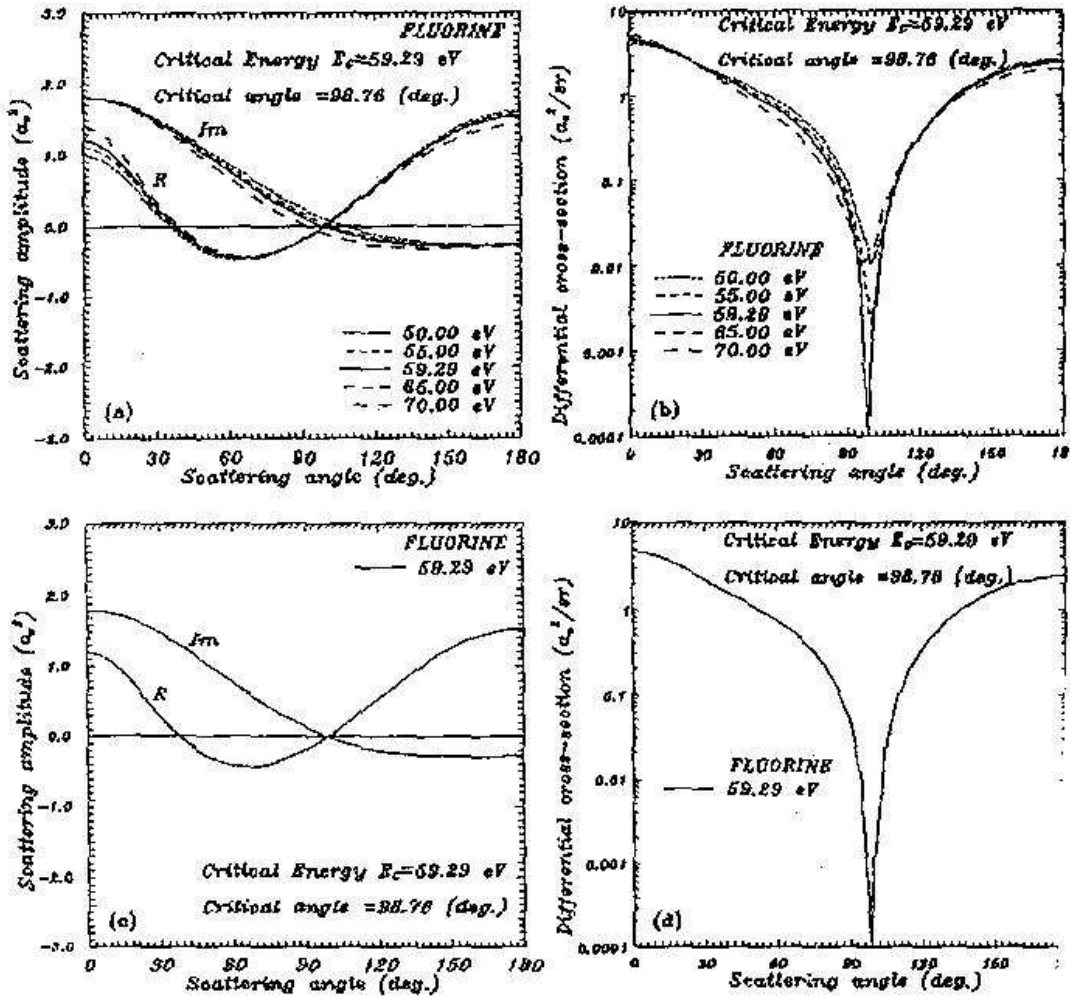
Figure(2) : Critical points results for Carbon atom.



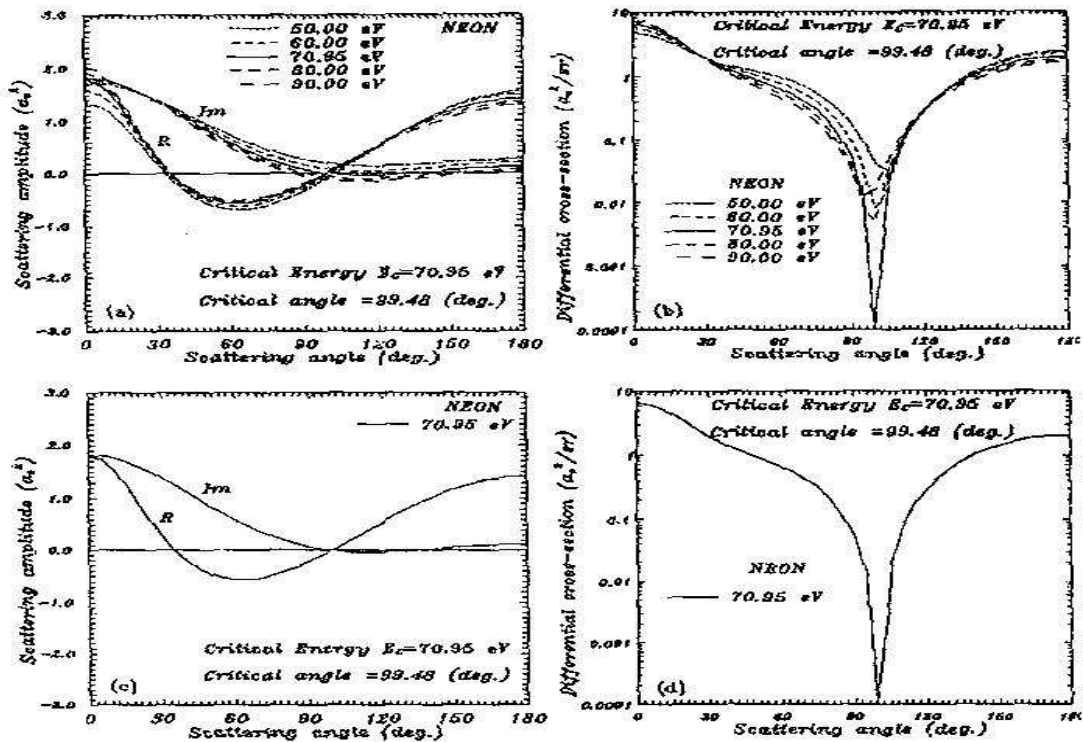
Figure(3) : Critical points results for Nitrogen. atom.



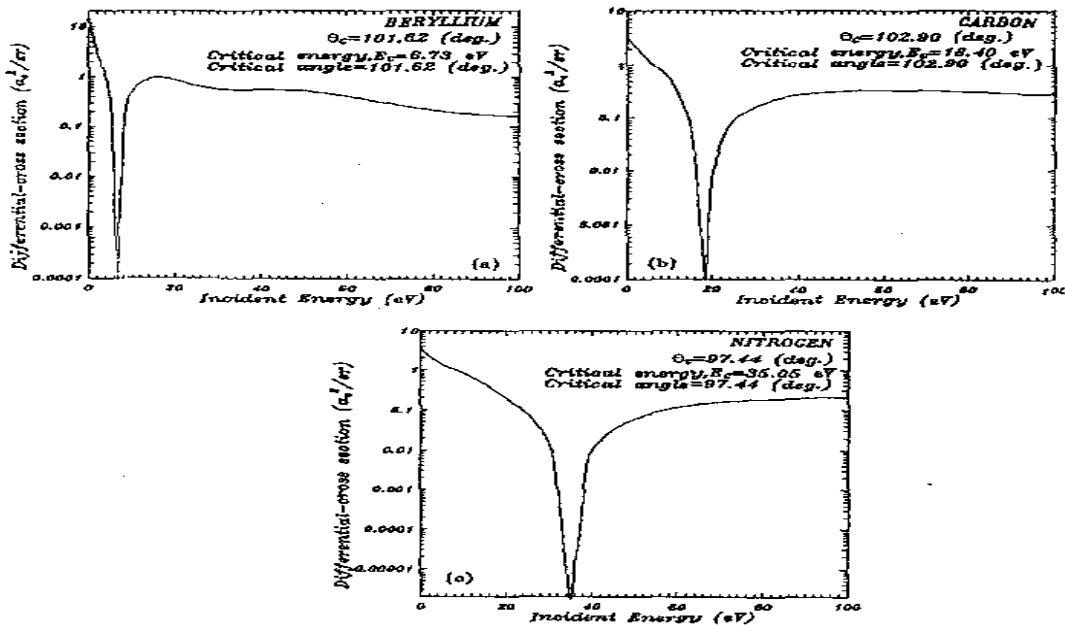
Figure(4) : Critical points results for Oxygen atom.



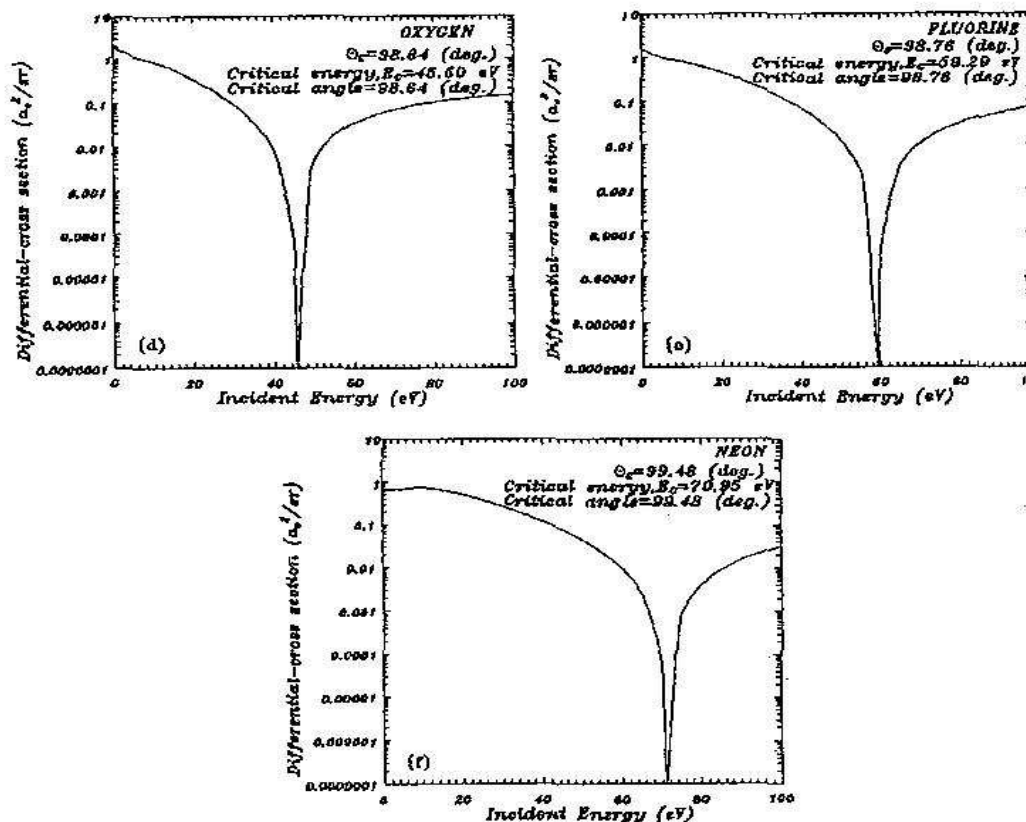
Figure(5) : Critical points results for Fluorine atom.



Figure(6) : Critical points results for Neon atom.



Figure(7) : The relation between the differential cross section (α_0^2 / sr) and incident electron energies (eV) , at constant critical angle θ_c (degree) , in the (0-100) eV energy range , for atoms (a) Beryllium (b) Carbon (c) Nitrogen (d) Oxygen (e) Fluorine (f) Neon.



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النقاط الحرجة لاستطارة الالكترونات من الذرات الخفيفة عند الطاقات الواطئة

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المستخلص :

ان طاقة الإلكترون التي يبلغ عندها المقطع العرضي التفاضلي قيمته الصغرى تدعى بالطاقة الحرجة . اما زاوية الاستطارة المقابلة لهذه الطاقة فتدعى بالزاوية الحرجة . لقد استخدمت الطريقة التي تتطلب ازاحات الطور عند ثلاث طاقات لاستحصاال المواضع الحرجة لاستطارة الإلكترون بواسطة الذرات ، وكذلك تمت الاستعانة بطريقة الموجة المجزئة وبأستخدام نموذج الجهد البصري لانجاز الحسابات. ولاكتشاف امكانية وجود النقاط الحرجة للذرات فأن المقاطع العرضية التفاضلية للاستطارة المرنة للالكترونات من الذرات الخفيفة التي عددها الذري من 4 الى 10 تم حسابها لمدى الطاقات (0 - 100) eV . كانت النتائج المستحصلة بأستخدام الطريقة المقترحة متوافقة بصورة جيدة مع النتائج المستخرجة حسابيا ومع نتائج باحثين آخرين .