



## CONTINUATION POWER FLOW OF AN UNBALANCED THREE-PHASE FOUR-WIRE DISTRIBUTION SYSTEM

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**Abstract:** A continuation power flow algorithm for an unbalanced three phase four wire distribution system is presented. The proposed algorithm is used to trace the loci of the voltage of the different phases of an unbalanced three-phase distribution system. The locus of the neutral voltage for a wide range of loading conditions is also obtained. The proposed technique is applied on a typical unbalance three phase radial Iraqi distribution system. The results obtained reveal the effectiveness of the continuation technique for exploring all the possible solutions and the determination of maximum loadability of the distribution feeder.

**Keywords:** Continuation technique; Four-wire system ; Maximum loadability; Distribution system; Power flow.

### سريان الحمل المتتابع لمنظومة توزيع ثلاثية الطور غير متزنة ذات أربعة أسلاك

**الخلاصة:** هذا البحث يقدم خوارزمية لحساب سريان الحمل المتتابع لمنظومة توزيع ثلاثية الطور ذات أربعة أسلاك غير متزنة. ان الخوارزمية المقترحة تستخدم لرسم المحال الهندسية لفولتيات الاطوار المختلفة لمنظومة توزيع ثلاثية الطور غير المتزنة. تم ايجاد المحل الهندسي لفولتية السلك المحايد لمدى واسع من ظروف التحميل. تم تطبيق الخوارزمية المقترحة على منظومة توزيع عراقية شعاعية ثلاثية الطور غير متزنة. لقد أظهرت النتائج فاعلية تقنية التتابع في استكشاف كل الحلول الممكنة لسريان الحمل وحساب أقصى تحميل لمغذي شبكة التوزيع.

## 1. Introduction

The main function of a distribution utility is ensuring a reliable electric power delivery to all consumers with a high quality. Among the main features of a well-designed distribution system are the constancy of the voltage, constancy of frequency and the symmetry of the phase voltages. The three phase four wire scheme has been adopted in the Iraqi distribution system from the last century. The operation with this scheme is often accompanied by asymmetrical three phase voltages and currents due to unbalance loading and unbalanced structure of the feeders [1].

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Depending on the degree of load imbalance, these unbalanced three phase voltages may increase the system losses, overload the phase line, neutral line and deteriorate the quality of the supply [2]. To this end, an effective technique to find the phase voltages and neutral voltage for a wide range of loading conditions is prerequisite for assessing system quality. A measurement evaluation of neutral currents and voltages has been conducted on three-phase four-wire multi grounded distribution system [3]. Many attempts have been reported to study the load flow for unbalanced three phase distribution system [4-7]. Also, several techniques have been proposed to mitigate this quality problem by load balancing [8-11] and other techniques [12]. An exploration to the inherent characteristics of multi grounded three-phase four-wire distribution systems under unbalanced conditions has been presented in [13],[14] propose an approach to determine a pair of Power-flow solutions for unbalanced three phase network. The approach has been applied on an ungrounded system by assuming constant impedance load. The interdependent between the multiple solutions of in unbalanced three phase network and the degree of network unbalance has been discussed in [15]. All these previous attempts have been carried out for a particular loading condition.

Continuation power flow has found to be an effective approach for unearthing all the stable and unstable solution [16].The main objective of the continuation technique is trace the solutions path of the load flow problem for a range of loading conditions. Continuation power flow is also providing valuable information related to the voltage stability of the system. The generated P-V curves allow the operator to predict the maximum loadability point. For the last two decades, a lot of research has been carried out to improve the continuation technique for a balanced three phase system [17-23]. Recently, the continuation technique has been applied to unbalanced three-phase distribution system [24-26]. However, the main focus of interest of these recent articles was the application of continuation technique analysis on an unbalanced three-phase distribution system excluding the neutral. Such analysis is deficient in finding out the variation of the neutral point voltage for a wide range of system asymmetry. The aim of this paper is to apply the continuation technique to an unbalanced distribution system by including the neutral wire into consideration. This paper is organized as follows: Section 2 reviews the power flow formulation of unbalanced three phase four wire system. The application of continuation technique to the unbalanced three phase four wire system is presented in section 3. Section 4 discusses the results obtained by applying the proposed algorithms on a typical Iraqi unbalanced distribution system. Finally, section 5 presents the conclusion.

## **2. Power Flow Formulation of unbalanced three phase four wire distribution system**

For N bus electrical power system. The steady-state behavior of the three-phase system is described by the following matrix equation:

$$[I^{abcn}] - [Y^{abcn}][V^{abcn}] = [0] \quad (1)$$

Where

$I^{abcn}$  is the vector of the net injected current at each phase  $p$  of each bus  $i$  including the neutral current  $I_n$

$Y^{abcn}$  is the bus admittance matrix in the phase domain

$V^{abcn}$  is the vector of the  $p$  phase voltages at each bus  $i$  including the neutral voltage  $V_n$

$$Y_{ik}^{pm} = G_{ik}^{pm} + jB_{ik}^{pm}$$

The size of the matrices in (1) are:

The current balance equation at phase  $p$  of bus  $i$

$$\text{size of } [I] = 4N \times 1$$

$$\text{size of } [V] = 4N \times 1$$

$$\text{size of } [Y] = 4N \times 4N$$

$$I_i^p = \frac{S_i^{p*}}{V_i^{p*}} = \sum_{k=1}^N \sum_{m=1}^4 Y_{ik}^{pm} V_k^m \quad (2)$$

$$S_i^p = P_i^p + jQ_i^p \quad (3)$$

by substituting (3) into the (2), and resolved the resulting equation into the following two real equations:

$$P_i^p = V_i^p \sum_{k=1}^N \sum_{m=1}^4 V_k^m \{G_{ik}^{pm} \cos \delta_{ik}^{pm} + B_{ik}^{pm} \sin \delta_{ik}^{pm}\} \quad (4)$$

$$Q_i^p = V_i^p \sum_{k=1}^N \sum_{m=1}^4 V_k^m \{G_{ik}^{pm} \sin \delta_{ik}^{pm} - B_{ik}^{pm} \cos \delta_{ik}^{pm}\} \quad (5)$$

Where  $i=1 \dots N$ ;  $p=1 \dots 4$ (number of phases and the neutral)

for  $N$  bus power system, there are  $8N$  real nonlinear algebraic equations similar to (4) & (5) for each bus. These equations are nonlinear function of the state variables vectors ( $|V^a|, |V^b|, |V^c|, |V^n|, \delta^a, \delta^b, \delta^c, \delta^n$ ). The conventional technique to solve these nonlinear algebraic equations is by using a numerical technique. Newton\_Raphson method was the most widely used method. A load flow solution by NR method can be recognized through expanding equations (4) and (5) by Taylor series. The power mismatch equations can be derived as:

$$\begin{bmatrix} \Delta P_i^a \\ \Delta P_i^b \\ \Delta P_i^c \\ \Delta P_i^n \\ \Delta Q_i^a \\ \Delta Q_i^b \\ \Delta Q_i^c \\ \Delta Q_i^n \end{bmatrix} = \begin{bmatrix} J_{\delta a}^{Pa} & J_{\delta b}^{Pa} & J_{\delta c}^{Pa} & J_{\delta n}^{Pa} & J_{|Va|}^{Pa} & J_{|Vb|}^{Pa} & J_{|Vc|}^{Pa} & J_{|Vn|}^{Pa} \\ J_{\delta a}^{Pb} & J_{\delta b}^{Pb} & J_{\delta c}^{Pb} & J_{\delta n}^{Pb} & J_{|Va|}^{Pb} & J_{|Vb|}^{Pb} & J_{|Vc|}^{Pb} & J_{|Vn|}^{Pb} \\ J_{\delta a}^{Pc} & J_{\delta b}^{Pc} & J_{\delta c}^{Pc} & J_{\delta n}^{Pc} & J_{|Va|}^{Pc} & J_{|Vb|}^{Pc} & J_{|Vc|}^{Pc} & J_{|Vn|}^{Pc} \\ J_{\delta a}^{Pn} & J_{\delta b}^{Pn} & J_{\delta c}^{Pn} & J_{\delta n}^{Pn} & J_{|Va|}^{Pn} & J_{|Vb|}^{Pn} & J_{|Vc|}^{Pn} & J_{|Vn|}^{Pn} \\ J_{\delta a}^{Qa} & J_{\delta b}^{Qa} & J_{\delta c}^{Qa} & J_{\delta n}^{Qa} & J_{|Va|}^{Qa} & J_{|Vb|}^{Qa} & J_{|Vc|}^{Qa} & J_{|Vn|}^{Qa} \\ J_{\delta a}^{Qb} & J_{\delta b}^{Qb} & J_{\delta c}^{Qb} & J_{\delta n}^{Qb} & J_{|Va|}^{Qb} & J_{|Vb|}^{Qb} & J_{|Vc|}^{Qb} & J_{|Vn|}^{Qb} \\ J_{\delta a}^{Qc} & J_{\delta b}^{Qc} & J_{\delta c}^{Qc} & J_{\delta n}^{Qc} & J_{|Va|}^{Qc} & J_{|Vb|}^{Qc} & J_{|Vc|}^{Qc} & J_{|Vn|}^{Qc} \\ J_{\delta a}^{Qn} & J_{\delta b}^{Qn} & J_{\delta c}^{Qn} & J_{\delta n}^{Qn} & J_{|Va|}^{Qn} & J_{|Vb|}^{Qn} & J_{|Vc|}^{Qn} & J_{|Vn|}^{Qn} \end{bmatrix} \begin{bmatrix} \Delta \delta_i^a \\ \Delta \delta_i^b \\ \Delta \delta_i^c \\ \Delta \delta_i^n \\ \Delta |V_i^a| \\ \Delta |V_i^b| \\ \Delta |V_i^c| \\ \Delta |V_i^n| \end{bmatrix} \tag{6}$$

Where the left hand side of (6) is the vector of power mismatch, which can be calculated as:

$$\begin{bmatrix} \Delta P_i^a \\ \Delta P_i^b \\ \Delta P_i^c \\ \Delta P_i^n \\ \Delta Q_i^a \\ \Delta Q_i^b \\ \Delta Q_i^c \\ \Delta Q_i^n \end{bmatrix} = \begin{bmatrix} P_i^{a\ sp} - P_i^{a\ cal} \\ P_i^{b\ sp} - P_i^{b\ cal} \\ P_i^{c\ sp} - P_i^{c\ cal} \\ P_i^{n\ sp} - P_i^{n\ cal} \\ Q_i^{a\ sp} - Q_i^{a\ cal} \\ Q_i^{b\ sp} - Q_i^{b\ cal} \\ Q_i^{c\ sp} - Q_i^{c\ cal} \\ Q_i^{n\ sp} - Q_i^{n\ cal} \end{bmatrix} \tag{7}$$

The continuation process is starting from a base case conventional three-phase load flow by the following algorithm:

1. Assume a guess values for the state variables (flat start  $|V^a|=|V^b|=|V^c|=1.0$ ;  $|V^n|=0.1$  pu;  $\delta^a=\delta^n=0$ ;  $\delta^b=-120^\circ$   $\delta^c=120^\circ$ ).
2. Evaluate the vector of three phase power mismatch and the elements of the three phase Jacobian matrix.
3. Calculate the vector of the change of the state variables.
4. Update the state variables at the end of iteration.
5. Check the absolute value of the elements of the vector of power mismatch, if it is less than a specified tolerance; calculate the line flow in each transmission line. Otherwise go to step 2

Once the base case solution is obtained by the previous algorithm, the loci of the solutions for other loading conditions can be traced by the following continuation technique based on prediction a new solution point and correcting the path of the solutions.

### 3. Formulation of the Continuation Technique

The main objective of a continuation process is to predict a new solution set of the state variables and correcting the solution path for a given loading scenario. In the continuation technique, the load factor is considered a new state variable.

Load flow equations (4) & (5) can be rewritten as:

$$f(\delta^a, \delta^b, \delta^c, \delta^n, |V|^a, |V|^b, |V|^c, |V|^n, \lambda) = 0 \quad (8)$$

$$g(\delta^a, \delta^b, \delta^c, \delta^n, |V|^a, |V|^b, |V|^c, |V|^n, \lambda) = 0 \quad (9)$$

Where  $\lambda$  is the loading factor which represent the continuous variations in the load. The load is obeying the following formula:

$$P_i^{p\ sp} = P_i^{p\ sp} - (1 + \lambda)P_i^{p\ o\ sp} \quad (10)$$

It is assumed that any incremental change in the load at any bus is accompanied by a corresponding power drawn from the root bus of the distribution system. At the end of the base case algorithm the solution of the following state vector is obtained:

$$[\delta^{p\ o} \ |V|^{p\ o} \ \lambda^o]^T$$

The continuation process consists of the following two parts:

### 3.1. Prediction a New Solution Vector

The first part is the prediction part by which a new solution point is estimated. The prediction can be recognized by using the tangent vector method. The tangent vector  $t$  can be evaluated the first derivative of both sides of equations (8) and (9) with respect to the state variables:

$$\begin{bmatrix} \frac{df}{d\delta^p} & \frac{df}{d|V|^p} & \frac{df}{d\lambda} \\ \frac{dg}{d\delta^p} & \frac{dg}{d|V|^p} & \frac{dg}{d\lambda} \end{bmatrix} \begin{bmatrix} d\delta^p \\ d|V|^p \\ d\lambda \end{bmatrix} = [0] \quad (11)$$

$$t = [d\delta^p \ d|V|^p \ \lambda]^T$$

Where the  $t$  is the tangent vector of order 4N. Equation (11) cannot be solved since the number of unknown variables is greater than the number of equations by one. Equation (11) can be solved by specifying an additional equation which describes the continuation parameter. The new system of equations is: Once equation 12 is solved for the tangent vector, the next solution point is estimated by the following formula:

$$\begin{bmatrix} \frac{df}{d\delta^p} & \frac{df}{d|V|^p} & \frac{df}{d\lambda} \\ \frac{dg}{d\delta^p} & \frac{dg}{d|V|^p} & \frac{dg}{d\lambda} \\ \mathbf{0} & e_k & \mathbf{0} \end{bmatrix} \begin{bmatrix} d\delta^p \\ d|V|^p \\ d\lambda \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mp \mathbf{1} \end{bmatrix} \quad (12)$$

$$\begin{bmatrix} \delta^{p*} \\ |V|^{p*} \\ \lambda^* \end{bmatrix} = \begin{bmatrix} \delta^{p\ o} \\ |V|^{p\ o} \\ \lambda^o \end{bmatrix} + \sigma \begin{bmatrix} d\delta^p \\ d|V|^p \\ d\lambda \end{bmatrix} \quad (13)$$

Where the left hand side of (13) is the vector of the predicted solution and  $\sigma$  is the step size which is used to refine the prediction near maximum loadability limit.

### 3.1. Correction by Iterative Scheme

The predicted equilibrium point is corrected by solving (8) and (9) iteratively. The correction process is based on a local parameterization. The parameterization can be realized by including one equation into the system of the power flow equations which specify one of the state variables ( $\delta, |V|, \lambda$ ). The new system of power flow equation to be solved after augmentation by the new equation is:

$$\begin{bmatrix} f(\delta^p, |V|^p, \lambda) \\ g(\delta^p, |V|^p, \lambda) \\ x_k \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \eta \end{bmatrix} \quad (14)$$

$$x_k = \eta \quad (15)$$

Equation (15) is the augmented equation which is specifying nothing but the actual value of one of the state variable. Applying Newton-Raphson iterative technique to the system of equations (14), the linearized equation to be solved :

$$\begin{bmatrix} \frac{df}{d\delta^p} & \frac{df}{d|V|^p} & \frac{df}{d\lambda} \\ \frac{dg}{d\delta^p} & \frac{dg}{d|V|^p} & \frac{dg}{d\lambda} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix} \begin{bmatrix} \Delta\delta^p \\ \Delta|V|^p \\ \Delta\lambda \end{bmatrix} = \begin{bmatrix} \Delta P^p \\ \Delta Q^p \\ \mathbf{0} \end{bmatrix} \quad (16)$$

The starting value of the iterative process of (16) should be the predicted point of the state variables.

## 4. Results and Discussion

The derived continuation algorithm is implemented in a code written in MATLAB. The proposed algorithm is applied to following two unbalanced power systems:

### 4.1 Two-Bus Power System

The proposed algorithm is firstly applied to a simple two bus unbalanced three-phase power system whose bus and line data is given in Appendix 1. Two cases are investigated. In case 1, it was assumed that the three phases of the network are symmetrical but the load at the three phases are unbalanced. In case 2, it was assumed the both the three phases of the network and the load at the three-phases are unbalanced. By assuming a load change scenario at bus 2, the produced P-|V| curves for case 1 is shown in Fig.1. It is clear that for each particular value of the power demand (represented by the load factor  $\lambda$ ) there are two solutions for the voltage at phase  $c$  (i.e.  $|V_c|$ ). The high voltage solution represented by the point (H.V.S.) and the low voltage solution represented by the point (L.V.S.). The high voltage solution can be

considered as the stable equilibrium point. It is the feasible solution which can be obtained by solving the system with the conventional load flow algorithm. The low voltage solution is the unstable and infeasible solution. The stability solution is confirmed by computing the determinant of the Jacobin matrix at each solution. A positive determinant indicates a stable solution while a negative determinant indicate unstable one.

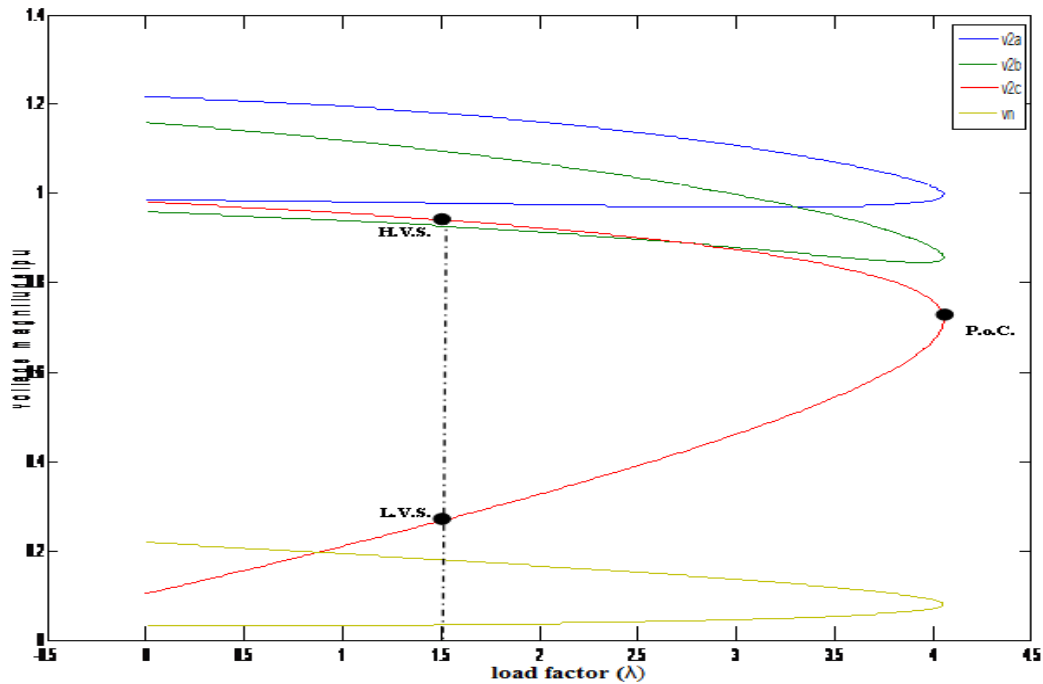


Fig.1 P-|V| curves of bus 2 for case .1

Where

$V_{2a}$  is the phase voltage of phase a to bus number 2

$V_{2b}$  is the phase voltage of phase b to bus number 2

$V_{2c}$  is the phase voltage of phase c to bus number 2

As the loading at the bus 2 is increased gradually (simulated by the incremental change of the loading factor  $\lambda$ ), the value of high voltage solution is decrease and the value of the low voltage solution increase. The two solution move closed to other and eventually collided at one point referred to as point of collapse (P.o.C.). It is impossible to obtain this point by conventional load flow analysis. The point of collapse corresponds to the maximum loadability point. The determinant value of the Jacobian at the point of collapse approximate to zero. It is clear from the Fig.1 that the bus voltage magnitude at the point of voltage collapse is below the rated voltage ,It's value around 0.72 pu . As shown from the same figure , the lower part of the P-|V| curve for phase c voltage ,which represents the locus of any stable solution is less than 0.72pu .the higher part of the PV curve for phase c voltage ,which represents the locus of any stable solution is greater than 0.72pu. This behavior is in sharp contrast to that of PV curve

for the voltage magnitude for phase a .the voltage magnitude of phase (a) at the point of collapse is ( 0.9975pu )which is around the rated voltage .Also the upper part of the PV curve of the phase voltage of phase a is represents the locus of unstable solution. The lower part of the same PV curve represent of the stable solution .

This behavior is in sharp contrast to that of PV curve for the voltage magnitude for phase (a) .the voltage magnitude of phase (a) at the point of collapse is ( 0.9975pu )which is around the rated voltage .Also the upper part of the PV curve of the phase voltage of phase (a) is represents the locus of unstable solution .The lower part of the same PV curve represent of the stable solution .It can be seen from the same figure, the behavior of the neutral voltage for the range of loading conditions.

It can be seen that the neutral voltage is increased with the loading . The maximum loadability limit is found to be 4.1 from the base case (base case of load power in each phase) .The loci of the voltage angle with a wide range of the loading condition is shown in Fig 2.

The produced the P-V curves for case 2 shown in Fig 3.from this figure it is clear that the behavior of the PV curve for the three phases is almost similar to that of case 1 .It can be concluded that the effect of unbalance loading in the three phases is greater than the effect of the asymmetry of the distribution network in the behavior of the P-V curves .

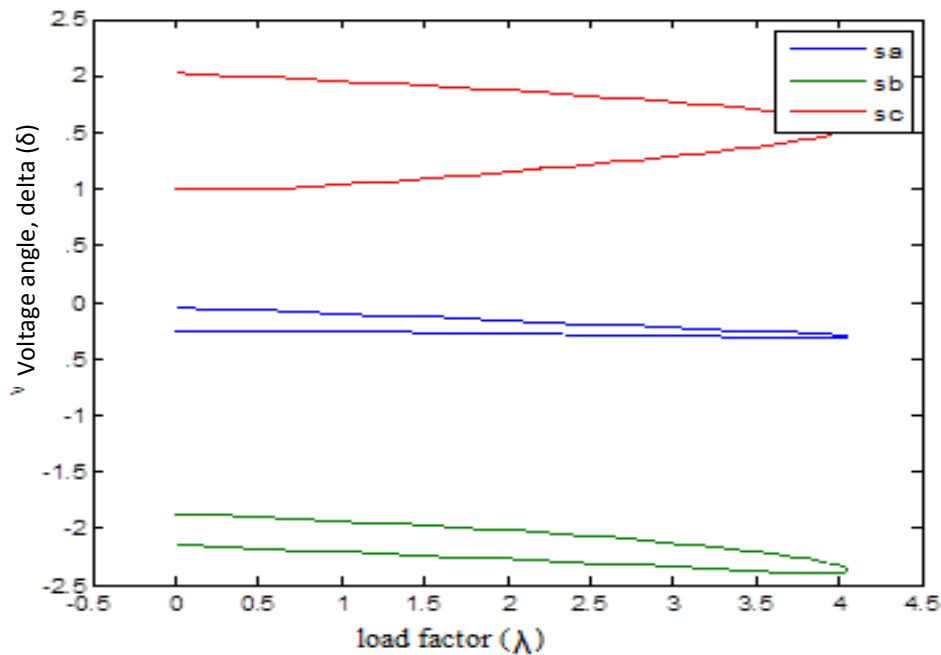


Fig. 2 The loci of the voltage angle versus load factor, ( $\delta_a$ ) is the delta of phase a, ( $\delta_b$ ) is the delta of phase b, ( $\delta_c$ ) is the delta of phase c.



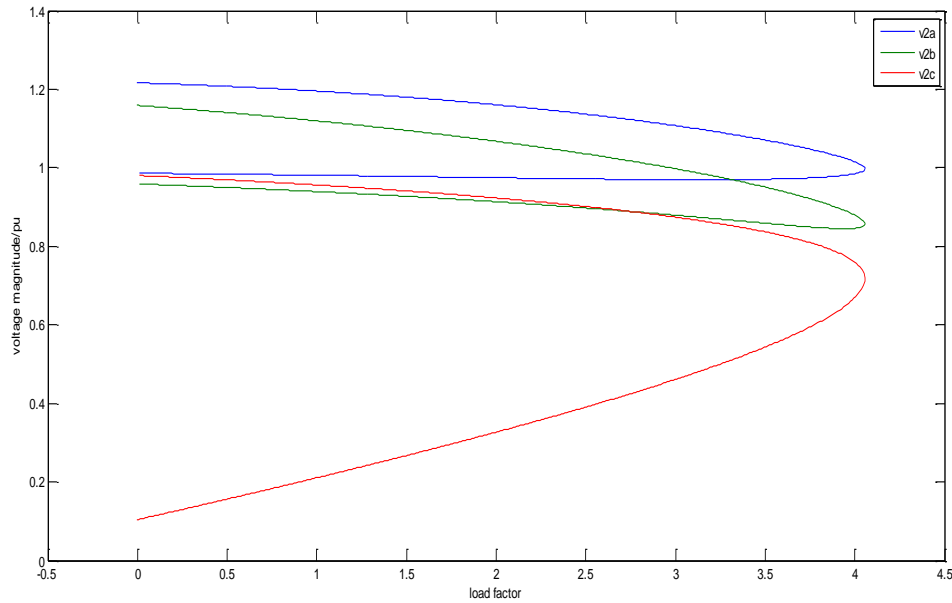


Fig. 3 P-|V| curves of Case.2

Also, the weakest bus can be identified by this method. The weakest bus is the bus whose voltage vary at a high rate of change with respect load parameter ( $\frac{dv}{d\lambda}$ ) and heavily loaded.

Fig.4 shows the locus of the power losses in the neutral wire with a wide range of the loading condition. It can be seen from figure that the losses is relatively low for the stable solutions while it is very high for the upper part of the P-V curve which represent the unstable solutions.

The loci of the phase and neutral currents (P-I curve) for the range of the loading condition is shown in Fig.5. It can be seen that the for the all phases the lower part of (P-I) curve represent the stable operating points .while the losses in neutral wire is high for the upper part of these curves which represent the unstable operating points.

#### 4.2 Nine-Bus Iraqi Radial Distribution System

In the second system, the proposed continuation algorithm is applied on a simple Iraqi three phase radial distribution system in Addujail district, Salahuddin province. The system shown in Figure 6 consists of nine bus bars which is fed from a root bus. The branch sections are assumed of equal length. The self and mutual impedance of each section are given in Appendix 2 and the bus data of three phase loads

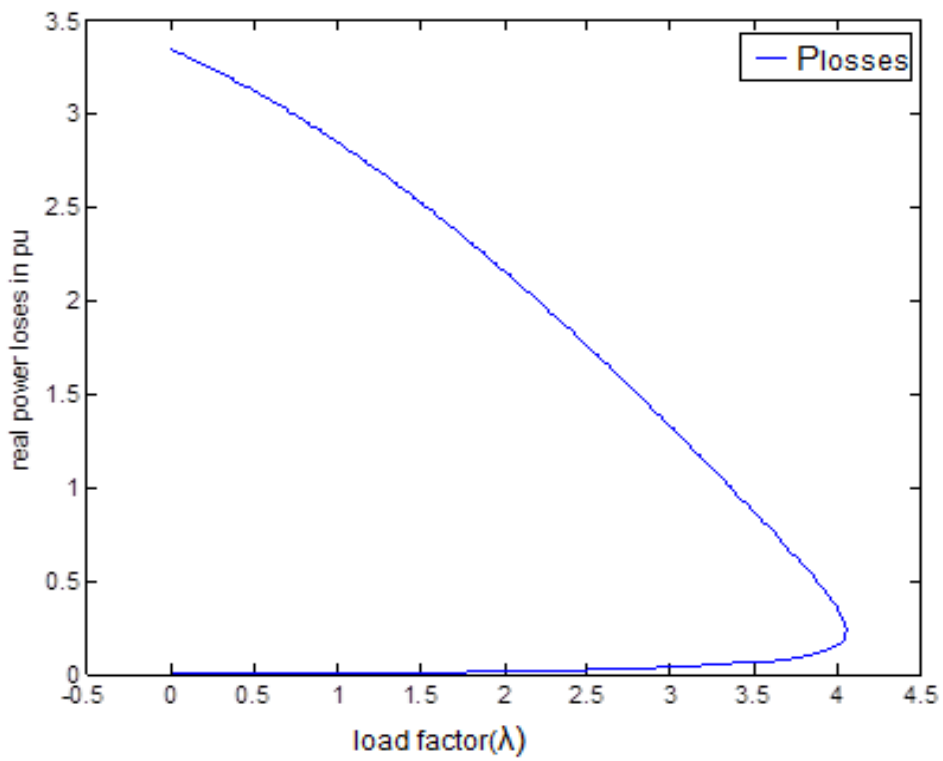


Fig.4 Power losses variations with the loading for case 2

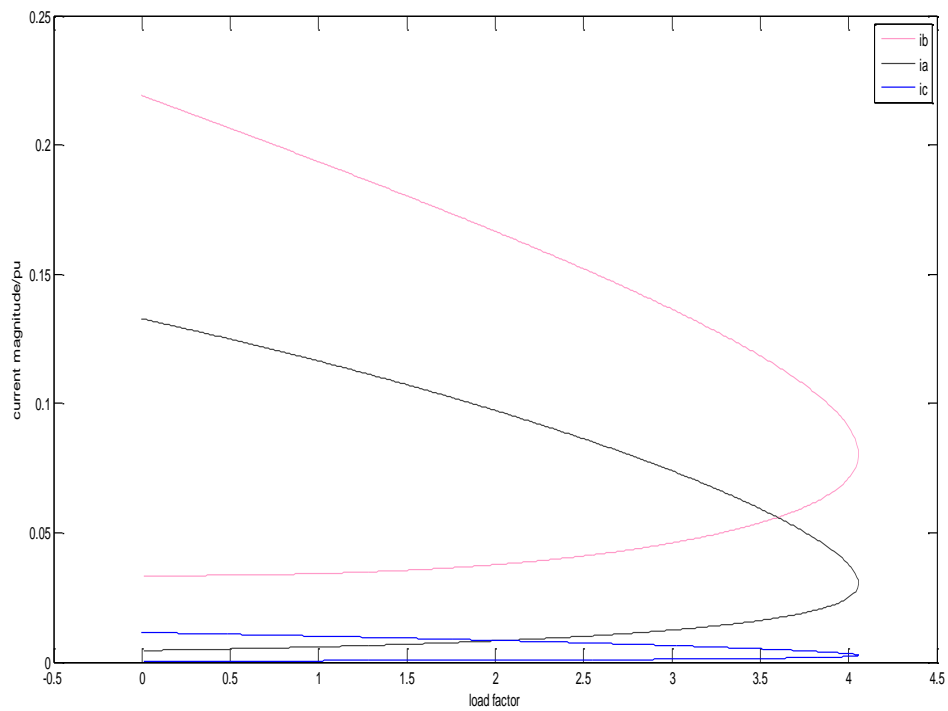


Fig.5 loci of the phase current with a wide range of loading conditions for case 2

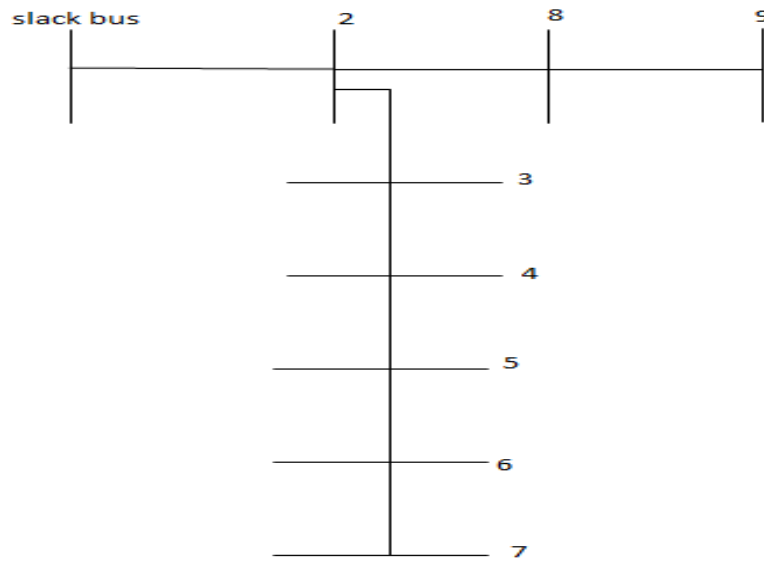


Fig. 6 Iraqi 9-bus radial distribution system

The base case solution of the load flow for the 9-bus distribution system is given in Table 1. It can be seen from the Fig.7 that the voltage magnitude declined for the busses far away from the root bus.

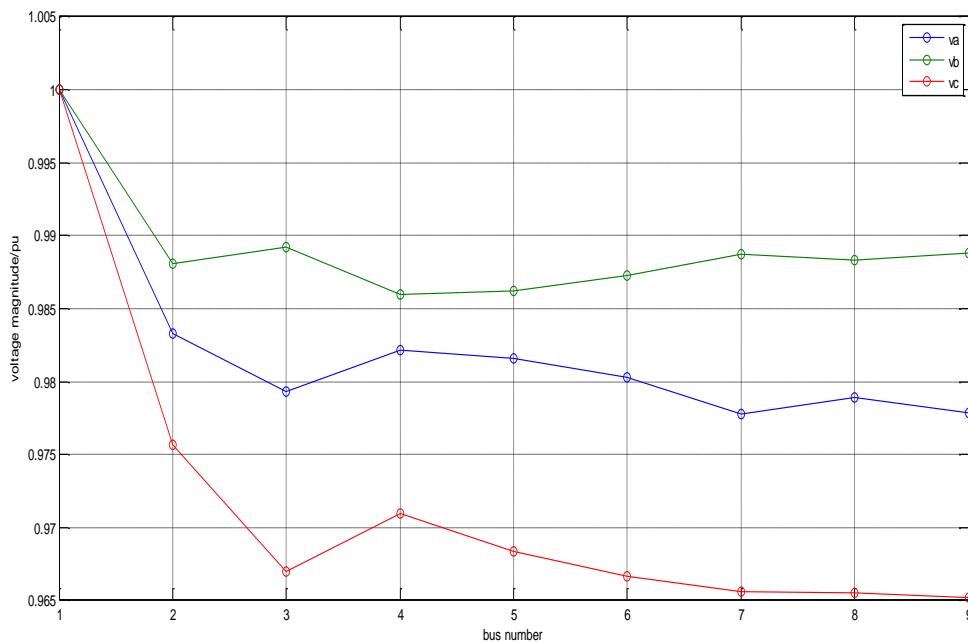


Fig.7 The voltage profile of the Iraqi 9-bus system with unbalance loading

Table 1 base case solution for the Iraqi 9-bus distribution system

Bus Number	$\delta_a$ (rad)	$\delta_b$ (rad)	$\delta_c$ (rad)	$V_a$ p.u	$V_b$ p.u	$V_c$ p.u
1	0	-2.094	2.094	1	1	1
2	-0.13	-2.22	1.9791	0.9833	0.9881	0.9757
3	-0.20	-2.29	1.9279	0.9793	0.9892	0.9670
4	-0.18	-2.28	1.9294	0.982	0.9860	0.9710
5	-0.21	-2.32	1.898	0.9816	0.9862	0.9684
6	-0.23	-2.33	1.885	0.9803	0.9873	0.9667
7	-0.23	-2.33	1.892	0.9778	0.9887	0.9656
8	-0.23	-2.35	1.891	0.9789	0.9883	0.9655
9	-0.255	-2.35	1.87	0.977	0.988	0.9652

The occurrence of voltage instability is very dependent on the maximum load at a particular bus. It is essential to know, how much a particular bus can be loaded and remain far away from the voltage collapse point. Figure 8 shows the produced P-|V| curves for the phase voltages at bus 9. It is evident from Fig.8 that the max loadability point is 0.42. This is the critical point at which the system may be prone to voltage collapse. Fig. 9 shows the family of the P-V curves for bus 4 and bus 7. The similarity of the behavior of the P-V curve for different buses is due to the assumption that the branch distribution are identical. If the load at bus 9 has been increased by 20%, the maximum loadability point of a system can be obtained by multiplying the value of load factor( $\lambda$ ) at the critical point times the increased load in the system added to the base load.

The initial load at bus 9=6512W. The final maximum loadability point =  $6512+0.42*6512=9247.04$ W

The maximum load at bus 9 can be allowed to pick up an addition load of 42% of the base load.

From the PV curves obtained previously, It can be drawn that the voltage collapse would first occur at the weakest phase of system. The P-V curve of weak phase have tracing clockwise direction.

The P-V curves of other phases may be tracing anti-clock direction which indicate that the lower half part is stable and the upper half part is unstable. The weakest phase is the phase which is heavily loaded or the phase whose voltage vary at a high rate of change with respect load parameter ( $\lambda$ ). The results for 3-ph 4-wire continuation power flow has been verified by solving the system with the conventional 3-ph power flow with the same loading increase as in continuation power flow. The values of voltages matched in both the cases. The determination of

critical point will help the operator to decide the loading of a system without stressing the system to its maximum limit. The value of  $\lambda$  obtained at the critical point times the increased load in a system with the addition of base loading gives the maximum loading point of a system. The curve for a balanced case is similar for all three phases while, it is different in case of unbalanced systems like case A and case B. In order to accelerate the tracing of P-|V| curve an adaptive predicted step size technique is used in the proposed algorithm. Another important note is that the maximum loadability limit is less than the thermal limit in distribution network. The thermal limit for the branches of the 9 bus distribution system in Fig.6 is found to be 52kVA but the maximum loadability point is 44kVA .

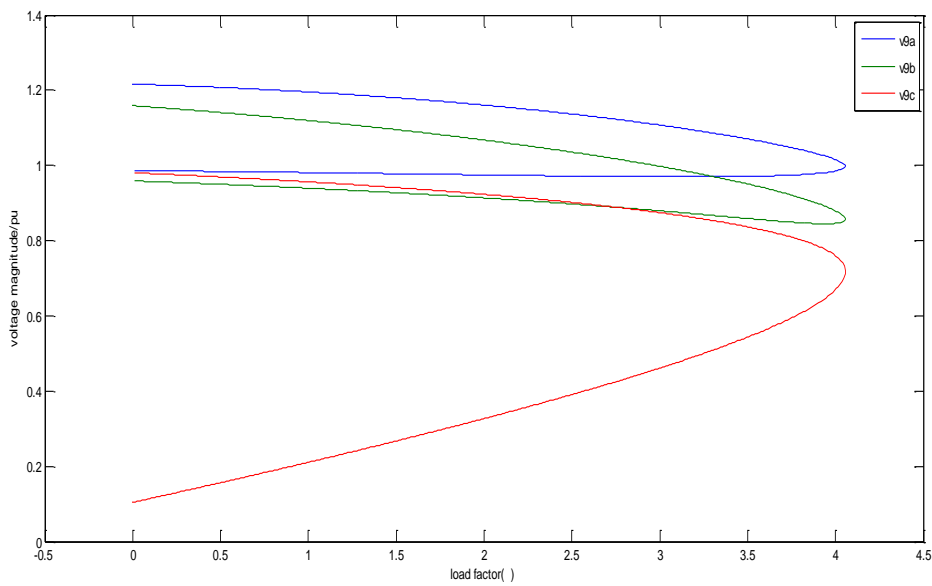


Fig.8 P-|V| curves for bus 9 of the 9-bus system

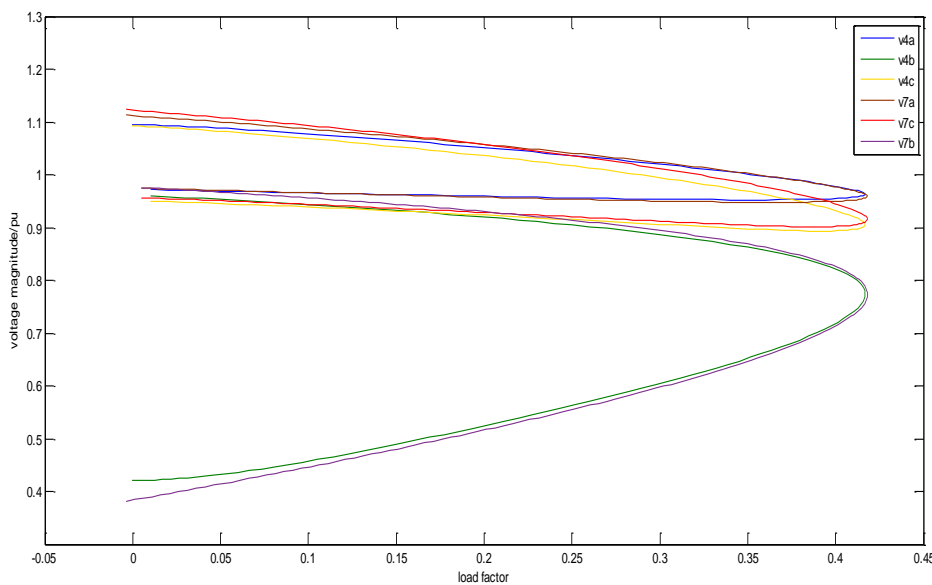


Fig.9 P-|V| curves for buses 4 and 7 of the 9bus system

## 5. Conclusions

A proposed algorithm to generate the P-|V| curves for an unbalanced three phase four wire distribution system was presented. The loci of the phase voltage and the neutral voltage for a wide range of loading conditions were traced. The important feature of the proposed algorithm was the identification of the maximum loadability point which is difficult to capture by conventional power flow analysis. The proposed continuation technique was applied to a typical Iraqi radial distribution system. The results obtained reveal the importance of the proposed technique in exploring all the possible solutions and obtaining the maximum loadability point. Also, the results reveal that even with distribution voltage level, the voltage stability limit impose an important constraint in the determining the power capability of the distribution feeder.

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### Appendix – 1

The line data and the bus data of the two bus system  
Bus admittance matrix of the two bus

$$Y = \begin{bmatrix} 4.02 - 26.32i & -0.394 + 5.441i & -2.072 + 7.314i \\ -0.394 + 5.441i & 2.791 - 23.57i & -0.952 + 6.525i \\ -2.0723 + 7.3142i & -0.9522 + 6.5255i & 4.48821 - 25.6987i \end{bmatrix}$$

### Appendix – 2

The self and mutual impedance of each branch of the Iraqi 9 bus distribution system

$$Z = \begin{bmatrix} 0.0190 + 0.0145i & 0.0021 + 0.0096i & 0.0021 + 0.0087i \\ 0.0021 + 0.0096i & 0.0190 + 0.0145i & 0.0021 + 0.0096i \\ 0.0021 + 0.0087i & 0.0021 + 0.0096i & 0.0190 + 0.0145i \end{bmatrix}$$

The bus data of the Iraqi 9 bus distribution system on a common base of 250 kVA

BUS	$S_a$	$S_b$	$S_c$
1	-	-	-
2	0.0176	0.0158	0.0088
3	0.01056	0.01144	0.0158
4	0.00704	0.01936	0.01144
5	0.01848	0.01056	0.01144
6	0.01408	0.0132	0.00704
7	0.0158	0.0158	0.01056
8	0.01232	0.01056	0.01408
9	0.01936	0.02024	0.02112