A New Image Inpainting Model's Code for Restoring Missing Regions of Digital Images Using Different Fractional Values

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Doi:10.29072/basjs.20230203

ARTICLE INFO

Keywords

Image inpainting. Isotropic equation. PDE-based inpainting method. Code of diffusion.

ABSTRACT

A modified isotropic model code was suggested for restoring missing areas in a digital image. The code was utilized on a set of RGB images with varying fractional values of mathematical coefficients. The researcher conducted tests using a mask produced based on the original clear images. Statistical quality metrics including MSE, PSNR, SSIM, and entropy are used to assess the quality of image inpainting. This method outperformed the current state-of-the-art PDEs inpainting methods.

Received 23 Jan 2023; Received in revised form 27 Mar 2023; Accepted 15 May 2023, Published 31 Aug 2023

1.Introduction

A great number of other modes of communication become obsolete by the image, which serves as a medium for expression and communication. On the other hand, there are instances in which certain information is missing from the image for a variety of reasons, including problems with storage and the effects of the weather [1]. Image inpainting is a technique that seeks to restore the damaged section of an image in order to rebuild it and provide a high-quality semantic approximation of the original image. This is accomplished by repairing the damaged part of the image. In the field of art, the concept of picture inpainting was developed. In the field of mathematics, the term "inpainting" refers to the process of filling in damaged regions by propagating information from their surrounding areas in the image [2]. Inpainting of ruined paintings was done by expert artists.

In the beginning, Masnou and Morel (1998) made some adjustments to the diffusion-based technique, which is characterized by the level lines-based disocclusion that generates a result that is marked by severe discontinuities [3]. An iterative approach that was based on PDEs and utilized the diffusion equation was presented by Bertalmio and colleagues in the year 2000. Using the isophote process, which permits missing spaces to be filled in concurrently in any direction without defining the area to be painted, this method was inspired by the procedures that museum craftsmen [4] utilize. These craftsmen automatically distribute pixels using the isophote process. In order to accomplish this, it is possible to incorporate a smoothness estimator into the computing process, and the information that is diffused is located along the isophote. In addition, Oliveira et al(2001) [5] offer a convolution-based algorithm for rapid image inpainting. In their approach, the to-be-inpainted areas are periodically convolved with a predefined diffusion mask. This model matches isotropic diffusion closely.

This method considers the center weight of the diffusion mask to be zero since the original image pixel to which it corresponds is unknown. The TV model was extended to the CDD (Curvature Driven Diffusion) model in 2001[6] [7] which included the isophote curvature information to handle curved structures more effectively. Then, Telea (2004) [8] developed a fast marching approach in which, in this category, image contents diffuse smoothly from boundary areas towards the interior of the missing area, however this model is more time-consuming than previously published models and has certain problems when applied to a wide area. In the year of

2007, Charles K. Chui and J. Wang investigated the total energy function in the wavelet domain [9]. Hadhoud et al. (2008) suggested a modification to the Oliveira approach that would shorten the amount of time required for implementation [10]. The algorithms that are based on convolution are quick, but they do not function well in images that have damaged edges and a high contrast. As a result of the modification of the convolution stage, this approach alters the Oliveira algorithm in order to reduce the amount of time spent inpainting and to enhance the overall quality of the results. This innovative method for digital inpainting is based on a mixture of wavelet decomposition that was developed. [11] Based on the work of C.H. Hagiwara etc. I would want to suggest surface-based inpainting as well as texture generation. The authors of [12] proposed a convolution-based inpainting method that utilised an adaptive kernel to enhance the processing of edge regions. This was accomplished by utilising the gradient of known pixels in the region of the missing pixel to compute weights in the convolving mask. It is necessary for the contribution of pixels that are next to the edges to be lower than that of pixels that are located in smooth areas since the gradient values in the edge areas are substantially higher. In accordance with the zero or small weights of the neighbours of missing pixels that have strong local gradients, edges will be conserved and preserved more effectively.

Consequently, the method can estimate the missing pixels while keeping the image's sharp edges. While performing convolution on images with damage using two mask functions. In 2011, H. Noori et al. [13] presented bilateral filtering, a convolution-based approach using an edge-preserving smoothing filter. By applying two Gaussian filters to a localized pixel neighborhood, one in the intensity domain and the other in the spatial domain that retains sharp edges, the bilateral filter replaces a pixel's value with the average weight of its neighbors in both space and range. The mask functions are useful for repairing the damaged regions. Chan, Raymond H, and others suggest in [14] an Alternating Direction Method for Image Inpainting (2011). Consequently, information appearing on a local pixel on an image contour diffuses the smoothness along the direction of the contour and not across borders, addressing the shortcomings of the most recent approaches by Bertalmio et al.[4] [3]. Smoothness will be deteriorated in both directions by pixels situated on uniform surfaces. Consequently, the integration of PDEs and isophotes results in an inpaint image with a structure of continuous progression. When compared to the sluggish construction of structures in border zones, these solutions are more expensive. For the inpainting work in 2019, Sridevi and Kumar suggested employing discrete Fourier transform (DFT) with fractional-order derivatives.[15].

The method employs a fractional-order nonlinear diffusion model that is difference curvature driven by Chen et al. [10] to manage gap areas and a fractional-order variational model to eliminate image noise and blurring. The edges of the image can be retained using this method during the process of image restoration. On the other hand, this method is dependent on human interaction for the manual selection of fractional orders, which may lead to a performance that is below par in areas that have been painted.

The diffusion model was used as the basis for the modification of the model code that was proposed in this paper. It is intended to improve the results that were achieved from the code of the diffusion model, which are characterized by blurring and a lack of clarity of details in the area that is absent, and the edges do not show. especially in cases where the areas are quite large. The numerical coefficients that were selected were those that had fractional values, as well as negative and positive values. When it came to repairing edges and displaying amazing details, the model code that was proposed produced better results.

2. Mathematical Diffusion Model

Diffusion-based inpainting is the first model was used to recover the image missing area by spreading information from the surrounding area of the missing region at the pixel point [14, 15]. It is possible to express the diffusion equation, which was referred to in[6] as:

$$\begin{cases} u_t = \Delta u, \quad t \geq 0 \\ u(0,x,y) = 0. \end{cases} \quad \ldots\ldots..1$$

3. Mathematical Modified diffusion Model

It is necessary to incorporate a number of algebraic expressions into the isotropic diffusion model in order to enhance the edge of restoration and reduce the blurring of the large missing area. In order to complete the modified isotropic model (1) [6], the first derivative of the image's missing region is included. The capability of the first derivative to extend the isophote line and construct edges contributes to the improvement of the recovery of sections that need to be recovered. The modified isotropic equation can be expressed using the formula that is presented below:

$$u_t = \Delta u + \alpha u_x + \beta u_y \,, \quad t \geq 0 \quad \ldots 2$$

Where u_x and u_y are the first derivatives of the image, the α and β are coefficients. Furthermore, the value of α and β play an important effect on the values of quality measures.

4.Code of Isotropic Model

In order to fill in the damaged and missing portions in the image, it is possible to describe the code that was used in the process of conducting the tests by making use of the isotropic model diffusion equation. With the understanding that the missing area can be filled in when T(i,j) represents the missing area and (i,j) is the position of the pixel, this is done for the purpose of achieving the desired result. By utilizing the diffusion model, the utilization of the information that is positioned in close proximity to the damaged area, and the utilization of the diffusion mechanism, the information is utilized to spread and fill the gap in the digital image. This is accomplished by the utilization of the diffusion model.

```
[T11,fg] = improv(T,m,n,step,TOL,alpha,R1,R2,H2,H1)

  n =50;m=50;
step= 10;
x = linspace(0,step,m);
y=linspace(0,step,n);
dy=1;
 dx=1;
TOL = 1e-2;
 T = zeros(n);
 T(1,1:n) = R1; %TOP
 T(m,1:n) = R2;  %BOTTOM
 T(1:m,1) = H1;  %LEFT
 T(1:m,n) = H2;  %RIGHT
alpha=2;
 dt = 1/(2*alpha*(1/dx^2+1/dy^2));
 error = 1; k = 0;
while error > TOL
    k = k+1;

    Told = T;
    for i = 2:m-1
        for j = 2:n-1

T(i,j)=dt*((Told(i+1,j)-2*Told(i,j)+Told(i-1,j))/dx^2+
(Told(i,j+1)- 2*Told(i,j)+Told(i,j-1))/dy^2)  + Told(i,j);




        end
    end
```

```
            error = max(max(abs(Told-T)));
     fg(k,:)=error;
end
T11=T;
end
```

In the experiments performed, the missing regions were represented (square-shaped) as a square matrix of degree NXN. The matrix elements represent the missing pixels, and using the finite difference method these elements are found.

5. Code of Modified Isotropic Model

When T1(i,j) is represent missing area and (i,j)is a position of pixel then the missing area can be founded by the **modified** diffusion model

```
[T11,fg] = improv(T,m,n,step,TOL,alpha,R1,R2,H2,H1)

  n =50;m=50;
step= 10;
x = linspace(0,step,m);
y=linspace(0,step,n);
dy=1;
 dx=1;
TOL = 1e-2;
 T = zeros(n);
 T(1,1:n) = R1; %TOP
 T(m,1:n) = R2;  %BOTTOM
 T(1:m,1) = H1;  %LEFT
 T(1:m,n) = H2;  %RIGHT
alpha=2;
 dt = 1/(2*alpha*(1/dx^2+1/dy^2));
 error = 1; k = 0;
while error > TOL
```

```
    k = k+1;
    Told = T;
    for i = 2:m-1
        for j = 2:n-1
Tx(i,j)=dt*((Told(i+1,j)-Told(i-1,j))/dx);
Ty(i,j)=dt*((Told(i,j+1)-Told(i,j-1))/dy);% DRIVETIV OF IMAGE

T1(i,j)=dt*((Told(i+1,j)-2*Told(i,j)+Told(i-1,j))/dx^2+
(Told(i,j+1)- 2*Told(i,j)+Told(i,j-1))/dy^2)  + Told(i,j);
 T(i,j)=T1(i,j)+ ALFAA *Tx(i,j)- BETA*Ty(i,j);

    % T(i,j)=T1(i,j);% diffusion code
        end
    end

    error = max(max(abs(Told-T)));
    fg(k,:)=error;
end
T11=T;
end
```

6. Code to find quality assessment

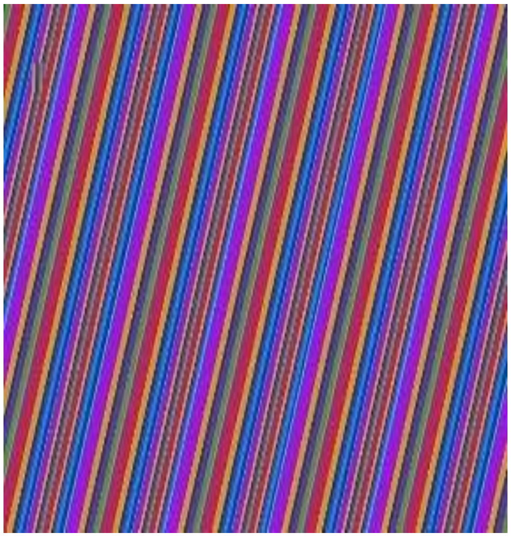
```
Mean square error (MSE)
MSE1 =sum(sum((S-J3).^2))/(m*n);
MSE2 =sum(sum((Ss-Js3).^2))/(m*n);
MSE3= sum(sum((Sss-Jss3).^2)) /(m*n);
Av_M=(MSE1+MSE2+MSE3)/3;
%%
Peak signal-to-noise ratio (PSNR)

PSNR1 = 10*log10(256*256/MSE1);

PSNR2 = 10*log10(256*256/MSE2);
PSNR3 = 10*log10(256*256/MSE3);
Av_P=(PSNR1+PSNR2+PSNR3)/3;
%%%
Structural similarity index measure (SSIM)
ss1 = ssim(S,J3) ;
ss2 = ssim(Ss,Js3) ;
ss3 = ssim(Sss,Jss3) ;
Av_SS=(ss1+ss2+ss3)/3;
%%%%
Entropy
ES1= entropy(J3);
ES2= entropy(Js3);
ES3= entropy(Jss3);
Av_E=(ES1+ES2+ES3)/3;
```

2.Experimental Results

For the purpose of evaluating the proposed change of the isotropic model code, a separate collection of ten natural images that were closed from the Berkeley Segmentation Dataset and Benchmarks 300 (BSDS300) was utilized [16]. Moreover, among the trials carried out were the selection of 10 distinct color stripes images from [17]. A comparison is made between the outcomes of the suggested method and the original isotropic method [6]. Moreover, virtual masks are utilized to create gaps in the image in order to assess the capability and efficacy of the proposed model through filling in the gaps. The suggested model code was tested via Matlab programming. To test how the α and β values affect the inpainted images. Selected a different value of α and β were studied. Which included fractional values such as 0.2, 0.8, 0.5, 0.75, 0.25 under the condition of $\alpha, \beta \in (0,1); \alpha + \beta \leq 1$ [6]. And studied the effect of the coefficient sign (negative or positive) on the results of filling in the missing space. Squares were the shape of the regions that were missing. During the testing procedure, virtual masks were utilized to manufacture gaps in the image. This was done in order to evaluate the capabilities and efficiency of the suggested model and determine whether or not it was successful in filling gaps. MATLAB was used to test the recommended model on striped images as well as random natural photos. This was done in order to put the software through its paces. When we examine Figure 1, we are able to observe the outcomes that occur when we combine the two terms u_x and u_y. Additionally, we can observe the outcomes that occur when we substitute alternative values (fractions) for α and β, while simultaneously altering the sign in every possible scenario where the two terms are either both positive or negative, or when one term is positive and the other term is negative. When the α values are 0.5 and - 0.5, and the β values are 0.5 and - 0.5, as depicted in Figure 1, the following occurs. When the α values are 0.5, -0.8, and 0.8, along with the β values being 0.5 and 0.2, Figure 2 illustrates the results. This figure illustrates the situations in which the α values are 0.5, -0.8, and 0.8, while the β values are 0.5 and 0.2.

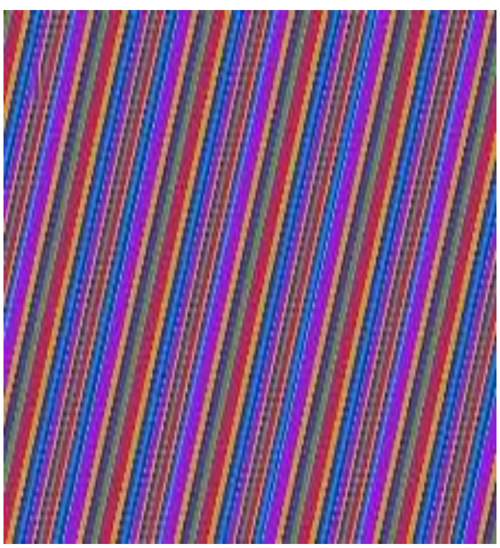


$\alpha = 0.5$

$\beta = -0.5$

$\alpha = -0.5$

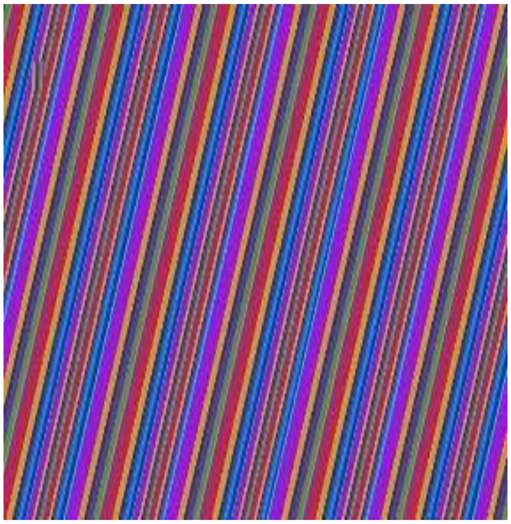
$\beta = 0.5$



$\alpha = -0.5$

$\beta = -0.5$

Fig .1: Effect of α and β values on color stripes images inpainting



$\alpha = 0.5$

$\beta = 0.5$

$\alpha = 0.8$

$\beta = 0.2$

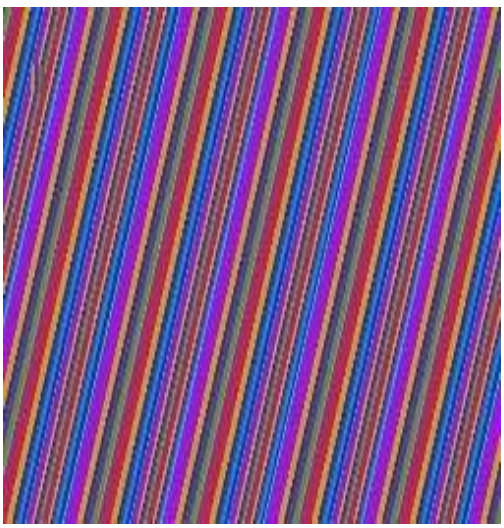
$\alpha = -0.8$

$\beta = 0.2$

Fig .2: Effect of α and β values on color stripes images inpainting

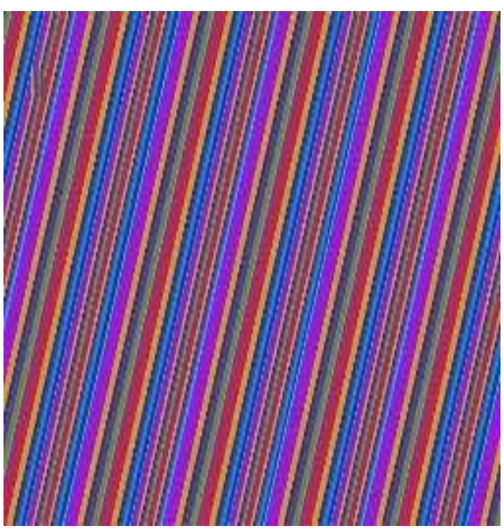
$\alpha = 0.8$

$\beta = -0.2$



$\alpha = -0.8$

$\beta = -0.2$



$\alpha = -0.2$

$\beta = 0.8$

Fig .3: Effect of α and β values on color stripes images inpainting

Table 1 includes the values of quality measures (MSE, PSNR, SSIM and entropy) for 10 color tape images. And time taken for implementation is measured in seconds.

Table 1: Quality assessment and time

	MSE	PSNR	SSIM	Entropy	Time
$\alpha = o.5$, $\beta = -0.5$	48.397	32.845	0.337	6.075	1.998
$\alpha = -o.5$, $\beta = 0.5$	55.974	31.093	0.397	5.435	2.1134
$\alpha = -o.5$, $\beta = -0.5$	60.529	30.737	0.380	5.411	2.387
$\alpha = o.5$, $\beta = 0.5$	52.948	31.556	0.406	5.2851	2.002
$\alpha = o.8$, $\beta = 0.2$	51.172	32.085	0.496	5.380	2.001
$\alpha = -o.8$, $\beta = 0.2$	52.274	31.597	0.405	5.2536	2.002
$\alpha = o.8$, $\beta = -0.2$	55.141	30.887	0.384	5.196	2.561
$\alpha = -o.8$, $\beta = -0.2$	57.349	31.008	0.385	5.245	2.994
$\alpha = o.2$, $\beta = 0.8$	48.265	32.018	0.407	5.204	1.899

Conclusions

In most cases, the modified properties model produces satisfactory outcomes when the portions that are missing are relatively modest. However, the efficiency with which the equation is redrawn is improved when the area is smaller. Both the inclination angle of the line and the values of the coefficients multiplied by the image derivative are factors that determine whether or not diagonal lines may be recovered during the process. It was possible for the modified model to partially restore edges, provide better clarity, and reduce blur in the area that was missing. Additionally, it was able to reveal more details than when the traditional isotropic model was used. The redesigned model is effective and continues to develop. The consequence of this is that the region surrounding the lost area will have a lower amount of tissue. The statistical findings of the quality metrics MSE, SSIM, PSNR, and Entropy provide sufficient evidence to support this assertion. There is a correlation between the point at which information travels from top to bottom, right to left, or vice versa and the presence of both positive and negative signals.

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كود لنموذج جديد لطلاء الصور لاستعادة المناطق المحذوفة من الصور الرقمية

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المستخلص

تم اقتراح كود نموذج معدل الخواص لاستعادة المناطق المفقودة من صورة رقمية، وتم تطبيق كود النموذج على مجموعة من صور RGB التي تم فيها اختيار قيم كسرية مختلفة للمعاملات الرياضية. تم إجراء الاختبارات باستخدام القناع الذي أنشأه الباحث بناءً على الصور الأصلية (الواضحة). من أجل تقييم جودة رسم الصورة، يتم استخدام مقاييس الجودة الإحصائية مثل MSE (متوسط الخطأ المربع)، PSNR (نسبة الإشارة إلى الضوضاء)، SSIM (مؤشر التشابه الهيكلي) والإنتروبيا. كان أداء هذه الطريقة متفوقًا على أحدث تقنيات PDEs (المعادلات التفاضلية الجزئية) في طرقالطلاء الداخلي .