

# **CUSUM Control Chart for Symlets Wavelet to Monitor Production Process Quality**

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DOI: [10.33899/iqjoss.2024.185240](https://stats.uomosul.edu.iq/article_185240.html), ©Authors, 2024, College of Computer Science and Mathematics, University of Mosul. This is an open access article under the CC BY 4.0 license [\(http://creativecommons.org/licenses/by/4.0/\)](http://creativecommons.org/licenses/by/4.0/).

#### **1. Introduction**

In today's competitive manufacturing landscape, ensuring consistent product quality is essential for maintaining competitiveness and meeting customer expectations. Traditional quality control methods, including the Cumulative Sum (CUSUM) chart, are valuable tools for identifying process deviations. However, in phase I of increasingly complex and variable production settings, these techniques may struggle to detect subtle aberrations or noise in the data [1]. CUSUM charts are particularly effective for detecting small shifts in process means that may not be easily detectable using traditional control charts like the Shewhart control chart [2]. There are different variations of CUSUM charts, including one-sided CUSUM and two-sided CUSUM, depending on whether you are interested in detecting shifts in one direction or both directions. Additionally, parameters such as the reference value, decision intervals, and smoothing constants need to be determined when constructing a CUSUM chart [3]. By using the tabular method, you can track the cumulative sums and make decisions based on the values directly from the table, rather than relying on graphical interpretation from a chart. This can be particularly useful in situations where a visual chart is not practical or when you prefer a more straightforward method of analysis [4]. De-noising data involves removing or reducing unwanted noise or disturbances from a dataset while retaining meaningful information [5]. The choice of de-noising

method depends on the characteristics of the data and the nature of the noise present. It's often beneficial to experiment with different methods and parameters to find the most suitable approach for a particular dataset [6]. A possible approach to overcoming these obstacles is wavelet analysis. Wavelet analysis is a powerful technique that breaks down signals into their constituent frequency components to extract meaningful information while removing noise [7]. Symlets are unique among wavelet families in that they can balance frequency localization and vanishing moments, which makes them perfect for applications involving feature extraction and signal denoising [8]. The CUSUM control chart and Symlets wavelet analysis are used in this article to provide a novel way to improve the monitoring of production process quality [9]. We want to get around the drawbacks of traditional quality control procedures by utilizing state-of-the-art signal processing techniques. We employ Symlets wavelets, specifically Symlets 1, 2, and 3, to effectively reduce noise in production data and retrieve valuable information while maintaining essential attributes necessary for assessing quality. The suggested method integrates Symlets wavelets (orders 1, 2, and 3) with wavelet shrinkage to obtain de-noised data. Noise is efficiently eliminated using a hard threshold rule in conjunction with discrete wavelet treatment [10]. The Tabular technique is then used to create a CUSUM chart utilizing the de-noised data that have been produced. We anticipate substantial improvements in monitoring production process quality efficiency through this integrated approach. By promptly identifying deviations and implementing proactive quality control measures, we aim to enhance product quality and improve the costeffectiveness and efficiency of manufacturing processes.

### **2. Quality Control Charts**

Quality control charts, vital in manufacturing and process industries, monitor process variability over time. They distinguish between common cause and special cause variations, aiding timely intervention [11]. Simple and intuitive, they plot sample statistics against control limits. Charts like X-bar, R-chart, and p-chart are commonly used. They aid process improvement by revealing trends and patterns. However, they may struggle with subtle deviations and rely on assumptions like normality. Enhancements include advanced statistical techniques and computational tools. In subsequent sections, we explore the Cumulative Sum (CUSUM) control chart and its integration with wavelet analysis for improved process monitoring and quality assurance [12].

## **2.1. CUSUM Control Chart**

CUSUM Control Charts are a powerful tool for detecting subtle shifts in process performance, offering greater sensitivity compared to traditional control charts. Introduced by E.S. Page in 1954, CUSUM charts track the cumulative sum of deviations from a target value over time. They are more effective at identifying changes in process parameters quickly and can detect minor deviations that may be missed by other methods [13]. The flexibility of CUSUM charts extends to various process types and data distributions, making them resilient and customizable. Different versions, such as one-sided, two-sided, and exponentially weighted moving average (EWMA) CUSUM charts, cater to different needs. Advanced versions like multivariate and adaptive CUSUM charts have also been developed for more complex scenarios [14]. Overall, CUSUM control charts enhance proactive process monitoring and control, ultimately leading to improved product quality and operational efficiency. While their visual approach, known as the V-Mask method, aids in understanding process behaviour, the Tabular method provides a simpler alternative [15].

#### **2.2. The Tabular method**

Instead of presenting the cumulative sums of deviations from a target value on a graph, the Tabular approach for CUSUM control charts computes and records them in a table format. For individuals who prefer a more direct approach to data interpretation, this method offers a more basic method for numerically analysing process data. Here are the steps to make a CUSUM chart [16]. Estimate the standard deviation of the data from the moving range control chart  $\sigma = \overline{R}/d_2$ . For univariate (r = 2),  $d_2 = 1.128$ 

- Calculate the reference value or allowable slack since the CUSUM chart monitors the small shifts. Generally, 0.5 to 1 sigma will be considered.  $k = 0.5 \sigma$ .
- Compute decision interval H, generally  $\pm 4 \sigma$  will be considered (some place  $\pm 5 \sigma$  also be used).
- Calculate the upper and lower CUSUM values for each individual i value.
- Upper CUSUM  $(UC_i) = Max [0, UC_{i-1}+x_i-Target value-k)$ .
- Lower CUSUM  $(LC_i) = Min [0, LC_{i-1}+x_i-Target value+k].$
- Draw all UC<sub>i</sub> & LC<sub>i</sub> values in the graph and also draw decision intervals (UCL and LCL).
- Check if any of the UCi values are above the UCL and any of the LCi values are below the LCL.
- Finally, take necessary action to eliminate the special causes if any of the points are out of control limits.

To find the plotting position for the new point in a cumulative sum chart, you can use the following formula.

$$
S_n = k + \frac{1}{\sigma_x} \sum_{i=1}^n (x_i - T) \tag{1}
$$

 $S_n$  is the plotting position, *k* is the reference value or the center line of the cumulative sum chart (if provided),  $x_i$ represents the new point you want to plot, *T* is the Target value and  $\sigma_x$  is the Standard Deviation.

#### **3. Wavelet Shrinkage**

Wavelet shrinkage is a technique used for signal denoising based on wavelet transforms. It involves shrinking or thresholding the wavelet coefficients to remove noise while preserving the essential features of the signal [17]. However, wavelet shrinkage also has limitations, such as the need to select appropriate thresholding parameters and the potential for introducing artefacts if the thresholding is not performed carefully. Additionally, wavelet shrinkage may not perform well for signals with complex noise structures or non-Gaussian noise [18].

#### **3.1 Symlets Wavelet**

Symlets (short for Symmetric Least Asymmetric) wavelet is a family of wavelets used in signal processing and data compression. They are similar to Daubechies wavelets but have more symmetry and fewer vanishing moments [19]. The Symlets (Sym) are variations on the (db) family of wavelets developed by Daubechies that are approximately symmetrical, orthogonal, and biorthogonal. The two wavelet families' characteristics are the same. Sym N has N as its order [20].

#### **3.2 Universal Thresholding**

The universal threshold approach provided by  $[21]$  is shown by the following formula.

$$
\delta^U = \hat{\sigma}_{MAD} \sqrt{2 \log \log (n)} \tag{2}
$$

Where  $\hat{\sigma}_{MAD}$  is the standard deviation estimator of details coefficients, and is equal to  $\frac{MAD}{0.6745}$ . The wavelet coefficients' median absolute deviation at the finest scale is known as MAD [22].

#### **3.3 Hard Threshold Rule**

Hard thresholding, a straightforward approach for thresholding and evaluating the proposition of (keep or kill), was proposed by Donoho and Johnstone. Wavelet de-noising was implemented by a simple approach called Hard Thresholding [23]. The wavelet coefficient is set to the vector  $Wn^{(H)}$  with element.

$$
Wn^{(H)} = \begin{bmatrix} 0 & \quad \text{if} \quad |Wn| \le \delta \\ Wn & \quad \text{if} \quad |Wn| > \delta \end{bmatrix} \tag{3}
$$

Exceeding  $\delta$  are left untouched, while smaller than or equal to  $\delta$  are eliminated or set to 0. Thus, the operation of hard thresholding is not continuous mapping.

#### **4. Proposed Chart**

To address data noise and configure the proposed chart, the following steps can be performed:

- Transforming data using Symlets wavelets:

Symlets wavelets with different orders (1, 2, and 3) transform the original data to discrete wavelet transformation coefficients.

- Estimating the threshold level using the Universal method: The Universal method is used to determine the threshold level to be used in the rasterization process [24].
- Application of hard rule:

A hard threshold rule is applied to the maximum overlap discrete wavelet transformation coefficients to refine denoise data.

- De-Noise data setting: After cleaning the data using a hard threshold rule, de-noise data can be used in creating the proposed chart.
- Create a CUSUM chart using the Tabular method:

The proposed chart employs wavelet shrinkage in data processing to obtain de-noise data through the use of Symlets wavelet with orders of (1, 2, and 3) to obtain maximum overlap discrete wavelet transformation coefficients, through which the threshold parameter is estimated using the universal method and then applied hard threshold rule to obtain de-noise data which will be relied upon in constructing the CUSUM chart using the Tabular method.

## **5. Application aspect**

To compare the classical and the proposed charts in terms of efficiency and accuracy of the estimated parameters (the average moving range (AMR) and  $\sigma$ ) and the difference between upper and lower control limits, in addition to calculating the number of points outside the limits of control to determine the sensitivity of the proposed charts to minor changes that may occur in the production process, the simulation study was done by simulating the CUSUM quality control chart, then the application for the real data. By designing a program in MATLAB (version 2022a) dedicated to this purpose (Appendix).

#### **5.1. Simulation study**

To generate data for the CUSUM chart, the standard uniform distribution function was used plus linearly equally spaced points between (0 and 0.5) and (0 and 1) for different samples (m= 20 and 25) observations. The data generation was also repeated (10,000) times, through which the classical CUSUM charts were formed using the Tabular method  $(r = 2, d_2 = 1.128, k = 0.5)$  and then the proposed charts were formed based on Symlets wavelets of orders  $(1, 2 \text{ and } 3)$ with Universal de-noising method and hard threshold rule. The average of the quality control limits (lower and upper limits), the average moving range (AMR), the difference between the control limits (Difference), the standard deviation (Sigma) and the number of points outside the control limits are summarised in Tables (1-4).

Chart	LCL	UCL	<b>AMR</b>	<b>Difference</b>	Sigma	No. of Point
Classical	$-1.1843$	1843.	0.3340	2.3687	0.2961	0.4843
Svm1	$-0.4394$	0.4394	0.1239	0.8788	0.1098	5.2791
Svm2	$-0.5666$	0.5666	0.1598	1.1332	0.1417	3.6841
Svm3	$-0.5938$	0.5938	0.1674	1.1875	0.1484	3.4662

**Table 1. Average simulation results at m = 20 and interval (0, 0.5)**









**Table 4. Average simulation results at m = 25 and interval (0, 1)**



Table .1-4 shows that all the proposed charts were better than the classical chart, depending on the difference, Sigma, which has lower values than its counterpart in the classical method, in addition, the average of points outside the control limits was greater than its counterparts, which indicates the sensitivity of the proposed chart in detecting minor changes that may occur in the production process. The first proposed chart (Sym1) was the best compared to the rest of the proposed charts for all simulation cases. Increasing the number of observations (from 20 to 25) and the interval  $((0, 0.5)$  to  $(0, 1)$ ) leads to an increase in the difference and Sigma as well as an increase in the number of points falling outside the control limits.

## **5.2. Real data**

CUSUM chart will be used in the control phase of DMAIC (Define, Measure, Analyze, Design, and Verify). DMAIC is a data-driven quality strategy used to improve processes. It is an integral part of a Six Sigma initiative, in general, it can be implemented as a standalone quality improvement procedure or as part of other process improvement initiatives. In a drug manufacturing unit, potassium content is one of the important parameters. The target potassium content is  $0.21\%$  in weight (wt) [25]. Therefore, the team has collected 15 batches from the production in a time interval to monitor the process mean shift. are shown in Table 5.





The original data for potassium 0.21% in wt and then the de-noise data using Symlets wavelets of orders (1, 2 and 3) with Universal de-noising method and hard threshold rule were plotted in Figure 1.



## **Figure 1. Real data and de-noise data**

The quality control limits (lower and upper limits), the average moving range (AMR), the difference between the control limits (Difference), the standard deviation (Sigma) and the number of points outside the control limits for the classical and proposed method are summarised in Table 6.



## **Table 6. Results of real data**

Table 6 shows that all the proposed charts were better than the classical chart, depending on the difference, Sigma, which has lower values than its counterpart in the classical method, in addition, the average of points outside the control limits was greater than its counterparts, which indicates the sensitivity of the proposed chart in detecting minor changes that may occur in the production process. The first proposed chart (Sym1) was the best compared to the rest of the proposed charts for real data. The points and quality control limits on the classical and proposed (Sym1) charts are plotted in Figures 2 and 3.



**Figure 2. Classical CUSUM Chart for Real Data**





It is evident from Figure 2 (classical chart) that from sample 13 onwards the process is going out of control (3 points) while from Figure 3 (proposed (Sym1) chart) from sample 12 onwards the process is going out of control (4 points). Hence, the proposed (Sym1) chart has more sensitivity than the classical chart in detecting minor changes that may occur in the production process.

#### **6. Conclusion & Recommendations**

Through the study of simulation and real data, the following main conclusions and recommendations were summarized:

#### **6.1. Conclusions**

- 1. All the proposed (Sym1, 2, and 3) charts were better than the classical chart for the CUSUM chart for all simulation cases and real data.
- 2. The proposed charts have more sensitivity than the classical CUSUM chart in detecting minor changes that may occur in the production process.

3. The first proposed chart (Sym1) was the best compared to the rest of the proposed charts for all simulation cases and real data.

## **6.2. Recommendations**

- 1. Using the proposed chart for Quality Control of the CUSUM.
- 2. Conducting a prospective study on the use of the Symlets Wavelet with a multivariate CUSUM chart.
- 3. Conducting a prospective study on the use of the other wavelets with a CUSUM chart.

## **Conflict of interest**

The author has no conflict of interest.

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## **Appendix**

clc, clear all, rng('default') for  $i=1:10000$  $m=25$ ; rnds = rand $(1,m)$ ; trnd = linspace $(0,.5,m)$ ; data = rnds+trnd; d1 = wdenoise(data,'Wavelet','sym1', 'DenoisingMethod','universal','ThresholdRule','hard'); data=d1;  $MR = abs(df(f(data)); AMR(j) = mean(MR); d2=1.128; Sigma(j) = AMR(j)/d2;$ K=.5\*Sigma(j); UCL(j)=4\*Sigma(j); LCL(j)=-4\*Sigma(j); T=mean(data); UCLx(1)=0;LCLx(1)=0; for  $i=2:m+1$  $UCLx(i)=max(0,UCLx(i-1)+data(i-1)-T-K);$   $LCLx(i)=min(0, LCLx(i-1)+data(i-1)-T+K);$ end  $UCLx(1)=[[:LCLx(1)=[]; UCLx; LCLx; NO=0;$ for  $i=1:m$ if  $UCLx(i) > UCL(j)$ ;  $NO=NO+1$ ; end end,  $NO1(j)=NO; D(j)= UCL(j)-LCL(j); end$ [mean(LCL) mean(UCL) mean(AMR) mean(D) mean(Sigma) mean(NO1)]

## **لوحة سيطرة المجموع المتراكم للمويجة سيملت لمراقبة جودة عملية اإلنتاج**

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**الخالصة :** في هذا البحث تم اقتراح إنشاء لوحة جديدة تمثل لوحة المويجات سيملت بالرتب ) ،1 ،2 3( للحصول على معامالت التحويل المتقطع للمويجات ذات الفائض العال، والتي من خلالها يتم تقدير معلمة قطع العتبة باستخدام الطريقة الشاملة ومن ثم طبقت قاعدة قطع العتبة الصلبة للحصول على بيانات ذات ضوضائية أقل والتي سيتم الاعتماد عليها في إنشاء لوحة المجموع التراكمي باستخدام الطريقة الجدولية. تم قياس ومقارنة كفاءة اللوحة المقترحة مع اللوحة التقليدية من خلال محاكاة عدة حالات وبيانات حقيقية وحساب الفرق بين حدود السيطرة والانحراف المعياري وعدد النقاط الواقعة خارج حدود السيطرة لتحديد حساسية اللوحة للتغييرات الطفيفة التي قد تحدث في عملية الإنتاج، وذلك باستخدام خوارزمية مصممة لهذا الغرض في برنامج ماتلاب. أظهرت نتائج البحث أن اللوحات المقترحة أكثر كفاءة وحساسية للتغيرات الطفيفة (التي قد تحدث) في عملية الإنتاج من اللوحات التقليدية. **الكلمات المفتاحية:** لوحة سيطرة المجموع المتراكم ، الطريقة الجدولية ، تقليص ضوضائية ال بيانات ، المويجة سيملت، التقليص المويجي.