On Artin cokernel of The Group $Q_{2p}\times C_5$ Where p> 2and p is prime number

حول النواة المشارك- آرتن للزمرة $Q_{2p}\times C_5$ عندما p>2 و p عدد اولي

Ass.prof.Naserr Rasool Mahmood
University of Kufa
Faculty of Education for Girls
Department of Mathematics
Naserr.mahmood@uokufa.edu.iq

Salah Hassoun jihadi
University of Kufa
Faculty of Education for Girls
Department of Mathematics
salah1978hassoun@gmail.com

Abstract:
The main purpose of This paper is to find Artin's character table $Ar(Q_{2p}\times C_5)$ when p>2 and p is a prime number.

المستخلص
الهدف الرئيسي من هذا البحث هو ايجادجدول شواخص ارتن ($Ar(Q_{2p}\times C_5$ عندما p>2 و p عدد اولي .

Introduction:

For a finite group G, The factor group $\overline{R}(G)/T(G)$ is called the Artin cokernel of G denoted by AC(G) and $\overline{R}(G)$ is denoted to the a belian group generated by Z-valued characters of G under the operation of pointwise addition, T(G) is a subgroup of $\overline{R}(G)$ which is generated by Artin's characters **.**

1-Preliminars : [1]

For each positive integer $m\geq 2$,The generalized Quaternion Group Q_{2m} of order 4m with two generators x and y satisfies $Q_{2m}=\{x^h y^k, 0\leq h\leq 2m-1, k=0,1\}$, Which has the following properties $\{x^{2m}=y^4=I, yx^m y^{-1}=x^{-m}\}$.

Let G be a finite group ,all the characters of group G induced from a principal character of cyclic subgroup of G are called Artin characters of G .

Artin characters of the finite group can be displayed in a table called Artin characters table of G which is denoted by Ar(G); The first row is Γ-conjugate classes ; The second row is The number of elements in each conjugate class, The third row is the size of the centralized $|C_G(CL_\alpha)|$ and other rows contains the values of Artin characters

Theorem:(1.2): [2]

The general form of Artin characters table of Cp^s when p is a prime number and s is a positive integer number is given by :-

$Ar(Cp^s)$=

Γ-classes	[1]	$[x^{p^{s-1}}]$	$[x^{p^{s-2}}]$	$[x^{p^{s-3}}]$	⋯	$[x]$
$\|CL_\alpha\|$	1	1	1	1	⋯	1
$\|C_{p^s}(CL_\alpha)\|$	p^s	p^s	p^s	p^s	⋯	p^s
φ'_1	p^s	0	0	0	⋯	0
φ'_2	p^{s-1}	p^{s-1}	0	0	⋯	0
φ'_3	p^{s-2}	p^{s-2}	p^{s-2}	0	⋯	0
⋮	⋮	⋮	⋮	⋮	⋮	
φ'_s	p^1	p^1	p^1	p^1	⋯	0
φ'_{s+1}	1	1	1	1	⋯	1

Table(1,1)

Example : (1.3)

We can write Artin characters table of the group C_p, p>2 and p is a prime number.

$Ar(C_p)$=

Γ- classes	[1]	[x]
$\|CL_\alpha\|$	1	1
$\|Cc_p(CL\alpha)\|$	p	p
φ'_1	p	0
φ'_2	1	1

Table(1,2)

Corollary : (1.4) :[2]

Let $m = p_1^{\alpha_1}.\, p_2^{\alpha_2} \ldots p_n^{\alpha_n}$ where g.c.d(p_i ,p_j) =1, if i≠j and p_i's are prime numbers, and α_i any positive integers for all $1 \le i \le n$, then :

$$\mathbf{Ar(C_m) = Ar(C_{p_1{}^{\alpha_1}}) \otimes Ar(C_{p_2{}^{\alpha_2}}) \otimes \ldots \otimes Ar(C_{p_n{}^{\alpha_n}})}.$$

Example :(1.5)

Consider the cyclic group C_{2p} .To find Artin characters table for it, we use corollary(1.4) as the following :

$Ar(C_{2p})=Ar(C_2) \otimes Ar(C_p)$=

Γ- classes	[1]	$[x^2]$	$[x^p]$	[x]
$\|CL_\alpha\|$	1	1	1	1
$\|Cc_{2p}(CL\alpha)\|$	2p	2p	2p	2p
φ'_1	2p	0	0	0
φ'_2	2	2	0	0
φ'_3	p	0	p	0
φ'_4	1	1	1	1

Table(1,4)

Example :(1.6)

Consider the cyclic group C_{10} .To find Artin characters table for it, we use corollary as the following :

$Ar(C_{10})=Ar(C_2) \otimes Ar(C_5)=$

Γ- classes	[1]	$[x^2]$	$[x^5]$	[x]
$\|CL_\alpha\|$	1	1	1	1
$\|C_{C_{10}}(CL_\alpha)\|$	10	10	10	10
φ_1'	10	0	0	0
φ_2'	2	2	0	0
φ_3'	5	0	5	0
φ_4'	1	1	1	1

Table(1,5)

Theorem(1.7):[1]

The Artin characters table of the Quaternion group Q_{2m} when m is an odd number is given as follows :

$Ar(Q_{2m})$ =

	Γ-Classes of C_{2m}								
Γ-Classes	χ^{2r}				χ^{2r+1}				$[y]$
$\|CL_\alpha\|$	1	2	…	2	1	2	…	2	2m
$\|C_{Q_{2m}}(CL_\alpha)\|$	4m	2m	…	2m	4m	2m	…	2m	2
Φ_1	2.Ar(C_{2m})								0
Φ_2									0
:									:
Φ_l									0
Φ_{l+1}	m	0	…	0	m	0	…	0	1

Table (1.6)

where $0 \le r \le m-1$, l is the number of Γ-classes of C_{2m} and Φ_j are the Artin characters of the quaternion group Q_{2m} , for all $1 \le j \le l+1$.

Example (1.8):

Consider the Quaternion group Q_{2p} ; p>2 and p is prim number.To find Artin characters table for it, we use theorem:(1.7): as the following :

$Ar(Q_{2p})=$

	Γ-classes of C_{2p}				
Γ- classes	[1]	$[x^2]$	$[x^p]$	[x]	[y]
$\lvert CL_\alpha \rvert$	1	2	1	2	2p
$\lvert C_{Q_{2p}}(CL\alpha) \rvert$	4p	2p	4p	2p	2
Φ_1	4p	0	0	0	0
Φ_2	4	4	0	0	0
Φ_3	2p	0	2p	0	0
Φ_4	2	2	2	2	0
Φ_5	p	0	p	0	1

Table(1,7)

Example (1.9):

Consider the Quaternion group Q_{10} .To find Artin characters table for it, we use theorem:(1.7): as the following :

$Ar(Q_{10})=$

	Γ-classes of C_{10}				
Γ- classes	[1]	$[x^2]$	$[x^5]$	[x]	[y]
$\lvert CL_\alpha \rvert$	1	2	1	2	10
$\lvert C_{Q_{10}}(CL\alpha) \rvert$	20	10	20	10	2
Φ_1	20	0	0	0	0
Φ_2	4	4	0	0	0
Φ_3	10	0	10	0	0
Φ_4	2	2	2	2	0
Φ_5	5	0	5	0	1

Table(1,8)

Theorem(1.8): [4]

Let H be a cyclic subgroup of G and $h_1, h_2, \ldots, h_m$ are chosen representatives for the m-conjugate classes of H contained in CL(g) in G, then :

$$\phi'(g) = \begin{cases} \dfrac{|C_G(g)|}{|C_H(g)|}\sum_{i=1}^{m}\varphi(h_i) & \text{if} \quad h_i \in H \cap CL(g) \\ 0 & \text{if} \quad H \cap CL(g) = \phi \end{cases}$$

Proposition(1,9).[3]

The number of all distinct Artin characters on group G is equal to the number of Γ-classes on G. Furthermore, Artin characters are constant on each Γ-classes .

2.The main results:

In this section we give the general form of Artin's characters table of the group $(Q_{2p}\times C_5)$,p>2 and p is prim numper .

Definition(2.1):

The group$Q_{2p}\times C_5$ is the direct product group of the quaternion groupQ_{2m} of order **4p** and the cyclic group C_5 of order 5, then the order of The group$(Q_{2p}\times C_5)$ is **20p** .

***Example (2.2)*:**

Let p=5 then $(Q_{2p}\times C_5)= (Q_{2.5}\times C_5)= (Q_{10}\times C_5)=\{ (1,I), (1,z), (1,z^2) ,(1,z^3) ,(1,z^4)$ $(x,I), (x,z), (x,z^2), (x,z^3) ,(x,z^4)$ $(x^2,I), (x^2,z), (x^2,z^2), (x^2,z^3), (x^2,z^4), \ldots ,(x^9,I), (x^9,z), (x^9,z^2), (x^9,z^3), (x^9,z^4), (y,I), (y,z), (y,z^2), (y,z^3), (y,z^4), (xy,I), (xy,z), (xy,z^2), (xy,z^3), (xy,z^4), (x^2y,I), (x^2y, z), (x^2y, z^2), (x^2y ,z^3), (x^2y ,z^4),\ldots, (x^9y ,I),(x^9y ,z), (x^9y ,z^2), (x^9y ,z^3), (x^9y ,z^4) \}$.

To find Artin's characters for this group, there are ten cyclic subgroups, which are : $<1,I>,<x^2,I>,<x^5,I>,<x,I>,<y,I>,<1,z>,<x^2,z>,<x^5,z>,<x,z>,<y,z>$,then there are ten Γ-Classes , we have ten distinct Artin's characters,.Let $g\in(Q_{10}\times C_5)$,g=(q,I)or g=(q,z),$q\in Q_{10}$,I,z $\in C_5$ and Let φ the principal character of H ,Φ_j Artin characters of Q_{10}, $1 \le j \le 5$, then by using theorem (1.8)

$$\Phi_j(g) = \begin{cases} \dfrac{|C_G(g)|}{|C_H(g)|}\sum_{i=1}^{m}\varphi(h_i) & \text{if} \quad h_i \in H \cap CL(g) \\ 0 & \text{if} \quad H \cap CL(g) = \phi \end{cases}$$

Case (I):- *If H is a cyclic subgroup of $(Q_{2p}\times \{I\})$,then:*

$H_1=<1,I>$

If g=(1,I) , $\Phi_{(1,1)}((1,I))=\frac{|C_{Q_{10}\times C_5}(g)|}{|C_{H_1}(g)|}\varphi\big((I\ ,1)\big) = \frac{100}{1}.1 = 100 = 5.20 = 5.\Phi_1(1)$,

since $H_1\cap CL(g)=\{ (1,I)\}$ and $\varphi(g)=1$, Otherwise $\Phi_{(1,1)}(g)= 0$ since $H_1 \cap CL(g) = \phi$

$H_2=<x^2,I>$

If g=(1,I)

$\Phi_{(2,1)}(g)=\frac{|C_{Q_{10}\times C_5}(g)|}{|C_{H_2}(g)|}\ \varphi(g) = \frac{100}{5}.1 = 20 = 5.4 = 5.\Phi_2(1)$

since $H_2\cap CL(g)=\{ (1,I)\}$ and $\varphi(g)=1$

If $g=(x^2,I)$

$\Phi_{(2,1)}((x^2,I))= \frac{|C_{Q_{10}\times C_5}(g)|}{|C_{H_2}(g)|}(\varphi(g)+\varphi(g^{-1})= \frac{50}{5}(1+1)=20=5.4=5.\Phi_2(x^2)$

since $H_2 \cap CL(g)=\{g,g^{-1}\}$ *and* $\varphi(g)=\varphi(g^{-1})=1$, *Otherwise* $\Phi_{(2,1)}(g)=0$ *since* $H_2 \cap CL(g)=\phi$

$H_3=\langle x^5,I\rangle$

If $g=(1,I)$

$\Phi_{(3,1)}((1,I))= \frac{|C_{Q_{10}\times C_5}(g)|}{|C_{H_3}(g)|}(\varphi(g))=\frac{100}{2}.1=50=5.10=5.\Phi_3(1)$ *since* $H_3 \cap CL(g)=\{(1,I)\}$ *and* $\varphi(g)=1$

If $g=(x^5,I)$, $\Phi_{(3,1)}((x^5,I))= \frac{|C_{Q_{10}\times C_5}(g)|}{|C_{H_3}(g)|}(\varphi(g))=\frac{100}{2}(1)=50=5.10=5.\Phi_3(x^5)$

since $H_3 \cap CL(g)=\{g\}$ *and* $\varphi(g)=1$, *Otherwise* $\Phi_{(3,1)}(g)=0$ *since* $H_3 \cap CL(g)=\phi$

$H_4=\langle x,I\rangle$

If $g=(1,I)$

$\Phi_{(4,1)}((1,I))= \frac{|C_{Q_{10}\times C_5}(g)|}{|C_{H_4}(g)|}(\varphi(g))=\frac{100}{10}.1=10=5.2=5.\Phi_4(1)$*since* $H_4 \cap CL(g)=\{(1,I)\}$ *and* $\varphi(g)=1$

If $g=(x^2,I)$, $\Phi_{(4,1)}((x^2,I))= \frac{|C_{Q_{10}\times C_5}(g)|}{|C_{H_4}(g)|}(\varphi(g)+\varphi(g^{-1})=\frac{50}{10}(1+1)=10=5.2=5.\Phi_4(x^2)$

since $H_4 \cap CL(g)=\{g,g^{-1}\}$ *and* $\varphi(g)=\varphi(g^{-1})=1$

If $g=(x^5,I)$, $\Phi_{(4,1)}((x^5,I))= \frac{|C_{Q_{10}\times C_5}(g)|}{|C_{H_4}(g)|}(\varphi(g))=\frac{100}{10}(1)=10=5.2=5.\Phi_4(x^5)$

since $H_4 \cap CL(g)=\{g\}$ *and* $\varphi(g)=1$

If $g=(x,I)$

$\Phi_{(4,1)}((x,I))= \frac{|C_{Q_{10}\times C_5}(g)|}{|C_{H_4}(g)|}(\varphi(g)+\varphi(g^{-1})=\frac{50}{10}(1+1)=10=5.2=5.\Phi_4(x)$

since $H_4 \cap CL(g)=\{g,g^{-1}\}$ *and* $\varphi(g)=\varphi(g^{-1})=1$, *Otherwise* $\Phi_{(4,1)}(g)=0$ *since* $H_4 \cap CL(g)=\phi$

$H_5=\langle y,I\rangle$

If $g=(1,I)$

$\Phi_{(5,1)}((1,I))= \frac{|C_{Q_{10}\times C_5}(g)|}{|C_{H_5}(g)|}(\varphi(g))=\frac{100}{4}.1=25=5.5=5.\Phi_5(1)$

since $H_5 \cap CL(g)=\{(1,I)\}$ *and* $\varphi(g)=1$

If $g=(x^5,I)$

$\Phi_{(5,1)}((x^5,I))= \frac{|C_{Q_{10}\times C_5}(g)|}{|C_{H_5}(g)|}(\varphi(g))=\frac{100}{4}(1)=25=5.5=5.\Phi_5(x^5)$ *since* $H_5 \cap CL(g)=\{g\}$ *and* $\varphi(g)=1$

If $g=(y,I)$ $\Phi_{(5,1)}((y,I))= \frac{|C_{Q_{10}\times C_5}(g)|}{|C_{H_5}(g)|}(\varphi(g)+\varphi(g^{-1})=\frac{10}{4}(1+1)=5=5.1=5.\Phi_5(y)$

since $H_5 \cap CL(g)=\{g,g^{-1}\}$ *and* $\varphi(g)=\varphi(g^{-1})=1$, *Otherwise* $\Phi_{(5,1)}(g)=0$ *since* $H_5 \cap CL(g)=\phi$

Case (II):- *If H is a cyclic subgroup of* $(Q_{2p}\times\{z\})$,*then:*

$H_{1.2}=\langle 1,z\rangle$, *If* $g=(1,I)$

$\Phi_{(1,2)}((1,I))= \frac{|C_{Q_{10}\times C_5}(g)|}{|C_{H_{1.2}}(g)|}(\varphi(g))=\frac{100}{5}.1=20=\Phi_1(1)$ *since* $H_{1.2} \cap CL(g)=\{(1,I)\}$ *and* $\varphi(g)=1$

If $g=(1,z)$ $\Phi_{(1,2)}((1,z))= \frac{|C_{Q_{10}\times C_5}(g)|}{|C_{H_{1.2}}(g)|}(\varphi(g))=\frac{100}{5}.1=20=\Phi_1(1)$ *since* $H_{1.2} \cap CL(g)=\{(1,z)\}$

and $\varphi(g)=1$ *Otherwise* $\Phi_{(1,2)}(g)=0$ *since* $H_{1.2} \cap CL(g)=\phi$

$H_{2.2}=\langle x^2,z\rangle$

If $g=(1,I)$

$\Phi_{(2,2)}((1,I))= \frac{|C_{Q_{10}\times C_5}(g)|}{|C_{H_{2.2}}(g)|}(\varphi(g))=\frac{100}{25}.1=4=\Phi_2(1)$ *since* $H_{2.2} \cap CL(g)=\{(1,I)\}$ *and* $\varphi(g)=1$

If $g=(1,z)$

$\Phi_{(2,2)}((1,z))= \frac{|C_{Q_{10}\times C_5}(g)|}{|C_{H_{2.2}}(g)|}(\varphi(g))=\frac{100}{25}.1=4=\Phi_2(1)$ *since* $H_{2.2} \cap CL(g)=\{(1,z)\}$ *and* $\varphi(g)=1$

If $g=(x^2,I)$, $\Phi_{(2,2)}((x^2,I))=\frac{|C_{Q_{10}\times C_5}(g)|}{|C_{H_{2.2}}(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{50}{25}(1+1)=4=\Phi_2(x^2)$

since $H_{2.2}\cap CL(g)=\{g,g^{-1}\}$ *and* $\varphi(g)=\varphi(g^{-1})=1$

If $g=(x^2,z)$

$\Phi_{(2,2)}((x^2,z))=\frac{|C_{Q_{10}\times C_5}(g)|}{|C_{H_{2.2}}(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{50}{25}(1+1)=4=\Phi_2(x^2)$

since $H_{2.2}\cap CL(g)=\{g,g^{-1}\}$ *and* $\varphi(g)=\varphi(g^{-1})=1$ *Otherwise* $\Phi_{(2,2)}(g)=0$ *since* $H_{2.2}\cap CL(g)=\phi$

$H_{3.2}=\langle x^5,z\rangle$

If $g=(1,I)$

$\Phi_{(3,2)}((1,I))=\frac{|C_{Q_{10}\times C_5}(g)|}{|C_{H_{3.2}}(g)|}(\varphi(g))=\frac{100}{10}.1=10=\Phi_3(1)$ *since* $H_{3.2}\cap CL(g)=\{(1,I)\}$ *and* $\varphi(g)=1$

If $g=(1,z)$

$\Phi_{(3,2)}((1,z))=\frac{|C_{Q_{10}\times C_5}(g)|}{|C_{H_{3.2}}(g)|}(\varphi(g))=\frac{100}{10}.1=10=\Phi_3(1)$ *since* $H_{3.2}\cap CL(g)=\{(1,z)\}$ *and* $\varphi(g)=1$

If $g=(x^5,I)$, $\Phi_{(3,2)}((x^5,I))=\frac{|C_{Q_{10}\times C_5}(g)|}{|C_{H_{3.2}}(g)|}(\varphi(g))=\frac{100}{10}(1)=10=\Phi_3(x^5)$ *since* $H_{3.2}\cap CL(g)=\{g\}$ *and* $\varphi(g)=1$

If $g=(x^5,z)$, $\Phi_{(3,2)}((x^5,z))=\frac{|C_{Q_{10}\times C_5}(g)|}{|C_{H_{3.2}}(g)|}(\varphi(g))=\frac{100}{10}(1)=10=\Phi_3(x^5)$ *since* $H_{3.2}\cap CL(g)=\{g\}$ *and* $\varphi(g)=1$, *Otherwise* $\Phi_{(3,2)}(g)=0$ *since* $H_{3.2}\cap CL(g)=\phi$

$H_{4.2}=\langle x,z\rangle$

If $g=(1,I)$

$\Phi_{(4,2)}((1,I))=\frac{|C_{Q_{10}\times C_5}(g)|}{|C_{H_{4.2}}(g)|}(\varphi(g))=\frac{100}{50}.1=2=\Phi_4(1)$ *since* $H_{4.2}\cap CL(g)=\{(1,I)\}$ *and* $\varphi(g)=1$

If $g=(1,z)$

$\Phi_{(4,2)}((1,z))=\frac{|C_{Q_{10}\times C_5}(g)|}{|C_{H_{4.2}}(g)|}(\varphi(g))=\frac{100}{50}.1=2=\Phi_4(1)$ *since* $H_{4.2}\cap CL(g)=\{(1,z)\}$ *and* $\varphi(g)=1$

If $g=(x^2,I)$

$\Phi_{(4,2)}((x^2,I))=\frac{|C_{Q_{10}\times C_5}(g)|}{|C_{H_{4.2}}(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{50}{50}(1+1)=2=\Phi_4(x^2)$ *since* $H_{4.2}\cap CL(g)=\{g,g^{-1}\}$ *and* $\varphi(g)=\varphi(g^{-1})=1$

If $g=(x^2,z)$

$\Phi_{(4,2)}((x^2,z))=\frac{|C_{Q_{10}\times C_5}(g)|}{|C_{H_{4.2}}(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{50}{50}(1+1)=2=\Phi_4(x^2)$ *since* $H_{4.2}\cap CL(g)=\{g,g^{-1}\}$ *and* $\varphi(g)=\varphi(g^{-1})=1$

If $g=(x^5,I)$

$\Phi_{(4,2)}((x^5,I))=\frac{|C_{Q_{10}\times C_5}(g)|}{|C_{H_{4.2}}(g)|}(\varphi(g))=\frac{100}{50}(1)=2=\Phi_4(x^5)$ *since* $H_{4.2}\cap CL(g)=\{g\}$ *and* $\varphi(g)=1$

If $g=(x^5,z)$

$\Phi_{(4,2)}((x^5,z))=\frac{|C_{Q_{10}\times C_5}(g)|}{|C_{H_{4.2}}(g)|}(\varphi(g))=\frac{100}{50}(1)=2=\Phi_4(x^5)$ *since* $H_{4.2}\cap CL(g)=\{g\}$ *and* $\varphi(g)=1$

If $g=(x,I)$

$\Phi_{(4,2)}((x,I))=\frac{|C_{Q_{10}\times C_5}(g)|}{|C_{H_{4.2}}(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{50}{50}(1+1)=2=\Phi_4(x)$ *since* $H_{4.2}\cap CL(g)=\{g,g^{-1}\}$ *and* $\varphi(g)=\varphi(g^{-1})=1$

If $g=(x,z)$

$\Phi_{(4,2)}((x,z))=\frac{|C_{Q_{10}\times C_5}(g)|}{|C_{H_{4.2}}(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{50}{50}(1+1)=2=\Phi_4(x)$ *since* $H_{4.2}\cap CL(g)=\{g,g^{-1}\}$ *and* $\varphi(g)=\varphi(g^{-1})=1$, *Otherwise* $\Phi_{(4,2)}(g)=0$ *since* $H_{4.2}\cap CL(g)=\phi$

$H_{5.2}=\langle y,z\rangle$

If g=(1,I)

$\Phi_{(5,2)}((1,I))= \frac{|C_{Q_{10}\times C_5}(g)|}{|C_{H_{5.2}}(g)|}$ $(\varphi(g))=\frac{100}{20}.1=5=\Phi_5(1)$ *since* $H_{5\cdot2}\cap CL(g)=\{(1,I)\}$ *and* $\varphi(g)=1$

If g=(1,z)

$\Phi_{(5,2)}((1,z))= \frac{|C_{Q_{10}\times C_5}(g)|}{|C_{H_{5.2}}(g)|}$ $(\varphi(g))=\frac{100}{20}.1=5=\Phi_5(1)$ *since* $H_{5\cdot2}\cap CL(g)=\{(1,z)\}$ *and* $\varphi(g)=1$

If $g=(x^5,I)$

$\Phi_{(5,2)}((x^5,I))= \frac{|C_{Q_{10}\times C_5}(g)|}{|C_{H_{5.2}}(g)|}$ $(\varphi(g))=\frac{100}{20}(1)=5=\Phi_5(x^5)$ *since* $H_{5\cdot2}\cap CL(g)=\{g\}$ *and* $\varphi(g)=1$

If $g=(x^5,z)$, $\Phi_{(5,2)}((x^5,z))= \frac{|C_{Q_{10}\times C_5}(g)|}{|C_{H_{5.2}}(g)|}$ $\varphi(g)=\frac{100}{20}(1)=5=\Phi_5(x^5)$ *since* $H_{5\cdot2}\cap CL(g)=\{g\}$ *and* $\varphi(g)=1$

If g=(y,I)

$\Phi_{(5,2)}((y,I))= \frac{|C_{Q_{10}\times C_5}(g)|}{|C_{H_{5.2}}(g)|}$ $(\varphi(g)+\varphi(g^{-1})=\frac{10}{20}(1+1)=1=\Phi_5(y)$ *since* $H_{5\cdot2}\cap CL(g)=\{g,g^{-1}\}$ *and* $\varphi(g)=\varphi(g^{-1})=1$

If g=(y,z)

$\Phi_{(5,2)}((y,z))= \frac{|C_{Q_{10}\times C_5}(g)|}{|C_{H_{5.2}}(g)|}$ $(\varphi(g)+\varphi(g^{-1})=\frac{10}{20}(1+1)=1=\Phi_5(y)$ *since* $H\cap CL(g)=\{g,g^{-1}\}$ *and* $\varphi(g)=\varphi(g^{-1})=1$ *Otherwise* $\Phi_{(5,2)}(g)=0$ *since* $H_{5\cdot2}\cap CL(g)=\phi$.

Then, the Artin characters table of $(Q_{10}\times C_5)$ is given in the following Table:-

$Ar(Q_{10}\times C_5)$ =

Γ-Classes	[1,I]	$[x^2,I]$	$[x^5,I]$	[x,I]	[y,I]	[1,z]	$[x^2,z]$	$[x^5,z]$	[x,z]	[y,z]
$\lvert CL_\alpha\rvert$	1	2	1	2	10	1	2	1	2	10
$\lvert C_{Q_{10}\times C_5}(CL_\alpha)\rvert$	100	50	100	50	10	100	50	100	50	10
$\Phi_{(1,1)}$	100	0	0	0	0	0	0	0	0	0
$\Phi_{(2,1)}$	20	20	0	0	0	0	0	0	0	0
$\Phi_{(3,1)}$	50	0	50	0	0	0	0	0	0	0
$\Phi_{(4,1)}$	10	10	10	10	0	0	0	0	0	0
$\Phi_{(5,1)}$	25	0	25	0	5	0	0	0	0	0
$\Phi_{(1,2)}$	20	0	0	0	0	20	0	0	0	0
$\Phi_{(2,2)}$	4	4	0	0	0	4	4	0	0	0
$\Phi_{(3,2)}$	10	0	10	0	0	10	0	10	0	0
$\Phi_{(4,2)}$	2	2	2	2	0	2	2	2	2	0
$\Phi_{(5,2)}$	5	0	5	0	1	5	0	5	0	1

Table(2,1)

Theorem (2.2):

The Artin's character table of the group $(Q_{2p}\times C_5)$ where ,p> 2 and p is prime number ; is given as follows:

$Ar(Q_{2p}\times C_5)$ =

	Γ-Classes of $Q_{2p}\times\{I\}$					Γ-Classes of $Q_{2p}\times\{z\}$				
Γ-Classes	[1,I]	$[x^2,I]$	$[x^p,I]$	[x,I]	[y,I]	[1,z]	$[x^2,z]$	$[x^p,I]$	[x,z]	[y,z]
$\|CL_\alpha\|$	1	2	1	2	2m	1	2	1	2	2m
$\|CQ_{10}\times C_5(CL_\alpha)\|$	20p	10p	20p	10p	10	20p	10p	20p	10p	10
$\Phi_{(1,1)}$	**5 Ar(Q_{2m})**					0				
$\Phi_{(2,1)}$										
$\Phi_{(3,1)}$										
$\Phi_{(4,1)}$										
$\Phi_{(5,1)}$										
$\Phi_{(1,2)}$	**Ar(Q_{2m})**					**Ar(Q_{2m})**				
$\Phi_{(2,2)}$										
$\Phi_{(3,2)}$										
$\Phi_{(4,2)}$										
$\Phi_{(5,2)}$										

Table(2,2)

Which is 10×10 matrix square.

Proof:-

Let $g\in(Q_{2p}\times C_5)$; g=(q,I) or g=(q,z), $q\in Q_{2p}$, I, $z\in C_5$

Case (I):

If H is a cyclic subgroup of $(Q_{2p}\times \{I\})$then:

1-H=$\langle(x,I)\rangle$ 2-H=$\langle(y,I)\rangle$

and φ the principal character of H , Φ_j Artin characters of Q_{2p} , 1<j<i+1 , then by using theorem (1.8): -

$$\Phi_j(g)=\begin{cases}\dfrac{|C_G(g)|}{|C_H(g)|}\sum_{i=1}^{m}\varphi(h_i) & \text{if } h_i\in H\cap CL(g)\\ 0 & \text{if } H\cap CL(g)=\phi\end{cases}$$

(i) If g =(1,I)

$\Phi_{(j,1)}(g)=\left|\frac{C_{Q_{2p}\times C_5(cL_\alpha)}}{|C_H(g)|}\right|\varphi((I\ ,1)) = \frac{20p}{|C_H((I\ ,1))|}.1 = \frac{5\,|Q_{2p}(1)|}{|C_{<x>}(1)|}$=5 $\Phi_j(1)$ since $H\cap CL(1,I)=\{(1,I)\}$ and φ (g)= 1

(ii) If g= $(x^p,I),g\in H$

$\Phi_{(j,1)}(g)=\left|\frac{C_{Q_{2p}\times C_5(cL_\alpha)}}{|C_H(g)|}\right| \varphi(g) = \frac{20p}{|C_H(x^p,1)|}.1 = \frac{5|Q_{2p}(x^p)|}{|C_{<x>}(x^p)|}.1 = 5\Phi_j(x^p)$ since $H \cap CL(g) = \{g\}$ and $\varphi(g)=1$

(iii) If $g \neq (x^p)$ and, $g \in H$ $\Phi_{(j,1)}(g)=\left|\frac{C_{Q_{2p}\times C_5(cL_\alpha)}}{|C_H(g)|}\right| (\varphi(g) + \varphi(g^{-1}) = \frac{10p}{|C_{<x>}(g)||}(1+1) = \frac{20p}{|C_{<x>}(g)|} = \frac{5|Q2p(q)|}{|C_{<x>}(q)|} = 5\Phi_j(q)$

since $H \cap CL(g)=\{g,g^{-1}\}$, g=(q,I), $q \in Q_{2p}$ and $q \neq x^p$

(iv) If $g \notin H$ $\Phi_{(j,1)}(g)= 0 = 5.\ 0 = 5\Phi_j(q)$ since $H \cap CL(g) = \phi$ and $q \in Q_{2p}$

2-IF $H = <(y,I)> = \{(1,I),(y,I),(y^2,I),(y^3,I)\}$

(i) If g = (1,I) $\Phi_{(l+1,1)}(g)=\left|\frac{C_{Q_{2p}\times C_5(cL_\alpha)}}{|C_H(g)|}\right| \varphi(g)= \frac{20p}{4}.1=5.p=5\Phi_{(i+1)}(1)$ since $H \cap CL(1,I) = \{(1,I)\}$ and $\varphi(g)=1$

(ii) If $g= (x^p,I) = (y^2,I)$ and $g \in H$ $\Phi_{(l+1,1)}(g)=\left|\frac{C_{Q_{2p}\times C_5(cL_\alpha)}}{|C_H(g)|}\right| \varphi(g)) = \frac{20p}{4}.1=5.p=5\Phi_{(i+1)}(x^p)$

since $H \cap CL(g)=\{g\}$ and $\varphi(g)=1$

(ii) If $g \neq (x^p,I)$ and $g \in H$, i.e.$\{g = (y,I)$or $g = (y^3,I)\}$

$\Phi_{(l+1,1)}(g) = \left|\frac{C_{Q_{2m}\times C_5(cL_\alpha)}}{|C_H(g)|}\right| (\varphi(g) + \varphi(g^{-1}) = \frac{10}{4}(1+1) = \frac{20}{4} = 5.1 = 5\Phi_{(i+1)}(y)$

since $H \cap CL(g)=\{g,g^{-1}\}$and $\varphi(g)=\varphi(g^{-1})=1$ Otherwise $\Phi_{(l+1,1)}(g)= 0$ since $H \cap CL(g) = \phi$

Case (II):

If H is a cyclic subgroup of ($Q_{2p}\times\{z\}$) then:

1-H=$\langle(x,z)\rangle$ 2-H=$\langle(y,z)\rangle$

and φ the principal character of H , Φ_j Artin characters of Q_{2p}, 1<j<i+1 , then by using theorem (1.8): -

$$\Phi_j(g) = \begin{cases} \frac{|C_G(g)|}{|C_H(g)|}\sum_{i=1}^{m}\varphi(h_i) & if \quad h_i \in H \cap CL(g) \\ 0 & if \quad H \cap CL(g) = \phi \end{cases}$$

1-H=$\langle x,z\rangle$

(i) If g =(1,I),(1,z)

$\Phi_{(j,2)}(g)=\left|\frac{C_{Q_{2p}\times C_5(cL_\alpha)}}{|C_H(g)|}\right| \varphi(g) = \frac{20p}{|C_H((I,1))|}.1 = \frac{5|Q_{2p}(1)|}{5|C_{<x>}(1)|}\varphi(1) = \Phi j(1)$

since $H \cap CL(g)=\{(1,I),(1,z)\}$ *and* $\varphi(g)=1$

(ii) g= (1,I),(x^p,I)(x^p,z) ,(1,z) ; $g \in H$

If g=(1,I),(1,z)

$\Phi_{(j,2)}(g)= \frac{|CQ_{2p}\times C_5(g)|}{|C_H(g)|}\varphi(g) = \frac{20p}{|C_H(g))|}.1 = \frac{5|Q_{2p}(1)|}{5|C_{<x>}(1)|}\varphi(1) = \Phi_j(1)$

since $H \cap CL(g) = \{g\}$ *and* $\varphi(g)=1$

If g=(x^p,I),(x^p,z)

$\Phi_{(j,2)}(g)=\left|\frac{C_{Q_{2p}\times C_5(cL_\alpha)}}{|C_H(g)|}\right| \varphi(g) = \frac{5|Q_{2p}(x^p)|}{5|C_{<x>}(x^p)|}\varphi(x^p) = \Phi_j(x^p)$ *since* $H \cap CL(g) = \{g\}$ *and* $\varphi(g)=1$

(iii) If $g \neq$ *(x^p,I),(x^p,z) and* $g \in H$

$\Phi_{(j,2)}(g)=\left|\frac{C_{Q_{2p}\times C_5(cL_\alpha)}}{|C_H(g)|}\right| (\varphi(g) + \varphi(g^{-1}) = \frac{10}{|C_{<x>}(g)|}(1+1) = \frac{5|Q2p(q)|}{5|C_{<x>}(q)|}\varphi(q) = \Phi j(q)$

since $H \cap CL(g)=\{g,g^{-1}\}$, $\varphi(g)=\varphi(g^{-1})=1$, g=(q,z), $q \in Q_{2p}$ *and* $q \neq x^p$

(iv) If $g \notin H$ $\Phi_{(j,2)}(g)= 0 = \Phi_j(q)$ *since* $H \cap CL(g) = \phi$ *and* $q \in Q_{2p}$

2-IF H = <y,z > $=\{(1,I),(y,I),(y^2,I),(y^3,I),(1,z),(y,z),(y^2,z),(y3,z),(1,z^2),(y,z^2),(y^2,z^2),(y^3,z^2),(1,z^3),(y,z^3),(y^2,z^3),(y^3,z^3)$, $(1,z^4),(y,z^4),(y^2,z^4),(y^3,z^4)\}$

(i) If g = (1,I),(1,z)

$\Phi_{(l+1,2)}(g)= \left|\frac{C_{Q_{2p}\times C_5(cL_\alpha)}}{|C_H(g)|}\right| \varphi(g) = \frac{20p}{20}.1 = p = \Phi_{l+1}(g)$

since $H \cap CL(g)=\{(1,I),(1,z)\}$ *and* $\varphi(g)=1$

(ii) If $g=(y^2,I) = (x^p,I), (y^2,z), (y^2,z^2), (y^2,z^3), (y^2,z^4)$ *and* $g \in H$

$\Phi_{(l+1,2)}(g)= \left|\frac{C_{Q_{2p}\times C_5(cL_\alpha)}}{|C_H(g)|}\right| \varphi(g) = \frac{20p}{20}.1 = p = \Phi_{l+1}(y)$ since $H \cap CL(g)=\{g\}$ *and* $\varphi(g) =1$

(ii) If $g \neq (x^p,I)$ *and* $g \in H$ i.e $g=\{(y,I),(y,z),(y,z^2),(y,z^3),(y,z^4)\}$ or $g =\{(y^3,I), (y^3,z), (y^3,z^2), (y^3,z^3), (y^3,z^4)\}$

$\Phi_{(l+1,2)}(g)= \left|\frac{C_{Q_{2m}\times C_5(cL_\alpha)}}{|C_H(g)|}\right| = (\varphi(g) + \varphi(g^{-1}) = \frac{10}{20}(1+1) = 1 = \Phi_{(i,1)}(y)$

since $H \cap CL(g)=\{g,g^{-1}\}$ *and* $\varphi(g)=\varphi(g^{-1})=1$

Otherwise $\Phi_{(l+1,2)}(g)= 0$ *since* $H \cap CL(g) = \phi$

Example (2.3):

To construct $Ar(Q_{14}\times C_5)$,p=7 , we use theorem:(1.7) as the following :-

$Ar(Q_{14})$=

Γ-Classes	[1]	[x^2]	[x^7]	[x]	[y]
$\mid CL_\alpha \mid$	1	2	1	2	14
$\mid CQ_{14}\times C_5(CL_\alpha)\mid$	28	14	28	14	2
Φ_1	28	0	0	0	0
Φ_2	4	4	0	0	0
Φ_3	14	0	14	0	0
Φ_4	2	2	2	2	0
Φ_5	7	0	7	0	1

Table(2,3)

Then by using theorem (2.2) Artin characters teble of the group $(Q_{14}\times C_5)$ is:-

$Ar(Q_{14}\times C_5)$=

Γ-Classes	[1,I]	[x^2,I]	[x^7,I]	[x,I]	[y,I]	[1,z]	[x^2,z]	[x^7,z]	[x,z]	[y,z]
$\mid CL_\alpha \mid$	1	2	1	2	10	1	2	1	2	14
$\mid CQ_{14}\times C_5(CL_\alpha)\mid$	140	70	140	70	10	140	70	140	70	10
$\Phi_{(1,1)}$	140	0	0	0	0	0	0	0	0	0
$\Phi_{(2,1)}$	20	20	0	0	0	0	0	0	0	0
$\Phi_{(3,1)}$	70	0	70	0	0	0	0	0	0	0
$\Phi_{(4,1)}$	10	10	10	10	0	0	0	0	0	0
$\Phi_{(5,1)}$	35	0	35	0	5	0	0	0	0	0
$\Phi_{(1,2)}$	28	0	0	0	0	28	0	0	0	0
$\Phi_{(2,2)}$	4	4	0	0	0	4	4	0	0	0
$\Phi_{(3,2)}$	14	0	14	0	0	14	0	14	0	0
$\Phi_{(4,2)}$	2	2	2	2	0	2	2	2	2	0
$\Phi_{(5,2)}$	7	0	7	0	1	7	0	7	0	1

Table(2,4)

References

[1] A.H.Abdul-Munem" On Artin Cokernel Of The Quaternion group Q_{2m} When m is an Odd number",M.Sc. , thesis , University of Kufa , 2008

[2] A.S. Abid , " Artin's Characters Table of Dihedral Group for Odd Number " , M.Sc. thesis , University of Kufa , 2006

[3] C.Curits and I.Reiner ," Methods of Representation Theory with Application to Finite Groups and Order " , John willy& sons, New York, 1981.

[4] I. M.Isaacs, "On Character Theory of Finite Groups ", Academic press, New york, 1976.