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Fuzzy Bridge Regression Model Estimating via Simulation

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Abstract

The main problem when dealing with fuzzy data variables is that it cannot be formed by a model that represents the data through the method of Fuzzy Least Squares Estimator (FLSE) which gives false estimates of the invalidity of the method in the case of the existence of the problem of multicollinearity. To overcome this problem, the Fuzzy Bridge Regression Estimator (FBRE) Method was relied upon to estimate a fuzzy linear regression model by triangular fuzzy numbers. Moreover, the detection of the problem of multicollinearity in the fuzzy data can be done by using Variance Inflation Factor when the inputs variable of the model crisp, output variable and parameters are fuzzed. The results were compared using standard mean squares error via simulated experiments and taking different sample sizes (20, 40, 80, and 160). The model's superiority was shown by achieving the least value of the mean squares error (MSE), which indicated by the fuzzy bridge regression model.

Paper type: Research paper.
Keywords: Fuzzy linear regression, Fuzzy least squares method, Fuzzy Bridge regression, multicollinearity, triangular fuzzy numbers, Variance Inflation Factor.

1.Introduction

There are many phenomena whose data is fuzzy uncertainty , that is, its value cannot be determined by a single value, and to represent this data by estimating a model that describes it and estimating its parameters, especially when the inputs are not distorted and the outputs and parameters are fuzzy with the existence of the problem of multicollinearity in the data as the traditional methods Like the fuzzy least squares estimation method, it does not give effective estimates because its conditions are not applied, and there are many methods that treat fuzzy data in the presence of the multicollinearity problem, which will be based on the fuzzy bridge regression model in this research so fuzzy bridge regression will be resorted to to take advantage of the inaccuracy in this type of data to build regression models and analyze them efficiently in order to make estimates and predictions instead of ignoring when the response variable response triangular fuzzy numbers and independent variables are crisp, and the mean squared error (MSE) criterion is used to compare the effects of fuzzy least squares estimation and fuzzy bridge regression.

1.1 Literature review

Abbas and Mohammed (2017) studied present new method of fuzzy regression, where the adaptive fuzzy regression model was estimated using the entropy function. Prevailing in fuzzy linear regression analysis, a new fuzzy method was adopted, which depends on the location and entropy functions. Two fuzzy numbers trapezoid and triangular with different membership curves instead of using membership functions that depend on the center and spread sets. Performance and study results showed the efficiency of using location and entropy functions to describe fuzzy numbers and their superiority over the use of member functions.

Karbasi et al. (2020) studied bridge regression based on the slope of lasso and ridges, when the outputs are fuzzy, and developed a bridge regression model in a fuzzy environment. To estimate the fuzzy coefficients of the proposed model, by presenting a hybrid scheme based on recurrent neural networks, the proposed neural network model was built based on some concepts of convex optimization and stability theory that ensures finding the approximate parameters of the proposed model, and showed higher accuracy of the proposed fuzzy bridge method compared to the others fuzzy regression models.

Bogdana and Milan (2022) studied the role of regression analysis is crucial in many disciplines. Addressing the fuzzy quadratic least square regression for observed data modeled by fuzzy numbers, we aim to emphasize how a methodology that does not fully comply with the extension principle may fail to predict fuzzy valued numbers. We also propose a solution approach that functions in full accordance with the extension principle, thus overcoming the shortcomings arising from the practice of splitting the optimization of a fuzzy number in independent optimizations of its components.

Shemail and Mohammed (2022) studied Response variables with fuzzy triangular numbers and the predictors are exact data and other fuzzy regression methods.

2.Materials and Methods

2.1 Fuzzy number

A fuzzy number $\widetilde{A}$ is defined as a fuzzy set on the real number line R and must satisfy the following conditions:

i. There is at least one element X_0 ∈R such that $\mu_{\tilde{A}} = 1$.

ii. $\mu\widetilde{A}_{(X)}$ is a continuous ordered pair.

iii. $\widetilde{A}$ It must be normal and concave or (convex).(Gebray & Reddy ,2014), (Nareshkumar & Ghurum,2014)

$x_1\, x_2 \in R, \mu_A(\lambda\, x_1 + (1-\lambda)x_2) \geq \min\{\mu_A(x_1), \mu_A(x_2)\}, \lambda \in [0,1]$

iv. Supp $\widetilde{A}$ is bounded in R.

2.2 The membership functions

It is a function that expresses the degree of affiliation or the degree of membership, which are real numbers within the closed period [0,1] and is expressed in the degree of membership or affiliation ($F_{(x)}$(M)) which represents the degree of belonging of the element from the variable x to the fuzzy group.(Nareshkumar & Ghurum,2014).

2.3 Triangular membership function

It is one of the most common and widely used membership functions. This function has three basic parameters (a, b, c). Its aim is to allocate the lowest cost to reach the optimal fuzzy trigonometric solution, so that any Triangular fuzzy number can be represented by three real numbers. (Barua & et al, 2013) Representation of the Triangular function as in the following formula:

$$\mu\tilde{A}_{(x_i)} = \begin{cases} \frac{(x-a)}{(b-a)} & a \leq x \leq b \\ 1 & x = b \\ \frac{(c-x)}{(c-b)} & b \leq x \leq c \end{cases} \qquad \cdots (1)$$

Since: a: the minimum , b: the middle limit (center) , c: the upper limit

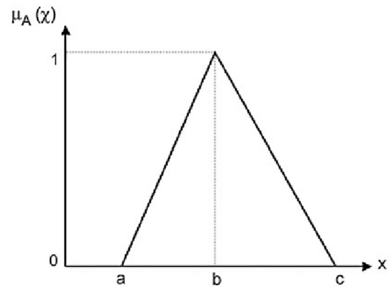

Figure 1. Triangular membership function (Barua & et al, 2013)

2.4 Fuzzy Linear Regression Model

The linear fuzzy regression model is used to estimate the significant relationship between the response variable and the independent variables in a fuzzy with a linear function, thus it is called Fuzzy Linear Regression (FLR). There are three types of fuzzy regression:

Uncertainty in fuzzy regression if the relationship between the independent variables and the dependent variable is fuzzy, or if the data themselves are fuzzy, leads to the following types of fuzzy regression. (Nassif & et al, 2019)

i. The relationship between fuzzy variables (fuzzy parameters). (Urkis & Mahdi, 2018)
ii. The data is fuzzy and the parameters are crisp ,The parameters are estimated using the Fuzzy Least Squares Method (FLSM) to estimate the parameters .(Choi &Yoon,2010)
iii. The data is fuzzy and the parameters are fuzzy.

In this research, a fuzzy model will be adopted, where the inputs are crisp and the outputs and parameters are fuzzy.

$$\tilde{Y} = f(x,\tilde{\beta}) = \tilde{\beta}_0 + \tilde{\beta}_1 x_1 + \tilde{\beta}_2 x_2 \ldots + \tilde{\beta}_n x_n + \varepsilon \qquad , i = 1,2,\cdots n \qquad \cdots (2)$$

$\tilde{\beta}$: Fuzzy parameters model, $\tilde{Y}$: Fuzzy dependent variable, $x_1, x_2 \ldots x_n$: Crisp independent variables.

ε: Fuzzy Random error.

2.5 Fuzzy Least Squares Estimation (FLS)

The parameters of the fuzzy regression model can be estimated based on the usual least squares formula with the fuzzy model, which is called the fuzzy least squares method, as in the following formula: (Kamile & Aysen, 2004)

$$\hat{\tilde{\beta}} = Min \sum_{i=1}^{n} (\tilde{y}_i - \tilde{\beta}_0 - \sum_{j=1}^{p} \tilde{\beta}_j x_{ij})^2 \quad , \tilde{\beta} = (\tilde{\beta}_0, \tilde{\beta}_1, \dots, \tilde{\beta}_n) \quad \cdots (3)$$

And by using a y n Lagrange multiplier, by deriving the above equation with respect to $\tilde{\beta}$, it results

$$\hat{\tilde{\beta}}^{FLSE} = \arg\min \sum_{i=1}^{n} d^2(\tilde{y}_i - \tilde{y}_i^{*}) \quad \cdots (4)$$

$$d^2(\tilde{y}_i, \tilde{y}_i^{*}) = [({y_i}^C - {y_i}^{*C})^2 + [({y_i}^C - {x_i}^L) - ({y_i}^{*C} - {y_i}^{*L})]^2 + [({y_i}^C - {y_i}^U) - ({y_i}^{*C} - {y_i}^{*U})]^2 \cdots (5)$$

After obtaining the parameters of the fuzzy regression model, it will be used to estimate the parameters of the fuzzy Ridge regression model.

2.6 Multicollinearity problem

Multicollinearity is a compound term of (Multi) multiple and (co) correlation and (linearity). The problem of linearity occurs when two or more independent variables are in a linear relationship so strong that it becomes difficult separate the effect of each variable from the dependent variable in applied reality, especially with regard to behavioral functions, as there is often a relationship between the independent variables, as a result of the influence of the economic variables on each other, since if there is no relationship between the independent variables, there is no need to use a Multiple linear regression model , then the general linear model can be replaced by (1-p) of the simple linear models (SLM).

The problem of multicollinearity occurs when the value of one of the independent variables is equal for all observations, or when the values of one of the independent variables depend on the values of one or more of the independent variables. (O'Brien, 2007).

2.7 Multicollinearity Test Method

There are metrics used to test for the existence of multicollinearity between the independent variables, and the variable inflation factor will be used for the multicollinearity problem in fuzzy data.

Variance Inflation Factor (VIF)

The variance inflation factor is used to test multicollinearity and to determine the independent variable responsible for it. This measure was proposed by researchers Farrar & Glauber (1967) and it was called (1970) by Marquardt Variance Inflation Factors and symbolized by the symbol (VIF). The mathematical formula of Variance Inflation Factors takes the following form (Farrar & Glauber ,1967)):

$$VIF_j = \frac{1}{1 - R_j^2} \quad , \text{j} = 1,2,3, \cdots \text{p}. \quad \cdots (6)$$

p : represents the number of fuzzy independent variables.

R_j^2: The coefficient of determination of the regression model of the independent variable x_j on the rest of the independent variables and that $0 \leq R_j^2 \leq 1$.

$$R_j^2 = \frac{\| \hat{x}_{ij} - \bar{x}_j \|^2}{\sum_{i=1}^{n} \| x_{ij} - \bar{x}_j \|^2} \quad \cdots (7)$$

To develop (VIF) to fit the fuzzy regression model for fuzzy variables, as in the following formula:

$$R_{Gj}^2 = \frac{\sum_{i=1}^{n} d^2(\hat{x}_{ij}, \bar{x}_j)}{\sum_{i=1}^{n} d^2(x_{ij}, \bar{x}_j)} \quad \cdots (8)$$

Where $d^2(\hat{x}_{ij}, \bar{x}_j)$ represents the distance between two fuzzy numbers between the real fuzzy number and the estimated fuzzy number.

$$VIF_{Gj} = \frac{1}{1-R_{Gj}^2} \quad \dots (9)$$

Gunst and Mason (1980) suggested, if: VIF> 5, this obscures the possibility of a problem of multicollinearity between the independent variables, and this would be a sufficient reason to discard the variable x_j from the analysis or to use another alternative method for fuzzy least squares in estimation, but if the variable $x_j$is independent from of the other, then ($R_j^2 = 0$) and thus the value of (VIF=1) means that there is no problem of multicollinearity between the independent variables, which will be adopted as a measure to detect multicollinearity in the fuzzy data. (Muniz, 2009)

2.8 Fuzzy Bridge Regression Estimator (FBRE)

This method is considered to be a development of Ridge Fuzzy Regression model, based on the fact that the model consists of the response variable, the explanatory variables, and the parameters are fuzzy. In order to develop the Ridge Regression method according to the following steps:

$$\text{Minimize} \sum_{i=1}^{n} (Y_i \sum_{j=0}^{p} \beta_j x_{ij}^2) \qquad \text{Subject to} \sum_{j=0}^{p} |\beta_j|^\gamma \le t \quad \dots (10)$$

Where t represents the positive parameter of the cut-off limit and the procrastination increases the
model's efficiency through the optimal choice γ by increasing the efficiency. If $\gamma \ge 1$, the model represents a convex problem. By setting the value of $\gamma = 0$, $\gamma = 1$, $\gamma = 2$ to improve the problem and by using fuzzy least squares regression model, lasso regression model and Ridge Regression method respectively to develop a new model with higher efficiency. (Delara, 2020).

2.9 Goodness of fit measure

To choose which model is better using the fuzzy least squares method or the Fuzzy Bridge regression method, it is done through MSE criterion. The results were obtained when comparing the original value with the estimated value.
Mean square of prediction error (MSE)

$$MSE = \frac{1}{n} \sum_{i=1}^{n} D_i^2(\tilde{y}_i, \hat{\tilde{y}}_i) \qquad i = 1,2,3,\dots,n \quad \dots (11)$$

$$D^2(\tilde{y}_i, \hat{\tilde{y}}_i) = |y_i^c - y_i^{*c}| + |y_i^l - y_i^{*l}| + |y_i^u - y_i^{*u}|$$

Where $\boldsymbol{D_i^2}$ represents the distance between two fuzzy numbers between the real fuzzy number and the estimated fuzzy number.
The smaller value is the best estimator of the model. (Davison, 1997)

3. Discussion of Results

3.1 Simulation

The R 4.2.1 statistical programming was used to write the simulation program. The written program includes four basic stages for estimating the fuzzy regression model, as follows
The first stage: At this stage, the parameter values are chosen on the applied side by taking the average value of the parameter (50,0.21,0.28,-0.08,-0.14,0.23,0.16), as follows:
Six independent variables were selected.
The fuzzy random error was generated, and the distribution was uniform.
Different default values for the standard deviation of random errors (σ = 0.5, 2) were chosen.
Different values of the correlation parameter between the independent variables (ρ = 0.8, 0.9, 0.95, 0.99) were chosen.

Different sample sizes (n=20, 40, 80, 160) were chosen.
q= (0.25, 0.5, 0.75) Bridge Parameter
Each experiment was repeated 1,000 times.
The second stage: Data generation
It is a very important stage to adopt the steps that follow it, as the explanatory variables are generated that have a correlation equal to ρ, the random error that follows the normal distribution with mean equal to zero, and the variance of $\hat{\sigma}^2$, and then finding the values of the dependent variable based on the generated value for each of the independent variables and random error.
Stage Three: Estimation
At this stage, the estimation process for the regression parameters is performed using the estimation methods of interest, as follows:
Fuzzy least squares Estimator (FLSE) method.
Fuzzy Bridge Regression Estimator (FBRE) method.
Fuzzy Bridge Regression Estimator 1(FBRE1) method.
Fuzzy Bridge Regression Estimator 2 (FBRE2) method.
Fuzzy Bridge Regression Estimator 3 (FBRE3) method.
The fourth stage: the stage of comparison between the methods
compare the different estimation methods for the models and find the best estimators, the mean square error (MSE) criterion was used equation (11).

The following results have been obtained for Table 1, Table 2 Table 3 and Table 4

Table 1. MSE model for different methods when ρ=(0.8)

σ	$\sigma = 0.5$				$\sigma = 2$			
Sample Size	**FLSE**	**FBRE1**	**FBRE2**	**FBRE3**	**FLSE**	**FBRE1**	**FBRE2**	**FBRE3**
	q=0	q=0.25	q=0.5	q=0.75	q=0	q=0.25	q=0.5	q=0.75
n=20	0.35524	0.26036	0.23656	0.21524	3.69028	2.8723	2.47309	2.16001
n=40	0.17888	0.16617	0.17865	0.16099	1.46581	1.03396	0.85283	0.73895
n=80	0.11766	0.11615	0.11242	0.10795	0.70104	0.46233	0.38799	0.34527
n=160	0.09333	0.01861	0.01621	0.00622	0.36991	0.2714	0.24614	0.2247

Table 2. MSE model for different methods when (ρ = 0.9)

σ	$\sigma = 0.5$				$\sigma = 2$			
Sample Size	**FLS**	**FBR1**	**FBR2**	**FBR3**	**FLS**	**FBR1**	**FBR2**	**FBR3**
	q=0	q=0.25	q=0.5	q=0.75	q=0	q=0.25	q=0.5	q=0.75
n=20	3.51022	0.40133	0.34349	0.30897	7.70039	6.60368	5.92221	5.2578
n=40	1.39251	0.23400	0.21843	0.1960	2.82618	2.16559	1.81929	1.56251
n=80	0.69516	0.18212	0.17678	0.1562	1.28446	0.88187	0.72021	0.62389
n=160	0.40135	0.14389	0.1406	0.1251	0.63453	0.41925	0.35852	0.32254

Table 3. MSE model for different methods when (ρ = 0.95)

σ	$\sigma = 0.5$				$\sigma = 2$			
Sample Size	**FLS**	**FBR1**	**FBR2**	**FBR3**	**FLS**	**FBR1**	**FBR2**	**FBR3**
	q=0	q=0.25	q=0.5	q=0.75	q=0	q=0.25	q=0.5	q=0.75
n=20	1.17773	0.74856	0.6244	0.55083	14.48492	12.92856	11.85233	10.64673
n=40	0.46634	0.31786	0.28099	0.25375	5.58186	4.6587	4.08009	3.54998
n=80	0.2533	0.22543	0.21429	0.19189	2.56292	1.96918	1.65634	1.42684
n=160	0.15962	0.18005	0.17475	0.15249	1.23451	0.8419	0.68441	0.59394

Table 4. MSE model for different methods when (ρ = 0.99)

σ	$\sigma = 0.5$				$\sigma = 2$			
Sample Size	**FLS**	**FBR1**	**FBR2**	**FBR3**	**FLS**	**FBR1**	**FBR2**	**FBR3**
	q=0	q=0.25	q=0.5	q=0.75	q=0	q=0.25	q=0.5	q=0.75
n=20	5.5524	4.46548	3.90198	3.40302	72.85769	70.34702	67.93737	64.1689
n=40	2.09189	1.49171	1.2152	1.03512	28.39853	26.94957	25.48581	23.52346
n=80	1.00224	0.65827	0.54153	0.47392	11.65059	10.48169	9.46618	8.36917
n=160	0.49029	0.33466	0.29772	0.2694	5.67868	4.7989	4.16052	3.58002

Discussion through what was obtained in a table**:**

- **i.** In all cases, the fuzzy Bridge regression method (FBR) is superior to the fuzzy least squares method (FLS).
- **ii.** The MSE value decreases with increasing sample size indicating that the two methods' estimators have consistency.
- **iii.** The MSE value increases by increasing the standard deviation.
- **iv.** Increasing the MSE value by increasing the correlation coefficient between the independent variables.
- **v.** The behaviour of the two methods converged when the sample size was increased.

The following figure 2 illustrates this:

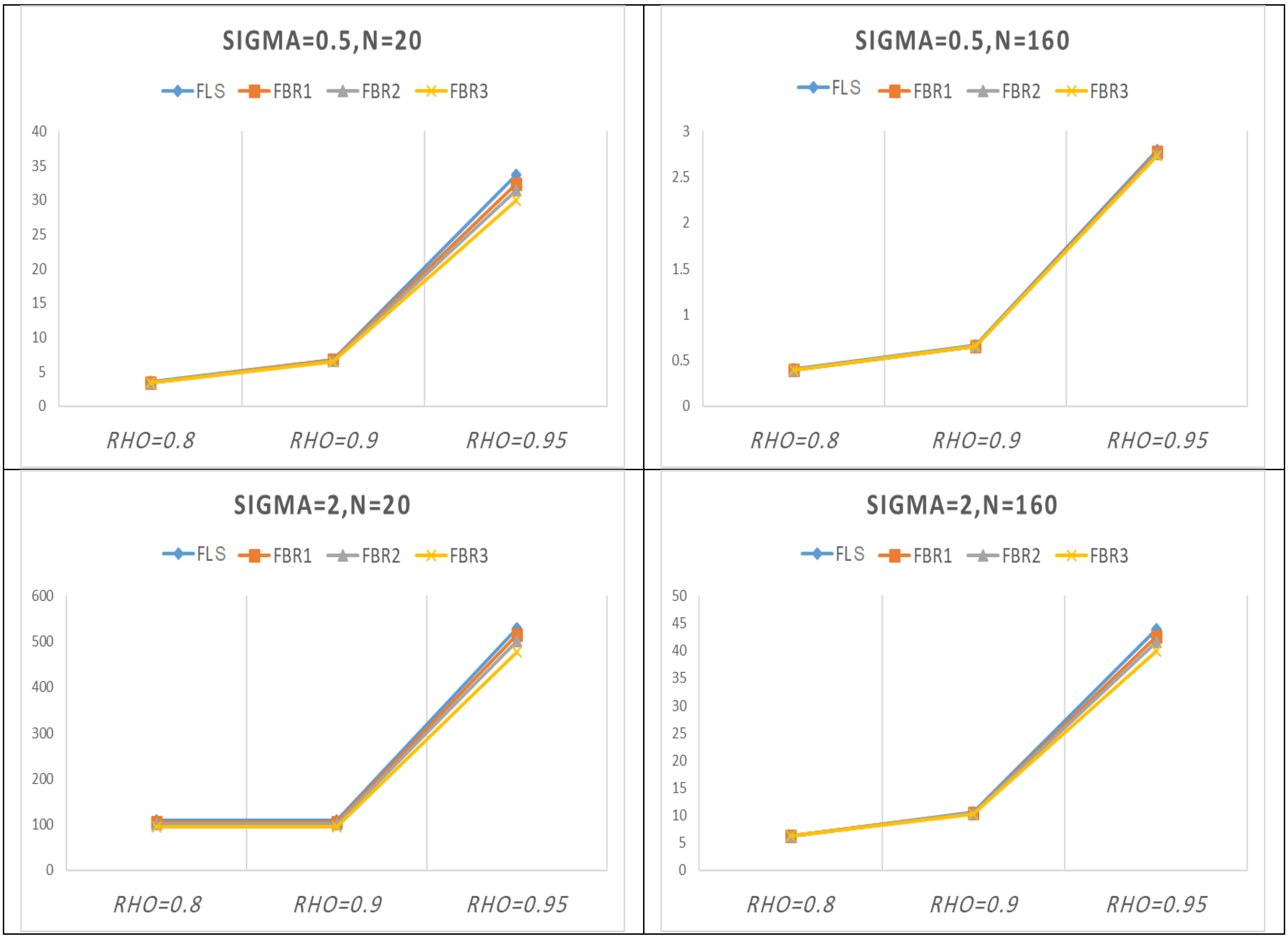

Figure 2. Plotting the MSE values by increasing the value of the correlation coefficient

As we can see from Fig 2. As the correlation coefficient's value increases, the MSE value of the Fuzzy Bridge Regression (FBR) method increases compared to the Fuzzy Least Squares (FLS) method.

4. Conclusions

Through the results presented previously, several conclusions were achieved as below.

i. Using of simulation, it was found that the fuzzy Bridge regression method is better than the least squares method.

ii. The method of estimating the parameters using fuzzy least squares gave less efficiency due to violating its conditions by observing the fuzzy least squares values.

iii. The fuzzy Bridge regression estimation method achieved a new efficiency higher than the fuzzy least squares method.

iv. With increasing the correlation coefficient, the MSE value of the Fuzzy Bridge Regression method (FBR) increases compared to the fuzzy Least Squares (FLS) method.

v. Convergence of the behaviour of the two methods when increasing the sample size.

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مستخلص البحث
ان وجود مشكلة التعدد الخطي في البيانات الضبابية سيودي الى تقدير غير كفؤ في انموذج يمثل البيانات الضبابية بالطرق التقليدية مثل استخدام طريقة المربعات الصغرى الضبابية عندما تكون المعلمات والمخرجات ضبابية والمدخلات اي المتغيرات المستقلة غير ضبابية وللتغلب على هذه المشكلة ، سيتم استخدام انحدار الجسر الضبابي واستخدام الدالة المثلثية عبر تمثيلها بالارقام الضبابية وعن طريق المحاكاة من خلال استخدام أحجام عينات مختلفة ، ستتم مقارنة النتائج بين طريقة المربعات الصغرى وطريقة انحدار الجسر الضبابي باستخدام معيار متوسط مربعات الخطأ (MSE) ومن خلال مقارنة النتائج تم التوصل الى أن أفضل انموذج حقق اقل متوسط مربعات الخطأ هو انموذج انحدار الجسر الضبابي في تقدير الانموذج وحل مشكلة الخطية المتعددة في البيانات حيث حقق أقل MSE من طريقة المربعات الصغرى لأحجام العينات المختلفة المختارة (20، 40، 80، 160).

نوع البحث: ورقة بحثية.
المصطلحات الرئيسة للبحث: الانحدار الخطي الضبابي ، طريقة المربعات الصغرى الضبابية ، انحدار الجسر الضبابي ، التعدد الخطي ، الأرقام الضبابية المثلثية ، ، عامل تضخم التباين.

***البحث مستل من رسالة ماجستير**