

**Effect of thickness on the fractal optical modulator for  
MgF<sub>2</sub>,LiF, Al<sub>2</sub>O<sub>3</sub> materials by testing  
modulation transfer function (MTF)**

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**Abstract:**

The optical modulator or chopper is an important component in optical systems. It is a device, which changes the angle between the vision line of the target and the coordinate of an electrical signal. This paper has been designed as a new model of the fractal optical modulator. A new optical modulator involves two cycles (inner and outer). Each cycle of them consists of eighteen sections. It is known that the sections of the optical modulator become two parts, nine sections considered are transparent and the other sections are opaque to light. There are two parameters have taken to design the optical modulator, first factor is the shape and second factor is the section number. A new optical modulator is made from three materials such as (Magnesium Fluoride (MgF<sub>2</sub>), Lithium Fluoride (LiF) and Sapphire (Al<sub>2</sub>O<sub>3</sub>) by using the fractal function. So an optical modulator is created by building a computer program using the visual basic language. For the importance of the modulation transfer function (MTF) in testing and evaluating optical systems, the dependent measurement is done to know the optical systems efficiency. It has been studied the optical systems with a circular aperture, where that function could be evaluated the image efficiency for point object of image plane at different magnitude of the wavelength and thickness. In this research, the fractal optical modulator has been designed of insulator material by using the fractal function. It has been evaluated the values of MTF at different values of the wavelength and thickness. Also, we studied the relation between spatial frequency and number of sections, as well.

**Keywords:** Optical modulator, MTF, Fractal, Spatial Frequency, Chopper.

## تأثير السمك على كفاءة قرص تضمين بصري مصنوع من مواد شبه موصلة

$MgF_2, LiF, Al_2O_3$  واختباره بواسطة دالة الانتقال المعدلة

فلاح حسن حنون      محمد لطيف جبار      عباس شويح علوان

### الخلاصة:

قرص التضمين البصري مركبة م همة في الانظمة البصرية، فمو عبارة عن ج هاز يقوم بتغيير الزاوية بين خط النظر الى ال هدف و المحور البصري الى اشارة لهر بائية. كما يقوم بتوليد اشارة ضوئية ذات تردد معين ) من خلال تقطيع الحزمة الضوئية ( اي حسب تصميم شكل القرص و كذلك حسب سرعة دوران القرص و بنفس الوقت يمكن أن يعمل عمل المرشحات الضوئية لأطوال موجية معينة حسب نوع المادة المصنوع منها قرص. التضمين البصري).

تم تصميم قرص تضمين بصري يتكون من دائرة داخلية و دائرة خارجية وكل من الدائرتين تتالف من ثمان عشر مقطعاً، و ذلك بواسطة استخدام البرنامج بلغة الفجول بيسك الاصدار 6.0. ولكل دائرة تم افتراض تسع مقاطع منفذة للضوء والتسع الاخرى معتمة بالنسبة للضوء الساقط على القرص البصري. وفي نفس الوقت فان التسع المنفذة تعتبر معتمة للأطوال الموجية غير المرغوب فيها من الطيف الكيرومغناطيسي وذلك بالاعتماد على معامل انكسار المادة المصنوع منها القرص. ان دالة الانتقال المعدلة MTF هي احدى الدوال المستخدمة في فحص مدى كفاءة الانظمة البصرية. حيث بواسطة الدالة يمكن ان نحسب كفاءة الصورة لجسم نقطي وكذلك اُستخدمت دالة الكسوريات في تصميم قرص من ثلاث مواد شبه موصلة ( $MgF_2, LiF, Al_2O_3$ ). ثم تم حساب قيم دالة الانتقال المعدلة (MTF) باخذ مستوى الصورة بالنسبة للجسم عند تغير السمك و الطول الموجي لكل مادة مدروسة في البحث و ايضا تناول دالة الانتقال المعدلة مع التردد الفراغي (الفضائي).

## 1. Introduction

Magnesium Fluoride is grown by vacuum Stockbarger technique in an ingots of various diameters. Magnesium Fluoride is a tough material and polished well.  $MgF_2$  is slightly birefringent and usually supplied with the optical axis perpendicular to the window faces. Lithium Fluoride is grown by vacuum Stockbarger technique in ingots approximately 100mm diameter. Lithium Fluoride cleaves easily and should be worked with highly care. So polishing, particularly steep radii, often causes the surface "rip-out". Whereas Sapphire is grown by a variety of methods. Verneuil and Czochralski methods are usually for standard grade Sapphire material. Higher quality Sapphire, particularly for electronic substrates is manufactured by Kyropulos growth which can be very pure with excellent UV transmission. Large thin sheets of Sapphire is made by ribbon growth. Sapphire is slightly birefringent, so the purpose of IR windows are usually cut in a random way from crystal, but for specific applications, an orientation is selected. Usually this is happened with the optical axis at 90 degrees to the surface plane which is known as "zero degree" material. So synthetic optical sapphire has no coloration [1].

The optical system is passed through several steps to be produced. The optical design is the first step that after this stage is completed, that the optical components manufacturing will be the next stage. Finally, the calculation and the testing of these components are the last steps. The optical design includes specifications for the radii of the surface curvature, the thickness, and the air spaces, the diameters of the various components, the type of glass to be used and the position of the stop. These parameters are well-known as "degrees of freedom" since the designer can change them to maintain the desired system. The image which is formed by these optical systems will approximately correct from the aberrations. But, there isn't an ideal image which corresponds to the dimensions of the object because of the wave nature of the light, is most affected by several factors like the type of illumination is used (coherent, incoherent, and partially coherent), the object shape (Point, Edge or Line) and the aperture shape[2]. Optical modulator any device used to modify any characteristic of an optical signal (light wave) for the purpose of conveying information. Optical choppers are mechanical devices that physically block a light beam of some type. Rotating optical modulators (choppers) are perhaps the most common form and are they produced by SciTech Instrument Ltd. A metal disc with slots etched into its mounted on a dc motor and rotated. The disc is placed in the light beam path which will then cause the beam to be periodically interrupted by the blocking part of the disc [3].

The term fractal was coined by Benoit Mandelbrot. The word fractal has two related meanings. In colloquial usage, it denotes a shape that is recursively constructed or self-similar, that is a shape

that shows similar at all scales of magnification and is often indicated to as "infinitely complex". In mathematics a fractal is a geometric object that satisfies a specific technical condition, namely having a Hausdroff dimension greater than is topological dimensions [4].

### 1.1. Optical Modulator

Optical modulator is a device, which changes the angle between the vision line of the target and coordinate of on electrical signal. It is used to provide directional information about the target, to suppress unwanted signals from background, and used for chopping the emitted light from the source. This will be done by choosing the best shape and size [2,4].

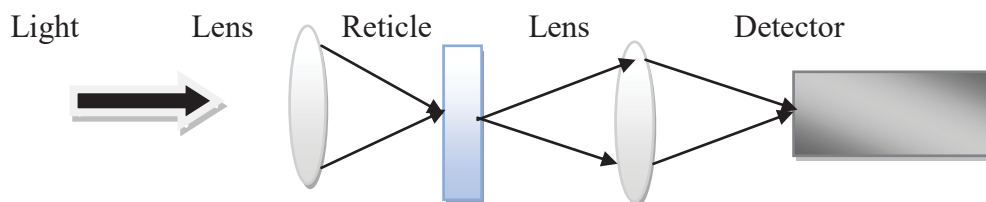


Fig.(1) The Position of the modulator in the optical system.

Reticle has been used and is still used in a multitude of operations from commercial applications or surveying to military uses of boresighting surveillance and fire control systems. The general case that most people is familiar with is the simple sight on a rifle or gun. There are as many types of reticle as there are uses for them. However, one type of reticle, commonly referred to as a spinning frequency modulated (FM) reticle, can be used to provide range and bearing information [5].

The minimum reticle consist of simple "cross-hairs", a crosshair is a shape superimposed on an image that are used for a device precise alignment. Crosshairs are most commonly a "+" shape, although many variations exist, including dots, spots, circles and chevrons. Most commonly associated with telescope sights for aiming firearms, crosshairs are also common to optical instruments used for astronomy and surveying and also popular with graphical user interfaces as a precision pointer. [3]

The optical modulator is a device used for chopping the light beam and the output signal has frequency which can be described by this relation:

$$F = n * \nu \quad (1)$$

Where:  $n$ : number of sectors,  $v$ : the velocity of angular (rotation speed),  $F$ : the spatial frequency.

The modulation operation in optical modulator depends on the movement between image object and optical modulator. It is classified in two types: -

1-Rotating Reticle Disk: In this type, the disk rotates around its axis, while the object images rotate within the disk area. Sometimes, the disk axis has been rotated around the optical axis of the Electro-Optical-System, in circular path. This type of disk is called (Nutating Reticle).

2-Stationary Reticle Disk: In the second type of this disk is stationary, while the image object has been rotated on the disk surface by using rotational optical system.

The optical modulator has two important operations in detection, tracking and guidance system, and this operation are to:-

1-Provide directional information about tracking and to suppress unwanted signals from the background. This operation is called (Spatial-Filtering).

2-Change the optical signal parameter, which is produced from the object, by designing suitable disk pattern.

The better efficiency of the optical modulator can be produced when the spot size is not larger than three times the object image size. The real efficiency is produced when the spot size is equal to the object image. When the object image approaches the optical system, its size will be increased[2,6]

## 1.2. AM and FM Optical Modulator

One of the optical modulator shapes is (Fan Shape), and sometimes called (Wagon Wheel), it is shown in Fig.(2) and it is used in many optical applications, In radiation measurement system, it is used as optical chopper. Therefore, it is used in optical modulation in most Tracking and Guidance systems.

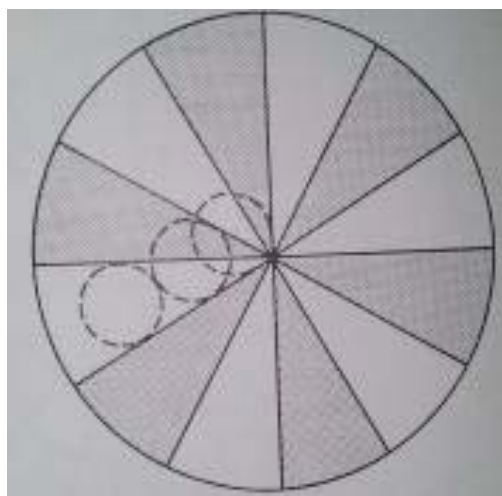


Fig. (2): wagonwheel patter

This type of optical modulator works in two modes:

The first mode, is when the optical modulator is rotated around its axis. Then, the incident radiation will be modulated in amplitude modulation (AM) .

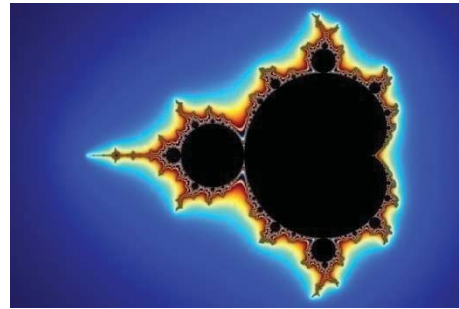
The second mode, is when the optical modulator is stationary, while the object scene rotates about the disk axis by nutating movement. Or the optical modulator center will be rotated about the optical axis of the tracking system. then the incident radiation will be modulated in frequency modulation (FM) [7,8].

### **1.3. Fractal Function**

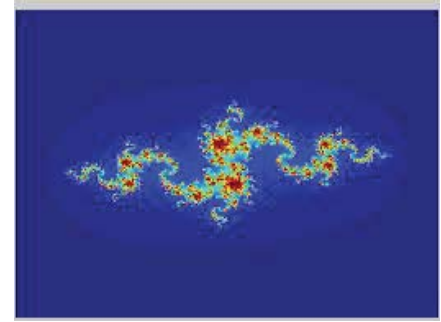
Euclidean geometry provides a first approximation to the structure of physical objects. It describes objects of simple shapes, point, line segments, ellipses, circles, boxes, and cubes that have a few characteristic sizes, with dimensions one, two, and three. This geometry is mainly oriented a round linear, integral system[9].

Nonlinear shapes and nonintegral systems are not easily described by the traditional Euclidean geometry. These shapes and systems need another geometry that is quite different from Euclidean geometry to describe and study these cases. Benoit Mandebort [10] suggested the existence of geometries near to the geometry of nature, known as fractal geometry.

Mandelbrot's famous and pioneering work with fractal geometry and his introduction to two new basic concepts including; first, self-similarity, which is to say that the fractal shapes are to be self-similar and independent of scales or scale. The general nature of the fractal irregular bumpy structure remain constant through successive magnifications such as is the case for coastlines and mountains. Each small portion when magnified can reproduce exactly a large portion. Fractal images to exist as the limit of both random and deterministic processes based upon the representation named Iterated Function System (IFS). Second, a fractal has non-integer dimension known as the fractal dimension, which allows scale independent measurement of such objects, and gives a numerical measure of the degree of boundary irregularity or surface roughness. The fractal dimensions one of the most important concepts in the study distribution. It is analogous to the concepts of length, area and volume in Euclidean Geometry [11]. From examples of nonlinear fractals: Mandelbrot set, and Julia set which they are shown in Fig. (3).



(a)-Mandelbrot set



(b)- Julia set

Fig. (3): Mandelbort and Julia sets

Now it is seen an alternative way to specify the dimension of a self-similar object. The dimension is simply the exponent of the number of self-similar pieces with magnification factor  $N$  in which the figure may be broken,

$$N = \left( \frac{L}{K} \right)^{D'} \quad (2)$$

$$D' = \frac{\text{Log}N}{\text{Log}\left(\frac{L}{K}\right)} \quad (3)$$

where  $D'$  is fractal dimension,  $N$ : number segment,  $L$ : length,  $K$ : length each piece.

#### 1.4. Iterated function system (IFS)

Fractals as they are normally called can be any number of dimensions, but are commonly computed and drawn in 2D. The fractal is made up of the union of several copies of itself, each copy being transformed by a function. This is the source of its self-similar fractal nature that [12]:

$$s = \bigcup_i f_{i(s)} \quad \text{Where } s \subset R^2 \text{ and } f_i: R^2 \rightarrow R^2 \quad (4)$$

with  $(i=1, 2, 3, 4, \dots, m)$ .

Sometimes each function  $f_i$  is required to be linear, or more accurately an affine transformation and hence can be represented by a matrix.

$$w \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix} = \begin{bmatrix} ax + by + e \\ cx + dy + f \end{bmatrix} \quad (5)$$

where

$(x, y)$ : a metric space,  $(e, f)$ : transformation parameters

$(a, b, c, d)$ : real numbers (in two-dimensional)

However, IFSs may also be built from non-line a function, including projective transformations, as well. One can describe a general construction for fractal that occurs in classical number theory, of which sierpinski triangle, von koch curve, and cantor set are examples.

### 1.5. Cantor Set

In order to understand the cantor set, the construction becomes with a line segment of length (1) which is subdivided into three sections, removing the middle third; then removing the middle third of the remaining segment and so on. So, the number of segments is increased to two and length is reduced to (2/3). The cantor set is simply the dust of pointing remain. The number of these points is infinite, but their total length is zero. As shown in fig.( 4)[13].

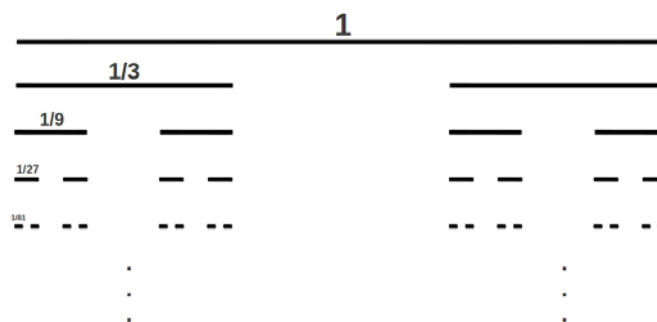


Fig.(4) Construction of the classical cantor set

### 1.6. OTF & MTF

Another method to specify the resolving power of an optical imaging system is by means of the optical transfer function (OTF). This function is defined as the contrast in the image of a sinusoidal grating with a given spatial frequency,

$$\omega = 2\pi/L \tag{6}$$

Let us assume that we form the image of an object containing a wide spectrum of spatial frequencies. Then analyze the frequency content of the image of this object. Then, the OTF is the ratio of the amplitude of a given spatial frequency in the image to the amplitude of the component with the same spatial frequency in the object. If the object contains all spatial frequencies of a constant amplitude, the OTF becomes the Fourier transforms into the image. Such an object is the point object and its image is point spread function (PSF). Hence, the OTF is simply the Fourier transforms from the point spread function. the optical transfer function  $F(\omega_x, \omega_y)$  may be obtained from the Fourier transform of the point spread function  $S(x,y)$  as follows:

$$F(\omega_x, \omega_y) = \iint S(X_F, Y_F) e^{i(\omega_x X_F + \omega_y Y_F)} dx_F dy_F \tag{7}$$



We see that in general this OTF is complex and, thus it has a real and an imaginary term. The modulus of the OTF refer to the modulation transfer function (MTF) and represents the contrast in the image of a sinusoidal periodic structure. The imaginary term receives the name of phase transfer function (PTF) and gives information about the spatial phase shifting or any contrast reversal (when the phase shift is  $180^\circ$ ) in the image [14].

The MTF is then the magnitude response for the imaging system to sinusoids of different spatial frequencies. This response can also be defined as the attenuation factor in modulation depth:

$$M = \frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}} \quad (8)$$

where  $A_{\max}$  and  $A_{\min}$  denote to the maximum and minimum values of the waveform that describe the object or image in  $W/cm^2$  versus position. The modulation depth is actually a measure of visibility or contrast. The effect of the finite-size impulse response of the optical system is to decrease the modulation depth of the image relative to that in the object distribution. This attenuation in modulation depth is a position function in the image plane. The MTF is the ratio of image-to-object modulation depth as a function of spatial frequency[15]:

$$MTF = \frac{M_{img}}{M_{obj}} \quad (9)$$

Or

$$MTF = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} \quad (10)$$

In summary, the MTF is a powerful tool used to characterize the imaging system's ability to reproduce signals as a function of spatial frequency. It is a fundamental parameter that determines where the performance limitations in optical and electro-optical systems occur, and the crucial components must be enhanced to yield a better overall image quality[15].

## 2. Results and Discussion

The optical modulator is an important component in optical system. The optical modulator is a disc from semiconductor materials which has a radius  $R_1$  and  $R_2$  where  $R_1$  refers to the radius inner of the circle and  $R_2$  indicates to the radius outer of the circle, and assumed the number of sectors is eighteen sectors. The chopper is designed by the computer program, which was written by using visual basic language. In the present study it is assumee that nine sectors opaque and nine sectors are transmitted alternating to the light as shown in Fig. (5). Fractal function was used to divide optical

modulator into very small segments of line, which is simply the dust of point. The circular aperture is a clear transparency aperture (100%).

if the incident lights are vertical to the chopper. The chopper is moving in a circular form. Hence, the light beam will make discrete circles according to the number of sectors. The distance from the light movement on all sectors of it is parting is an arc from the circumference of the total circle. Thus, the light form will through one revolution of the radius (the point of beam incidence of light on the sector). It is considered that the arc of a sector, which moves on the opaque sector are the required distance only. The resultant circumference of the circle was divided among the total number of sectors.

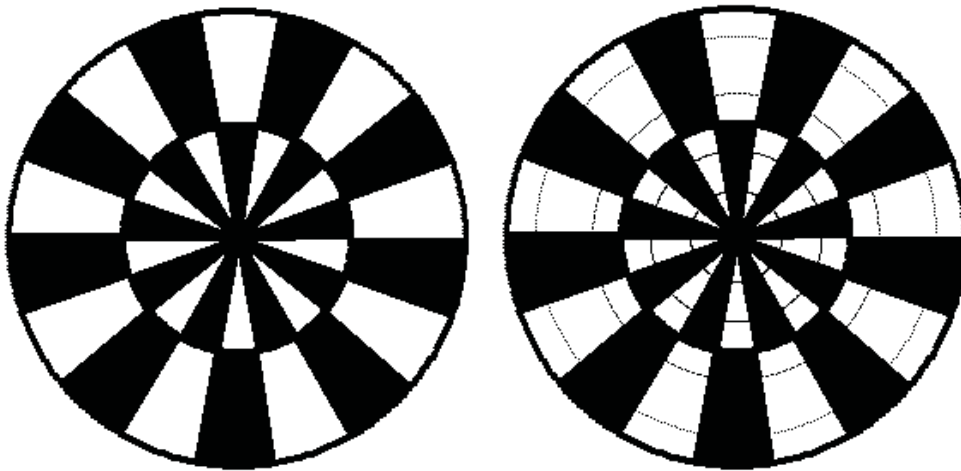


Fig.(5) optical modulator and fractal optical modulator (design by searchers)

Figures (6),(7),(8) show the behavior modulation transfer function (MTF) with wave-lengths of various thicknesses of Magnesium Fluoride ( $MgF_2$ ), Lithium Fluoride (LiF) and Sapphire ( $Al_2O_3$ ) respectively. All figures to reveal that the MTF is decreasing by increasing the thickness when the same wavelength is used.

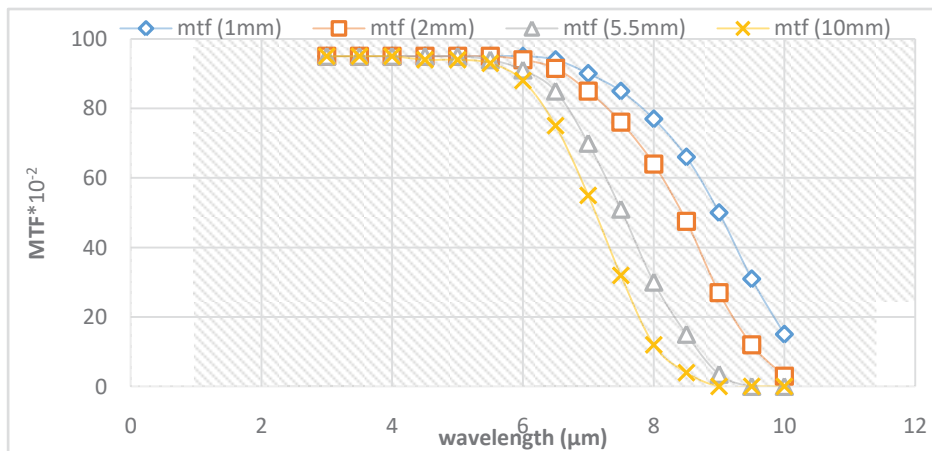


Fig.(6) wavelengths versus MTF for  $MgF_2$  at different thicknesses

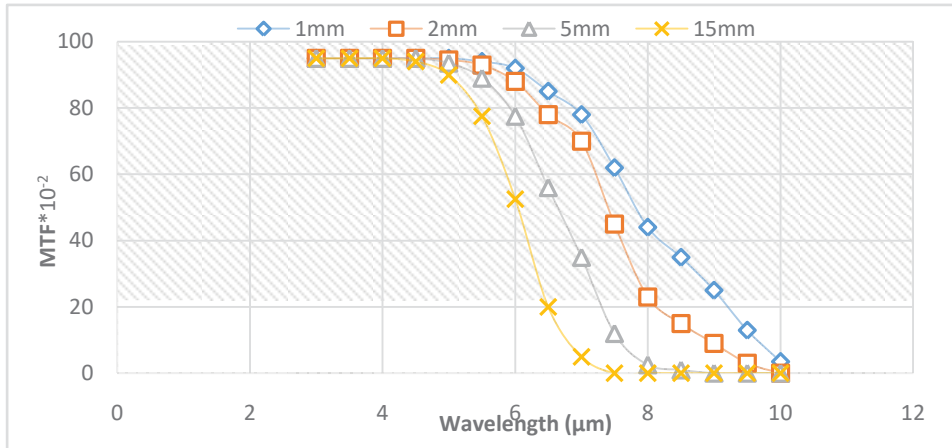


Fig.(7) wavelengths versus MTF for LiF at different thicknesses

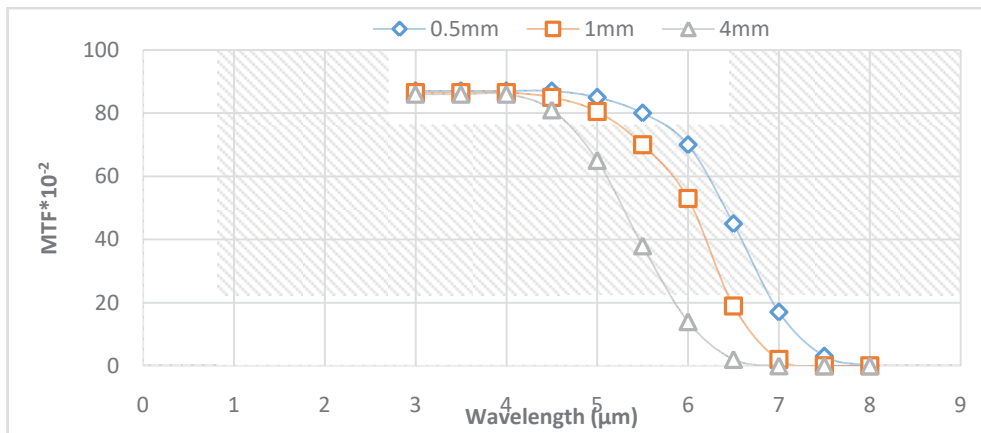


Fig.(8) wavelengths versus MTF for AL<sub>2</sub>O<sub>3</sub> at different thicknesses

A comparison of the behavior of MTF at thickness of 1mm is shown figure (9). It is clear that both MgF<sub>2</sub> and LiF have the same variation whereas AL<sub>2</sub>O<sub>3</sub> behaves slightly different where the gradient of decreasing is sharp with increasing wavelengths.

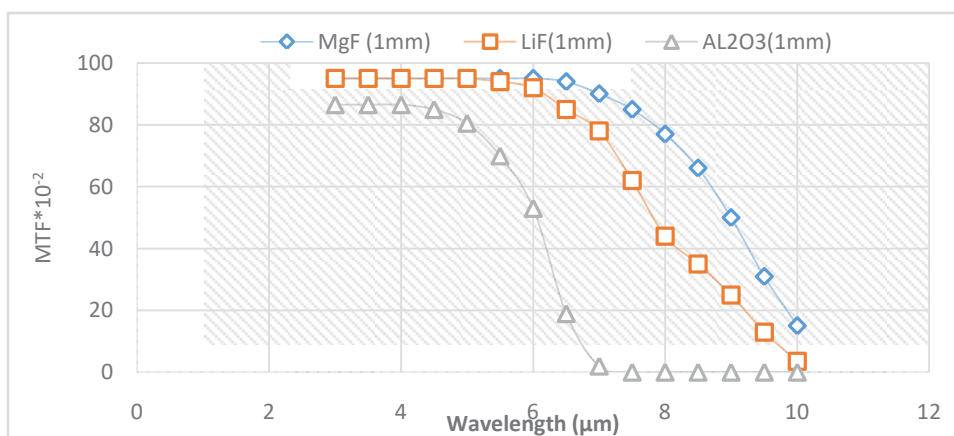


Fig.(9) wavelengths versus MTF for different materials at (1mm)

The unit of spatial frequency will be in (Rev/s) which depends on the velocity, and the number of sectors. Equations 1 and 10 have been simulated utilizing visual basic environment in order to obtain values of MTF and spatial frequency. The result of these equations has been plotted as shown in figures (10-15). The obtained behavior in these figures is agreement with the other published paper [16]. The relation between MTF and spatial frequency (f) for the three materials under the study has been plotted in fig. (10) at 1mm thickness. Figures 11 and 12 show the same behavior of MgF<sub>2</sub> and LiF at 5 and 2 mm, respectively. The relation of (MTF) with spatial frequency for the three materials individually at different thickness. Eventually, it is clear that the general behavior of these figures show a transition point at a value of (2) which agrees with other studies, “The first inflection point always gets at the spatial frequency value equal 2 (Rev/s) which are compatible with the MTF value for theoretical real optical system theoretically” [17]. The spatial frequency is very small at the maximum value of MTF, and the spatial frequency begins increase with decreasing of MTF values.

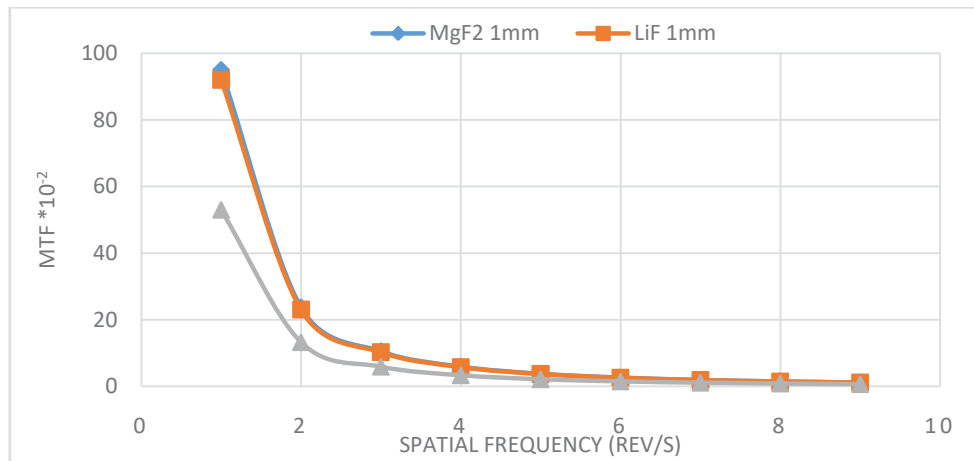


Fig.(10)spatial frequency versus MTF at thickness 1mm and wavelength 6 μm

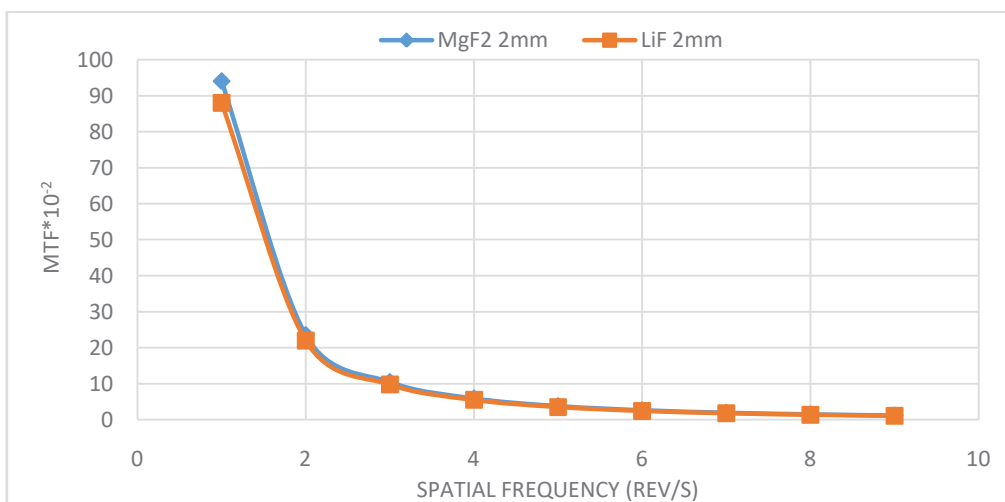


Fig.(11) spatial frequency versus MTF at thickness 2mm and wavelength 6 μm

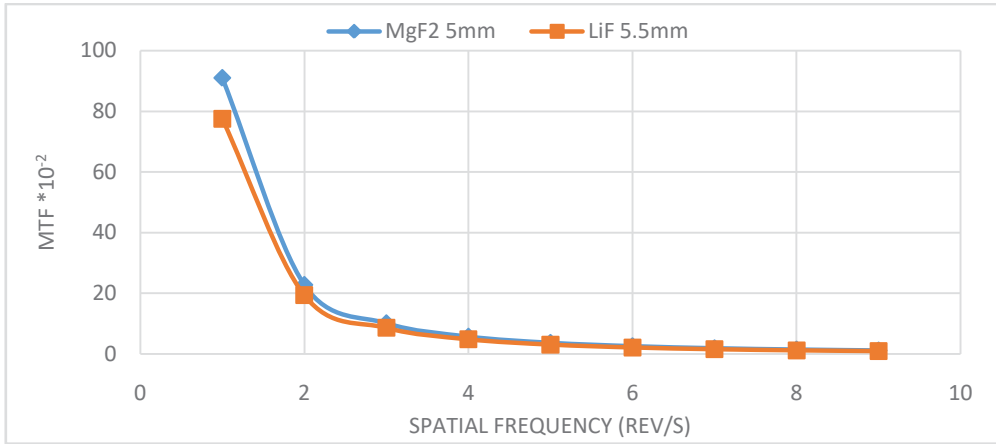


Fig.(12) spatial frequency versus MTF at thickness (5,5.5)mm and wavelength 6  $\mu$ m

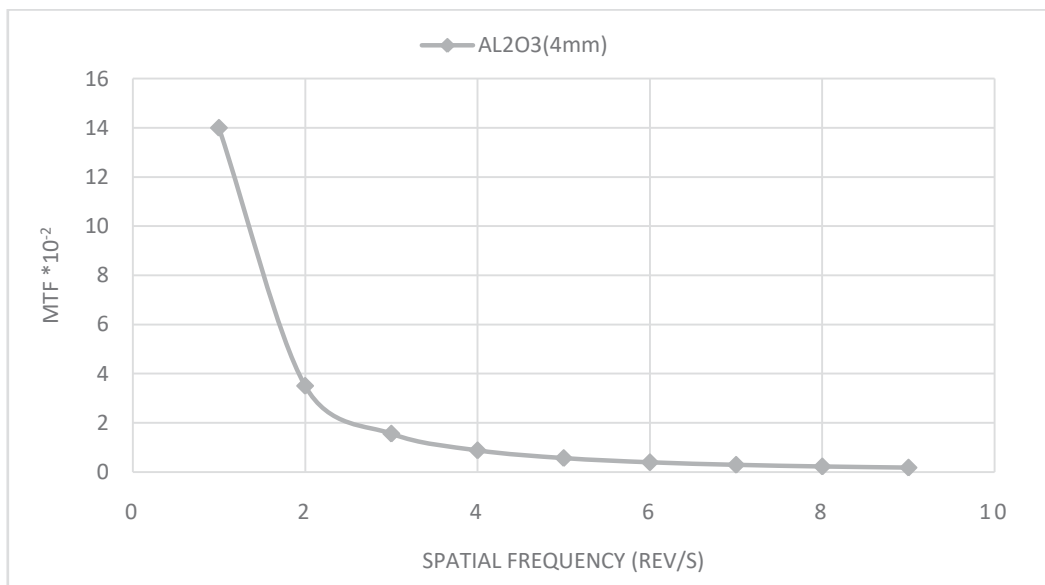


Fig.(13) spatial frequency versus MTF at thickness 4mm and wavelength 6  $\mu$ m

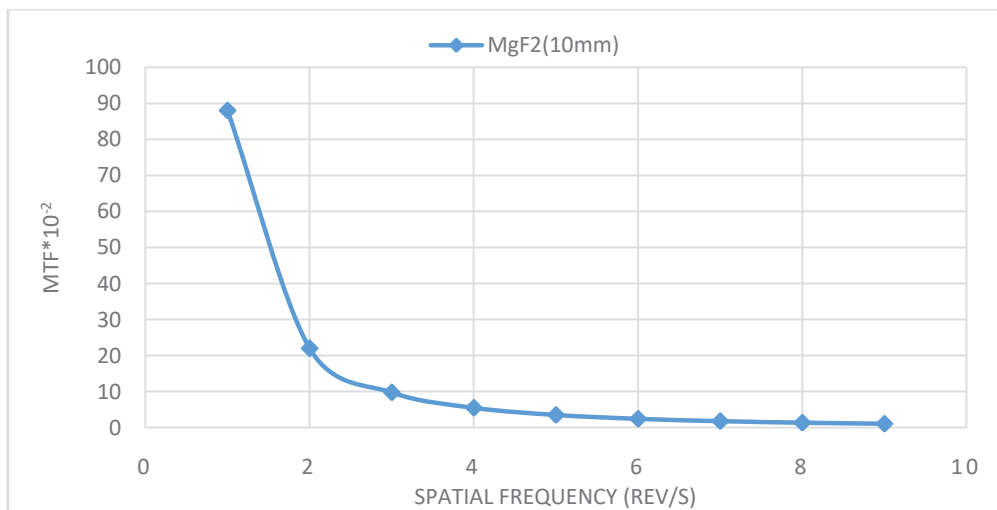


Fig.(14) spatial frequency versus MTF at thickness 10mm and wavelength 6  $\mu$ m

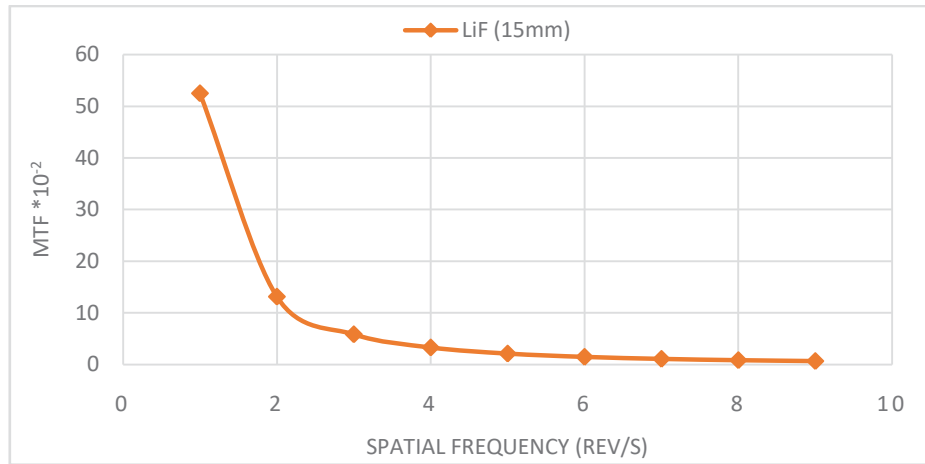


Fig.(15) spatial frequency versus MTF at thickness 15mm and wavelength 6  $\mu\text{m}$

### 3. Conclusion

It has been designed a new model for fractal optical modulator which consists of inner circle and outer circle throughout the simulation in visual basic 6.0. The comparison between the values of MTF at different thickness of values for different types, showed that at big values of thicknesses the value of MTF becoming low (with increasing the thickness, the MTF will decrease), and this means that the detector has low efficiency. The high inflection points for MTF changes of material thickness and type of fractal optical modulator changes. It has been visualized that the values of thickness are increased versus the spatial frequency is increased, i.e. that fractal optical modulator works well. There is a considerable increase in Modulation Transfer Function (MTF) with decreasing of the spatial frequency. It is very important in the optical fractal modulator, when it is designed from a specific material such special filter. The  $\text{Al}_2\text{O}_3$  has a lower value of MTF. Finally, the best material may be used to make the optical modulator is Magnesium Fluoride because it must to value larger of MTF than other materials

### 4. Acknowledgements

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