

## **Fractal Image Compression using Quantum PSO**

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### **Abstract:**

The self-similarity property is used by Fractal Image Compression (FIC) of a natural image. FIC performs two processes encoded and decoded. The encoding time is very large for most existing algorithms to obtain the coded image. A new particle swarm optimization (PSO) technique for fast fractal encoding is proposed by many types of research to reduce the encoding time. In this paper, Optimizing PSO by Quantum to reduce the encoding time. Quantum particle swarm optimization (QPSO) technique can speed up the fractal encoder and preserve the image.

**Keywords:** Fractal image compression; quantum particle swarm optimization; particle swarm optimization; encoding time.

## ضغظ الصور باستخدام الفراكتل و PSO الكمية

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### الخلاصة:

استخدمت طريقة ضغظ الصور الكسورية خاصية التشابه الذاتي لضغظ الصور. الطريقة تتم بمرحلتين الترميز وفك الترميز حيث مرحلة الترميز تتطلب وقت طويل جدا للحصول على الصورة المضغوطة. لذلك اقترحت طريقة جديدة باستخدام PSO لتسريع من قبل عدة باحثين لتقليل وقت الترميز. في هذا البحث استخدمه PSO مع الكوانتم لتقليل الوقت. تقنية QPSO تستطيع تسريع الترميز مع الحفاظ على معالم الصورة

## 1- Introduction

The idea of fractal image compression (FIC) was originally introduced by Barnsley[1] and the first practical FIC scheme was realized by Jacquin in 1992 [2]. It is based on the partitioned iteration function system (PIFS) which utilized the self-similarity property in the image to achieve the purpose of compression [3]. The encoding process of the fractal image compression is time-consuming. The reason is that most of the encoding time is spent on a lot of computations of the similarity measure. Hence one of the main research directions for fractal image compression is focused on how to reduce the encoding time. In the past, FIC with some classification methods were usually adopted, in which ranges and domains were classified in the pre-processing step. At each search entry, only domains with a similar class were examined[4-6]. Domain pool reduction is another technique to reduce the encoding time. A common method was to restrict the search space to those blocks whose spatial location was near the range location [7]. Wavelet transform is used to decompose the original image to various frequency sub-bands in which the attributes can be extracted from the wavelet coefficients belonging to different sub-bands. The distribution of wavelet coefficients can be used in context-based multiscale classification of document image[8]. The fast and efficient algorithm [9] was applied to triangular mesh to approximate surface data using wavelet transform coefficients. It directly determined local area complexity in an image and divides square cells depending on complexity. In [10], the authors implemented a hybrid image classification method combining wavelet transform, rough set approach, and artificial neural.

The particle swarm optimization (PSO) is focused gradually, which is one of the commonly used evolutionary algorithms[11-15]. PSO is an optimization algorithm having origins from evolutionary computation together with social psychology principle. Essentially, PSO is dependent on stochastic processes and also uses the concept of fitness similar to the negative of the cost of Genetic Algorithm (GA).

A quantum-inspired version of the PSO algorithm (QPSO) was proposed very recently [16]. The QPSO algorithm permits all particles to have a quantum behavior instead of

the classical Newtonian dynamics that was assumed so far in all versions of the PSO. Thus, instead of the Newtonian random walk, some sort of “quantum motion” is imposed in the search process. When the QPSO is tested against a set of benchmarking functions, it demonstrated superior performance as compared to the classical PSO but under the condition of large population sizes. One of the most attractive features of the new algorithm is the reduced number of control parameters. Strictly speaking, there is only one parameter required to be tuned in the QPSO.

## 2- The Classical PSO

It will be very instructive to review first the basics of the PSO method in order to introduce the quantum version. In the Standard PSO model, each individual is treated as a volume-less particle in the D-dimensional space, with the position and velocity of *i*th particle represented as

$$V_i(t + 1) = w \times V_i(t) + c_1 \times \text{random} \times (P_i - X_i(t)) + c_2 \times \text{random} \times (P_g - X_i(t)) \quad (1)$$

$$X_{i(t+1)} = X_i(t) + V_i(t + 1) \quad (2)$$

where

- $c_1$  and  $c_2$  are positive constant.
- random is a random number in the range of [0,1].
- Parameter  $w$  is the inertia weight introduced to accelerate the convergence speed of the PSO.
- The vector  $P_i = (P_{i1}, P_{i2}, P_{i3}, \dots, P_{iD})$  is the best previous position (the position giving the best fitness value) of particle *i* called pbest, and the vector  $P_g = (P_{g1}, P_{g2}, P_{g3}, \dots, P_{gD})$  is the position of the best particle among all the particles in the population and called gbest.

The steps involved here is the population size is first determined, and the velocity and position of each particle are initialized. Each particle moves according to [17], and the fitness is then calculated. Meanwhile, the best positions of each swarm and particles are recorded. Finally, as the stopping criterion is satisfied, the best position of the swarm is the final solution. The main steps are given as follows:

1. Set the swarm size. Initialize the velocity and the position of each particle randomly.
2. For each  $i$ , evaluate the fitness value of  $x_i$  and update the individual best position  $P_i$ , if better fitness is found.
3. Find the new best position of the whole swarm. Update the swarm best position  $x_i$ . if the fitness of the new best position is better than that of the previous swarm.
4. If the stopping criterion is satisfied, then stop.
5. For each particle, update the position and the velocity according (1)and (2).  
Go to step2.

### 3-Quantum Particle Swarm Optimization

The QPSO algorithm allows all particles to move under quantum-mechanical rules rather than the classical Newtonian random motion. In the classical environment, all bees are flying toward the optimum location. The particles are then attracted to this location through the optimization process. Such attraction leads to the global optimum. From equation (1) it is easy to see that is nothing but a random average of the global and local bests of the particles of the swarm. In quantum mechanics, the governing equation is the general time-dependent Schrödinger equation.

In Quantum-behaved Particle Swarm Optimization (QPSO), the particle moves according to the following equation:

$$mbest = \frac{1}{M} \sum_{i=1}^M P_i = \left( \frac{1}{M} \sum_{i=1}^M P_{i1}, \frac{1}{M} \sum_{i=1}^M P_{i2}, \dots, \frac{1}{M} \sum_{i=1}^M P_{id} \right) \quad (3)$$

$$p_{id} = \varphi * P_{id} + (1 - \varphi) * P_{gd}, \quad \varphi = rand() \quad (4)$$

$$X_{id} = p_{id} \pm \alpha * |mbest_d - X_{id}| * \ln(1/u), \quad u = Rand() \quad (5)$$

Where

- $mbest$  is the mean best position among the particles.
- $P_{id}$ , a stochastic point between  $P_{id}$  and  $P_{gd}$ , is the local attractor on the  $d$ th dimension of the  $i$ th particle.
- $\Phi$  and  $u$  are a random number distributed uniformly on  $[0,1]$ .
- $A$  is a parameter of QPSO that is called Contraction-Expansion Coefficient.

The Quantum-behaved Particle Swarm Optimization (QPSO) algorithm is described as follows.

1. Initialize an array of particles with arandom position inside the problem space.
2. Determine the mean best position among the particles by Eq(3)
3. Evaluate the desired objective function (for example minimization) for each particle and compare with the particle's previous best values: If the current value is less than the previous best value, then set the best value to the current value. That is, if  $f(X_i) < f(P_i)$  then  $X_i = P_i$ .
4. Determine the current global position minimum among the particle's best positions. That is:  $g = \arg \min_{1 \leq i \leq M} (f(P_i))$  M is the population size).
5. Compare the current global position to the previous global: if the current global position is less than the previous global position; then set the global position to the current global.
6. For each dimension of the particle, get a stochastic point between  $P_{id}$  and  $P_{gd}$ : Eq(4)
7. Attain the new position by stochastic equation: Eq(5)
8. Repeat steps (2)-(7) until a stop criterion is satisfied OR a pre-specified number of iterations are completed.

#### **4- Fractal Image Compression**

The fundamental idea of fractal image compression is the Iteration Function System (IFS) in which the governing theorems are the Contractive Mapping Fixed-Point Theorem and the Collage Theorem [3]. For a given gray level image of size  $N \times N$ , let the range pool R be the set of the  $(N/L)^2$  non-overlapping blocks of size  $L \times L$  which is the size of encoding unit. Let the counteractivity of the fractal coding be a fixed quantity of 2. Thus, the domain pool makes up the set of  $(N - 2L + 1)^2$  overlapping blocks of size  $(2L \times 2L)$ . For the case of  $256 \times 256$  image with  $8 \times 8$  coding size, the range pool contains 1024 blocks of size  $8 \times 8$  [ $(256/8) \times (256/8) = 1024$ ], and the domain pool contains 58081 blocks of size  $16 \times 16$  [ $(256 - 16 + 1) \times (256 - 16 + 1) = 58081$ ].

For each range block r in R, one searches in the domain pool D to find the best match, i.e., the most similar domain block. The parameters describing this fractal affine

transformation form the fractal compression code of  $r$ . At each search entry, the domain block is first down-sampled to  $8 \times 8$ . In fractal coding, it is also allowed a contrast scaling  $p$  and a brightness offset  $q$  on the transformed blocks. Thus, the fractal affine transformation  $\Phi$  of  $u(x, y)$  in  $D$  can be expressed as:

$$\Phi \begin{bmatrix} x \\ y \\ u(x, y) \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & 0 \\ 0 & 0 & p \end{bmatrix} \begin{bmatrix} x \\ y \\ u(x, y) \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \\ q \end{bmatrix} \quad (6)$$

Where the  $2 \times 2$  sub-matrix  $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$  is one of the Dihedral transformations and  $(t_x, t_y)$  is the coordinate of the domain block in the domain pool. In each search entry, there are eight separate MSE computations required to find the index  $d$  such that:

$$d = \operatorname{argmin}\{MSE((p_k u_k + q_k), v): \quad k = 0, 1, \dots, 7\} \quad (7)$$

Where

$$MSE(u, v) = \frac{1}{L^2} \sum_{i,j=0}^{L-1} (u(i, j) - v(i, j))^2 \quad (8)$$

Here,  $p_k$  and  $q_k$  can be computed directly as

$$p_k = \frac{[L^2 \langle u_k, v \rangle - \sum_{i=0}^{L-1} \sum_{j=0}^{L-1} u_k(i, j) \sum_{i=0}^{L-1} \sum_{j=0}^{L-1} v(i, j)]}{[L^2 \langle u_k, u_k \rangle - (\sum_{i=0}^{L-1} \sum_{j=0}^{L-1} u_k(i, j))^2]} \\ < u_k, v \rangle = \sum_{i=0}^{L-1} \sum_{j=0}^{L-1} u_k(i, j) \cdot v(i, j) \\ < u_k, u_k \rangle = \sum_{i=0}^{L-1} \sum_{j=0}^{L-1} u_k(i, j)^2 \quad (9)$$

$$q_k = \frac{1}{L^2} \left[ \sum_{i=0}^{L-1} \sum_{j=0}^{L-1} v(i, j) - p_k \sum_{i=0}^{L-1} \sum_{j=0}^{L-1} u_k(i, j) \right] \quad (10)$$

As  $u$  runs over all of the 58081 domain blocks in  $D$  to find the best match, the terms  $t_x$  and  $t_y$  in (6) can be obtained. Together with  $d$  and the specific  $p$  and  $q$  corresponding this  $d$ , the affine transformation (6) is found in the given range block  $v$ . In practice,  $t_x$ ,  $t_y$ ,  $d$ ,  $p$ , and  $q$  can be encoded using 8, 8, 3, 5, and 7 bits, respectively, which are regarded as the compression code of  $v$ . Finally, as  $v$  runs over all of the 1024 range blocks in  $R$ , the encoding process is completed. To decode, one first makes up the 1024 affine transformations from the compression codes and chooses any image as the initial one. Then, one performs the 1024 affine transforms on the image to obtain a new image and proceeds recursively. According to Partitioned Iteration Function Theorem (PIFS), the sequence of images will converge. The stopping criterion of the

recursion is designed according to user's application. The final image is the retrieved image of fractal coding.

### 5 - Fractal Image Compression using Quantum PSO

Fractal Image Compression is used to search the near-best matches so as to speed up the encoder. Since the evaluated value of the sub-optimum is close to that of the best match, the quality of the retrieved image can be preserved. As discussed in Section 4, the parameters  $t_x, t_y, d, p$ , and  $q$  constitute the fractal code. In the proposed method, we encode the particle as  $(t_x, t_y)$ , which is the position of the domain block. The quantities  $p$  and  $q$  can be calculated from (9) and (10), and  $k$  is searched separately. At each search entry best index,  $d$  in (7) can be obtained. The fitness value of a particle is defined as the minus of the minimal MSE produced  $-MSE((p_d u_d + q_d), v)$ . When the stopping criterion is satisfied, the final  $p_g$  with the corresponding  $d, p$ , and  $q$  is the fractal code of the given range block  $v$ . The steps of encoding a range block using QPSO are summarized as follows:

Algorithm 1: Fractal Image Compression using Quantum PSO.

**Input:**  $v$  range and Pool domain  $D$ .

Output: the contrast scaling  $p_k$  and brightness offset  $q_k$ .

- 1: Initialize the parameters of QPSO
- 2: Initialize an array of particles with the random position.
- 3: Calculate the  $m_{best}$  by Eq(3).
- 4: For each particle  $(t_x, t_y)$ , update the particle by Eq (4) and Eq (5).
- 5: Get the domain block at  $(t_x, t_y)$  in the image. Sub-sample the block and denote it by  $u$ .
- 6: Calculate  $p_k$  and  $q_k$ .
- 7: Find the fitness of the particle corresponding to the best parameter  $d$  by Eq (7).
- 8: If the current particle < previous best particle then set the best value to the current value.
- 9: Determine the current global position minimum among the particle's best positions.
- 10: Repeat steps(3-9) until a stop criterion is satisfied OR a pre-specified number of iterations are completed
- 11: Compute  $p_k$  and  $q_k$  by Eq (9) and Eq (10).



12: Return  $p_k$  and  $q_k$

### 6- Experimental Results

This section experiments the proposed algorithm and compares the proposed algorithm with the performance of PSO in fractal image compression. The proposed algorithm is examined on images Lena, Pepper and Baboon with the size of  $256 \times 256$  and grayscale. The size of range blocks is considered as  $8 \times 8$  and the size of domain blocks is considered as  $16 \times 16$ . In order to compare the quality of results, the PSNR test is performed:

$$PSNR = 10 \times \log \left( \frac{255^2}{\frac{1}{m \times n} \sum_{i=1}^n \sum_{j=1}^m (f(i, j) - g(i, j))^2} \right) \quad (11)$$

Where  $m \times n$  is the size of image. In our experiments, the population size of the swarm is set to be 10 and the maximum number of iterations is set to be 10. Table 1 shows the experimental results on the proposed method, Full Search method and PSO method. According to Tables 1, the proposed algorithm improves the performance of fractal image compression for all the experimental results.

Table 1. Comparison of proposed method with full search and PSO method

| Image  | Method      | PSNR  | Time |
|--------|-------------|-------|------|
| Lena   | Full Search | 28.91 | 3135 |
|        | PSO         | 28.65 | 64   |
|        | QPSO        | 26.41 | 59   |
| Pepper | Full Search | 29.84 | 3145 |
|        | PSO         | 28.35 | 64   |
|        | QPSO        | 26.31 | 58   |
| Baboon | Full Search | 20.15 | 2966 |
|        | PSO         | 19.77 | 64   |
|        | QPSO        | 18.02 | 59   |

### 7- Conclusion

In this paper, a quantum particle swarm optimization (QPSO) in the fractal image compression has been successfully introduced to minimize time. The premature convergence of the traditional PSO algorithm is weakened and the convergence of it is also accelerated. Furthermore, the performance to accomplish the global optimization is also improved subject to the chaos method. Such a method can speed up the encoder and also preserve the image quality. The simulation shows that the encoding time of our method faster than that of the full search method and traditional PSO, whereas the retrieved Lena image quality is still relatively acceptable.

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