Journal of College of Education for pure sciences(JCEPS)
Web Site: http://eps.utq.edu.iq/ Email: com@eps.utq.edu.iq
Volume 7, Number 1, January 2017

Characterization of Some Intuitionistic Fuzzy Subsets of Intuitionistic Fuzzy IdealTopological Spaces

Amneh KareemYousif

Amneh.almusawi@yahoo.com

Department of Mathematics , College of Education for Pure Sciences , University of Thi-Qar.

Abstract

In this paper we study some subsets of intuitionistic fuzzy ideal topological space like intuitionistic fuzzy regular-I-closed set, intuitionistic fuzzy A_I −set, intuitionistic fuzzy f_I −set and intuitionistic fuzzy-I-locally closed set. Also, we characterizeτ −codense intuitionistic fuzzy ideals in terms of intuitionistic fuzzy A_I −set, intuitionistic fuzzy regular-I-closed set and intuitionistic fuzzy-I-locally closed set and prove some result about them.

Keywords: Intuitionistic fuzzy ideal topological spaces, τ −codense intuitionistic fuzzy ideals, Intuitionistic fuzzy A_I −set.

Journal of College of Education for pure sciences(JCEPS)
Web Site: http://eps.utq.edu.iq/ Email: com@eps.utq.edu.iq
Volume 7, Number 1, January 2017

وصف بعض المجوعات الضبابية الحدسية الجزئية من الفضاءات المثالية الضبابية الحدسية

آمنة كريم يوسف
Amneh.almusawi@yahoo.com
قسم الرياضيات ,كلية التربية للعلوم الصرفة, جامعة ذي قار

الخلاصة

في هذا البحث درسنا بعض المجموعات الجزئية من الفضاء التبولوجي المثالي الضبابي الحدسي مثل المجموعة الضبابية الحدسية المنظمة I المغلقة , المجموعة A_I الضبابية الحدسية, المجموعة f_I الضبابية الحدسية والمجموعة Iالمغلقة المحلية الضبابية الحدسية. وكذلك قمنا بتمييز المثاليات الضبابية الحدسية $\tau-$codense من ناحية المجموعات الضبابية الحدسية المنظمة I المغلقة , المجموعات A_I الضبابية الحدسية, المجموعات f_I الضبابية الحدسية والمجموعات Iالمغلقة المحلية الضبابية الحدسية وبرهنا بعض النتائج عليها.

1.Introduction

The concept of intuitionistic fuzzy sets and their operations introduced by Atanassov[1] in 1986 as a generalization of fuzzy sets which published by L.A. Zadeh[8] in 1965. Later many researchers worked on this set as Coker and Saadati [2, 4] who defined the notion of intuitionistic fuzzy topology and studied the basic concepts of intuitionistic fuzzy point [4]. Gain P.K. and et al [3] in 2012 are studied and characterized some fuzzy subsets like fuzzy regular I-closed set, fuzzy A_I −set and others and characterized τ −codense fuzzy ideals on fuzzy A_I −set, fuzzy regular-I-closed set and fuzzy-I-locally closed set. In this paper, we extended those ideas of fuzzy ideal topological space in intuitionistic fuzzy ideal topological spaces and we generalized some concepts of intuitionistic fuzzy ideal topological space which initiated by Salama [5] and proved some results about them.

2.Preliminaries

Definition(2.1) [1] Let X be a non-empty set. An intuitionistic fuzzy set (IFS) A is an object having the form:

A=$\{\langle x, \mu_A(x), v_A(x)\rangle, x \in X\}$, where the functions $\mu_A: X \longrightarrow I$ and $v_A: X \longrightarrow I$ denote the degree of membership and the degree of non-membership of each element $x \in X$ to the set A, respectively, and

$0 \leq \mu_A(x) + v_A(x) \leq 1$for each $x \in X$.

Definition(2.2) [1]$\tilde{0}$ =$\{\langle x, 0,1\rangle, x \in X\}$

$\tilde{1}$ =$\{\langle x, 1,0\rangle, x \in X\}$

are the intuitionistic fuzzy sets corresponding to empty set and the entire universe respectively .

Definition(2.3) [1]Let X be a non-empty set and let A and B are IFSs in the form A=$\{\langle x,\mu_A(x),v_A(x)\rangle, x\in X\}$, B=$\{\langle x,\mu_B(x),v_B(x)\rangle, x\in X\}$.

Then:

1) $A\subseteq B$if and only if $\mu_A(x)\leq\mu_B(x)$ and $v_A(x)\geq v_B(x)$ for all $x\in X$.

2) $A=B$if and only if $A\subseteq B$ and $B\subseteq A$.

3)$A^c=\{\langle x,v_A(x),\mu_A(x)\rangle, x\in X\}$.

4)$A\cap B=\{min\{\mu_A(x),\mu_B(x)\}, max\{v_A(x),v_B(x)\}, x\in X\}$.

5) $A\cup B=\{max\{\mu_A(x),\mu_B(x)\}, min\{v_A(x),v_B(x)\}, x\in X\}$.

Definition(2.4) [1] Let $\{A_i, i\in J\}$ be an arbitrary family of IFSs in X, then

1)$\bigcap_i A_i=\left\{\langle x,\ \wedge_i\mu_{A_i}(x), \vee_i v_{A_i}(x)\rangle, x\in X\right\}$

2) $\bigcup_i A_i=\left\{\langle x, \vee_i\mu_{A_i}(x), \wedge_i v_{A_i}(x)\rangle, x\in X\right\}$.

Definition(2.5)[2, 4]An intuitionistic fuzzy topology (IFT for short) on a nonempty set X is a family τ of an intuitionistic fuzzy sets in X satisfying the following conditions:

1)$\tilde{0},\tilde{1}\in\tau$

2) For any $G_1,G_2\in\tau$, then $G_1\cap G_2\in\tau$

3) For any family $\{G_i, i\in J\}\subseteq\tau$,then$\bigcup G_i\in\tau$.

The pair (X,τ) is called intuitionistic fuzzy topological space (IFTS for short),and any intuitionistic fuzzy set in τ is known as an intuitionistic fuzzy open set (IFOS for short) in X.

Definition(2.6)[2] The complement A^c of an intuitionistic fuzzy open set in(X,τ) is called an intuitionistic fuzzy closed set (IFCS for short) in X.

Definition(2.7)[2] Let (X,τ) is an intuitionistic fuzzy topological space and A=$\{\langle x,\mu_A(x),v_A(x)\rangle, x\in X\}$ be an intuitionistic fuzzy set in X. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure of A are defined as follows:

$cl(A)=\bar{A}=\cap\{K: K\ is\ an\ IFCS\ in\ X\ and\ A\subseteq K\}$

$int(A)=A^{\circ}=\cap\{G: G\ is\ an\ IFOS\ in\ X\ and\ G\subseteq A\}$

Definition(2.8) [2] Let X be a nonempty set, an intuitionistic fuzzy point denoted by $\chi_{(\alpha,\beta)}$ is an intuitionistic fuzzy set A={$\langle x, \mu_A(x), \nu_A(x)\rangle, x \in X$} such that

$\mu_A(y) = \begin{cases} \alpha & if\ y = x \\ 0 & e.w \end{cases}$ and $\nu_A(y) = \begin{cases} \beta & if\ y = x \\ 1 & e.w \end{cases}$

Where $x \in X$ is a fixed point, and constant $\alpha, \beta \in I$, satisfy $\alpha + \beta \leq 1$.

Definition(2.9) [5] Let X be a nonempty set and L a nonempty family of intuitionistic fuzzy sets. We call L is an intuitionistic fuzzy ideal (IFL for short) on X if:

1)$A \in I$ and $B \leq A \Longrightarrow B \in L$.

2)$A, B \in L \Longrightarrow A \vee B \in L$.

Let (X, τ) is an intuitionistic fuzzy topological space and L is an intuitionistic fuzzy ideal, then (X, τ, I) is said to be an intuitionistic fuzzy ideal topological space.

Definition(2.10)[5] Let (X, τ) is an intuitionistic fuzzy topological space and L be is an intuitionistic fuzzy ideal on X, then the intuitionistic fuzzy local function $A^*(L, \tau)$ of A is the union of all intuitionistic fuzzy points (IFP for short) $\chi_{(\alpha,\beta)}$ such that if $U \in N_{\chi_{(\alpha,\beta)}}$ and

$A^*(L, \tau)$ =∨ {$\chi_{(\alpha,\beta)} \in$ IFP: $A \wedge U \notin L$ for any $U \in N_{\chi_{(\alpha,\beta)}}$

$A^*(L, \tau)$is called an intuitionistic fuzzy local function of A with respect to τ and L which it will be denoted by $A^*(L, \tau)$ or simply $A^*(L)$.

Theorem(2.11) [5] Let (X, τ) is an intuitionistic fuzzy topological space and L be an intuitionistic fuzzy ideal on X. Then for any intuitionistic fuzzy sets A, B of X, the following statements are satisfied:

1)$A \subseteq B \Longrightarrow A^*(L, \tau) \subseteq B^*(L, \tau)$.

2)$A^* = cl(A^*) \subseteq cl(A)$.

3)$A^{**} \subseteq A^*$.

4)$(A \vee B)^* = A^* \vee B^*$.

5)$(A \wedge B)^* \subseteq A^* \wedge B^*$.

Lemma (2.12) Let (X, τ,I) is an intuitionistic fuzzy ideal topological space and $U \in \tau$, then $U \wedge A^* \subseteq (U \wedge A)^*$.

Definition(2.13)[6, 7] An intuitionistic fuzzy set $A=\{\langle x, \mu_A(x), \nu_A(x)\rangle, x \in X\}$ in an intuitionistic fuzzy topological space (X, τ) is said to be an:

1) Intuitionistic fuzzy semiopen set (IFSOS in short) if $A \subseteq cl\big(int(A)\big)$.

2) Intuitionistic fuzzy preopen set (IFPOS in short) if $A \subseteq int\big(cl(A)\big)$.

3) Intuitionistic fuzzy α –open set (IFα OS in short) if $A \subseteq int\Big(cl\big(int(A)\big)\Big)$.

4) Intuitionistic fuzzy β open set (IFβOS in short) If $A \subseteq cl\Big(int\big(cl(A)\big)\Big)$.

5) Intuitionistic fuzzy regular closed set (IFRCS in short) if
$A = cl\big(int(A)\big)$.

6) Intuitionistic fuzzy regular open set (IFROS in short) if $A = int\big(cl(A)\big)$.

Definition(2.14) An intuitionistic fuzzy set $A=\{\langle x, \mu_A(x), \nu_A(x)\rangle, x \in X\}$ in an intuitionistic fuzzy topological space (X, τ) is said to be an:

1) Intuitionistic fuzzy locally closed set if $A = U \wedge V$, where $U \in \tau$ and V is an intuitionistic fuzzy closed set (IFLC(X)) .

2) Intuitionistic fuzzy A-set if$A = U \wedge V$, where $U \in \tau$ and V is an intuitionistic fuzzy regular closed set (IFA(X)) .

Definition(2.15) An intuitionistic fuzzy set $A=\{\langle x, \mu_A(x), \nu_A(x)\rangle, x \in X\}$ in an intuitionistic fuzzy ideal topological space (X, τ, I) is said to be an:

1) Intuitionistic fuzzy I $-$open if $A \subseteq int(A)$ (IFIO for short).

2) Intuitionistic fuzzy $* -$dense $-$ in $-$ itselfif $A \subseteq A^*$.

3) Intuitionistic fuzzy $\tau^* -$ closedif $A^* \subseteq A$.

4) Intuitionistic fuzzy $* -$perfectif $A = A^*$.

5) Intuitionistic fuzzy $\alpha - I -$ open if $A \subseteq int\Big(cl^*\big(int(A)\big)\Big)$ (IF$_\alpha$IO in short).

6) Intuitionistic fuzzy semi-I-open if $A \subseteq cl^*\big(int(A)\big)$ (IFSIOS in short).

7) Intuitionistic fuzzy pre-I-open if $A \subseteq int\big(cl^*(A)\big)$ (IFPIOS in short).

Definition(2.16) In an intuitionistic fuzzy ideal topological space(X, τ, I), I is said to be $\tau -$codense if $I \wedge \tau = \{\tilde{0}\}$.

Lemma(2.17) For an intuitionistic fuzzy ideal topological space (X, τ, I), I is said to be τ −codense if and only if $\tilde{1} = \tilde{1}^*$.

Lemma(2.18) For an intuitionistic fuzzy ideal topological space (X, τ, I), then the following are equivalent:

1) I is τ −codense

2)$\tilde{1} = \tilde{1}^*$.

3) For every $A \in \tau, A \subseteq A^*$.

4) For every $A \in IFSO(X), A \subseteq A^*$.

5) For every intuitionistic fuzzy regular closed set C, $C = C^*$.

Proof:$1 \Leftrightarrow 2$ follows from lemma(2.17)

$2 \Longrightarrow 3$

Let $A \in \tau$, since $\tilde{1} = \tilde{1}^*$ by using lemma(2.17)

$\Longrightarrow$ I is τ −codense

$$\Longrightarrow I \wedge \tau = \{\tilde{0}\}$$

$$\Longrightarrow A \notin I$$

$$\Longrightarrow A \wedge U \notin I \qquad \forall U \in N_{\chi_{(\alpha,\beta)}}$$

$$\Longrightarrow A \in A^*$$

Thus, $A \subseteq A^*$.

$$3 \Longrightarrow 4$$

Let $A \in$ IFSO(X)

$\Longrightarrow$there exists an intuitionistic fuzzy set H in X such that

$H \subseteq A \subseteq cl(H)$.

For any intuitionistic fuzzy subset H in X , by Theorem(2.11)

We have $H^* = cl(H^*) \subseteq cl(H)$.

Since H is intuitionistic fuzzy open, $H \subseteq H^* \Longrightarrow cl(H) \subseteq cl(H^*)$

and so $H^* = cl(H^*) = cl(H)$.

Therefore, $A \subseteq cl(H) = cl(H^*) = H^* \subseteq A^*$.

$\Longrightarrow A \subseteq A^*$.

$4 \Longrightarrow 5$ Let C is an intuitionistic fuzzy regular closed, then C is an intuitionistic fuzzy semiopen and intuitionistic fuzzy closed.

Since $C \in IFSO(X) \Longrightarrow C \subseteq C^*$

Since C is an intuitionistic fuzzy closed

$C = cl^*(C) = C \vee C^* \Longrightarrow C^* \subseteq C$.

Hence, $C^* = C$.

$$5 \Longrightarrow 1$$

Since $\tilde{1}$ is an intuitionistic fuzzy regular closed

$\Longrightarrow$by hypothesis $\tilde{1} = \tilde{1}^*$

$\Longrightarrow$by lemma(2.17) we get I is τ $-$codense.

Lemma(2.19): Let (X, τ, I) be an intuitionistic fuzzy ideal topological space. If an intuitionistic fuzzy subset A of X is an intuitionistic fuzzy $*$ $-$dense in itself, then $A^* = cl(A) = cl^*(A)$.

Proof: Since A is $*$ $-$dense in itself, $A \subseteq A^*$.

By Theorem(2.11)for any intuitionistic fuzzy subset A of X ,

$A^* = cl(A^*) \subseteq cl(A)$ (1)

Since $A \subseteq A^* \Longrightarrow cl(A) \subseteq cl(A^*)$(2)

From (1) and (2) we get $cl(A) = cl(A^*)$

$\Longrightarrow A^* = cl(A^*) = cl(A)$.

Also, $cl(A) = cl^*(A)$

Hence, $A^* = cl(A) = cl^*(A)$.

Lemma(2.20): Let (X, τ, I) be an intuitionistic fuzzy ideal topological space and let $\Delta = \{A: A$ is an intuitionistic fuzzy subsetof X and $A \subseteq A^*\}$.

Then, $\Delta \wedge I = \{\tilde{0}\}$.

Proof: Let $A \in \Delta \wedge I \Longrightarrow A \in \Delta$ and $A \in I$

$$A \in I \Longrightarrow A^* = \tilde{0}$$

$A \in \Delta \Longrightarrow A \subseteq A^* = \tilde{0}$

Therefore, $A = \tilde{0}$.

Hence, $\Delta \wedge I = \{\tilde{0}\}$.

3.Intuitionistic fuzzy regular I-closed set, intuitionistic fuzzy A_I-set, intuitionistic fuzzy I-locally closed set and intuitionistic fuzzy f_I-set.

Definition(3.1): An intuitionistic fuzzy subset A of an intuitionistic fuzzy ideal topological space (X, τ, I) is said to be intuitionistic fuzzy regular I-closed if $A = (int(A))^*$.

Definition(3.2): An intuitionistic fuzzy subset A of an intuitionistic fuzzy ideal topological space (X, τ, I) is said to be

1) Intuitionistic fuzzy A_I −set if $A = U \wedge V$ where $U \in \tau$ and V is an intuitionistic fuzzy regular I-closed set ($IFA_I(X)$).

2) Intuitionistic fuzzy I-locally closed set if $A = U \wedge V$ where $U \in \tau$ and V is an intuitionistic fuzzy $*$ −perfect set (IFILC(X)).

Lemma(3.3): Let (X, τ, I) be an intuitionistic fuzzy ideal topological space. An intuitionistic fuzzy subset A of X is an intuitionistic fuzzy I-locally closed if and if $A = U \wedge A^*$ for some $U \in \tau$.

Proof: Let A is an IFILC set in (X, τ, L)

$\Longrightarrow A = U \wedge V, U \in \tau$ and V is an intuitionistic fuzzy $*$ −perfect set

$$\Longrightarrow V = V^*$$

Since $A \subseteq V$

ByTheorem(2.11)$\Longrightarrow A^* \subseteq V^*$.

Also, by lemma(2.12)$A^* = (U \wedge V)^* \supseteq U \wedge V^* = U \wedge V = A$

$\Longrightarrow \mathrm{A} \subseteq A^*$

$\Longrightarrow \mathrm{A} = A \wedge A^* = (U \wedge V) \wedge A^* = U \wedge (V \wedge A^*) = U \wedge A^*$

$\Longrightarrow \mathrm{A} = U \wedge A^*$.

Theorem(3.4): Every intuitionistic fuzzy A_I −set in (X, τ, I) is an intuitionistic fuzzy I-locally closed.

Proof: Let A is an intuitionistic fuzzy A_I −set in (X, τ, I).

$\Longrightarrow \mathrm{A} = U \wedge V$where $U \in \tau$ and V is an intuitionistic fuzzy regular I-closed.

$\Longrightarrow V = (int(V))^*$

Since $int(V) \subseteq V \Longrightarrow (int(V))^* \subseteq V^*$

$\Longrightarrow V = \left(int(V)\right)^{*} \subseteq V^{*} \Longrightarrow V \subseteq V^{*}(1)$

Since $V = \left(int(V)\right)^{*} \Longrightarrow V^{*} = \left(\left(int(V)\right)^{*}\right)^{*} \subseteq \left(int(V)\right)^{*} = V$

$\Longrightarrow V^{*} \subseteq V(2)$

From (1) and (2) we get $V = V^{*}$

Therefore, V is an intuitionistic fuzzy $*$ $-$perfect set.

Hence A is an intuitionistic fuzzy I-locally closed set.

Theorem(3.5): Let A is an intuitionistic fuzzy open set in (X, τ, I). A is an intuitionistic fuzzy A_I $-$set if and only if A is an intuitionistic fuzzy I-locally closed set.

Proof: ($\Longrightarrow$) Let A intuitionistic fuzzy A_I $-$set in (X, τ, I).

$\Longrightarrow$by Theorem(3.4) A is an intuitionistic fuzzy I-locally closed set.

($\Longleftarrow$) Let A is an intuitionistic fuzzy I-locally closed set and fuzzy open set.

$\Longrightarrow$ A $= U \wedge A^{*}$for some $U \in \tau$

$\Longrightarrow$ A $\subseteq A^{*}$

Since $int(A^{*}) \subseteq A^{*}$

$\Longrightarrow \left(\left(int(A)\right)^{*}\right)^{*} \subseteq (A^{*})^{*} \subseteq A^{*} \Longrightarrow \left(\left(int(A)\right)^{*}\right)^{*} \subseteq A^{*}(1)$

Since A $\leq A^{*}$ and A is an intuitionistic fuzzy open set

$$\Longrightarrow A^{*} = \left(int(A)\right)^{*} \subseteq \left(\left(int(A)\right)^{*}\right)^{*}$$

$\Longrightarrow A^{*} \subseteq \left(\left(int(A)\right)^{*}\right)^{*}(2)$

From (1) and (2) we get $A^{*} = \left(\left(int(A)\right)^{*}\right)^{*}$

$\Longrightarrow$ $A^{*}$is an intuitionistic fuzzy regular I-closed set.

Hence A is an intuitionistic fuzzy A_I $-$set.

Theorem(3.6): Let (X, τ) is an intuitionistic fuzzy topological space and A is an intuitionistic fuzzy subset of X then A is an intuitionistic fuzzy A-set if and only if A is an intuitionistic fuzzy semiopen and intuitionistic fuzzy locally closed set.

Proof: Let A is an intuitionistic fuzzy A-set in (X, τ)

$\Longrightarrow A = U \wedge F$where$U \in \tau$ and F is an intuitionistic fuzzy regular closed

$\Longrightarrow$F is an intuitionistic fuzzy closed.

$\Longrightarrow$F is an intuitionistic fuzzy locally closed.

Since $A = U \wedge F \Longrightarrow int(A) = int(U \wedge F)$

$\Longrightarrow int(A) = U \wedge int(F)$.

So, $A = U \wedge cl\big(int(F)\big) \subseteq cl\big(U \wedge int(F)\big) = cl\big(int(A)\big)$

$A \subseteq cl\big(int(A)\big)$.

Therefore, A is an intuitionistic fuzzy semiopen.

Conversely, let A is an intuitionistic fuzzy semiopen and intuitionistic fuzzy locally closed set.

$\Longrightarrow A \subseteq cl\big(int(A)\big)andA = U \wedge cl(A),\ U \in \tau$

$\Longrightarrow cl(A) \subseteq cl\left(cl\big(int(A)\big)\right) = cl(A) \subseteq cl\big(int(A)\big)$(1)

Since, $int(A) \subseteq A \ \ \Longrightarrow cl\big(int(A)\big) \subseteq cl(A)$(2)

From (1) and (2) we get $cl(A) = cl\big(int(A)\big)$.

Therefore, $cl(A)$ is an intuitionistic fuzzy regular closed.

Hence, A is an intuitionistic fuzzy A-set.

Definition(3.7): An intuitionistic fuzzy subset A of IFITS (X, τ, I) is said to be intuitionistic fuzzy f_I-set if $A \subseteq \big(int(A)\big)^*$.

The family of all intuitionistic fuzzy f_I-set in X denoted by IFf_I(X).

Theorem(3.8): If A is an intuitionistic fuzzy A_I −set in IFLTS (X, τ, I) then the following holds:

1)A and $int(A)$ are *-dense-in-itself.

2)$A^* = cl(A) = cl^*(A)$and$\big(int(A)\big)^* = cl\big(int(A)\big)$.

3) A is an intuitionistic fuzzy f_I-set.

4)$A^* = \big(int(A)\big)^* = \Big(\big(int(A)\big)^*\Big)^* = (A^*)^*$.

5)$A^*$and$\big(int(A)\big)^*$ are intuitionistic fuzzy *-perfect and intuitionistic fuzzy I-locally closed sets.

6)$A^*$is an intuitionistic fuzzy regular-I-closed.

Proof: **1)**Let A is an intuitionistic fuzzy A_I −set in IFLTS (X, τ, I)

$\Longrightarrow A = U \wedge V$,where$U \in \tau$ and V is an intuitionistic fuzzy regular-I-closed set.

$\Longrightarrow V = \left(int(V)\right)^*$

$\Longrightarrow A = U \wedge V = U \wedge \left(int(V)\right)^* \subseteq \left(U \wedge int(V)\right)^* = \left(int(A)\right)^* \subseteq A^*$

$\Longrightarrow A \subseteq A^*$

Since $int(A) \subseteq A \subseteq \left(int(A)\right)^* \subseteq A^*$

$\Longrightarrow int(A) \subseteq \left(int(A)\right)^*$ and $A \subseteq A^*$

Therefore, A and $int(A)$ are intuitionistic fuzzy *-dense-in-itself.

2) by lemma(2.4) , $A^* = cl(A) = cl^*(A)$

and $\left(int(A)\right)^* = cl\left(int(A)\right)$.

3) From(1), $A \subseteq \left(int(A)\right)^*$

So, A is an intuitionistic fuzzy f_I-set.

4) From(1), $int(A) \subseteq A \subseteq \left(int(A)\right)^* \subseteq A^*$

And so $\left(int(A)\right)^* \subseteq A^* \subseteq \left(\left(int(A)\right)^*\right)^* \subseteq \left(int(A)\right)^* \subseteq A^*$

Therefore, $A^* = \left(int(A)\right)^* = \left(\left(int(A)\right)^*\right)^* = (A^*)^*$.

5) From(4) $A^* = (A^*)^*$ and $\left(int(A)\right)^* = \left(\left(int(A)\right)^*\right)^*$

Then, A^* and $\left(int(A)\right)^*$ are intuitionistic fuzzy *-perfect and hence are intuitionistic fuzzy I-locally closed sets.

6) From(4), $A^* = \left(int(A)\right)^*$

Let $B = \left(int(A)\right)^*$

$\Longrightarrow \left(int(B)\right)^* = \left(int\left(int(A)\right)^*\right)^* = \left(int(A^*)\right)^* \supseteq \left(int(A)\right)^* = B$

$\Longrightarrow B \subseteq \left(int(B)\right)^*$(1)

Since $int(B) \subseteq B \Longrightarrow int(B)^* \subseteq B^* = \left(\left(int(A)\right)^*\right)^* \subseteq \left(int(A)\right)^* \subseteq B$

$\Longrightarrow \left(int(B)\right)^* \subseteq B$(2)

From(1) and (2), we obtain $\left(int(B)\right)^* = B$

Then B is an intuitionistic fuzzy regular I-locally closed set and so A^* is an intuitionistic fuzzy regular I-locally closed set.

Theorem(3.9) In any IFLTS (X, τ, I), $IFA_I(X) \wedge I = \{\tilde{0}\}$.

Proof: Let $A \in IFA_I(X) \wedge I \implies A \in IFA_I(X)$.

By Theorem(3.8), A is an intuitionistic fuzzy *-dense-in-itsehf

$\implies A \subseteq A^*$

By lemma(2.20), $IFA_I(X) \wedge I = \{\tilde{0}\}$.

Theorem(3.10) An intuitionistic fuzzy subset A of an IFLTS (X, τ, I) is an intuitionistic fuzzy A_I −set if and if A is both intuitionistic fuzzy f_{I-}set and intuitionistic fuzzy I-locally closed set.

Proof: Let A is an IFA_I −set

By Theorem(3.4), A is an intuitionistic fuzzy I-locally closed set.

Also, $A = U \wedge V$ where $U \in \tau$ and V is an intuitionistic fuzzy regular-I-closed set

$\implies V = \left(int(V)\right)^*$

$\Rightarrow int(A) = int(U \wedge V) = U \wedge \left(int(V)\right) \implies \left(int(A)\right)^* = (U \wedge int(V))^*$

Since $A = U \wedge V$

$\implies A = U \wedge \left(int(V)\right)^* \subseteq \left(U \wedge int(V)\right)^* = \left(int(A)\right)^*$

$\implies A \subseteq \left(int(A)\right)^*$

Hence, A is an intuitionistic fuzzy f_{I-}set.

Conversely, let A is both intuitionistic fuzzy f_{I-}set and intuitionistic fuzzy I-locally closed set.

$\implies A \subseteq \left(int(A)\right)^* \implies A^* \subseteq \left(\left(int(A)\right)^*\right)^* \subseteq \left(int(A)\right)^* \subseteq A^*$

$\implies A^* = \left(int(A)\right)^*$

By Theorem(3.8), A^* is an intuitionistic fuzzy regular $-I-$ closed set.

Since A is an intuitionistic fuzzy I-locally closed set.

By lemma(3.3), $A = U \wedge A^*$ for some $U \in \tau$.

Since A^* is an intuitionistic fuzzy regular $-I-$ closed set,

Then A is an intuitionistic fuzzy A_I −set.

Theorem(3.11) Let (X, τ, I) be an intuitionistic fuzzy ideal topological space and let $A \in \tau$, then A is intuitionistic fuzzy f_I −set if and only if A is an intuitionistic fuzzy A_I −set.

Proof: let $A \in \tau$ and A is anintuitionistic fuzzy f_I $-$set

$\Longrightarrow A \subseteq \left(int(A)\right)^* \subseteq A^*andA^* = \left(int(A)\right)^*$

By Theorem(3.8), A^* is an intuitionistic fuzzy regular $-I-$ closed set.

Since $A = A \wedge A^*$

$\Longrightarrow A$is an intuitionistic fuzzy A_I $-$set.

Conversely, let A is an intuitionistic fuzzy A_I $-$set.

By Theorem(3.10), A is an intuitionistic fuzzy f_I $-$set.

Theorem(3.12) In any intuitionistic fuzzy ideal topological space (X, τ, I), $IFf_I(X) \wedge I = \{\tilde{0}\}$.

Proof: Let $A \in IFf_I(X) \wedge I$

$\Longrightarrow A \in IFf_I(X)$and $A \in I$.

$A \in I \Longrightarrow A^* = \tilde{0}$.

$A \in IFf_I(X) \Longrightarrow A \subseteq \left(int(A)\right)^* \subseteq A^* = \tilde{0} \Longrightarrow A = \tilde{0}$

$\Longrightarrow IFf_I(X) \wedge I = \{\tilde{0}\}$.

Theorem(3.13) Let (X, τ, I) be an intuitionistic fuzzy ideal topological space and A is an intuitionistic fuzzy subset of X. Then A is an intuitionistic fuzzy regular-I-closed set if and only if A is both intuitionistic fuzzy f_I $-$set and intuitionistic fuzzy τ^* $-$closed set.

Proof: Let A is an intuitionistic fuzzy regular-I-closed in (X, τ, I).

$\Longrightarrow A = \left(int(A)\right)^* \Longrightarrow A \subseteq \left(int(A)\right)^* \Longrightarrow A$is an intuitionistic fuzzy f_I $-$set.

Since $int(A) \subseteq A \Longrightarrow \left(int(A)\right)^* \subseteq A^*$

$\Longrightarrow A = \left(int(A)\right)^* \subseteq A^* \Longrightarrow A \subseteq A^*$

$\Longrightarrow A^* = \left(\left(int(A)\right)^*\right)^* \subseteq \left(int(A)\right)^* = A \Longrightarrow A^* \subseteq A$

$\Longrightarrow A^* = A$

Then A is an intuitionistic fuzzy τ^* $-$closed set.

Conversely, let A is both intuitionistic fuzzy f_I $-$set and intuitionistic fuzzy τ^* $-$closed set.

$\Longrightarrow A \subseteq \left(int(A)\right)^*$and $A^* \subseteq A$.

Since $int(A) \subseteq A \Longrightarrow \left(int(A)\right)^* \subseteq A^*$

$$\Longrightarrow \left(int(A)\right)^* \subseteq A^* \subseteq A \subseteq \left(int(A)\right)^*$$

Therefore, $A = \left(int(A)\right)^*$

Hence A is an intuitionistic fuzzy regular-I-closed set.

Theorem(3.14) Every intuitionistic fuzzy f_I −set of an intuitionistic fuzzy ideal topological space (X, τ, I) is an intuitionistic fuzzy semi-I-open set.

Proof: Let A is an intuitionistic fuzzy f_I −set.

$\Longrightarrow A \subseteq \left(int(A)\right)^* \subseteq cl^*\left(int(A)\right)$

$\Longrightarrow A \subseteq cl^*\left(int(A)\right)$.

Then A is an intuitionistic fuzzy semi-I-open set.

Theorem(3.15) For an intuitionistic fuzzy ideal topological space (X, τ, I), I is τ −codense if and if $A \in \tau$, then $A \in IFA_I(X)$.

Proof: Let I is τ −codense and $A \in \tau$

By lemma(2.18), for any $A \in \tau$, $A \subseteq A^* \Longrightarrow A = A \wedge A^*$.

Since $A \in \tau \Longrightarrow \left(int(A)\right)^* = A^*$

By Theorem(3.8), A^* is an intuitionistic fuzzy regular-I-closed set.

$\Longrightarrow A$is an intuitionistic fuzzy A_I −set.

Conversely, if we have $A \in \tau \Longrightarrow A \in IFA_I(X)$

By Theorem(3.9), $IFA_I(X) \wedge I = \{\tilde{0}\} \Longrightarrow I \wedge \tau = \{\tilde{0}\}$.

Therefore, I isτ −codense.

Corollary(3.16) Let (X, τ, I) be an intuitionistic fuzzy ideal topological space. Then the following statements are equivalent:

1) I isτ −codense.

2) $\tau = \mathrm{IFPIO(X)} \wedge \mathrm{IF}A_I(X)$.

3) $\tau = \mathrm{IF}_\alpha\mathrm{(X)} \wedge \mathrm{IF}A_I(X)$.

4) $A \in \tau \Longrightarrow A \in IFA_I(X)$.

Proof: obvious.

Theorem(3.17) Let (X, τ, I) be an intuitionistic fuzzy ideal topological space. Then I is τ −codense if and only if IFRIC(X)=IFRC(X).

Proof: Let I isτ −codense.

By lemma(2.18), $A \in \tau$, $A \subseteq A^*$ $\Longrightarrow$ A is an intuitionistic fuzzy *-dense-in-itself .

Let $A \in IFRIC(X) \Leftrightarrow A = \big(int(A)\big)^*$

$\Leftrightarrow \big(int(A)\big)^* = cl\big(int(A)\big)$by lemma(2.19)

$\Longrightarrow A = cl\big(int(A)\big) \Leftrightarrow A \in$ IFRC(X).

Conversely, let IFRIC(X)=IFRC(X)

Since $\tilde{1}$ is an IFRC $\Longrightarrow$ $\tilde{1}$ is an IFRIC

$\tilde{1} = \big(int(\tilde{1})\big)^* = \tilde{1}^*$ $\Longrightarrow$by lemma(2.18), I is τ −codense.

Theorem(3.18) Let (X, τ, I) be an intuitionistic fuzzy ideal topological space. Then I is τ −codense if and only if IFA_I(X)=IFA(X).

Proof: Let I isτ −codense.

Let $A \in$ IFA_I(X) $\Longrightarrow A = U \wedge V$ where $A \in \tau$ and V is an IFRIC set

$$\Longrightarrow V = \big(int(V)\big)^*$$

$\Longrightarrow cl(V) = cl\Big(\big(int(V)\big)^*\Big) = \big(int(V)\big)^* = V$byTheorem(2.11).

Also, by Theorem(2.11),$\big(int(V)\big)^* \subseteq cl\big(int(V)\big)$

$\Longrightarrow V = \big(int(V)\big)^* \subseteq cl\big(int(V)\big) \subseteq cl(V) = V$

$\Longrightarrow V = cl\big(int(V)\big)$

$\Longrightarrow$ $V$is an IFRC(X).

$A \in$ IFA(X).

Let $B \in$ IFA(X)

$\Longrightarrow B = S \wedge T$where$S \in \tau$ and T $\in$ IFRC(X)

$\Longrightarrow$by Theorem(3.17), $T \in IFRIC(X)$

$\Longrightarrow B \in$ IFA_I(X).

Conversely, let IFA_I(X)=IFA(X).

Since $\tilde{1}$ is an IFA-set $\Longrightarrow$ $\tilde{1}$ is an IFA_I −set.

by Theorem(3.4) $\Longrightarrow \tilde{1} \subseteq \tilde{1}^* \Longrightarrow \tilde{1}$ is *-dense-in-itself.

Then by lemma(2.18), I is τ −codense.

Theorem(3.19) For an intuitionistic fuzzy ideal topological space (X, τ, I), $IFILC(X) \wedge I = \{\tilde{0}\}$.

Proof: $A \in$ IFILC(X).

$\Longrightarrow A = U \wedge V$ where $A \in \tau$ and V is an IF *-perfect set

$\Longrightarrow V = V^*$.

Since $A \subseteq V \Longrightarrow A^* \subseteq V^*$

$\Longrightarrow A^* = (U \wedge V)^* \supseteq U \wedge V^* = U \wedge V = A$

$\Longrightarrow A \subseteq A^*$

$\Longrightarrow$by lemma(2.20), $IFILC(X) \wedge I = \{\tilde{0}\}$.

Theorem(3.20) Let (X, τ, I) be an intuitionistic fuzzy ideal topological space. Then the following statements are equivalent:

1) I is τ −codense.

2) $\tau =$ IFPIO(X) $\wedge$ IFILC(X).

3) $\tau =$ IF$_\alpha$IO(X) $\wedge$ IFILC(X).

4) $A \in \tau \Longrightarrow A \in$ IFILC(X).

Proof: 1 $\Longrightarrow$ 2, Let I is τ −codense.

From corollary(3.16), $\tau =$ IFPIO(X) $\wedge$ IF$A_I(X)$.

By Theorem(3.5), $\tau =$ IFPIO(X) $\wedge$ IFILC(X).

2 $\Longrightarrow$ 3, $\tau =$ IFPIO(X) $\wedge$ IFILC(X).

Let $A \in \tau \Longrightarrow A \in$ IFPIC(X) $\Longrightarrow A \subseteq int(cl^*(A))$.

Since $A \in \tau \Longrightarrow A = int(A)$

$$\Longrightarrow A \subseteq int\left(cl^*(int(A))\right)$$

$\Longrightarrow A \in$ IF$_\alpha$IO(X)

$\Longrightarrow \tau =$ IF$_\alpha$IO(X) $\wedge$ IFILC(X)

3 $\Longrightarrow$ 4and2 $\Longrightarrow$ 4 are obvious.

$4 \Longrightarrow 1$let$A \in \tau \Longrightarrow A \in$ IFILC(X).

By Theorem(3.19), $IFILC(X) \wedge I = \{\tilde{0}\} \Longrightarrow \tau \wedge I = \{\tilde{0}\}$.

Therefore, I isτ −codense.

$1 \Longrightarrow 3$letI is τ −codense and $A \in \tau$

By lemma(2.18), $A \in \tau \Longrightarrow A \subseteq A^*$

Since $int(A) = A \Longrightarrow cl(int(A)) = cl(A)$

$\Longrightarrow cl^*(int(A)) = cl^*(A)$

$\Longrightarrow int\left(cl^*(int(A))\right) = int(cl^*(A))$

Since $A \subseteq cl(A) \Longrightarrow A^* \subseteq cl^*(A) \Longrightarrow A \subseteq A^* \subseteq cl^*(A)$

$\Longrightarrow int(A) \subseteq int(A^*) \subseteq int(cl^*(A)) = int\left(cl^*(int(A))\right)$.

$$\Longrightarrow A \subseteq int\left(cl^*(int(A))\right)$$

Hence $A \in$ IF$_\alpha$IO(X).

Since $A \subseteq A^* \Longrightarrow A = A \wedge A^*$

$A \subseteq A^* \Longrightarrow A^* \subseteq (A^*)^* \subseteq A^* \Longrightarrow (A^*)^* = A^*$

$\Longrightarrow A^*$is an IF *-perfect set.

$\Longrightarrow A \in$ IFILC(X).

Conversely, let $A \in$ IFILC(X)and $A \in$ IF$_\alpha$IO(X).

$A \in$ IFILC(X) $\Longrightarrow A = U \wedge A^*$for some $U \in \tau$.

$A \in$ IF$_\alpha$IO(X) $\Longrightarrow A \subseteq int\left(cl^*(int(A))\right) \subseteq int(cl^*(A)) = int(cl^*(U \wedge A^*)) \subseteq$

$int(cl^*(A^*))$.

$\Longrightarrow A \subseteq int(cl^*(A^*)) = int(A^*)$

$\Longrightarrow A \subseteq int(A^*)$

Since $A \subseteq U \Longrightarrow A \subseteq U \wedge int(A^*) = int(U \wedge A^*) = int(A)$

$$\Longrightarrow A \subseteq int(A)$$

Since $int(A) \subseteq A$

Then $int(A) = A$ and therefore $A \in \tau$.

References

[1] Atanassov K. T. .Intuitionistic Fuzzy Sets.Fuzzy sets and Systems. 20. 1. 87-96. (1986).

[2] Coker D..An Introduction to Intuitionistic Fuzzy Topological Spaces.Fuzzy sets and Systems. 88. 81-89. (1997).

[3] Gain P. K. and et al. Characterization of Some Fuzzy Subsets of Fuzzy Ideal Topological Spaces and Decomposition of Fuzzy Continuity. International Journal of Fuzzy Mathematics and Systems. 2. 2. 149-161.(2012).

[4] Saadati R. and Park J. H. .On the Intuitionistic Fuzzy Topological Spaces.Chaos Solitons and Fractals. 27. 331-344. (2006).

[5] Salama A. A. and Albowi S.A. .Intuitionistic fuzzy Ideal Topological Spaces. Advances in fuzzy Mathematics. 7. 1. 51-60. (2012).

[6] Santhi R and Sakthivel K. Strongly Alpha Generalized Closed Sets in Intuitionistic Fuzzy Topological Spaces. Int. J. Pure Appl. Technol. 3.50-58. (2011).

[7] Vaishnavy V and JayanthiD . Regular Generalized Semipreopen Sets in Intuitionistic Fuzzy Topological Spaces. International of Journal Engineering Research and Technology (IJERT). 4. 890-892. (2015)

[8] Zadeh L.A. .Fuzzy Sets.Information and Control. 8. 338-353. (1965).