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On Intuitionistic Fuzzy Closure and Intuitionistc Fuzzy Interior Defined by Intuitionistic Fuzzy Semi-pre Set

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Abstract

In this paper, we introduce the concepts of closure and interior defined by an intuitionistic gradation of openness and intuitionistic fuzzy semi-peropen. We also introduce the concepts of weakly gp.semi-premaps

Keyword: Intuitionistic Fuzzy Semi-peropen, Intuitionistic Fuzzy Closure, Intuitionistic Fuzzy Interior and Weakly gp.semi-premap.

الخلاصة

في هذا البحث تمكنا من ايجاد تعريف جديد للانغلاق الحدسي الضبابي والانفتاح الحدسي الضبابي بواسطةالتدرج الانفتاحي الحدسي و المجموعة الحدسية الضبابية شبه-قبل المفتوحة ,ثم اوجدنا تعريف الداله الحدسيه الضبابيه شبه-قبل المفتوحة الضعيفة وناقشنا عدة علاقات معا للانغلاق الحدسي الضبابي والانفتاح الحدسي الضبابي.

1. Introduction

In 1965, L.Zadeh[11] introduced the concept of fuzzy set . In 1968, Change[4] introduced the concept of fuzzy topology on set X by axiomatizing, a collection T of fuzzy subsets of X. In [5], Chattopadyay et al. introduced the concept of fuzzy topology redefined by a gradation of openness and investigated some fundamental properties. M.Demirci[7] introduced the concepts of fuzzy closure and fuzzy interior in the fuzzy topological space, and obtained some properties of them. Atanassov introduced the concept of intuitionistic fuzzy set which is a generalization of fuzzy set in Zadeh's sense [3]. D¸ coker introduced the concept of intuitionistic fuzzy topological spaces [6] by using the intuitionistic fuzzy sets, which is an extended concept of fuzzy topological spaces in Chang's sense. In 2002, Mondal and Samanta introduced the concept of intuitionistic gradations of openness [9] which is a generalization of the concept of gradation of openness defined by Chattopadyay. In this paper, we introduce the concepts of closure and interior defined by intuitionistic gradation of openness and intuitionistic fuzzy semi-peropen set, We also introduce the concepts of weakly gp.semi-premaps

2.Preliminaries

Let X be a set and I = [0, 1] be the unit interval of the real line. I^X will denote the set of all fuzzy sets of X. $\underline{0}$ and $\underline{1}$ will denote the characteristic functions of φ and X, respectively.

Definition 2.1 [9]:

An intuitionistic gradation of openness (IGO for short) on set X an order pair $(T, T^*)$of mapping from $I^X$to I such that:

(IGO1) $T(A) + T^*(A) \leq 1, \forall A \in I^X$,

(IGO2) $T(\underline{0}) = T(\underline{1}) = 1, T^*(\underline{0}) = T^*(\underline{1}) = 0$, (IGO3)$T(A_1 \wedge A_2) \geq T(A_1) \wedge T(A_2)$ $and T^*(A_1) \wedge (A_2) \leq T^*(A_1) \vee T^*(A_2)$ for each $A_i \in I^X, i = 1$

(IGO4) $T(\vee_{i\in\Gamma} A_i) \geq \wedge_{i\in\Gamma} T(A_i) and T^*(\vee_{i\in\Gamma} A_i) \leq \vee_{i\in\Gamma} T^*(A_i)$

for each $A_i \in I^X, i \in \Gamma$.

The triple(X, T, T^*) is called an intuitionistic fuzzy topological space (IFTS for short). $T$and T^*may be interpreted as gradation of openness and gradation of non-openness, respectively.

Definition 2.2 [9]:

Let X be a nonempty set and be an order pair $(F, F^*)$of mapping from $I^X$to I such that:

(IGC1) $F(A) + F^*(A) \leq 1, \forall A \in I^X$,

(IGC2) $F(\underline{0}) = F(\underline{1}) = 1, F^*(\underline{0}) = F^*(\underline{1}) = 0$,

(IGC3)$F(A_1 \vee A_2) \geq F(A_1) \wedge F(A_2)$ $and F^*(A_1) \vee (A_2) \leq F^*(A_1) \vee F^*(A_2)$

for each $A_i \in I^X, i = 1,2,$

(IGC4) $F(\wedge_{i\in\Gamma} A_i) \geq \wedge_{i\in\Gamma} F(A_i) and F^*(\wedge_{i\in\Gamma} A_i) \leq \vee_{i\in\Gamma} F^*(A_i)$for each $A_i \in I^X, i \in \Gamma$.

The triple(X, F, F^*) is called an intuitionistic fuzzy topological space (IFTS for short). $F$and F^*may be interpreted as gradation of closeness and gradation of non-closeness, respectively.

Example: 2.3:

Let$X$={a, b}, define subsets $\mu_1, \mu_2 \in I^X$as follows:

$$\mu_1(a) = 0.2 \mu_1(b) = 0.5$$

$$\mu_2(a) = 0.7 \mu_2(b) = 0.3$$

$$F(\beta) = \begin{cases} 0 & if \beta = \underline{0}, \underline{1} \\ 1/2 & if \beta = \mu_1 \\ 1 & otherwise \end{cases} \quad F^*(\beta) = \begin{cases} 1 & if \beta = \underline{0}, \underline{1} \\ 1/2 & if \beta = \mu_1 \\ 0 & otherwise \end{cases}$$

Then(F,F^*)is IGC onX

Definition 2.4 [9]:

Let X be a nonempty set , if $(T, T^*)$is an IGO on X, then the pair$(F_T , F^*_{T^*})$ defined by

$F_T(A) = T(A^c)$, $F^*_{T^*}(A) = T^*(A^c)$ whereA^c denotes the complement of A, is an IGC on X and if (F, F^*) is an IGC on X , then the pair $(T_F , T^*_{F^*})$ defined by $T_F(A) = F(A^c)$, $T^*_{F^*}(A) = F^*(A^c)$ is IGO on X

Definition 2.5 [2]:

Let(X, T, T^*) be an intuitionistic fuzzy topological space, define an operator$C_{T,T^*}: I^X \times I_0 \times I_1 \to I^X$ by:

$C_{T,T^*}(\lambda, r, s) = \wedge \{\mu \in I^X: \lambda \leq \mu, T(\underline{1} - \mu) \geq r, T^*(\underline{1} - \mu) \leq s\}$.

Theorem 2.6[2]:

Let(X, T, T^*) be an intuitionistic fuzzy topological space.

for $\lambda, \mu \in I^X$,$r, r_1 \in I_0$s,$s_1 \in I_1$ the operatorC_{T,T^*} satisfies the following conditions:

(C1) $C_{T,T^*}(\underline{0}, r, s) = \underline{0}$.

(C2)$\lambda \leq C_{T,T^*}(\lambda, r, s)$.

(C3) $C_{T,T^*}(\lambda, r, s) \vee C_{T,T^*}(\mu, r, s) = C_{T,T^*}(\lambda \vee \mu, r, s)$.

(C4)$C_{T,T^*}(\lambda, r, s) \leq C_{T,T^*}(\lambda, r_1, s_1)$ if $r \leq r_1$ and$s \geq s_1$.

(C5)$C_{T,T^*}(C_{T,T^*}(\lambda, r, s), r, s) = C_{T,T^*}(\lambda, r, s)$

The pair (X, C) is called an intuitionistic fuzzy closure space.

Definition 2.7[1]:

Let (X, T, T^*) be an intuitionistic fuzzy topology. Define an operator $I_{T,T^*}: I^X \times I_0 \times I_1 \to I^X$ by:

$$I_{T,T^*}(\lambda, r, s) = \vee \{\mu \in I^X: \lambda \geq \mu, T(\mu) \geq r, T^*(\mu) \leq s\}$$

Theorem 2.8[1]:

Let(X, T, T^*) be an intuitionistic fuzzy topological space.

for $\lambda, \mu \in I^X$,$r, r_1 \in I_0$,s,$s_1 \in I_1$, the operator $I_{T,T^*}(\lambda, r, s)$satisfies the following conditions:

(I1)$I_{T,T^*}(\underline{1} - \lambda, r, s) = \underline{1} - C_{T,T^*}(\lambda, r, s)$,

(2) For$\lambda, \mu \in I^X, r, r_1 \in I_0 and s, s_1 \in I_1$, then :

(i1), $I_{T,T^*}\left(\underline{1},r,s\right)=1$;

(i2) $\lambda \geq I_{T,T^*}(\lambda,r,s)$;

(i3) $I_{T,T^*}(\lambda,r,s) \wedge I_{T,T^*}(\mu,r,s) = I_{T,T^*}(\lambda \wedge \mu,r,s)$;

(i4) $I_{T,T^*}(\lambda,r,s) \geq I_{T,T^*}(\lambda,r_1,s_1) if r \leq r_1$ and $s \geq s_1$;

(i5) $I_{T,T^*}\left(I_{T,T^*}(\lambda,r,s),r,s\right) = I_{T,T^*}$.

Example 2.9:

Let $X=\{a,b,c\}$ and $\mu_1,\mu_2,\mu_3 \in I^X$ defined as follows:

$\mu_1(a)=0.3 \mu_1(b)=0.3 \quad \mu_1(c)=0.3 \mu_2(a)=0.4 \mu_2(b)=0.3 \mu_2(c)=0.3$

$\mu_3(a)=0.6 \mu_3(b)=0.4 \mu_3(c)=0.3$

We define $T,T^*: I^X \to I$ as follows:

$$T(\lambda)=\begin{cases} 1 if \lambda=\underline{0},\underline{1} \\ 0.6 if \lambda=\mu_1 \\ 0 \quad \text{otherwise} \end{cases} \quad T^*(\lambda)=\begin{cases} 0 \quad if \lambda=\underline{0},\underline{1} \\ 0.3 if \lambda=\mu_1 \\ 1 \text{otherwise} \end{cases}$$

Then (T,T^*) is an intuitionistic gradation of openness (IGO)

$I_{T,T^*}(\lambda,r,s)=\underline{1}$, where $r = 0.6$ $ands = 0.3$.

Definition 2.10:

Let (X,T,T^*) be an intuitionistic fuzzy topological space,

1) A fuzzy set $\lambda \in I^X$ is said to be an intuitionistic fuzzy semi- preopen set(IFspos for short) if and only if there exist $r \in I_0, s \in I_1$

Such that: $\lambda \leq C_{T,T^*}(I_{T,T^*}\left(C_{T,T^*}(\lambda,r,s),r,s\right),r,s)$

2) A fuzzy set $\lambda \in I^X$ is said to be an intuitionistic fuzzy semi-preclosed set(IFspcs for short) if and only if there exist $r \in I_0, s \in I_1$

Such that: $I_{T,T^*}(C_{T,T^*}\left(I_{T,T^*}(\lambda,r,s),r,s\right),r,s) \leq \lambda$.

Example 2.11:

Let $X=\{a,b,c\}$ and $\mu_1,\mu_2,\mu_3 \in I^X$ defined as follows:

$\mu_1(a)=0.3 \mu_1(b)=0.3 \quad \mu_1(c)=0.3$

$\mu_2(a)=0.4 \mu_2(b)=0.3 \mu_2(c)=0.3$

$\mu_3(a)=0.6 \quad \mu_3(b)=0.4 \mu_3(c)=0.3$

We define $T,T^*: I^X \to I$ as follows:

$$T(\lambda)=\begin{cases}1 & if\lambda=\underline{0},\underline{1}\\ 1/2 & if\lambda=\mu_1\\ 0 & \text{otherwise}\end{cases}\qquad T^*(\lambda)=\begin{cases}0 & if\lambda=\underline{0},\underline{1}\\ 1/2 & if\lambda=\mu_1\\ 1 & \text{otherwise}\end{cases}$$

Then (X,T,T^*) is an intuitionistic fuzzy topology , let $r=1/2$ and $s=1/3$ then μ_3 is an intuitionistic fuzzy semi-preopen

Definition 2.12 :

Let (X,T,T^*) be an IFTS and A∈ I^X then the closure(resp. interior) of A denoted by $\overline{A_{sp}}$ (resp.A°_{sp}) and is defined by

$$\overline{A_{sp}}=\wedge\{K\in I^X, F_T(K)>0 and F^*_{T^*}(K)\le F^*_{T^*}(A), A\le K, K\in IFspcs\}$$

$$A^\circ_{sp}=\vee\{K\in I^X, T(K)>0 and T^*(K)\le T^*(A), \mathrm{K}\le \mathrm{A}, \mathrm{A}\in \mathrm{IFspos}\}$$

Theorem 2.13

Let (X,T,T^*) be an IFTS and A,B ∈ I^X,then:

1. $F^*_{T^*}(\overline{A_{sp}})\le F^*_{T^*}(A)$
2. $T^*\left(A^\circ_{sp}\right)\le T^*(A)$
3. A≤B and $F^*_{T^*}(B)\le F^*_{T^*}(A)\Rightarrow \overline{A_{sp}}\le\overline{B_{sp}}$

   A≤B and $T^*(B)\le T^*(A)\Rightarrow A^\circ_{sp}\le B^\circ_{sp}$.

Proof: 1- from definition 2.4 and 2.12 , we have

$$F^*_{T^*}\left(\overline{A_{sp}}\right)=F^*_{T^*}(\wedge\{K\in I^X, F_T(K)>0 and F^*_{T^*}(K)\le F^*_{T^*}(A), A\le K, K\in IFspcs\})$$

$$\le\vee\{F^*_{T^*}(K): F_T(K)>0 and F^*_{T^*}(K)\le F^*_{T^*}(A), \mathrm{K}\le\mathrm{A}, \mathrm{K}\in\mathrm{IFspos}\}\le F^*_{T^*}(A)$$

2- It similar to (1)

3- Since $A\le B$ and $F^*_{T^*}(B)\le F^*_{T^*}(A)$

Let L be any element of

$$\{K\in I^X, F_T(K)>0 and F^*_{T^*}(K)\le F^*_{T^*}(B), B\le K, K\in IFspcs\}$$

Then it is also in

$$\{K\in I^X, F_T(K)>0 and F^*_{T^*}(K)\le F^*_{T^*}(A), A\le K, K\in IFspcs\}$$

Hence $\overline{A_{sp}} \leq \overline{B_{sp}}$

4- the proof is similar to that of (3).

Theorem 2.14:

Let (X,T,T^*) be an IFTS and A∈ I^X then

1. $(\overline{A_{sp}})^c = (A^c)^\circ_{sp}$
2. $\overline{A_{sp}} = ((A^c)^\circ_{sp})^c$
3. $(A^\circ_{sp})^c = (\overline{A^c})_{sp}$
4. $A^\circ_{sp} = ((\overline{A^c})_{sp})^c$.

Proof:

1- From definition 2.4,we have

$(\overline{A_{sp}})^c = (\wedge\{K \in I^X, F_T(K) > 0 and F^*_{T^*}(K) \leq F^*_{T^*}(A), A \leq K, K \in IFspcs\})^c$

$= \vee\{K^C : K \in I^X, T(K^C) > 0 and T^*(K^C) \leq T^*(A^c), K^C \leq A^C, K^C \in \text{IFspos}\}$

$= \vee\{V \in I^X, T(V) > 0 and T^*(V) \leq T^*(A^C), V \leq A^C, V \in \text{IFspos}\} = (A^c)^\circ_{sp}$

The statement 2,3,4 are easily obtain from (1)

Theorem 2.15:

Let (X,T,T^*) be an IFTS and A,B ∈ I^X,then:

1. $\overline{\underline{0}_{sp}} = \underline{0}$
2. A⊆ $\overline{A_{SP}}$
3. $\overline{A_{SP}} \subseteq \overline{(\overline{A_{SP}})_{SP}}$
4. $\overline{A_{SP}} \cap \overline{B_{SP}} \subseteq \overline{(A \cup B)_{SP}}$

Proof:

1 and 2are easily obtain from definition 2.12

With respect to 3) we have A⊆ $\overline{A_{SP}}$ from 2)⟹ $\overline{A_{SP}} \subseteq \overline{(\overline{A_{SP}})_{SP}}$

4) for every A, B ∈ I^X,by the definition 2.4 and definition 2.12

$\overline{(A \cup B)_{SP}} = \wedge\{K \in I^X, F_T(K) > 0 and F^*_{T^*}(K) \leq F^*_{T^*}(A \cup B), A \cup B \leq K, K \in IFspcs\}$

$\supseteq \wedge\{K \in I^X, F_T(K) > 0 and F^*_{T^*}(K) \leq F^*_{T^*}(A) \vee F^*_{T^*}(B), A \cup B \leq K, K \in IFspcs\}$

$$\supseteq \wedge [\{K \in I^X, F_T(K) > 0 and F^*_{T^*}(K) \le F^*_{T^*}(A), A \le K, K \in IFspcs\}]$$
$$\cup \{K \in I^X, F_T(K) > 0 and F^*_{T^*}(K) \le F^*_{T^*}(B), B \le K, K \in IFspcs\}$$
$$\supseteq \wedge [\{K \in I^X, F_T(K) > 0 and F^*_{T^*}(K) \le F^*_{T^*}(A), A \le K, K \in IFspcs\}]$$
$$\cap \{K \in I^X, F_T(K) > 0 and F^*_{T^*}(K) \le F^*_{T^*}(B), B \le K, K \in IFspcs\}$$
$$\supseteq \overline{A_{SP}} \cap \overline{B_{SP}}$$

Theorem 2.16: Let (X,T,T^*) be an IFTS and A , B ∈ I^X,then:

1. $\underline{(1)}^\circ_{sp} = \underline{1}$
2. $A^\circ_{sp} \subseteq A$
3. $(A^\circ_{sp})^\circ_{sp} \subseteq A^\circ_{sp}$
4. $(A \cap B)^\circ_{sp} \subseteq A^\circ_{sp} \cap B^\circ_{sp}$

Proof : it similar to the proof of theorem 2.15

Theorem 2.17:

Let (X,T,T^*) be an IFTS and A,B ∈ I^X,then:

1. T(A)> 0 $\Longrightarrow A^\circ_{sp} = A$
5. A⊆ $\overline{A_{SP}}$
6. $\overline{A_{SP}} \subseteq \overline{(\overline{A_{SP}})_{SP}}$
7. $\overline{A_{SP}} \cap \overline{B_{SP}} \subseteq \overline{(A \cup B)_{SP}}$

Proof: 1 and 2are easily obtain from definition 2.4

3) A⊆ $\overline{A_{SP}}$ from 2)$\Longrightarrow \overline{A_{SP}} \subseteq \overline{(\overline{A_{SP}})_{SP}}$

4) for every A, B ∈ I^X, by definition1.1and 1.5

$$\overline{(A \cup B)_{SP}} = \wedge \{K \in I^X, F_T(K) > 0 and F^*_{T^*}(K) \le F^*_{T^*}(A \cup B), A \cup B \le K, K \in IFspcs\}$$
$$\supseteq \wedge \{K \in I^X, F_T(K) > 0 and F^*_{T^*}(K) \le F^*_{T^*}(A) \vee F^*_{T^*}(B), A \cup B \le K, K \in IFspcs\}$$

$$\supseteq \wedge [\{K \in I^X, F_T(K) > 0 and F^*_{T^*}(K) \le F^*_{T^*}(A), A \le K, K \in IFspcs\}]$$
$$\cup \{K \in I^X, F_T(K) > 0 and F^*_{T^*}(K) \le F^*_{T^*}(B), B \le K, K \in IFspcs\}$$
$$\supseteq \wedge [\{K \in I^X, F_T(K) > 0 and F^*_{T^*}(K) \le F^*_{T^*}(A), A \le K, K \in IFspcs\}]$$
$$\cap \{K \in I^X, F_T(K) > 0 and F^*_{T^*}(K) \le F^*_{T^*}(B), B \le K, K \in IFspcs\}$$
$$\supseteq \overline{A_{SP}} \cap \overline{B_{SP}}$$

Theorem 2.18: Let (X,T,T^*) be an IFTS and A , B ∈ I^X,then:

5. $\underline{(1)}^{\circ}_{sp} = \underline{1}$

6. $A^{\circ}_{sp} \subseteq A$

7.$(A^{\circ}_{sp})^{\circ}_{sp} \subseteq A^{\circ}_{sp}$

8.$(A \cap B)^{\circ}_{sp} \subseteq A^{\circ}_{sp} \cap B^{\circ}_{sp}$

Proof : it similar to proof theorem 2.17

Theorem 2.19:

Let (X,T,T^*) be an IFTS and A,B ∈ I^X,then:

1.T(A)> 0 $\Longrightarrow A^{\circ}_{sp} = A$

2.$F_T(A) > 0 \Longrightarrow \overline{A_{sp}} = A$

Proof: 1) let T(A)> 0 ,then

$$A \in \{K \in I^X, F_T(K) > 0 and F^*_{T^*}(K) \leq F^*_{T^*}(A), A \leq K, K \in IFspcs\}$$

So A⊆ A°_{sp},thus we get

A=A°_{sp} (by theorem 2.17)

2) it is similar to (1)

3.weakly gp.semi-premap.

In this section, we introduce the concept ofweakly gp.semi-pre mappingand investigate some properties of them.

Definition3.1[10]:

Let (X,T,T^*) and (Y,σ,σ^*) be two IFTS amapping f : X→ Y is weakly gp.map if for every$A \in I^Y \sigma(A) \geq 0, T\big(f^{-1}(A)\big) \geq 0$ and $T^*(f^{-1}(A)) \leq \sigma^*(A)$

It is obvious that every gp.map is weakly gp.map from the above definition but we can show that the converse is not always true from the following example.

Example3.2:

Let x=I and N be the set of all natural numbers for each n∈ N.We consider $\mu_n \in I^X$such that

$\mu_n = 0.5$

Define T,T^*:$I^X \rightarrow I$ by

$$T(\lambda) = \begin{cases} 1 & if\lambda = \underline{0}, \underline{1} \\ \frac{1}{n+2} & if\lambda = \mu_n \\ 0 & otherwise \end{cases} \quad T^*(\lambda) = \begin{cases} 0 & if\lambda = \underline{0}, \underline{1} \\ \frac{1}{n+2} & if\lambda = \mu_n \\ 1 & otherwise \end{cases}$$

Define σ,$\sigma^*: I^X \to I$ by

$$\sigma(\lambda) = \begin{cases} 1 & if\lambda = \underline{0}, \underline{1} \\ \frac{1}{n+2} & if\lambda = \mu_n \\ 0 & otherwise \end{cases} \quad \sigma^* = \begin{cases} 0 & if\lambda = \underline{0}, \underline{1} \\ \frac{1}{n+2} & if\lambda = \mu_n \\ 1 & otherwise \end{cases}$$

(X,T,T^*) , (Y,σ,σ^*) are IGO

Let f: (X,T,T^*)→ (Y,σ,σ^*)

Then f is weakly gp-map but not a gp-map

Since for each fuzzy set μ_n

We get $T^*\left(f^{-1}(\mu_n)\right) \leq \sigma^*(\mu_n)$,but $\sigma(\mu_n)$ is not less than T($f^{-1}(\mu_n)$)

<u>Definition3.3:</u>

Let (X,T,T^*) and (Y,σ,σ^*) be two IFTSs then a mapping f : X→ Y is weakly gp.semi-pre mapping if for each A∈ I^Y,$\sigma(A) \geq 0, T\left(f^{-1}(A)\right) \geq 0$ and $T^*(f^{-1}(A)) \leq \sigma^*(A)$such that $f^{-1}(A) \in IFSPOS$

<u>Theorem3.4:</u>

Let (X,T,T^*) and (Y,σ,σ^*) be two IFTSs then amapping f : X→ Y is weakly gp.semi-pre mapping iff for every A∈ I^Y,$F_\sigma(A) > 0 \Rightarrow F_T(f^{-1}(A)) > 0 and F^*_{T^*}(f^{-1}(A)) \leq F^*_{\sigma^*}(A)$such that $f^{-1}(A) \in IFsposc$

Proof:

Suppose f is weakly semi-pre mapping and let $F_\sigma(A) > 0$for A∈ I^Y then $F_\sigma(A^{cc}) = \sigma(A^c) > 0$, it follows,$T\left(f^{-1}(A^c)\right) > 0$and $T^*\left(f^{-1}(A^c)\right) \leq \sigma^*(A^c)$,thus we get $F_T\left(f^{-1}(A)\right) > 0 and F^*_{T^*}(f^{-1}(A)) \leq F^*_{\sigma^*}(A)$,the converse is obvious.

<u>Theorem 3.5:</u>

Let (X,T,T^*) and (Y,σ,σ^*) be two IFTSs then amapping f : X→ Y is weakly gp.semi-pre mapping iff for every A∈ I^Y,$F_\sigma(A) \leq F_T\left(f^{-1}(A)\right)$ and$F^*_{T^*}\left(f^{-1}(A)\right) \leq F^*_{\sigma^*}(A)$ such that$f^{-1}(A) \in Ifsposc$

proof: the proof is similar to that theorem3.4.

Theorem 3.7:

Let (X,T,T^*) and (Y,σ,σ^*) be two IFTSs.Then amapping f : X→ Y is weakly gp.semi-pre mapping,then we have:

1. $f(\overline{A_{sp}}) \le \overline{f(A_{sp}})$, for every A∈ I^X
2. $\overline{(f^{-1}(A))_{sp}} \le f^{-1}(\overline{A_{sp}})$, for every A∈ I^Y
3. $f^{-1}(A^{\circ}_{sp}) \le (f^{-1}(A))^{\circ}$, for every A∈ I^Y

Proof:

1) Let A∈ I^X, then by definition 2.12and theorem 3.4,we have

$$f^{-1}(\overline{f(A_{sp})}) =$$

$$f^{-1}[\cap \{u \in I^Y: F_\sigma(A) > 0 and F^*_{\sigma^*}(u) \le F^*_{\sigma^*}(f(A)), f(A) < u: u \in Ifspsc\}]$$

$$\ge \cap \{f^{-1}(u) \in I^X: F_T(f^{-1}(u)) > 0\ and F^*_{T^*}(f^{-1}(u)) \le F^*_{\sigma^*}(u), A < f^{-1}(u): f^{-1}(u) \in Ifspsc\}$$

Since $F_T(f^{-1}(u)) > 0$, it follows $\overline{A_{sp}} \le \overline{(f^{-1}(u))_{sp}}$(from theorem 1.6)

$=f^{-1}(u)$ and so

$$\cap \{f^{-1}(u) \in I^X: F_T(f^{-1}(u)) > 0 and F^*_{T^*}(f^{-1}(u)) \le F^*_{\sigma^*}(u), A < f^{-1}(u): f^{-1}(u) \in Ifspsc\} \ge \overline{A_{sp}}$$

Consequently , we get $f(\overline{A_{sp}}) \le \overline{f(A_{sp}})$

2) It follows from(1) $f(\overline{f^{-1}(A_{sp})}) \le \overline{f(f^{-1}(A_{sp}))}$

$$f(\overline{f^{-1}(A_{sp})}) \le \overline{A_{sp}}$$

$$\overline{f^{-1}(A_{sp})} \le f^{-1}(\overline{A_{sp}})$$

3) It is obtained by (2)and theorem(2.19)

Corollary 3.7:

Let (X,T,T^*) and (Y,σ,σ^*) be two IFTSs , then a mapping f : X→ Y is weakly gp.semi-pre mapping, then we have:

1. $f(\overline{A_{sp}}) \le \overline{f(A)}_{sp}$, for every A∈ I^X.
2. $\overline{(f^{-1}(A))_{sp}} \le f^{-1}(\overline{A_{sp}})$, for every A∈ I^Y.
3. $f^{-1}(A^{\circ}_{sp}) \le (f^{-1}(A))^{\circ}_{sp}$, for every A∈ I^Y.

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