Journal of College of Education for pure sciences(JCEPS) Web Site: http://eps.utq.edu.iq/ Email: eps_tqr@yahoo.com Volume 7, Number 2, May 2017 Theoretical Study of The transmittance Effect of Fractal

Optical

Modulator For CaF₂

Mohammed L. Jabbar

Mohammed25382@gmail.com

Physics Department, College of Science, Thi-Qar University

Abstract:

Optical modulator is an important component in optical system. It is a device, which changes the angle between the vision line to the target and coordinate to electrical signal. The optical modulator modulates the optical signal by a frequency depending on the shape and number of sectors. The optical modulator takes various circular shapes due to the need for it. Through this study we have designed the fractal optical modulator consisting of eighteen sector , by building a computer program (SAM1) using visual basic language. In this paper assume that nine sectors one opaque and the other nine sectors are transmitted for the light. One may consider these nine sectors also as opaque for the other regions of electro-magnetic wave spectrum.

For the importance of the modulation transfer function (MTF) in testing and evaluating optical systems, it becomes the dependent measurement to know the optical systems efficiency. It has been studied for optical systems with circular aperture, where that function could evaluated the image efficiency for point object in image plane at different magnitude of transmittance.

In this research, the fractal optical modulator has been designed of semiconductor material by using the fractal function. Then evaluating the values of MTF at different values of refractive index and transmittance.

The best modulation of Calcium Fluoride CaF_2 material optical modulator was when transmittance T = 89%. Where the value of Modulation Transfer Function (MTF) is maximum at refractive index $\eta_{\lambda=0.400\mu m}$ (CaF₂)= 1.4419. The worst case of modulation was at transmittance T = 36%. Where the value MTF is minimum at refractive index $\eta_{\lambda=0.161\mu m}$ (CaF₂)= 1.5490.

Web Site: http://eps.utq.edu.iq/ Email: eps_tqr@yahoo.com Volume 7, Number 2, May 2017

دراسة نظرية لتأثير النفاذية على قرص تضمين بصري كسوري من مادة تنائي فلوريد الكالسيوم م.م. محم د لطيف جب ار <u>Mohammed25382@gmail.com</u> قسم الفيزياء , كلية العلوم , جامعة ذي قار

الخلاصة

قرص التضمين البصري (Optical Modulator) عنصر مهم في المنظومات البصرية , وهو عبارة عن آله (Device) يقوم بتحويل الزاوية بين خط النظر إلى الهدف, والمحور البصري إلى إشارة كهربائية, ويأخذ عدة أشكال دائرية مختلفة وذلك بحسب الحاجة إليه. قرص التضمين يستعمل لتضمين الاشارة الضوئية ويعتمد على شكل القرص المصمم و عدد المقاطع.

في هذا البحث تم استخدام الحاسوب لتصميم قرص تضمين بصري كسوري باستخدام لغة الفيجول بيسك 6.0حيث يتكون القرص من ثمان عشر مقطع متساوية, تسع منها معتمة و التسع المتبقية تكون شفافة ومنفذة للضوء انحصر علم الرياضيات لعدة قرون في العالم الخطي , ولكن مع تطور العلم وجب على العلماء وعلماء الرياضيات أن يتفهموا الأنظمة غير الخطية وما ينبثق منها من العلوم الحديثة ومن ضمنها الهندسة الكسورية.

ولمعرفة كفاءة النظام البصري يتم اختباره بواسطة دالة الانتقال المعدلة MTF. وللنظام المصمم في هذا البحث كانت الفتحة المستخدمة دائرية الشكل و المادة المصنوع منها القرص شبة موصلة حيث تم حساب قيم دالة الانتقال المعدلة عند قيم مختلفة من الاطوال الموجية. وقد وجد ان افضل حالة تضمين للقرص البصري الكسوري المصنوع من ثنائي فلوريد الكالسيوم عندالطول الموجي 0.16 مايكرومتر حيث يمتلك اعظم نفاذية 8%. كذلك وجد اقل قيمة للنفاذية 36% التي تقابل الطول الموجي 0.161 مايكرومتر.

Web Site: http://eps.utq.edu.iq/ Email: eps_tqr@yahoo.com Volume 7, Number 2, May 2017

1. Introduction

The production of the optical system has passed through several stages, the optical design is the first one, after this stage is completed, the optical components manufacturing will be the next stage and then, the evaluation and the testing of these components will be the last stage before the lens is being used. The optical design includes specification for the radii of the surfaces curvature, the thickness, the air spaces, the diameters of the various components, the type of glass to be used and the position of the stop. These parameters are known as "degrees of freedom" since the designer can change them to maintain the desired system.

The image that is formed by these optical systems will be approximately corrected from the aberrations. But there isn't ideal image which corresponds to the object dimensions because of the wave nature of the light, which is mostly affected by several factors like the type of illumination that is used (incoherent, coherent and partially coherent), the object shape (Point, Line or Edge) and the aperture shape [1].

Optical modulator: any device used to modify any characteristic of an optical signal (light wave) for the purpose of conveying information[2]. Optical choppers are mechanical devices that physically block a light beam of some type. Rotating optical modulators (choppers) are perhaps the most common form and are they produced by SciTech Instrument Ltd. A metal disc with slots etched into it's mounted on a dc motor and rotated. The disc is placed in the path of light beam which will then cause the beam to be periodically interrupted by the blocking part of the disc [3].

The term fractal was coined in 1975 by Benoit Mandelbrot, from the Latin fractious, meaning "broken" or "fractured" The word fractal has two related means. In colloquial usage, it denotes a shape that is recursively constructed or self-similar, that is, a shape that appears similar at all scales of magnification and is therefore often referred to as "infinitely complex". In mathematics a fractal is a geometric object that

36

Volume 7, Number 2, May 2017

satisfies a specific technical condition, namely having a Hausdroff dimension greater than is topological dimension [4].

2. Optical Modulator

Optical modulator is a device ,which changes the angle between the vision line to the target and coordinate to electrical signal[4]. An optical modulator is used to provide directional information for target , and to suppress unwanted signal from background [1]. It is a device used for chopping the emitted light from the source. And this will be done by choosing the best shape and size . The optical modulator takes many various circular shapes due to its need[1].

Fig. (1) The Position of the modulator in the optical system.

The optical modulator is called in many different names :-

- 1-Optical modulator.
- 2-Chopper.
- 3-Raster.
- 4-Reticle.

5- In case special is crosshair.

The term reticle has been defined as "a pattern located in the focal plane of an instrument to measureor locate a point in an image." Reticles have been used and are still used in a multitude of operations from commercial applications or surveying to military applications of boresighting surveillance and fire control systems. The general case that most people are familiar with is the simple sight on a rifle or gun. There are as many types of reticle as there are uses for them. However, one type of reticle, commonly referred to as a spinning frequency modulated (FM) reticle, can be used to provide range and bearing information.[5]

The word reticle is from the Latin meaning (net) a network or grid of lines displayed in an optical instrument.

The minimum reticle consists of simple "cross-hairs" [6], A crosshair is a shape superimposed on an image that is used for precise alignment of a device. Crosshairs are most commonly a "+" shape, though many variations exist, including dots, spots, circles and chevrons. Most commonly associated with telescope sights for aiming firearms, crosshairs are also common in optical instruments used for astronomy and

Volume 7, Number 2, May 2017

surveying and also popular in graphical user interfaces as a precision pointer. The crosshair was invented by Robert Hook and date to the 17th century.

It is sometimes called chopper. I.e. the optical modulator is a device used for chopping the light beam and the output signal has frequency figure (2) which can be describe by relation

 $\mathbf{F}=\mathbf{n}\mathbf{v} \tag{1}$

Where

n : the number of sector.

v : the angular velocity (rotation speed).

F : the spatial frequency.

The modulation operation in optical modulator depends on the movement between image object and optical modulator. In this concept the optical modulator can be classified in two types :-

1-Rotating Reticle Disk :- In this type the disk rotates about its axis ,while the object image rotates within the disk area. Sometimes the disk axis has been rotated about the optical axis of the Electro-Optical-System, in circular path. This type of disk is called (Nutating Reticle).

2-Stationary Reticle Disk :- In this type the disk is stationary, while the image object has been rotated on the disk surface by using rotational optical system.

The optical modulator has two important operations in detection, tracking and guidance system ,and this operations is to:-

1-Provide directional information about tracking and to suppress unwanted signal from background .This operation is called (Spatial-Filtering).

2-Change the optical signals parameter ,which is produced from the object, by designing suitable disk pattern.

The modulation can be done by using two types of mode Active and Passive modes . The two operations can be applied in the active mode in the same time, while just the second operation can be applied in the passive mode.

The better efficiency of the optical modulator can be produced when the spot size is not larger than three times the object image size. The real efficiency is produced when the spot size is equal to the object image. When the object image approaches the optical system, its size will be increased. [1,7]

3. AM and FM Optical Modulator

One of the optical modulator shapes is (Fan Shape), and sometimes called (Wagon Wheel), it is shown in Fig.(3) and it is used in many optical applications, In radiation measurement system, it is used as optical chopper. Therefore it is used in optical modulation in most Tracking and Guidance systems. This type of optical modulator works in two modes:

The first , is when the optical modulator is rotated around its axis, then the incident radiation will be modulated in amplitude modulation AM.

The second, is when the optical modulator is stationary ,while the object scene rotates about the disk axis by nutating movement. Or the optical modulator center will be rotated about the optical axis of the tracking system. then the incident radiation will be modulated in frequency modulation **FM** .[7, 8]

4. Fractal Function

Euclidean geometry provides a first approximation to the structure of physical object. It describes objects of simple shapes, point, line segment, ellipses, circles, boxes, and cubes that have a few characteristic sizes, with dimensions one, two, and three. This geometry is mainly oriented a round linear, integral system.[9]

Non linear shapes and nonintegral systems are not easily described by traditional Euclidean geometry. These shapes and systems need another geometry that is quite different from Euclidean geometry to describe and study these cases. Benoit Mandebort [10] suggested the existence of geometries near to the geometry of nature, known as fractal geometry.

Mandelbort, coined the term "Fractal" to describe object that are very "fractured" as a clouds, mountains, coaslines, leaves, sun,.....etc.

Mandelbrot's famous and pioneering work with fractal geometry and his introduction of two new basic concepts including ; first, self-similarity, which is to say that the fractal shapes are to be self-similar and independent of scale or scaling. The general nature of the fractal irregular bumpy structure remains constant through successive magnifications such as is the case for coastlines and mountains. Each small portion when magnified can reproduce exactly a large portion. Fractal images exists as the limit of both random and deterministic processes based upon the representation called Iterated Function System (IFS).Second, a fractal has non-integer dimension known as the fractal dimension, which allows scale independent measurement of such objects,

and gives a numerical measure of the degree of boundary irregularity or surface roughness. The fractal dimensions one of the most important concepts in the study distribution. It is analogous to the concepts of length, area and volume in Euclidean Geometry [11]. And from examples on nonlinear fractals: Mandelbrot set, and Julia set which they shown in figure (4).

Now it is seen an alternative way to specify the dimension of a self-similar object. The dimension is simply the exponent of the number of self-similar pieces with magnification factor N into which the figure may be broken.

$$\mathbf{N} = \left(\frac{\mathbf{L}}{\mathbf{K}}\right)^{\mathbf{D}'} \tag{2}$$

$$D' = \frac{LogN}{Log\left(\frac{L}{K}\right)}$$
(3)

Where

 D^{\setminus} is fractal dimension, N: number segment, L: length, K: length each piece

5. Iterated function system (IFS)

fractals as they are normally called can be any number of dimensions, but are commonly computed and drawn in 2D.The fractal is made up of the union of several copies of itself, each copy being transformed by a function. This is the source of its self-similar fractal nature [12]. Formally,

$$s = \bigcup_{i} f_{i(s)}$$
 Where $s \subset R^2$ and $f_i : \mathbb{R}^2 \to \mathbb{R}^2$ (4)

And (i=1, 2, 3, 4....m).

Sometimes each function f_i is required to be linear, or more accurately an affine transformation and hence can be represented by a matrix.

$$w\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix} = \begin{bmatrix} ax + by + e \\ cx + dy + f \end{bmatrix}$$
(5)

Where

(x, y): a metric space, (e, f): transformation parameters

(a, b, c, d): real numbers (in two-dimensional)

However, IFSs may also be built from non-line a function, including projective transformations.

One can describe a general construction for fractal that occurs in classical number theory, of which sierpinski triangle, von koch curve, and cantor set are examples.

6. Cantor Set

In order to understand the cantor set, the construction becomes with a line segment of length (1) which is subdivided into three sections, removing the middle third; then removing the middle third of the remaining segment and so on. So, the number of segment is increased to two and length is reduced to (2/3). The cantor set is simply the dust of point the remain. The number of these points is infinite, but their total length is zero. As shown in fig(5)

7. OTF & MTF

Another method to specify the resolving power of an optical imaging system is by means of the optical transfer function (OTF). This function is defined as the contrast in the image of a sinusoidal grating with a given spatial frequency,

$$\omega = 2\Pi/L \tag{6}$$

Let us assume that we form the image of an object containing a wide spectrum of spatial frequencies and then analyze the frequency content in the image of this object. Then, the OTF is the ratio of the amplitude of a given spatial frequency in the image to the amplitude of the component with the same spatial frequency in the object. If the object contains all spatial frequencies with a constant amplitude, the OTF becomes the Fourier transform of the image. Such an object is a point object and its image is point spread function (PSF). Hence, the OTF is simply the Fourier transform of the point spread function.

the optical transfer function $F(\omega_x, \omega_y)$ may be obtained from the Fourier transform of the point spread function S(x,y) as follows:

$$F(\omega_x, \omega_y) = \iint S(X_F, Y_F) e^{i(wx XF, wy YF)} dx_F dy$$
(7)

We see that in general this OTF is complex and, thus it has a real and an imaginary term. The modulus of the OTF is called the modulation transfer function (MTF) and represents the contrast in the image of a sinusoidal periodic structure. The imaginary term receives the name of phase transfer function (PTF) and gives information about

the spatial phase shifting or any contrast reversal (when the phase shift is 180°) in the image. [13]

The MTF is then the magnitude response of the imaging system to sinusoids of different spatial frequencies. This response can also be defined as the attenuation factor in modulation depth:

$$M = \frac{Amax - Amin}{Amax + Amin}$$
(8)

where Amax and Amin refer to the maximum and minimum values of the waveform that describe the object or image in W/cm2 versus position. The modulation depth is actually a measure of visibility or contrast. The effect of the finite-size impulse response (i.e., not a delta function) of the optical system is to decrease the modulation depth of the image relative to that in the object distribution. This attenuation in modulation depth is a function of position in the image plane. The MTF is the ratio of image-to-object modulation depth as a function of spatial frequency:[14]

$$MTF = \frac{Mimg}{Mobj}$$
(9)

Or

$$MTF = \frac{Imax - Imin}{Imax + Imin}$$
(10)

In summary, the MTF is a powerful tool used to characterize imaging system's ability to reproduce signals as a function of spatial frequency. It is a fundamental parameter that determines where the limitations of performance in optical and electro-optical systems occur, and which crucial components must be enhanced to yield a better overall image quality. It guides system design, and predicts system Performance.[14]

7. Discussion and result

The optical modulator is an important component in optical system. The optical modulator is a disc from Calcium Fluoride CaF_2 which has a radius R, and assumes the number of sector is (eighteen sector), The computer program (SAM1) was written by using visual basic language. In the present study assume that nine sectors opaque and nine sectors are transmitted alternating to the light as shown in figure (6). The transmitting sectors might be consider as opaque to the other wavelength. Fractal function was used to divide optical modulator into very small segment of line, it's

simply the dust of point. The circular aperture was a clear transparence aperture (100%).

Assuming that the incident light is a perpendicular to the chopper. The chopper is moveable in a circular form. Hence the light beam will make discrete circles according to the number of sectors. The distance of the light movement on all sectors of it is part is an arc from a circumference of the total circle. Thus, the light form will through one revolution of the radius (the point of beam incidence of light on the sector). It is considered that arc of sector which moves on the opaque sector is the required distance only. The resultant circumference of the circle was divided on the total number of sector.

The unit of spatial frequency will be in (Rev/s) which depends on the velocity, and the number of sectors. The relation of frequency is shown in equation (1). Also the Modulation Transfer Function (MTF) is evaluated using the relation(10).

In general, the range of transmittance for Calcium Fluoride CaF₂ was found between (36% to 89%) was related to the wavelength between (0.161 to 0.400)µm; therefore, the minimum transmittance at the refractive index n_{CaF2} = 1.5490 (λ =0.161µm). The maximum transmittance for Calcium Fluoride CaF₂ at the refractive index n_{CaF2} = 1.4419 (λ =0.400µm). The MTF has a maximum value when the transmittance at maximum value.

The first inflection point always gets at the spatial frequency value equal=2(Rev/s) which is compatible to the MTF value for theoretical real optical system theoretically [15]. The high inflection point for MTF changes with transmittance changes, and the MTF curve changes with transmittance of material and vice versa. The curves are similar behavior to its shape, but is varied according to the position inflection point for MTF. It has been noted that the value of MTF for Calcium Fluoride has two inflection points at the spatial frequency =2 and 4 (Rev/s). The curves showed, the spatial frequency changes between 1 to 2 (Rev/s) the value MTF is quickly gradient. When the spatial frequency changes from 2 to 4 (Rev/s) the gradient in value MTF becomes less than the first case. Then, the gradient in the values MTF makes the spatial frequency changes. The spatial frequency is very small at the maximum value of MTF, and the spatial frequency begins to increase with decreasing the MTF values.

8. conclusion

- The comparison between the values for MTF at different values for transmittances, showed that at values large for transmittance the value of MTF become large (at increasing the transmittance will be increasing the MTF).
- 2- There is a clear relation between the intensity increases and decreasing the spatial frequency for all cases (of transmittance).
- 3- There is a considerable increase in Modulation Transfer Function (MTF) by decreasing the spatial frequency for all materials.
- 4- It is very important the optical fractal modulator, when it is designed from a specific material such as Calcium Fluoride (CaF₂) as special filter will generate frequency.
- 5- The best modulation of Calcium Fluoride (CaF₂) material optical modulator was when transmittance T = 89%. Where the value of Modulation Transfer Function (MTF) is maximum at refractive index $\eta_{\lambda=0.400\mu m}$ (CaF₂)= 1.4419. The worst case of modulation was at transmittance T = 36%. Where the value MTF is minimum at refractive index $\eta_{\lambda=0.161\mu m}$ (CaF₂)= 1.5490.
- 6- Any searcher can be use same model and material but wave length different.As a wavelength in ultraviolet or visible



Fig. (1) The Position of the modulator in the optical system[4]



Figure (2): The output signal from the modulator

Web Site: http://eps.utq.edu.iq/ Email: eps_tqr@yahoo.com Volume 7, Number 2, May 2017



Figure (3): wagonwheel pattern[8]



(a)-Mandelbrot set









Fig.(5) Construction of the classical cantor set



Fig.(6) optical modulator (design by searcher)



Fig. (7)The supposed optical modulator

Table(1):Data for	Calcium Fluoride	CaF_2 when transmittanc	e (36%)

Ν	1	2	3	4	5	6	7	8	9	10
R*10 ⁻ 2	250	500	750	1000	1250	1500	1750	2000	2250	2500
MTF *10 ⁻²	36	18	12	9	7.2	6	5.1429	4.5	4	3.6
S	0	1	2	3	4	5	6	7	8	9



Figure (8) The MTF with the spatial frequency at transmittance 36%

Table(2):Data	for C	CaF ₂ wh	hen transn	nittance	(47%)
---------------	-------	---------------------	------------	----------	-------

10	9	8	7	6	5	4	3	2	1	N
25 00	225 0	2000	1750	150 0	12 50	10 00	750	500	2 5 0	R* 10 ⁻ 2

Volume 7, Number 2, May 2017												
ſ			5				11				М	
	4.	5.2	э. 97	6.71	7.8	9.	7	15.6	22.5	4	TF	
	7	222	87 5	43	333	4	./	667	25.5	7	*1	
			5				5				0-2	
ſ	9	8	7	6	5	4	3	2	1	0	S	





Figure (9) The MTF with the spatial frequency at transmittance 47%.

Ν	1	2	3	4	5	6	7	8	9	1 0
R * 1 0 ⁻ 2	2 5 0	5 0 0	750	1 0 0 0	1 2 5 0	1500	1750	2 0 0 0	225 0	2 5 0 0
M T F * 1 0 [°] 2	6 4	3 2	21. 333 3	1 6	1 2 8	10. 666 7	9.1 442 8	8	7.1 11 1	6 4
s	0	1	2	3	4	5	6	7	8	9

Table(3):Data for CaF_2 when transmittance (64%)





Figure (10) The MTF with the spatial frequency at transmittance 64%.

Ν	1	2	3	4	5	6	7	8	9	1 0
R *1 0 ⁻²	2 5 0	50 0	7 5 0	100 0	1 2 5 0	15 00	1750	200 0	2250	2 5 0 0
M T F *1 0 ⁻²	7 5	3 7. 5	2 5	18 .7 5	1 5	1 2. 5	10.7 143	9. 37 5	8.3 333	7 5
s	0	1	2	3	4	5	6	7	8	9

Table(4):Data for CaF₂ when transmittance (75%)



Figure (11) The MTF with the spatial frequency at transmittance 75%.

Ν	1	2	3	4	5	6	7	8	9	1 0
R	2	5		1	1			20	225	2
*	5	0	750	0	2	1500	1750	20	223	5
1	0	0		0	5			00	0	0

Table(5):Data for CaF_2 when transmittance (82%).

			V	Jume /	, Numi	per z, ivia	2017			
0 ⁻ 2				0	0					0
M T F * 1 0 [.] 2	8 2	4	27. 333 3	2 0 5	1 6 4	13. 666 7	11. 714 3	1 0. 2 5	9.1 11 1	8 2
s	0	1	2	3	4	5	6	7	8	9





Figure (12) The MTF with the spatial frequency at transmittance 82%

Ν	1	2	3	4	5	6	7	8	9	1 0
R * 1 0 ⁻ 2	2 5 0	5 0 0	750	10 00	1 2 5 0	1500	1750	200 0	225 0	2 5 0 0
M T F * 1 0 [.] 2	8 5	4 2 5	28. 333 3	2 1. 2 5	1 7	14. 166 7	12. 142 8	10 .6 25	9. 44 44	8 5
s	0	1	2	3	4	5	6	7	8	9

Table(6):Data for CaF_2 when transmittance (85%).

Web Site: http://eps.utq.edu.iq/Email: eps_tqr@yahoo.comVolume 7, Number 2, May 2017



Figure (13) The MTF with the spatial frequency at transmittance 85%.

N	1	2	3	4	5	6	7	8	9	1 0
R * 1 0 ⁻ 2	2 5 0	5 0 0	750	10 00	1 2 5 0	1500	1750	200 0	225 0	2 5 0 0
M T F * 1 0 [°] 2	8	4 4 5	29. 66 67	2 2. 2 5	1 7 8	14. 83 34	12. 71 43	11 .1 25	98 .8 89	8 9
s	0	1	2	3	4	5	6	7	8	9

Table(7):Data for CaF_2 when transmittance (89%)



Figure (14) The MTF with the spatial frequency at transmittance 89%.

Reference

[1] A.S.Abdula "calculation of MTF for optical Disk Modulator by using Fractal function" University of Technology Baghdad ,April 2008A.D.

Web Site: http://eps.utq.edu.iq/ Email: eps_tqr@yahoo.com Volume 7, Number 2, May 2017

[2] http://www.atis.org/tg2k/-signal.html.

[3] http://www.scitec.uk.com/optical-chopper/310cd.

[4] W.F.Ali AL-ERYANI " Design optical Modulator By Using Fractal function Geometry" AL-Mustansiryah University. Jul. 2004A.D.

[5] Ronald G.Driggers, Carl .E.Halford, Glenn D.Boreman & et.al. " Parameters of spinning FM reticles" Applied Optics / Vol.30 No. 7/ 1 March 1991

[6] http://en.wikipedia.org/wiki/crosshair#]

[7] A. H.Al- Hamdani " دراسة تصميمية لمنظومة توجيه ليزريه تستخدم قرص تضمين بصري " University of Technology Baghdad

[8] W.L.Wolfe & G.J.Zissis "The Infrared Handbook" (IRIA center), Environmental Research Institute of Michigan, 1978

[9] M.A.H.Kadhim and K.Patel," Engineering Material", University of Technology Baghdad, part 1, 1978.

[10] L. L. Kazmerski, "Polycrystalline and Amorphous Thin-film and Devices" Academic press, New York, 1980.]

[11] M. Frame, B. Mandelbort and N. Neger, "Fractal Geometry", Yale University, 28 April 2006].

[12] http://Fractal/wikipedia/the free encyclopedia.html.

[13] D. Malacara Z. Malacara "Handbook of Optical Design" Second Edition, Centro de Investigaciones en Oprica, A.C. Ledn, Mexico, 2004

[14] A. Daniels "Field Guide to Infrared Systems" Bellingham, Washington USA

[15] D. A. Dewolf, A. J. Distasio, R. D. Faulkner, R.W. Fitts, P.D. Huston, T.T. Lewis, G.D. Kissinger, W.D. Lindley, C.A. Meyer, A.G. Nekut "Electro-Optics Handbook".