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δ- Soft open set in Soft topological Space

دلتا المفتوحة المنعمة في الفضاءات التبولوجية الناعمة

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Abstract :

In this paper we introduce a new type of an open set in topological space (δ- soft open set in soft in soft topological space). Then we use it to define some relate notions and prove some important theorem in topology using this new concept.

الخلاصة :

يؤسس هذا البحث العلاقة بين الفضاءات التبولوجية الناعمة وبين المجموعة المفتوحة الناعمة في الفضاءات الناعمة δ- soft open set in soft in soft topological space بالاستناد الى هذه العلاقة يقدم هذا البحث بعض التعاريف ذات صيغة جديدة وكذلك بعض النظريات المهمة التي تخص هذا الموضوع وبعض الملاحظات .

Introduction

This paper consists basic definitions relate to soft topology such as (soft open set , soft interior , soft closed set , soft limit point and soft neighborhood) with new definition δ-Soft open set .

Now let us define the base stone definition of our work as let $(F_A , \tilde{\tau})$ be a soft topological space and $F_B \tilde{\subseteq} F_A$ then F_B is said to be δ-Soft open set in soft topological space iff $F_B \tilde{\subseteq} F_B^{\simeq\circ}$.

The following generalization is formulated using $\propto$-open set in topological space and δ-Soft open set in soft topological space .

Also in this article we prove some theorems relate to these notions .

Basic Definitions

1.1 Definition [1] :

A soft F_A on the universe U is define by the set of ordered pairs $F_A = \{(x, f_A(x)) : x \in E, f_A(x) \in p(U)\}$.

Where $f_A : E \longrightarrow p(U)$ suchthat $f_A(x) = \emptyset$ if $x \notin A$ and $p(U)$ is the power set of U .

The value of $f_A(x)$ may be arbitrary . some of them may be empty some may have non empty intersection .

The set of all soft sets over U will be denoted by $S(U)$.

1.2 Example:

Suppose that there are six houses in the universe

U= $\{h_1, h_2, h_3, h_4, h_5, h_6\}$ under consideration, and that

E= $\{x_1, x_2, x_3, x_4, x_5\}$ is a set of decision parameters. The x_i (i= 1,2,3,4,5) stand for the parameters "expensive", "beautiful", "wooden", "cheap", and "in green surroundings", respectively.

Consider the mapping f_A given by "houses (.)", where (.) is to be filled in by one of the parameters $x_i \in E$, for instance f_A (e_1) mean "houses (expensive)", and its functional value is the set $\{h \in U : h \text{ is an expensive house}\}$.

Suppose that A= $\{x_1, x_3, x_4\} \subseteq E$ and f_A (x_1) = $\{h_2, h_4\}$, f_A(x_3) = U, and f_A (x_4) = $\{h_1, h_3, h_5\}$, then, we can view the Soft set F_A as consisting of the following collection of approximations:

$F_A = \{(x_1, \{h_2, h_4\}), (x_3, U), (x_4, \{h_1, h_3, h_5\})\}$

1.3 Definition : [1]

Let $F_A \in S(U)$ the soft power set of F_A is defined by
$\tilde{p}(F_A) = \{F_{A_i}: F_{A_i} \tilde{\subseteq} F_A, i \in I \subseteq N\}$ and its cardinality is defined by

$$|\tilde{p}(F_A)| = 2^{\sum_{x \in E} |f_A(x)|}$$

Where $|f_A(x)|$ is the cardinality of $f_A(x)$

1.4 Example :

Let $U = \{u_1, u_2, u_3\}$, $E = \{x_1, x_2, x_3\}$, $A = \{x_1, x_2\} \subseteq E$ and $F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_2, u_3\})\}$. Then

$F_{A_1} = \{(x_1, \{u_1\})\}$
$F_{A_2} = \{(x_1, \{u_2\})\}$
$F_{A_3} = \{(x_1, \{u_1, u_2\})\}$
$F_{A_4} = \{(x_2, \{u_2\})\}$
$F_{A_5} = \{(x_2, \{u_3\})\}$
$F_{A_6} = \{(x_2, \{u_2, u_3\})\}$
$F_{A_7} = \{(x_1, \{u_1\}), (x_2, \{u_2\})\}$
$F_{A_8} = \{(x_1, \{u_1\}), (x_2, \{u_3\})\}$
$F_{A_9} = \{(x_1, \{u_1\}), (x_2, \{u_2, u_3\})\}$
$F_{A_{10}} = \{(x_1, \{u_2\}), (x_2, \{u_2\})\}$
$F_{A_{11}} = \{(x_1, \{u_2\}), (x_2, \{u_3\})\}$
$F_{A_{12}} = \{(x_1, \{u_2\}), (x_2, \{u_2, u_3\})\}$
$F_{A_{13}} = \{(x_1, \{u_1, u_2\}), (x_2, \{u_2\})\}$
$F_{A_{14}} = \{(x_1, \{u_1, u_2\}), (x_2, \{u_3\})\}$
$F_{A_{15}} = F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_2, u_3\})\}$
$F_{A_{16}} = F_\emptyset$

Are all soft subset of F_A-so $|\tilde{p}(F_A)| = 2^4 = 16$

1.5 Definition [1]

Let $F_A \in S(U)$. A soft topology on F_A , denoted by $\tilde{\tau}$ is a collection of soft subsets of F_A having the following properties

i- $F_\emptyset, F_A \in \tilde{\tau}$
ii- $\{F_{A_i} \tilde{\subseteq} F_A : i \in I \subseteq \mathbb{N}\} \subseteq \tilde{\tau} \Longrightarrow \tilde{\bigcup}_{i \in I} F_{A_i} \in \tilde{\tau}$
iii-$\{F_{A_i} \tilde{\subseteq} F_A : 1 \le i \le n, n \in \mathbb{N}\} \subseteq \tilde{\tau} \Longrightarrow \bigcap_{i=1}^{n} F_{A_i} \in \tilde{\tau}$

The pair $(F_A, \tilde{\tau})$ is called a soft topological space .

1.6 Example :

Let us consider the soft subset of F_A that are given in (Example 1.3) Then $\tilde{\tau}_1 = \{F_\emptyset, F_A\}$, $\tilde{\tau}_2 = \tilde{p}(F_A)$, and $\tilde{\tau}_3 = \left\{F_\emptyset, F_A, F_{A_2}, F_{A_{11}}, F_{A_{13}}\right\}$ are soft topologies on F_A .

1.7 Definition : [1]

Let $(F_A, \tilde{\tau})$ be a soft topological space , Then every element of $\tilde{\tau}$ is called a soft open set . clearly $F_\emptyset$, F_A are soft open set .

1.8 Definition : [1]

Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B \tilde{\subseteq} F_A$ Then the soft interior of F_B , denoted F_B° is define as the soft union of all soft open subsets of F_B .

Note that F_B° is the biggest soft open set is contain by F_B .

1.9 Example :

Let us consider the soft topology

$\tilde{\tau}_3 = \left\{F_\emptyset, F_A, F_{A_2}, F_{A_{11}}, F_{A_{13}}\right\}$,

If $F_B = F_{A_{12}} = \{(x_1, \{u_2\}) , (x_2, \{u_2, u_3\}) \}$

then $F_B^\circ = F_\emptyset \tilde{\cup} F_{A_2} \tilde{\cup} F_{A_{11}} = F_{A_{11}}$

$= F_\emptyset \tilde{\cup} \{(x_1, \{u_2\})\} \tilde{\cup} \{(x_1, \{u_2\}) , (x_2, \{u_3\})\}$

$= \{(x_1, \{u_2\}) , (x_2, \{u_3\})\}$

1.10 Definition : [1]

Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B \tilde{\subseteq} F_A$ Then the soft closure of F_B , denoted $\bar{F}_B$ is define as the soft intersection of all soft closed superset of F_B .

Note that $\bar{F}_B$ is the smallest soft closed set of F_B .

1.11 Example :

Let us consider the soft topology

$\tilde{\tau}_3 = \left\{F_\emptyset, F_A, F_{A_2}, F_{A_{11}}, F_{A_{13}}\right\}$,

If $F_B = F_{A_9} = \{(x_1, \{u_1\}) , (x_2, \{u_2, u_3\}) \}$ then

$F_{A_2} = \{(x_1, \{u_2\}) \} \Longrightarrow F_{A_2}^{\tilde{c}} = \{(x_1, \{u_2\}), (x_2, U), (x_3, U)\}$

and $F_\emptyset^{\tilde{c}} = F_{\tilde{E}}$ are soft closed super set of F_B .

Hence $\bar{F}_B = F_{A_2}^{\tilde{c}} \tilde{\cap} F_{\tilde{E}} = F_{A_2}^{\tilde{c}}$

1.12 Definition :[3]

Let $(F_A, \tilde{\tau})$ be a soft topological space and $\propto \in F_A$ There is a soft open set F_B such that $\propto \in F_B$, Then F_B is called a soft open neighborhood (or soft neighborhood) of $\propto$, These f of all soft neighborhoods of $\propto$ denoted $\tilde{v}(\propto)$ is called $\tilde{v}(\propto) = \{F_B, F_B \in \tilde{\tau}, \propto \in F_B\}$.

1.13 Example :

Let us consider the $(F_A, \tilde{\tau}_3)$ in Example (1.5) and $\propto = (x_1, \{u_1, u_2\}) \in F_A$ Then $\tilde{v}(\propto) = \left\{F_A, F_{A_{13}}\right\}$.

1.14 Definition : [3]

Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B \tilde{\subseteq} F_A$ and $\propto \in F_A$. If every neighborhood of $\propto$ soft intersects F_B in some points other then $\propto$ it self , then $\propto$ is called a soft limit point of F_B , The set of all limit point of F_B is denoted by $\acute{F}_B$.

In other words , if $(F_A, \tilde{\tau})$ is a soft topological space F_B , $F_C \tilde{\subseteq} F_A$ and $\propto \in F_A$ then $\propto \in \acute{F}_B \Leftrightarrow F_C \tilde{\cap} \left(F_B \tilde{\setminus} \{\propto\}\right) \neq F_\emptyset$ for all $F_C \in \tilde{v}(\propto)$.

A new definition for soft topology

1.15 Definition :

Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B \tilde{\subseteq} F_A$ Then F_B is said to be δ-Soft open set in soft topological space iff $F_B \tilde{\subseteq} F_B^{\tilde{\simeq}\circ}$.

1.16 Example :

Let us consider the soft topology

$\tilde{\tau}_3 = \{F_\emptyset, F_A, F_{A_2}, F_{A_{11}}, F_{A_{13}}\}$,

If $F_B = F_{A_{12}} = \{(x_1, \{u_2\}), (x_2, \{u_2, u_3\})\}$

then $F_B^\circ = F_\emptyset \tilde{\cup} F_{A_2} \tilde{\cup} F_{A_{11}}$

$= F_\emptyset \tilde{\cup} \{(x_1, \{u_2\})\} \tilde{\cup} \{(x_1, \{u_2\}), (x_2, \{u_3\})\}$

$= \{(x_1, \{u_2\}), (x_2, \{u_3\})\}$

$F_{A_{11}}^{\tilde{c}} = \{(x_1, \{u_1, u_3\}), (x_2, \{u_1, u_2\}), (x_3, U)\}$

$F_B^{\tilde{\simeq}\circ} = F_{A_{11}}^{\tilde{c}} \tilde{\cap} F_E = F_{A_{11}}^{\tilde{c}}$

$F_B^{\tilde{\simeq}\circ} = F_{A_{11}} = \{(x_1, \{u_2\}), (x_2, \{u_3\})\}$

2.1 Some Theorem of δ-soft open set in soft topological space

2.1.1 Theorem :

Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B \tilde{\subseteq} F_A$ Then δ-Soft interior of $F_B = \tilde{\cup}\{F_G, F_G$, δ-Soft open set , $F_G \tilde{\subseteq} F_B\}$.

***Proof* :** Let $\alpha \in \delta$-Soft interior of F_B iff F_B is δ-Soft neighborhood of F_B , $\propto \in \delta$-Soft interior of F_B iff there exists δ-Soft open set of F_G such that $\propto \in F_G \tilde{\subseteq} F_B$, $\propto \in \delta$-Soft interior of F_B iff $\alpha \in \tilde{\cup}\{F_G : F_G$ is δ-Soft open set$\}$.

Hence δ-Soft interior of $F_B = \tilde{\cup}\{F_G : F_G$ is δ-Soft open set $F_G \tilde{\subseteq} F_B\}$.

2.1.2 Theorem :

Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B \tilde{\subseteq} F_A$ Then

(a) δ-Soft interior of F_B is an δ-Soft open set .

(b) δ-Soft interior of F_B is the largest δ-Soft open set contained in F_B .

(c) F_B is δ-Soft open set iff δ-Soft interior of $F_B = F_B$.

Proof :

(a) Let α beany or bitrary point of δ-Soft interior of F_B Then α is an δ-Soft interior point of F_B . Hence by definition , F_B is a δ-Soft nbd. of α . Then there exist an δ-Soft open set $F_G \ni \alpha \in F_G \subset F_B$,since F_G is δ-Soft open , it is δ-Soft nbd. of each of its points and so F_B is also a δ-Soft nbd. of each point of F_G . It follows that every point of F_G is an δ-Soft interior point of F_B so that $F_G \subset \delta$-Soft interior of F_B , Thus it is shown that to each $\alpha \in$ δ-Soft interior of F_B There exists an δ-Soft open set F_G such that $\alpha \in F_G \subset \delta$-Soft interior of F_B . Hence δ-Soft interior of F_B is a δ-Soft nbd. of each of its points and con sequently δ-Soft interior of F_B is δ-Soft open set .

(b) Let F_G beany δ-Soft open subset of A and let $\alpha \in F_G$ so that $\alpha \in F_G \subset F_B$. Since F_G is δ-Soft open , F_B is a δ-Soft nbd. of α and con sequently α is an δ-Soft interior point of F_B . Hence $\alpha \in \delta$-Soft interior of F_B . Thus we have shown that $\alpha \in F_B \Longrightarrow \alpha \in \delta$-Soft interior of F_B and so $F_F \subset \delta$-Soft interior of $F_B \subset F_B$ Hence δ-Soft interior of F_B contains every δ-Soft open sub set of F_B and it is therefore is the largest δ-Soft open sub set of F_B .

(c) Let $F_B = \delta$-Soft interior of F_B By (a) δ-Soft interior of F_B is an δ-Soft open set and therefore F_B is also δ-Soft open .

Conversely let F_B be δ-Soft open , then F_B is surely identical with the largest δ-Soft open sub set of F_B . But By (b) δ-Soft interior of F_B is the largest δ-Soft open sub set of F_B .

Hence $F_B = \delta$-Soft interior of F_B

2.1.3 Theorem :

Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B \tilde{\subseteq} F_A$ Then δ-Soft interior of F_B equals these f of all those pair element of F_B which are not δ-Soft limit point of $\propto -F_B$.

2.1.4 Theorem :

Let $(F_A, \tilde{\tau})$ be a soft topological space and F_B , F_C be any subsets of F_A then

(a) δ-Soft interior of $F_B = F_B$, δ-Soft interior of $F_\emptyset = F_\emptyset$
(b) δ-Soft interior of $F_B \tilde{\subseteq} F_B$.
(c) If $F_B \tilde{\subseteq} F_C$, then δ-Soft interior of $F_B \tilde{\subseteq}$ δ-Soft interior of F_C .
(d) δ-Soft interior of $(F_B \tilde{\cap} F_C) =$ δ-Soft interior of $F_B \tilde{\cap}$ δ-Soft interior of F_C .
(e) δ-Soft interior of $F_B \tilde{\cup}$ δ-Soft interior of $F_C \tilde{\subseteq}$ δ-Soft interior of $(F_B \cup F_C)$.
(f) δ-Soft interior of (δ-Soft interior of $F_B) =$ δ-Soft interior of F_B .

***Proof* :**

(a) Since F_A and $F_\emptyset$ are δ-Soft open sets , so by above theorem δ-Soft interior of $F_B = F_B$, δ-Soft interior of $F_\emptyset = F_\emptyset$.
(b) If $\propto \in$ δ-Soft interior of F_B , then F_B δ-Soft neighborhood of F_B , so $\propto \in F_B$, Hence δ-Soft interior of $F_B \tilde{\subseteq} F_B$.
(c) Let $\propto \in$ δ-Soft interior of F_B , then F_B δ-Soft neighborhood of F_B , since $F_B \tilde{\subseteq} F_C$, so F_C is also a δ-Soft neighborhood of F_B , this implies $\propto \in$ δ-Soft interior of F_C , hence δ-Soft interior of $F_B \tilde{\subseteq}$ δ-Soft interior of F_C .
(d) Since $F_B \tilde{\cap} F_C \tilde{\subseteq} F_B$, and $F_B \tilde{\cap} F_C \tilde{\subseteq} F_C$, we have by theorem (c) δ-Soft interior of $(F_B \tilde{\cap} F_C) \tilde{\subseteq}$ δ-Soft interior of F_B and δ-Soft interior of $(F_B \tilde{\cap} F_C) \tilde{\subseteq}$ δ-Soft interior of F_C .

Hence

δ-Soft interior of $(F_B \tilde{\cap} F_C) \tilde{\subseteq}$ δ-Soft interior $F_B \tilde{\cap}$ δ-soft interior of F_C ……………………… (1)

Let $\propto \in$ δ-Soft interior $F_B \tilde{\cap}$ δ-Soft interior of F_C , then $\propto \in$ δ-Soft interior F_B and δ-Soft interior of F_C, hence $\propto$ is δ-Soft interior point of each the set F_B and F_C .

Since F_B and F_C are δ-soft neighborhood of F_B , so the intersection $F_B \cap F_C$ is also δ-soft neighborhood of F_B .

Hence δ-soft interior $(F_B \tilde{\cap} F_C)$, thus $\propto \in$ δ-Soft interior $F_B \cap$ δ-Soft interior of F_C , $\propto \in$ δ-Soft interior $(F_B \tilde{\cap} F_C)$

Hence δ-Soft interior $F_B \tilde{\cap}$ δ-soft interior of $F_C \tilde{\subseteq}$ δ-Soft interior $(F_B \tilde{\cap} F_C)$ ………………… (2)

From (1) and (2) we get

δ-Soft interior of $(F_B \tilde{\cap} F_C) =$ δ-Soft interior of $F_B \tilde{\cap}$ δ-Soft interior of F_C .

(e) So by (c) we have $F_B \tilde{\subseteq} F_B \tilde{\cup} F_C$, then δ-Soft interior $F_B \tilde{\subseteq}$ δ-Soft interior $(F_B \tilde{\cup} F_C)$
$F_C \tilde{\subseteq} F_B \tilde{\cup} F_C$, then δ-Soft interior $F_C \tilde{\subseteq}$ δ-Soft interior $(F_B \tilde{\cup} F_C)$
Hence δ-Soft interior of $F_B \tilde{\cup}$ δ-Soft interior of $F_C \tilde{\subseteq}$ δ-Soft interior of $(F_B \cup F_C)$.

2.1.5 Theorem :

Let $(F_A, \tilde{\tau})$ be a soft topological space and F_B , F_C be any subsets of F_A then

(a) δ-Soft cl $(F_\emptyset) = F_\emptyset$.
(b) $F_B \tilde{\subseteq}$ δ-Soft cl $(F_\emptyset)$.
(c) If $F_B \tilde{\subseteq} F_C$, then δ-Soft cl $(F_B) \tilde{\subseteq}$ δ-Soft cl (F_C) .
(d) δ-Soft cl $(F_B \tilde{\cap} F_C) \tilde{\subseteq}$ δ-Soft cl $(F_B) \tilde{\cap}$ δ-Soft cl (F_C) .
(e) δ-Soft cl (δ-Soft cl $(F_B)) =$ δ-Soft cl (F_B) .

***Proof* :**

(a) Since $F_\emptyset$ is δ-Soft closed , we have δ-Soft cl $(F_\emptyset) = F_\emptyset$.
(b) By above theorem (a) , hence $F_B \tilde{\subseteq}$ δ-Soft cl (F_B) .

(c) By (b) , $F_C \tilde{\subseteq} \delta$-Soft cl (F_C) , since $F_B \tilde{\subseteq} F_C$, then $F_B \tilde{\subseteq} \delta$-Soft cl (F_C) But δ-Soft cl (F_C) is Soft closed set , thus δ-Soft cl (F_C) is δ-Soft closed set containing F_B , since δ-Soft cl (F_B) is the smallest δ-Soft closed set containing F_B , we have δ-Soft cl $(F_B) \tilde{\subseteq} \delta$-Soft cl (F_C) .

(d) Since $F_B \tilde{\subseteq} F_B \tilde{\cup} F_C$ and $F_C \tilde{\subseteq} F_B \tilde{\cup} F_C$
We have δ-Soft cl $(F_B) \tilde{\subseteq} \delta$-Soft cl $(F_B \cup F_C)$ and δ-Soft cl (F_C) by (c) .

Hence δ-Soft cl $(F_B) \tilde{\cup}$ δ-Soft cl (F_C) δ-Soft cl $(F_B \tilde{\cup} F_C)$ …………………………………... (1)
Since δ-Soft cl (F_B) and δ-Soft cl (F_C) are ∝-Soft closed set , and δ-Soft cl $(F_B) \tilde{\cup}$ δ-Soft cl (F_C) is also δ-Soft closed set and by (b)
$F_B \tilde{\subseteq} \delta$-Soft cl (F_B) , $F_C \tilde{\subseteq} \delta$-Soft cl (F_C)
This implies that $F_B \tilde{\cup} F_C \tilde{\subseteq} \delta$-Soft cl $(F_B) \tilde{\cup}$ δ-Soft cl (F_C) .
Thus δ-Soft cl $(F_B) \tilde{\cup}$ δ-Soft cl (F_C) is δ-Soft closed set containing $F_B \tilde{\cup} F_C$, since δ-Soft cl $(F_B \tilde{\cup} F_C)$ is the smallest δ-Soft closed set containing $F_B \tilde{\cup} F_C$.
Therefore δ-Soft cl $(F_B \tilde{\cup} F_C) \tilde{\subseteq} \delta$-Soft cl $(F_B) \tilde{\cup}$ δ-Soft cl (F_C)……………………………………………… (2)
From (1) and (2) , we have
δ-Soft cl $(F_B \tilde{\cup} F_C) = \delta$-Soft cl $(F_B) \cup$ δ-Soft cl (F_C)

(e) Since $F_B \tilde{\cap} F_C \tilde{\subseteq} F_B$, then δ-Soft cl $(F_B \tilde{\cap} F_C) \tilde{\subseteq} \delta$-Soft cl (F_B) by (c) and $F_B \tilde{\cap} F_C \tilde{\subseteq} F_C$, then δ-Soft cl $(F_B \tilde{\cap} F_C) \tilde{\subseteq} \delta$-Soft cl (F_C) by (c) Hence δ-Soft cl $(F_B \tilde{\cap} F_C) \tilde{\subseteq} \delta$-Soft cl $(F_B) \tilde{\cap}$ δ-Soft cl (F_C)

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