

## **Some Results of Soft Expert GE-Metric Spaces**

### **بعض نتائج الفضاءات المترية-GE المتخصصة الناعمة**

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#### **Abstract**

In this paper , we study the concept soft expert *GE*-metric mapping by using soft expert real sets has been defined, then we gave definition of soft expert *GE*-metric space. Also we introduce some concepts as a soft expert *GE* -ball, soft expert *GE*-continuity and studied some properties of these concepts .

#### **الخلاصة**

في هذا البحث، درسنا مفهوم الدالة المترية-GE المتخصصة الناعمة باستخدام المجاميع الحقيقية المتخصصة الناعمة والتي تم تعريفها، ثم اعطينا تعريف الفضاء المترية-GE المتخصصة الناعمة. كذلك قدمنا بعض المفاهيم كالكرة-GE المتخصصة الناعمة، الاستمرارية-GE المتخصصة الناعمة ودرسنا بعض خصائص هذه المفاهيم.

#### **1.Introduction**

In [2] Molodtsov start the theory of soft sets as a new mathematical device to deal with uncertainties. However in 2006 the connotation of G-metric space was introduced [8].

Next, this soft metric spaces [4,5] . Zorlutuna [14] also studied on soft topological spaces. Das and Samahta [6] introduced the notations of soft real set and soft real number and give their characteristic .Mustafa and Sims [9] introduce a new structure of generalized metric spaces which are called G-metric spaces as a generalization of metric space  $(X, \mathbb{d})$  to extend and introduce a new fixed point theory for different mapping in this new composition. Some authors [1,3, 10,11,12,13] have proved some fixed point theorems in these spaces. Further the subject of soft expert metric spaces has also not been studied too. For this cause, using the definition of soft expert set given by [7] .

In the present paper, we introduce the idea of soft expert generalization metric space and some of their characteristic. Last, we give the relation between the soft expert metric space and soft expert generalization metric space. We discuss soft expert  $\mathbb{G}$ -open and soft expert  $\mathbb{G}$ -closed and get its basic feature.

#### **2) Preliminaries**

First, we recall some definitions and results. Let  $\mathbb{U}$  be universe,  $\mathbb{E}$  a set of parameters and  $X$  a set of experts. Let  $\mathbb{O}$  be a set opinions,  $\mathbb{z} = \mathbb{E} \times \mathbb{O} \times X$  and  $\mathbb{A} \subseteq \mathbb{z}$  .

##### **Definition 2.1 [15]**

A pair  $(\mathbb{F}, \mathbb{A})$  is called a soft expert set over  $\mathbb{U}$ , where  $\mathbb{F}$  is mapping given by  $\mathbb{F}: \mathbb{A} \rightarrow \mathbb{P}(\mathbb{U})$  where  $\mathbb{P}(\mathbb{U})$  denotes the power set of  $\mathbb{U}$ .

##### **Definition 2.2 [15]**

For two soft expert sets  $(\mathbb{F}, \mathbb{A})$  and  $(\tilde{\mathbb{Y}}, \mathbb{p})$  over  $\mathbb{U}$ ,  $(\mathbb{F}, \mathbb{A})$  is called a soft expert subset of  $(\tilde{\mathbb{Y}}, \mathbb{p})$  if

1-  $\mathbb{A} \subseteq \mathbb{p}$

2-  $\forall \epsilon \in \mathbb{p}, \tilde{\mathbb{Y}}(\epsilon) \subseteq \mathbb{F}(\epsilon)$ .

This relationship is denoted by  $(\mathbb{F}, \mathbb{A}) \subseteq (\tilde{\mathbb{Y}}, \mathbb{p})$ .

In this case  $(\tilde{\mathbb{Y}}, \mathbb{p})$  is called a soft expert super set of  $(\mathbb{F}, \mathbb{A})$  .

**Definition 2.3 [15]**

Two soft expert set  $(\mathbb{F}, \hat{A})$  and  $(\tilde{Y}, \hat{p})$  over a common universe  $\hat{U}$ , are said to be equal if  $(\mathbb{F}, \hat{A})$  is a soft expert subsets of  $(\tilde{Y}, \hat{p})$  and  $(\tilde{Y}, \hat{p})$  is a soft expert subset of  $(\mathbb{F}, \hat{A})$ .

**Definition 2.4[15]**

Let  $\mathbb{R}$  be the set of real numbers and  $\mathbb{B}(\mathbb{R})$  be collection of all non-empties bounded subsets of  $\mathbb{R}$  and  $\hat{E}$  set parameters and  $\mathbb{X}$  a set of expert.

Let  $\hat{O}$  be a set opinions,  $\mathbb{z} = \hat{E} \times \hat{O} \times \mathbb{X}$  and  $\hat{A} \subseteq \mathbb{z}$ . Then a mapping  $\mathbb{F}: \hat{A} \rightarrow \mathbb{B}(\mathbb{R})$  is called a soft expert real set. It is denoted by  $(\mathbb{F}, \hat{A})$ . If especially  $(\mathbb{F}, \hat{A})$  is a singleton soft expert set, then identifying  $(\mathbb{F}, \hat{A})$  with the corresponding soft expert element, it will be called a soft expert real number and denoted  $\mathbb{k}, \mathbb{m}, \mathbb{m}$  such that  $\mathbb{k}(\mathbb{e}) = \mathbb{e}, \forall \mathbb{e} \in \hat{A}$  etc.

For example  $\mathbb{I}$  soft expert real number  $\mathbb{I}(\mathbb{e}) = \mathbb{I} \quad \forall \mathbb{e} \in \hat{A}$ .

**Definition 2.5 [15]**

For two soft expert real number,

1.  $\mathbb{K} \leq \mathbb{m}$  if  $\mathbb{K}(\mathbb{a}) \leq \mathbb{m}(\mathbb{a}) \quad \forall \mathbb{a} \in \hat{A}$  ;
2.  $\mathbb{K} \geq \mathbb{m}$  if  $\mathbb{K}(\mathbb{a}) \geq \mathbb{m}(\mathbb{a}) \quad \forall \mathbb{a} \in \hat{A}$  ;
3.  $\mathbb{K} < \mathbb{m}$  if  $\mathbb{K}(\mathbb{a}) < \mathbb{m}(\mathbb{a}) \quad \forall \mathbb{a} \in \hat{A}$  ;
4.  $\mathbb{K} > \mathbb{m}$  if  $\mathbb{K}(\mathbb{a}) > \mathbb{m}(\mathbb{a}) \quad \forall \mathbb{a} \in \hat{A}$  .

**Definition 2.6 [15]**

A soft expert set  $(\mathbb{P}, \hat{A})$  over  $\hat{U}$  is said to be soft expert point if there is exactly one  $\mathbb{e} \in \hat{A}$ , such that  $\mathbb{P}(\mathbb{a}) = \{\mathbb{x}\}$  for some  $\mathbb{x} \in \hat{U}$  and  $\mathbb{P}(\mathbb{a}') = \emptyset \quad \forall \mathbb{a}' \in \hat{A} \setminus \{\mathbb{a}\}$ . It will be denoted by  $x_{\mathbb{a}}$ .

**Definition 2.7 [15]**

Two soft expert points  $x_{\mathbb{a}}, y_{\mathbb{a}}$  are said to be equal if  $\mathbb{a}' = \mathbb{a}$ , and

$$\mathbb{P}(\mathbb{a}) = \mathbb{P}(\mathbb{a}')$$

i.e.  $x = y$ . Thus  $x_{\mathbb{a}} = y_{\mathbb{a}} \leftrightarrow \text{or } \mathbb{a}' \neq \mathbb{a}$ .

**Theorem 2.8 [15]**

The union of any collection of soft expert points can be considered as soft expert set and every soft expert set can express as union of all soft expert points belonging to it

$$(\mathbb{F}, \hat{A}) = \coprod_{x_{\mathbb{a}} \in (\mathbb{F}, \hat{A})} x_{\mathbb{a}}$$

Let  $\hat{U}$  be absolute soft expert set i.e.  $\mathbb{F}(\mathbb{a}) = \hat{U}, \mathbb{a} \in \hat{A}$ , where,  $(\mathbb{F}, \hat{A}) = \hat{U}$  and  $\text{SEX}(\hat{U})$  be collection of all soft expert point  $\hat{U}$  and  $\mathbb{R}(\hat{A})$  denoted the set of all non-negative soft expert real numbers.

**3- Main Result**

We will introduce soft expert **GE**-metric function by using soft expert real sets has been defined, and study their properties.

**Definition 3.1**

A mapping  $\mathbb{G}: \text{SEX}(\hat{U}) \times \text{SEX}(\hat{U}) \times \text{SEX}(\hat{U}) \rightarrow \mathbb{R}(\hat{A})$  is said to be a soft expert generalized metric on the soft expert set  $\hat{U}$  if it satisfies the following conditions :-

- 1-  $\mathbb{G}(x_{\mathbb{a}_1}, y_{\mathbb{a}_2}, z_{\mathbb{a}_3}) = 0$  if  $x_{\mathbb{a}_1} = y_{\mathbb{a}_2} = z_{\mathbb{a}_3}$ .
- 2-  $\mathbb{G}(x_{\mathbb{a}_1}, x_{\mathbb{a}_1}, y_{\mathbb{a}_2}) > 0$  for all  $x_{\mathbb{a}_1}, y_{\mathbb{a}_2} \in \hat{U}$  with  $x_{\mathbb{a}_1} \neq y_{\mathbb{a}_2}$
- 3-  $\mathbb{G}(x_{\mathbb{a}_1}, x_{\mathbb{a}_1}, y_{\mathbb{a}_2}) \leq \mathbb{G}(x_{\mathbb{a}_1}, y_{\mathbb{a}_2}, z_{\mathbb{a}_3})$  for all  $x_{\mathbb{a}_1}, y_{\mathbb{a}_2}, z_{\mathbb{a}_3} \in \hat{U}$  with  $y_{\mathbb{a}_2} \neq z_{\mathbb{a}_3}$
- 4-  $\mathbb{G}(x_{\mathbb{a}_1}, y_{\mathbb{a}_2}, z_{\mathbb{a}_3}) = \mathbb{G}(x_{\mathbb{a}_1}, z_{\mathbb{a}_3}, y_{\mathbb{a}_2}) = \mathbb{G}(y_{\mathbb{a}_2}, z_{\mathbb{a}_3}, x_{\mathbb{a}_1}) = \dots$

5-  $\mathbb{G}(x_{a_1}, y_{a_2}, z_{a_3}) \leq \mathbb{G}(x_{a_1}, a', a') + \mathbb{G}(a', y_{a_2}, z_{a_3})$  for all  $x_{a_1}, y_{a_2}, z_{a_3}, a' \in \widehat{U}$

The soft expert set  $\widehat{U}$  with soft expert **GE** -Metric on  $\widehat{U}$  is called a soft expert **GE** -Metric space and is denoted by  $(\widehat{U}, \mathbb{G}, \widehat{A})$ .

**Example 3.2**

Let  $\widehat{U} \subseteq \mathbb{R}$  be a nonempty set and  $E \subseteq \mathbb{R}$  be the nonempty set of parameters and  $X$  a set of experts . Let  $\widehat{O}$  be a set opinions,  $\mathbf{z} = \widehat{E} \times \widehat{O} \times X$  and  $\widehat{A} \subseteq \mathbf{z}$  . Let  $\widehat{U}$  be the absolute soft expert set i.e.  $\mathbb{F}(\mathfrak{e}) = \widehat{U} \forall \mathfrak{e} \in \widehat{A}$  where  $(\mathbb{F}, \widehat{A}) = \widehat{U}$  . Let  $\mathbb{k}$  denoted the soft expert real number such that  $\mathbb{k}(\mathfrak{e}) = \mathfrak{e}, \forall \mathfrak{e} \in \widehat{A}$  .

Define  $\mathbb{G}: \text{SEX}(\widehat{U}) \times \text{SEX}(\widehat{U}) \times \text{SEX}(\widehat{U}) \rightarrow \mathbb{R}(\widehat{A})$  by  $\mathbb{G}(x_{a_1}(\alpha), y_{a_2}(\beta), z_{a_3}(\delta)) = |x_{a_1} - y_{a_2} - z_{a_3}| - |\bar{\alpha} - \bar{\beta} - \bar{\delta}|$  for all  $x_{a_1}, y_{a_2}, z_{a_3}, \bar{\alpha}, \bar{\beta}, \bar{\delta} \in \widehat{U}$  . Then  $\mathbb{G}$  is soft expert **GE** -metric space on  $\widehat{U}$  .

**Solve**

- 1) It is clear  $\mathbb{G}(x_{a_1}(\alpha), y_{a_2}(\beta), z_{a_3}(\delta)) \geq 0$  for all  $x_{a_1}, y_{a_2}, z_{a_3} \in \widehat{U}$  .
- 2)  $\mathbb{G}(x_{a_1}(\alpha), y_{a_2}(\beta), z_{a_3}(\delta)) = 0$  , then  $|x_{a_1} - y_{a_2} - z_{a_3}| - |\bar{\alpha} - \bar{\beta} - \bar{\delta}| = 0 \Rightarrow x_{a_1} = y_{a_2} = z_{a_3}$  and  $\bar{\alpha} = \bar{\beta} = \bar{\delta} \Rightarrow x_{a_1}(\alpha) = y_{a_2}(\beta) = z_{a_3}(\delta)$  .
- 3)  $\mathbb{G}(x_{a_1}(\alpha), y_{a_2}(\beta), z_{a_3}(\delta)) = |x_{a_1} - y_{a_2} - z_{a_3}| - |\bar{\alpha} - \bar{\beta} - \bar{\delta}| \geq |-y_{a_2}| - |-\bar{\beta}| = |x_{a_1} - x_{a_1} - y_{a_2}| + |\bar{\alpha} - \bar{\alpha} - \bar{\beta}| = \mathbb{G}(x_{a_1}(\alpha), x_{a_1}(\alpha), y_{a_2}(\beta))$  .
- 4)  $\mathbb{G}(x_{a_1}(\alpha), y_{a_2}(\beta), z_{a_3}(\delta)) = |x_{a_1} - y_{a_2} - z_{a_3}| - |\bar{\alpha} - \bar{\beta} - \bar{\delta}| = \mathbb{G}(x_{a_1}(\alpha), z_{a_3}(\delta), y_{a_2}(\beta))$

Also, we have  $\mathbb{G}(x_{a_1}(\alpha), y_{a_2}(\beta), z_{a_3}(\delta)) = \mathbb{G}(y_{a_2}(\beta), z_{a_3}(\delta), x_{a_1}(\alpha)) = \dots$

- 5)  $\mathbb{G}(x_{a_1}, w_a, w_a) + \mathbb{G}(w_a, y_{a_2}, z_{a_3}) = |x_{a_1} - w_a - w_a| - |\alpha - \delta - \delta| + |w_a - y_{a_2} - z_{a_3}| - |\alpha - \beta - \delta| \geq |x_{a_1} - y_{a_2} - z_{a_3}| - |\alpha - \beta - \delta| = \mathbb{G}(x_{a_1}, y_{a_2}, z_{a_3})$  .

**Example 3.3**

Let  $(\widehat{U}, \widehat{d}, \widehat{A})$  be a soft expert metric space. Define  $\mathbb{G}: \text{SEX}(\widehat{U}) \times \text{SEX}(\widehat{U}) \times \text{SEX}(\widehat{U}) \rightarrow \mathbb{R}(\widehat{A})$  by  $\mathbb{G}(x_{a_1}, y_{a_2}, z_{a_3}) = \mathbb{G}(x_{a_1}, y_{a_2}) + \mathbb{G}(y_{a_2}, z_{a_3}) + \mathbb{G}(z_{a_3}, x_{a_1})$ . Then  $\mathbb{G}$  is soft expert **GE** -metric space.

**Lemma 3.4**

In a soft expert **GE**-metric space , we have  $\mathbb{G}(x_{a_1}, x_{a_1}, y_{a_2}) = \mathbb{G}(y_{a_2}, y_{a_2}, x_{a_1})$

**Proof:**

By third stipulation of a soft expert **GE** -metric space we get

$$\begin{aligned} \mathbb{G}(x_{a_1}, x_{a_1}, y_{a_2}) &\leq \mathbb{G}(x_{a_1}, x_{a_1}, x_{a_1}) + \mathbb{G}(x_{a_1}, x_{a_1}, x_{a_1}) + \mathbb{G}(y_{a_2}, y_{a_2}, x_{a_1}) \\ &= \mathbb{G}(y_{a_2}, y_{a_2}, x_{a_1}) \quad \dots(1) \end{aligned}$$

And

$$\begin{aligned} \mathbb{G}(y_{a_2}, y_{a_2}, x_{a_1}) &\leq \mathbb{G}(y_{a_2}, y_{a_2}, y_{a_2}) + \mathbb{G}(y_{a_2}, y_{a_2}, y_{a_2}) + \mathbb{G}(x_{a_1}, x_{a_1}, y_{a_2}) \\ &= \mathbb{G}(x_{a_1}, x_{a_1}, y_{a_2}) \quad \dots\dots(2) \end{aligned}$$

From the equation (1) and (2), we have

$$\mathbb{G}(x_{a_1}, x_{a_1}, y_{a_2}) = \mathbb{G}(y_{a_2}, y_{a_2}, x_{a_1}).$$



**Proposition 3.5**

Let  $(\widehat{U}, \mathbb{G}, \widehat{A})$  be a soft expert **GE**-metric space. The function  $\mathbb{d}: \text{SEX}(\widehat{U}) \times \text{SEX}(\widehat{U}) \rightarrow \mathbb{R}(\widehat{A})$  defined by  $\mathbb{d}(x_{a_1}, y_{a_2}) = \mathbb{G}(x_{a_1}, y_{a_2}, y_{a_2})$  satisfies the following properties

- 1)  $\mathbb{d}(x_{a_1}, y_{a_2}) = 0$  if only if  $x_{a_1} = y_{a_1}$
- 2)  $\mathbb{d}(x_{a_1}, y_{a_2}) \leq \mathbb{d}(x_{a_1}, z_{a_3}) + \mathbb{d}(z_{a_3}, y_{a_2})$

For any points  $x_{a_1}, y_{a_2}, z_{a_3} \in \widehat{U}$ .

**Proof**

- 1- The proof (1) follows promptly from the properties (1) in Definition 3.1
- 2- Let  $x_{a_1}, y_{a_2}, z_{a_3}$  be any points in  $\widehat{U}$  using the property (5) in Definition 3.1, we have

$$\mathbb{d}(x_{a_1}, y_{a_2}) = \mathbb{G}(x_{a_1}, y_{a_2}, y_{a_2}) \leq \mathbb{G}(x_{a_1}, z_{a_3}, z_{a_3}) +$$

$$\mathbb{G}(z_{a_3}, y_{a_2}, y_{a_2}) = \mathbb{d}(x_{a_1}, z_{a_3}) + \mathbb{d}(z_{a_3}, y_{a_2}).$$

So that, (2) holds. ■

**Definition 3.6**

Let  $(\widehat{U}, \mathbb{G}, \widehat{A})$  be a soft expert **GE** -metric space and let  $\{v_m\}$  be a sequence in  $\widehat{U}$ . We say that  $\{v_m\}$  is:

- a) A soft expert **GE** -Cauchy a sequence if for  $\epsilon > 0$  there is  $m \in \mathbb{N}$  such that  $\mathbb{G}(v_m, v_m, v_l) < \epsilon$  for all  $m, m, l > N$
- b) a soft expert **GE** -convergent sequence if for any  $\epsilon > 0$  there exists a natural number  $N \in \mathbb{N}$  such that  $\mathbb{G}(v, v_m, v_m) < \epsilon$  for all  $m, m > N$ . we denote this by  $v_m \rightarrow v$  as  $m \rightarrow \infty$  or by  $\lim_{m \rightarrow \infty} (v_m) = v$
- c) A soft expert **GE** -complete if and only if each soft expert **GE** Cauchy sequence in  $\widehat{U}$  is convergent.

**Definition 3.7**

Let  $(\widehat{U}, \mathbb{G}, \widehat{A})$  be a soft expert **GE** -metric space. For  $a \in \widehat{U}$  and  $\epsilon'$  be a non-negative soft expert real number.

$$(\text{SEX}(\mathbb{B}_{\mathbb{G}})) (a, r) = \{ x_{a_1} \in \widehat{U} : \mathbb{G}(a, x_{a_1}, x_{a_1}) < r \} \leq \text{SEX}(\widehat{U})$$

is called the soft expert **GE**- open ball with center  $a$  and radius  $r$ .

$$(\text{SEX}(\mathbb{B}_{\mathbb{G}})) (a, r) = \{ x_{a_1} \in \widehat{U} : \mathbb{G}(a, x_{a_1}, x_{a_1}) \leq r \} \leq \text{SEX}(\widehat{U})$$

is called the soft expert **GE** -closed ball with center  $a$  and radius  $r$ .

**Definition 3.8**

Let  $(\widehat{U}_1, \mathbb{G}_1, \widehat{A}_1)$  and  $(\widehat{U}_2, \mathbb{G}_2, \widehat{A}_2)$  be two soft expert **GE** -metric spaces .

A function  $f: \widehat{U}_1 \rightarrow \widehat{U}_2$  is said to be soft expert **GE** - continuous at a soft expert element  $a \in \widehat{U}$  if and only if the following criterion is satisfied:

given any real number  $\epsilon'$  satisfy in  $\epsilon' > 0$  there exist some  $\epsilon' > 0$  there exist some  $\delta > 0$  such that  $v_{a_1}, z_{a_2} \in \widehat{U}_1$  and

$$\widehat{U}_1(a, v_{a_1}, z_{a_2}) < \delta \text{ implies that } \widehat{U}_2(f(a), f(v_{a_1}), f(z_{a_2})) < \epsilon'.$$

**Example 3.9**

Let  $(\widehat{U}, \mathbb{G}, \widehat{A})$  be a soft expert **GE** -metric space and  $x_0 \in \widehat{U}$  be a soft expert point, then  $f(x) = \widehat{U}(x, x_0)$  is **GE**- continuous. Indeed

$$\widehat{U}(\widehat{U}(x_{a_1}, x_0), \widehat{U}(\widehat{U}(y_{a_2}, x_0)), \widehat{U}(\widehat{U}(z_{a_3}, x_0))) = [\widehat{U}(x_{a_1}, x_0) - \widehat{U}(y_{a_2}, x_0) - \widehat{U}(z_{a_3}, x_0)] \leq \widehat{U}(x_{a_1}, y_{a_2}, z_{a_3}).$$

**proposition 3.10**

Given soft expert **GE** -metric space  $(W_1, \mathbb{G}_1, \widehat{A})$  and  $(W_2, \mathbb{G}_2, \widehat{A})$  . A function  $f: W_1 \rightarrow W_2$  is a soft expert **GE** - continuous if and only if  $f^{-1}(N)$  soft expert **G**-open in  $W_1$  such that  $N$  soft expert **GE**-open in  $W_2$  .

**Proof**

Suppose first that  $f$  is soft expert **GE**-continuous and that  $N \subseteq W_2$  is soft expert **GE**-open if  $f^{-1}(N) = \emptyset$  it is soft expert **GE**-open moreover , let  $x_{a_1} \in f^{-1}(N)$  , so that  $f(x_{a_1}) \in N$  . Then  $SEP(\mathbb{B}_{\mathbb{G}}(f'(x_{a_1}))) \subseteq N$  for some  $\epsilon' > 0$ . But  $f$  is soft expert **GE**-continuous so there exists  $\delta$  for which  $f(SEX(\mathbb{B}_{\delta'}(x_{a_1}))) \subseteq SEX(\mathbb{B}_{\epsilon'}(f(x_{a_1})))$  and therefore

$$f(SEX(\mathbb{B}_{\delta'}(x_{a_1}))) \subseteq N.$$

Thus  $SEX(\mathbb{B}_{\delta'}(x_{a_1})) \subseteq f^{-1}(N)$  and  $f^{-1}(N)$  is soft expert **GE**-open

Conversely

Assume that  $N$  soft expert **GE**-open in  $W_2$  implies  $f^{-1}(N)$  soft expert **GE**-open in  $W_1$  .Let  $x_{a_1} \in W_1$  and consider the soft expert **GE**-open

$SEX(\mathbb{B}_{\epsilon'}(f(x_{a_1})))$  , then  $f^{-1}(SEX(\mathbb{B}_{\epsilon'}(f(x_{a_1}))))$  is soft expert **GE**-open in  $W_1$  by presumption and include the point  $x_{a_1}$ ,so there exists  $SEX(\mathbb{B}_{\delta'}(x_{a_1})) \subseteq f^{-1}(SEX(\mathbb{B}_{\epsilon'}(f(x_{a_1}))))$

Thus  $f(SEX(\mathbb{B}_{\delta'}(x_{a_1}))) \subseteq SEX(f(x_{a_1}))$  and  $f$  is soft expert **GE** - continuous .

**proposition 3.11**

Given soft expert **GE** -metric space  $(W_1, \mathbb{G}_1, \widehat{A})$  and  $(W_2, \mathbb{G}_2, \widehat{A})$  .  $f: W_1 \rightarrow W_2$  is soft expert **GE** - continuous if and only if  $f^{-1}(M)$  soft expert **GE**-closed in  $W_1$  such that  $M$  is soft expert **GE**-closed in  $W_2$  .

**Proof:-**

Assume first that  $f$  is soft expert **GE**-continuous, and that  $M \subseteq W_2$  is soft expert **GE**-closed. Then  $W_2 / M$  is soft expert **GE**-open and  $(f^{-1}(W_2 / M)) \subseteq W_1$  is soft expert **GE**-open by theorem (3.10). But  $f^{-1}(W_2 / M) = W / f^{-1}(M)$ , so  $f^{-1}(M)$  is soft expert **GE**- closed.

Conversely, assume that  $M$  soft expert **GE**-closed in  $W_2$ , implies  $f^{-1}(M)$  soft expert **GE**- closed in  $W_1$ . If  $N \subseteq W_2$  is soft expert **GE**- open.

Then  $(W_2 / N)$  is soft expert **GE**- closed, but  $f^{-1}(W_2 / N) = W_1 / f^{-1}(N)$  so  $f^{-1}(N) \subseteq W_1$  is soft expert **GE**- open. Thus,  $f$  is soft expert **GE**- continuous by theorem (3.10). ■

**Proposition 3.12**

Let  $(X, \mathbb{G}_1, \widehat{A})$ ,  $(Y, \mathbb{G}_2, \widehat{A})$  and  $(Z, \mathbb{G}_3, \widehat{A})$  be soft expert **GE**- metric space and let  $f : X \rightarrow Y$  and  $g : Y \rightarrow Z$  be soft expert **GE**- continuous mapping . Then the composition function  $g \circ f : X \rightarrow Z$  is soft expert **GE**- continuous .

**Proof**

Let  $U \subseteq Z$  be soft expert **GE**-open . Then  $g^{-1}(U) \subseteq Y$  is soft expert **GE**-open , since  $g$  is soft expert **GE**- continuous , so  $f^{-1}(g^{-1}(U)) = (g \circ f)^{-1}(U)$  is soft expert **GE**-open in  $X$  , but  $f$  is soft expert **GE**- continuous .

Hence  $g \circ f$  is soft expert **GE**- continuous . ■

**Definition 3.13**

A mapping  $f: \widehat{U}_1 \rightarrow \widehat{U}_2$  is called soft expert **GE**- uniformly continuous at a soft expert element  $a \in \widehat{U}_1$  if for every  $\epsilon > 0$  there is  $\delta > 0$  such that  $v_{a_1}, z_{a_2} \in \widehat{U}_1$  and  $\widehat{U}_1(a, v_{a_1}, z_{a_2}) < \delta$  implies that  $\widehat{U}_2(f(a), f(v_{a_1}), f(z_{a_2})) < \epsilon$ .

**Lemma 3.14**

Let  $(\widehat{U}_1, \mathbb{G}, \widehat{A})$  and  $(\widehat{U}_2, \mathbb{G}, \widehat{A})$  be a soft expert **GE** -metric space and let  $f: \widehat{U}_1 \rightarrow \widehat{U}_2$  uniformly continuous mapping . If  $\{v_m\}$  is a Cauchy sequence in  $\widehat{U}_1$ , then  $\{f(v_m)\}$  a soft expert **GE** - Cauchy sequence in  $\widehat{U}_2$ .

**Proof**

Assume  $\epsilon > 0$  . Then by soft expert **GE** -uniform continuity there is a  $\delta > 0$  such that  $\widehat{U}_2(f(a), f(v_{a_1}), f(z_{a_2})) < \epsilon$  for all  $a, v_{a_1}$  and  $z_{a_2}$  in  $\widehat{U}_1$  satisfying  $\widehat{U}_1(a, v_{a_1}, z_{a_2}) < \delta$  . Since  $\{v_m\}$  is soft expert **GE** - Cauchy , there exists  $N \geq 1$  such that for all  $m, m', l \geq N$ , we have  $\widehat{U}_1(v_m, v_{m'}, v_l) < \delta$  . Then  $\widehat{U}_2(f(v_m), f(v_{m'}), f(v_l)) < \epsilon$  for all  $m, m', l \geq N$ , so  $\{f(v_m)\}$  is soft expert **GE** - Cauchy sequence in  $\widehat{U}_2$ . ■

**Definition 3.15**

Let  $f$  be a mapping from a soft expert **GE** -metric space Let  $(\widehat{U}_1, \mathbb{G}, \widehat{A})$  and into a soft expert **GE** -metric space  $(\widehat{U}_2, \mathbb{G}, \widehat{A})$ . We say that  $f$  is a soft expert **GE** -isometric if  $\widehat{U}_1(x_{a_1}, y_{a_2}, z_{a_3}) = \widehat{U}_2(f(x_{a_1}), f(y_{a_2}), f(z_{a_3}))$  for any  $x_{a_1}, y_{a_2}, z_{a_3} \in \widehat{U}_1$

**Theorem 3.16**

Let  $f: \widehat{U}_1 \rightarrow \widehat{U}_2$  be a soft expert **GE**- isometry then it is soft expert **GE**- injective and soft expert **GE** -uniformly continuous moreover, it's inverse  $f^{-1}(f(\widehat{U}_2)) \rightarrow (\widehat{U}_1)$  .

**Proof**

Let  $f: \widehat{U}_1 \rightarrow \widehat{U}_2$  be a soft expert **GE**- isometric .Let  $\epsilon > 0$ , choose  $\delta = \epsilon > 0$ .  
 Let  $x_{a_1}, y_{a_2}, z_{a_3} \in \widehat{U}_1$  be such that  $\widehat{U}_1(x_{a_1}, y_{a_2}, z_{a_3}) < \delta$  .  
 Then  $\widehat{U}_2(f(x_{a_1}), f(y_{a_2}), f(z_{a_3})) = \widehat{U}_1(x_{a_1}, y_{a_2}, z_{a_3}) < \delta = \epsilon$   
 Hence  $f$  is soft expert **GE** - uniformly continuous. Next, let  $x_{a_1}, y_{a_2}, z_{a_3} \in \widehat{U}_1$  be such that  $(f(x_{a_1}) = f(y_{a_2}) = f(z_{a_3}))$  .Thus  $\widehat{U}_1(x_{a_1}, y_{a_2}, z_{a_3}) = \widehat{U}_2((f(x_{a_1}), f(y_{a_2}), f(z_{a_3}))) = 0$   
 This shows that  $x_{a_1} = y_{a_2} = z_{a_3}$  , hence  $f$  is soft expert **GE**- injective.  
 To see that  $f^{-1}$  is a soft expert **GE**- isometry , let  $r_{a_1}, q_{a_2}, p_{a_3} \in f(\widehat{U}_1)$  and let  $x_{a_1}, y_{a_2}, z_{a_3} \in \widehat{U}_1$  be such that  $f(x_{a_1}) = r_{a_1}, f(y_{a_2}) = q_{a_2}, f(z_{a_3}) = p_{a_3}$  thus  $\widehat{U}_2(r_{a_1}, q_{a_2}, p_{a_3}) = \widehat{U}_2(f(x_{a_1}), f(y_{a_2}), f(z_{a_3})) = \widehat{U}_1(x_{a_1}, y_{a_2}, z_{a_3}) = \widehat{U}_1(f^{-1}(r_{a_1}), f^{-1}(q_{a_2}), f^{-1}(p_{a_3}))$   
 Hence  $f^{-1}$  is a soft expert **GE**- isometry. ■

**Definition 3.17**

A mapping  $f: \widehat{U}_1 \rightarrow \widehat{U}_2$  is soft expert **GE**- contractive if there exist  $0 \leq \mathbb{H} < 1$  that satisfies  $\widehat{U}_1(f(x_{a_1}), f(y_{a_2}), f(z_{a_3})) \leq \mathbb{H} \widehat{U}_1(x_{a_1}, y_{a_2}, z_{a_3})$  for all  $x_{a_1}, y_{a_2}, z_{a_3} \in \widehat{U}_1$

**Theorem 3.18**

Suppose that  $(\widehat{U}, \mathbb{G}, \widehat{A})$  is a soft expert **GE**-metric space  $f: \widehat{U}_1 \rightarrow \widehat{U}_2$  is a soft expert **GE**-contractive operator. Then  $f$  is soft expert **GE**-continuous .

**Proof**

Suppose that there exist a constant  $0 \leq \mathbb{H} < 1$  such that  $\widehat{U}_1(f(x_{a_1}), f(y_{a_2}), f(z_{a_3})) \leq \mathbb{H} \widehat{U}_1(x_{a_1}, y_{a_2}, z_{a_3})$   
 Let us take  $\{v_m\}_{m \in \mathbb{N}}$  in  $\widehat{U}_1$  which convergent to  $v$ . Then we can write  $v_m \rightarrow 0, m \rightarrow \infty$  this implies that  $\widehat{U}_1(f(x_{a_1}), f(y_{a_2}), f(z_{a_3})) \rightarrow 0$  .  
 And then  $\widehat{U}_1(v_m) \rightarrow \widehat{U}_1(f(v))$  as  $m \rightarrow \infty$  therefore  $f$  is soft expert **GE**- continuous. ■

**Definition 3.19**

A sub set  $\mathbb{M}$  of a soft expert **GE** - metric space is called soft expert **GE** -compact if every sequence in  $\mathbb{M}$  has a subsequence that convergent in  $\mathbb{M}$  .

**Theorem 3.20**

Let  $(\widehat{\mathbb{U}}, \mathbb{G}, \widehat{\mathbb{A}})$  is soft expert **GE** - compact soft expert **GE**- metric space  $f: \widehat{\mathbb{U}}_1 \rightarrow \widehat{\mathbb{U}}_2$  is an operation that satisfies the following in quality ;

$$\widehat{\mathbb{U}} (f(x_{a_1}), f(y_{a_2}), f(z_{a_3})) < \widehat{\mathbb{U}} (x_{a_1}, y_{a_2}, z_{a_3}) \text{ for all } x_{a_1}, y_{a_2}, z_{a_3} \in \widehat{\mathbb{U}} \text{ with } x_{a_1} \neq y_{a_2}$$

Then  $F$  has exactly one fixed point  $x_0 \in \widehat{\mathbb{U}}$  .

**Proof**

It is clear that  $x_{a_0}$  is the minimum of the map  $x_{a_1} \rightarrow \widehat{\mathbb{U}}(x_{a_1}, x_{a_1}, f(x_{a_1}))$  and then we have  $x_{a_0} = f(x_{a_0})$  . In fact ,  $\widehat{\mathbb{U}}(f(f(x_{a_0})), f(f(x_{a_0})), f(x_{a_0})) < \widehat{\mathbb{U}}(f(x_{a_0}), (f(x_{a_0})), (x_{a_0})) = \widehat{\mathbb{U}}(x_{a_0}, x_{a_0}, f(x_{a_0}))$  whis is a contradiction

Suppose that we have  $x_{a_1}, y_{a_2}, z_{a_3} \in \widehat{\mathbb{U}}$  with  $x_{a_1} = f(x_{a_1}), y_{a_2} = f(y_{a_2}), z_{a_3} = f(z_{a_3})$

Then the inequality  $\widehat{\mathbb{U}}(f(x_{a_1}), f(y_{a_2}), f(z_{a_3})) < \widehat{\mathbb{U}}(x_{a_1}, y_{a_2}, z_{a_3})$  is equivalent to  $\widehat{\mathbb{U}}(x_{a_1}, y_{a_2}, z_{a_3}) < \widehat{\mathbb{U}}(x_{a_1}, y_{a_2}, z_{a_3})$  this a contradiction .hence  $x_{a_1} = y_{a_2}$ . ■

**Theorem 3.21**

A soft expert **GE** -closed subset  $\mathbb{P}$  of a soft expert **GE** -compact set  $\mathbb{M}$  is soft expert **GE** - compact.

**Proof**

Assume that  $\{v_n\}$  is soft expert **GE** -sequence in  $\mathbb{P}$  . We must show that  $\{v_n\}$  has a soft expert **GE** -subsequence converging to a point in  $\mathbb{P}$  . Since  $\{v_n\}$  is also a soft expert **GE** -sequence in  $\mathbb{M}$  , and  $\mathbb{M}$  is soft expert **GE** -compact , there is a soft expert **GE** -subsequence in  $\{v_{n_k}\}$  converging to a point  $a \in \mathbb{M}$  . Since  $\mathbb{P}$  is soft expert **GE** -closed,  $a \in \mathbb{P}$  , and hence  $\{v_n\}$  has a soft expert **GE** -subsequence converging to a point in  $\mathbb{P}$  . ■

**Proposition 3.22**

Assume that  $f: \widehat{\mathbb{U}}_1 \rightarrow \widehat{\mathbb{U}}_2$  is a soft expert **GE** -continuous function between two soft expert **GE** -metric space . If  $\mathbb{M} \subset \widehat{\mathbb{U}}_1$  is soft expert **GE** -compact , then  $f(\mathbb{M})$  is a soft expert **GE** -compact sub set  $\widehat{\mathbb{U}}_2$  .

**Proof**

Let  $\{w_n\}$  be a soft expert **GE** -sequence in  $(\mathbb{M})$  ; we shall show that  $\{w_n\}$  has a soft expert **GE** -subsequence converging to a point in  $(\mathbb{M})$  . Since  $w_n \in f(\mathbb{M})$  , we can for each  $n$  find an soft expert element  $v_n \in \mathbb{M}$  such that  $f(v_n) = w_n$  . Since  $\mathbb{M}$  is soft expert **GE**- compact , the soft expert **GE** -sequence  $\{v_n\}$  has a soft expert **GE** -subsequence  $\{v_{n_k}\}$  converging to a point  $v \in \mathbb{M}$  . But then  $\{w_{n_k}\} = \{f(v_{n_k})\}$  is a soft expert **GE** -subsequence in  $\{w_n\}$  converging to  $w = f(v) \in f(\mathbb{M})$  .

**Conclusion**

Soft set theory is a tool for solving problems with uncertainty. In the present paper we extend the concept soft expert **GE**-metric mapping by using soft expert real sets has been defined, then we gave definition of soft expert **GE**-metric space . In fact , we can discuss the relationship of soft expert **GE** -ball, soft expert **GE**-continuity and soft expert **GE**-compact etc. ■

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