Certain properties of λ – closed set

بعض خصائص المجموعة المغلقة - λ

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Abstract:
The aim of this paper is to study the notion of λ – closed, sets in topological spaces by given and proved some of their properties .

المستخلص:
الهدف الرئيسي في هذا البحث هو دراسة مفهوم المجموعات المغلقة - λ .بواسطه تقديم وبرهان بعض من خصائصها .

1- Introduction:

C – sets are one of the important definitions for studying topological spaces . In fact ,H. Maki [1] introduced the concept of C – set (which he call a subset A of a topological spaces (X, τ) is called a C- set if A = ker (A) , where ker (A) = the intersection of all open sets containing A, if A is open set , then A = ker (A), but the converse is not necessarily true) . Also ,M.Ganster and I.L. Reilly [3] . introduced the concepts of λ - closed sets , the subset A of a topological space (X, τ) is called λ - closed sets if and only if A =L∩ F, where F is closed in X and L is a C – set ,that is L = ker (L) and they prove every closed set is λ - closed set . in this works , several properties of λ - closed sets are proved .

2- Basic Definitions:

In this section, we recall and introduce the basic definitions needed in this work.

Definition (2.1):[1]

(i) - Let (X, τ) be a topological space let $A \subseteq X$, we say that A is a C- set if A = ker (A) where ker (A) = the intersection of all open sets containing A if A is open , then A is a C – set ·

(ii) - if X is an Alexandrof space (that is the arbitrary intersection of open sets is open) [2]· then every C- set is open ·

Definition (2.2):[3]

Let (X, τ) be a topological space let $W \subseteq X$ we say that W is λ - closed if $W = A \cap F$ where A is a C – set and F is a closed set .

Remarks and Examples (2.3):

1- Every closed set is λ - closed (because if W is closed then $W = X \cap W$, X is a C – set hence W is λ - closed) .

2 - Every C- set is λ - closed because if W is a C – set then $W = W \cap X$ but X is closed hence W is λ - closed .

3 - If X is a T1- space then every subset of X is a C – set , hence every subset of X is λ - closed .

Definition (2.4):[3].

Let (X , T) be a topological space. Let $A \subseteq X$, we say that A is locally closed if $A = W1 \cap W2$ where W1 is open and W2 is closed every open set is locally closed also every closed set is locally closed .

Remark (2.5):

Every locally closed is λ - closed because if A is locally closed then $A = W1 \cap W2$ where W1 is open and W2 is closed but every open set is a C– set hence A is λ - closed .

3. Main Results:

In this section, we state and prove several properties of λ- closed sets First , we need the following lemma .

Lemma (3.1) :[3]

Let (X, T) be a topological space. Let $A \subseteq X$,then the following statements are equivalent

1- A is λ - closed
2- $A = L \cap \overline{A}$ where L is a C- set
3- $A = \ker(A) \cap \overline{A}$

before , we state our first result we recall the following definition.

Definition (3.2)[6]:

Let (X, T) be a topological space. Let $A \subseteq X$, we say that A is g – closed if $A \subseteq U \rightarrow \overline{A} \subseteq U$ where U is open in X every closed set is g – closed .

Remark (3.3)

Let (X, T) be a topological space, let $A \subseteq X$, if A is g – closed then $\overline{A} \subseteq \ker(A)$.

Proposition (3.4):

Let (X, T) be a topological space, let $A \subseteq X$, then the following statements are equivalent .

1- A is closed .
2- A is locally closed and g – closed .
3- A is λ - closed and g – closed .

Proof:

$1\rightarrow 2$ clear ,[3].
$2\rightarrow 3$ clear because every locally closed is λ- closed
$3\rightarrow 1$ if A is g - closed then $\overline{A} \subseteq \ker(A)$ (Remark (3.3) using Lemma (3.1) (part (3) $A = \ker(A) \cap \overline{A}$ (A is λ- closed) now $\overline{A} \subseteq \ker(A) \cap \overline{A} = A$ but $A \subseteq \overline{A}$ then $A = \overline{A}$ then A is closed .

Proposition (3.5):[4,3]

Let (X, T) be a topological space then the following statement equivalent

1- X is a T_0 – space .
2- Every singleton $\{x\}$ is λ - closed set before, we state the next result , we recall the following definition .

Definition (3.6):[5]

Let (X, T) be a topological space we say that X is a $T_{1/2}$ - space if every singleton $\{x\}$is either open or closed so $T_1 \rightarrow T_{1/2} \rightarrow T_0$ ·

Proposition (3.7):[3,5]
Let (X,T) be a topological space then the following statements are equivalent
1- X is a T½ - space .
2- every subset of X is λ - closed .

Definition (3.8):[4]

Let (X,T) be topological space we say that X is a T¼ - space if given a finite set $F \subseteq X$ and given $y \notin F$, then $\exists W \ni F \subseteq W$ and $y \notin W$ and W is either open or closed

Remark (3.9):
We have the following implications
T1 → T½ → T¼ → T0

Proposition (3.10):
Let (X,T) be a topological space then the following statements are equivalent
1- X is T¼
2- Every finite subset of X is λ - closed

Proposition (3.11):
Finite union of λ - closed sets need not be λ - closed .

Proof:
Suppose every finite union of λ - closed sets is λ - closed ,then we get that every T0 – space is a T¼ - space which is a contradiction
We explaine this as follows
Let X be a T0 – space then every singleton {x} is λ - closed which implies that every finite set is λ-closed which implies that X is a T¼ - space by proposition (3..10)

Proposition (3.12):
Arbitrary intersection of λ - closed sets is λ - closed

Proof:
Let $\{A_\propto \mid \propto \in \ \}$ be any collection of λ - closed sets in X now by lemma (3.1) $A_\propto = W_\propto \cap F_\propto$ where $W_\propto$ is a C- set and $F_\propto$ is closed in X but arbitrary intersections of C – set is also a C – set [1] hence $\bigcap_{\alpha \in \Omega} A_\propto$ is λ - closed

4. References:

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