Fuzzy Translation and fuzzy multiplication of CI-algebras

الترجمات الضبابية والضرب الضبابي إلى جبر -CI

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Abstract:

In this paper, we give definition of fuzzy translation of CI-algebra, fuzzy extension ,fuzzy multiplication of CI-algebra and homomorphism of fuzzy translation and fuzzy multiplication of fuzzy

CI-ideal of CI-algebras and proved certain results based on the fuzzy CI-ideal of CI- algebra.

Keywords: fuzzy CI-ideal ,fuzzy translation, fuzzy extension, fuzzy multiplication, homomorphism of CI-algebras.

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الملخص:

في هذا البحث نعطي تعريف الترجمة الضبابية إلى الجبر من النوع CI ,التوسعيات الضبابية, الضرب الضبابي إلى الجبر من النوع CI والتشاكل إلى كل من الترجمة الضبابية والضرب الضبابي إلى الجبر من النوع CI وأثبتنا نتائج معينة على أساس المثالية الضبابية من النوع CI إلى الجبر من النوع CI

1- Introduction

The introduction of a lot of concepts as types of algebra in year 1966 and study fuzzy him in 1965 and then the studies on these concepts have evolved it has been Some of which circulating to other types for reach to new algebra types of which CI-algebra in 2009 which is the subject of our study, where be generalization of BE-algebra, the introduction of the concept of fuzzy translation and fuzzy multiplication on this kind of your new idea for CI-algebras as[1],[2] , the concepts ideal and filter of CI-algebra , and discussed the some of the properties of CI-algebras by [3]. Meng B.L, show the concept of an CI-algebra as a generation of a BE-algebra by[4]. Sithar Selvam P.M., and others introduce the notion of anti fuzzy subalgebra of CI-algebras and studied some of its properties under homomorphism and cartesian products by [5].

In this paper we define a fuzzy translation and fuzzy multiplication of CI-algebras and look for some of their properties accurately by using the concepts of fuzzy CI-ideal and fuzzy subalgebra. We prove to, that if homomorphism of fuzzy translation or fuzzy multiplication of fuzzy CI-ideal of CI-algebras is fuzzy translation or fuzzy multiplication of CI-algebras.

2- Preliminaries

In this section, we mention basic definitions that we need in this paper.

Definition 2.1 [4]:

Let(X;∗, 0) be a set with a binary operation (∗) and a constant (0).Then (X;∗, 0)is called a **CI-algebra** if it satisfies the following axioms: for all x, y ,z ∈X,

(1) $x * x = 0$,

(2) $0 * x = x$,

(3) $x * (y * z) = y * (x * z)$.

For brevity we also call X a **CI-algebra**, We can define a binary relation (≤) by putting $x \leq y$ if and only if $y * x = 0$.

Proposition 2.2 [3]:
Let (X;$*$, 0) be a CI-algebra, then the following hold :for any x, y $\in$ X,
(CI$_1$) $y * ((y * x) * x) = 0$,
(CI$_2$) $(x * 0) * (y * 0) = (x * y) * 0$.

Definition 2.3 [5]:
Let (X;$*$, 0) be a CI-algebra and S be a nonempty subset of X .Then S is called a **subalgebra** of X if, $x * y$ ∈S, for any x, y ∈S.

Definition 2.4[6] :
Let (X;$*$, 0) be a CI-algebra and I be a nonempty subset of X. I is called a **CI-ideal** of X if it satisfies:
i. 0∈I,
ii. $x * (y * z)$ ∈I and y ∈I imply $x * z \in$ I , for all x, y, z ∈X .

Definition 2.5[7] :
Let X be a nonempty set, a fuzzy subset μ of X is a function μ: X →[0,1].

Definition 2.6[7] :
Let μ and ν be a fuzzy subsets on X. Define the fuzzy subset $\mu \cap \nu$ as follows:
$(\mu \cap \nu)(x) = \min\{\mu(x), \nu(x)\}$, for all x∈X.
In general, if $\{\mu_i : i\epsilon\Lambda\}$ is a family of fuzzy subsets in X, then :
$\bigcap_{i\epsilon\Lambda} \mu_i(x) = \inf\{\mu_i(x) : i\epsilon\Lambda\}$, for all x∈X.

Definition 2.7[7] :
Let μ and ν be a fuzzy subsets on X. Define the fuzzy subset $\mu \cup \nu$ as follows:
$(\mu \cup \nu)(x) = \max\{\mu(x), \nu(x)\}$, for all x∈X.
In general, if $\{\mu_i : i\epsilon\Lambda\}$ is a family of fuzzy subsets in X, then :
$\bigcup_{i\epsilon\Lambda} \mu_i(x) = \sup\{\mu_i(x) : i\epsilon\Lambda\}$, for all x∈X.

Definition 2.8 [5]:
Let X be a CI-algebra. A fuzzy subset μ of X is said to be a **fuzzy subalgebra** of X if it satisfies:
$\mu\,(x * y) \;\geq\; min\,\{\mu\,(x), \mu(y)\,\}$,for all x , y ∈ X.

Definition 2.9[6]:
Let X be a CI-algebra. A fuzzy subset μ of X is said to be a **fuzzy CI-ideal** of X if it satisfies:
1. $\mu(0) \geq \mu(x)$, for all x∈ X,
2. $\mu(x * z) \geq min\{\mu(x * (y * z)), \mu(y)\,\}$, for all x, y, z∈ X.

Proposition 2.10[6] :
Let (X;$*$, 0) by a CI-algebra , then every fuzzy CI-ideal of X is a fuzzy subalgebra of X.

3- Fuzzy Translations of fuzzy CI-ideals.

We study the relations among fuzzy translation, fuzzy extension of CI-ideal of CI-algebra X as [1], [8].

In what follows let (X; $*$,0) denote a CI-algebra, and for any fuzzy subset μ of X, we denote T = 1 − sup{μ(x) | x ∈ X}.

Definition 3.1 [1], [8]:
Let X be a nonempty set and μ be a fuzzy subset of X and let α∈ [0,T]. A mapping μ_α^T : X →[0,1] is called a **fuzzy translation** of μ if it satisfies: $\mu_\alpha^T(x) = \mu(x) + \alpha$, for all x ∈ X.

Theorem3.2 :

Let μ be a fuzzy subset of CI-algebra X and μ_α^T is fuzzy translation of μ for $\alpha \in [0,T]$. μ is a fuzzy CI-ideal of X if and only if μ_α^T is a fuzzy CI-ideal of X.

proof:

($\Longrightarrow$) Assume μ be a fuzzy CI-ideal of X and let $\alpha \in [0,T]$. For all x, y, z $\in$ X we have

1. since $\mu(0) \geq \mu(x)$ Then $\mu_\alpha^T(0) = \mu(0) + \alpha \geq \mu(x) + \alpha = \mu_\alpha^T(x)$.
2. $\mu_\alpha^T(x * z) = \mu(x * z) + \alpha \geq min\{\mu(x * (y * z)), \mu(y)\} + \alpha$ $= min\{\mu(x * (y * z)) + \alpha, \mu(y) + \alpha\} = min\{\mu_\alpha^T(x * (y * z)), \mu_\alpha^T(y)\}$. Hence μ_α^T is a fuzzy CI-ideal of X .

($\Longleftarrow$) Assume the fuzzy translation μ_α^T is a fuzzy CI-ideal of X for some $\alpha \in [0,T]$.

Let x, y, z $\in$ X, we have

1- $\mu(0) + \alpha = \mu_\alpha^T(0) \geq \mu_\alpha^T(x) = \mu(x) + \alpha \Longrightarrow \mu(0) \geq \mu(x)$.

2- $\mu(x * z) + \alpha = \mu_\alpha^T(x * z) \geq min\{\mu_\alpha^T(x * (y * z)), \mu_\alpha^T(y)\}$

$= min\{\mu(x * (y * z)) + \alpha, \mu(y) + \alpha\} = min\{\mu(x * (y * z)), \mu(y)\} + \alpha$

$\Longrightarrow \mu(x * z) \geq min\{\mu(x * (y * z)), \mu(y)\}$. Hence μ is a fuzzy CI-ideal of X .⌂

Proposition 3.3:

Let the fuzzy translation μ_α^T of μ be a fuzzy CI-ideal of X for $\alpha \in [0,T]$. If $z \leq y$ then $\mu_\alpha^T(z) \geq \mu_\alpha^T(y)$.

Proof:

Let x, y ,z $\in$ X be such that $z \leq y$. Then $y * z = 0$ and hence $\mu_\alpha^T(z) = \mu_\alpha^T(0 * z)$ $\geq min\{\mu_\alpha^T(0 * (y * z), \mu_\alpha^T(y)\} = min\{\mu_\alpha^T(0), \mu_\alpha^T(y)\} = \mu_\alpha^T(y)$. ⌂

Definition 3.4 [1],[8]:

Let μ_1 and μ_2 be fuzzy subsets of a set X. If $\mu_1(x) \leq \mu_2(x)$ for all x $\in$ X, then we say that μ_2 is **a fuzzy extension of** μ_1 .

Definition 3.5:

Let μ_1 and μ_2 be fuzzy subsets of X. Then μ_2 is called **a fuzzy extension CI-ideal** of μ_1 if the following assertions are valid:

(I_1) μ_2 is a fuzzy extension of μ_1.

(I_2) If μ_1 is a fuzzy CI-ideal of X, then μ_2 is a fuzzy CI-ideal of X.

Proposition 3.6 :

Let μ be a fuzzy CI-ideal of X and let $\alpha, \gamma \in [0,T]$. If $\alpha \geq \gamma$, then the fuzzy translation μ_α^T of μ is a fuzzy extension CI-ideal of the fuzzy translation μ_γ^T of μ.

Proof:

Let μ be a fuzzy CI-ideal of X then by Theorem (3.2), the fuzzy translation μ_γ^T of μ and the fuzzy translation μ_α^T of μ are fuzzy CI-ideals of X, for all $\alpha, \gamma \in [0,T]$ since $\alpha \geq \gamma$, $\mu(x) + \alpha \geq \mu(x) + \gamma$, for all x $\in$ X. Therefore $\mu_\alpha^T(x) \geq \mu_\gamma^T(x)$. Hence μ_α^T is a fuzzy extension CI-ideal of μ_γ^T. ⌂

Proposition 3.7 :

For every fuzzy CI-ideal μ of X and $\gamma \in [0,T]$, the fuzzy translation μ_γ^T of μ is a fuzzy CI-ideal of X. If ν is a fuzzy extension CI-ideal of μ_γ^T, then there exists $\alpha \in [0,T]$ such that $\alpha \geq \gamma$ and $\nu(x) \geq \mu_\alpha^T(x)$ for all $x \in X$.

Proof:

Let μ be a fuzzy CI-ideal of X and $\gamma \in [0,T]$, then by Theorem (3.2), μ_γ^T is a fuzzy CI-ideal of X . Let ν be a fuzzy extension CI-ideal of μ_γ^T, therefore $\nu(x) \geq \mu_\gamma^T(x)$ $\forall$ $x \in X$. Then choose $\alpha = \gamma + min\{\nu(x) - \mu_\gamma^T(x)\}$. Clearly $\alpha \in [0,T]$ such that $\alpha \geq \gamma$. Then μ_α^T is a fuzzy translation μ and $\nu(x) \geq \mu_\alpha^T(x)$. Hence ν is also a fuzzy extension CI-ideal of the fuzzy translation μ_α^T.⌂

The following example illstrates proposition (3.7)

Example 3.8 :

Let X ={0, a, b} be a set with a binary operation (∗) defined by the following table:

∗	0	a	b
0	0	a	b
a	0	0	b
b	b	b	0

Then (X ;∗,0) is a CI-algebra by[6]. Define a fuzzy subset μ of X by:

X	0	a	b
μ	0.8	0.7	0.6

Then μ is a fuzzy CI-ideal of X and T =1- 0.8= 0.2. If we take γ = 0.11, then the fuzzy CI-ideal translation μ_γ^T of μ is given by :

X	0	a	b
μ_γ^T	0.91	0.81	0.71

Let ν be a fuzzy subset of X defined by:

X	0	a	b
ν	0.98	0.83	0.72

then ν is clearly a fuzzy extension CI-ideal of the fuzzy CI-ideal translation μ_γ^T of μ . But ν is not a fuzzy CI-ideal translation μ_α^T of μ for all $\alpha \in [0,T]$, since α in 0 ≠ α in 1 . Take α = 0.12, then α = 0.12 > 0.11 = γ and the fuzzy CI-ideal translation $\mu_\alpha^T$of μ is given as follows:

X	0	a	b
μ_α^T	0.92	0.82	0.72

Note that $\nu(x) \geq \mu_\alpha^T(x)$ for all $x \in X$, and hence ν is a fuzzy extension CI-ideal of the fuzzy CI-ideal translation μ_α^T of μ.

Proposition 3.9 :

Let μ be a fuzzy CI-ideal of a CI-algebra X and $\alpha\in[0,T]$. Then the fuzzy translation μ_α^T of μ is a fuzzy extension CI-ideal of μ.

Proof :

If μ is a fuzzy CI-ideal of X, then by Theorem (3.2), the fuzzy translation μ_α^T of μ is also a fuzzy CI-ideal of X, for all $\alpha\in[0,T]$. Now $\mu_\alpha^T(x)=\mu(x)+\alpha\geq\mu(x)$,for all $x\in X$. Hence , the fuzzy translation μ_α^T is a fuzzy extension CI-ideal of μ. ⌂

A fuzzy extension CI-ideal of a fuzzy CI-ideal μ may not be represented as a fuzzy CI-ideal translation μ_α^T of μ , that is , the converse of Proposition (3.9) is not true in general , as shown by the following example.

Example 3.10 :

Let X = {0,a,b} be a set with a binary operation (∗) defined by the following table:

∗	0	a	b
0	0	a	b
a	0	0	b
b	0	a	0

Then (X;∗,0) is a CI-algebra by [6]. Define a fuzzy subset μ of X by:

X	0	a	b
μ	0.8	0.6	0.7

Then μ is a fuzzy CI-ideal of X . Let ν be a fuzzy subset of X defined by:

X	0	a	b
ν	0.97	0.76	0.72

Then ν is a fuzzy extension CI-ideal of μ . But it is not the fuzzy translation μ_α^T of μ for all $\alpha\in[0,T]$,since α in 0 ≠ α in 1.

Proposition 3.11 :

The intersection of any set of fuzzy CI-ideals translation of CI-algebra X is also fuzzy CI-ideal translation of X.

Proof:

Let $\{(\mu_\alpha^T)_i : i\in\Lambda\}$ be a family of fuzzy CI-ideals translation of CI-algebra X , then for any x, y, z ∈ X , i ∈Λ,

1- $(\bigcap_{i\in\Lambda}(\mu_\alpha^T)_i)(0)=\inf((\mu_\alpha^T)_i\,(0))=\inf(\mu_i\,(0)+\alpha)\geq\inf(\mu_i(x)+\alpha)$
$=\inf((\mu_\alpha^T)_i\,(x))=((\bigcap_{i\in\Lambda}(\mu_\alpha^T)_i)(x).$

2- $(\bigcap_{i\in\Lambda}(\mu_\alpha^T)_i)(x*z)\ =\inf((\mu_\alpha^T)_i\,(x*z))=\ \inf(\mu_i(x*z)+\alpha)$
$\geq\inf\big(\min\{\mu_i\,(x*(y*z)),\mu_i(y)\}\big)+\alpha$
$=\ \inf(\min\{\mu_i\,(x*(y*z))+\alpha,\mu_i(y)+\alpha\})$
$=\min\{\inf(\mu_i\,(x*(y*z))+\alpha)\,,\inf(\mu_i\,(y)+\alpha)\}$
$=\min\{(\bigcap_{i\in\Lambda}\mu_i)(x*(y*z))+\alpha\,,(\bigcap_{i\in\Lambda}\mu_i)(y)+\alpha\}$
$=min\,\{(\bigcap_{i\in\Lambda}(\mu_\alpha^T)_i)(x*(y*z))\,,((\bigcap_{i\in\Lambda}(\mu_\alpha^T)_i)(y)\}.$ ⌂

Proposition 3.12 :

The intersection of any set of fuzzy extension CI-ideals of a fuzzy CI-ideal μ of X is a fuzzy extension CI-ideal of μ.

Proof:

Let $\{\mu_i : i \in \Lambda\}$ be a family of fuzzy extension CI-ideals of a fuzzy CI-ideal μ of X ,Then $\mu_i(x) \geq \mu(x)$ ∀ i ∈ Λ, x ∈ X since μ is a fuzzy CI-ideal of X. μ_i are fuzzy CI-ideals of X ∀ i ∈ Λ. Then $\cap_{i\in\Lambda}\ \mu_i$ is also a fuzzy CI-ideal of X, by Theorem(5.15) of [6].

Now $(\cap_{i\in\Lambda}\ \mu_i)(x) = inf_{i\in\Lambda}(\mu i(x)) \geq inf\ (\mu(x)) = \mu(x)$. Hence $\cap_{i\in\Lambda} \mu_i$ is a fuzzy extension CI-ideal of μ. ⌂

Clearly, the union of fuzzy extension CI-ideal of a fuzzy subset μ of X is not a fuzzy extension CI-ideal of μ as seen in the following example.

Example 3.13:

Let X = {0,a,b,c,d} be a set with binary operation (∗) defined by the following table:

∗	0	a	b	c	d
0	0	a	b	c	d
a	0	0	b	b	d
b	0	a	0	a	d
c	0	0	0	0	d
d	d	d	d	d	0

Then (X; ∗,0) is a CI-algebra by[3].

Let μ , ν and δ be fuzzy subsets of X defined by :

X	0	a	b	c	d
μ	0.7	0.6	0.5	0.6	0.5
ν	0.9	0.8	0.7	0.7	0.6
δ	0.9	0.8	0.6	0.8	0.6

Respectively. Then ν and δ are fuzzy extension CI-ideals of μ obviously, the union ν ∪ δ is a fuzzy extension of μ, but it is not a fuzzy extension CI-ideals of μ since (ν ∪ δ) (a∗c) =(ν ∪ δ)(b) = 0.7 < 0.8 = min{(ν ∪ δ)(a∗(c∗c)), (ν ∪ δ)(c)}=min{(ν ∪ δ)(0), (ν ∪ δ)(c)}= min{0.9, 0.8}.

Definition 3.14 [9]:

For a fuzzy subset μ of X, α∈ [0,T] and t ∈ [0,1] with $t \geq \alpha$,Let $U_\alpha(\mu; t) = \{x \in X : \mu(x) \geq t - \alpha\}$.

Theorem 3.15 :

Let μ be a fuzzy subset of a CI-algebra X and α∈[0,T]. Then μ_α^T is a fuzzy CI-ideal translation of X if and only if $U_\alpha(\mu; t)$ is a CI-ideal of X ∀ t∈Im (μ) with t ≥ α.

Proof :

(⟹) Assume μ is a fuzzy CI-ideal of X, then by Theorem (3.2) μ_α^T is a fuzzy CI-ideal translation of X. Let t∈Im(μ) be such that t ≥ α.

1- Now $\mu(0) + \alpha = \mu_\alpha^T(0) \geq \mu_\alpha^T(x) = \mu(x) + \alpha \geq t\ \forall\ x \in U_\alpha(\mu; t)$ this establishes that $0 \in U_\alpha(\mu; t)$

2- Let $(x * (y * z)), y \in U_\alpha(\mu; t) \Longrightarrow \mu(x * (y * z)) \geq t - \alpha$ and $\mu(y) \geq t - \alpha \Longrightarrow$

$\mu(x*(y*z)) + \alpha \geq t$ and $\mu(y) + \alpha \geq t \Rightarrow \mu_\alpha^T(x*(y*z)) \geq t$ and $\mu_\alpha^T(y) \geq t$.Now $\mu_\alpha^T(x*(y*z)) \geq min\{\mu_\alpha^T(x*(y*z)), \mu_\alpha^T(y)\} = t$ which implies $\mu(x*z) + \alpha \geq t \Rightarrow \mu(x*z) \geq t - \alpha \Rightarrow x*z \in U_\alpha(\mu; t)$ .Hence $U_\alpha(\mu; t)$ is a CI-ideal of X .

(⟸) suppose that $U_\alpha(\mu; t)$ is CI-ideal of X. for every t ∈ Im(μ) with t > α.

1- If there exists x ∈ X such that $\mu_\alpha^T(0) < t \leq \mu_\alpha^T(x)$, then $\mu(x) \geq t - \alpha$ but $\mu(0) < t - \alpha$. This shows that $x \in U_\alpha(\mu; t)$ and $0 \notin U_\alpha(\mu; t)$. This is a contradiction, and so $\mu_\alpha^T(0) \geq \mu_\alpha^T(x)$ for all x ∈ X.

2- Now assume that there exist x, y, z ∈ X such that $\mu_\alpha^T(x*z) < \gamma \leq min\{\mu_\alpha^T(x*(y*z)), \mu_\alpha^T(y)\}$. Then $\mu(x*(y*z)) \geq \gamma - \alpha$ and $\mu(y) \geq \gamma - \alpha$, but $\mu(x*z) < \gamma - \alpha$. Hence $(x*(y*z)) \in U_\alpha(\mu; \gamma)$ and $y \in U_\alpha(\mu; \gamma)$, but $(x*z) \notin U_\alpha(\mu; \gamma)$, this is a contradiction. Therefore, $\mu_\alpha^T(x*z) \geq min\{\mu_\alpha^T(x*(y*z)), \mu_\alpha^T(y)\}$, for all x ,y, z ∈ X. Hence μ_α^T is a fuzzy CI-ideal translation of X. ⌂

Corollary 3.16 :

Let μ be a fuzzy subset of a CI-algebra X and α∈[0,T]. Then μ is a fuzzy CI-ideal of X if and only if $U_\alpha(\mu; t)$ is a CI-ideal of X ∀ t∈Im(μ) with t ≥ α.

Proof :

By Theorem (3.2) and Theorem (3.15) .⌂

Proposition 3.17 :

Let μ be a fuzzy CI-ideal of a CI-algebra X and let α ∈ [0,T], then the fuzzy translation μ_α^T of μ is a fuzzy subalgebra of X.

Proof:

Since μ be a fuzzy CI-ideal of a CI-algebra X , then by Proposition (2.10), μ be a fuzzy subalgebra of a CI-algebra X and let α ∈ [0,T] and x, y∈ X. Then,

$\mu_\alpha^T(x*y) = \mu(x*y) + \alpha \geq min\{\mu(x), \mu(y)\} + \alpha = min\{\mu(x) + \alpha, \mu(y) + \alpha\}$
$= min\{\mu_\alpha^T(x), \mu_\alpha^T(y)\}$. Hence μ_α^T is a fuzzy subalgebra translation of X. ⌂

In general, the converse of Proposition (3.17) is not true. As the following example shows;

Example 3.18 :

Let X = {0,a,b,c} be a set with binary operation (∗) defined by the following table:

∗	0	a	b	c
0	0	a	b	c
a	0	0	a	a
b	0	0	0	a
c	0	0	a	0

Then (X; ∗,0) is a CI-algebra by[3]. Define a fuzzy subset μ of X by:

X	0	a	b	c
μ	0.8	0.7	0.6	0.5

Then μ is not fuzzy CI-ideal of X, since μ(0∗c)= μ(c)=0.5<0.7=min{μ(0∗(a∗c),μ(a)} =min {μ (a),μ (a)}, and T=0.2 .But if we take α=0.1 the fuzzy translation μ_α^T of μ is given as follows:

X	0	a	b	c
μ_α^T	0.9	0.8	0.7	0.6

Then μ_α^T is a fuzzy subalgera of CI-algebra X.

Proposition 3.19 :

If the fuzzy translation μ_α^T of μ is a fuzzy CI-ideal of X, α ∈[0,T] then μ is a fuzzy subalgebra.

Proof:

Since μ_α^T be a fuzzy CI-ideal of a CI-algebra X , then by Proposition (2.10) μ_α^T be a fuzzy subalgebra of a CI-algebra X and let α ∈ [0,T] and x, y ∈ X. Then $\mu(x * y) + \alpha$
$= \mu_\alpha^T(x * y) \geq min\{\mu_\alpha^T(x), \mu_\alpha^T(y)\} = min\{\mu(x) + \alpha, \mu(y) + \alpha\} = min\{\mu(x), \mu(y)\} + \alpha.$

Hence μ is a fuzzy subalgebra of X. ⌂

In general, the converse of Proposition (3.19) is not true. As the following example shows;

Example 3.20:

Let X = {0,a,b,c} be a set with binary operation (∗) defined by the following table:

∗	0	a	b	c
0	0	a	b	c
a	0	0	0	c
b	0	b	0	c
c	0	0	0	0

Then (X; ∗,0) is a CI-algebra. Define a fuzzy subset μ of X by:

X	0	a	b	c
μ	0.7	0.6	0.5	0.4

Then μ is a fuzzy subalgera of X, and T=0.2 .But if we take α=0.02 the fuzzy translation $\mu_\alpha^T$of μ is given as follows:

X	0	a	b	c
μ_α^T	0.72	0.62	0.52	0.42

Then μ_α^T is not a fuzzy CI-ideal of X. since μ_α^T (b∗a) =μ_α^T (b)=0.52< 0.62=min{μ_α^T (b∗(a∗a)), μ_α^T (a)}=min{μ_α^T (0), μ_α^T (a)}=min{0.72 , 0.62 }.

Proposition 3.21 :

If the fuzzy translation μ_α^T (x) of μ is a fuzzy CI-ideal ,then it satisfies the condition
$\mu_\alpha^T((y * x) * x) \geq \mu_\alpha^T(y)$.

Proof :

Let x , y ∈ X, then
$\mu_\alpha^T((y * x) * x) \geq \min\{\mu_\alpha^T((y * x) * (y * x)), \mu_\alpha^T(y)\} = \min\{\mu_\alpha^T(0), \mu_\alpha^T(y)\}$
$= \mu_\alpha^T(y)$. ⌂

Definition 3.22[5]:

Let (X; ∗, 0) and (Y; ∗́,0́) be CI-algebras. A mapping f : (X;∗ ,0) ⟶ (Y;∗́, 0́) is said to be a **homomorphism** if $f(x * y) = f(x) \acute{*} f(y)$ for all x, y ∈ X.

Definition 3.23 [10]:

If β is a fuzzy subset of Y , then the fuzzy subset μ = β o f in X (i.e the fuzzy subset defined by μ (x) = β (f(x)) for all x ∈ X) is called the pre-image of β under f .

Proposition 3.24:

Let f:X → Y be a homorphism of CI-algebra X in to a CI-algebra Y and μ_α^T be a fuzzy translation of μ, then the pre-image of μ_α^T denoted by f^{-1} $(\mu_\alpha^T)$is defined as { f^{-1} (μ_α^T)}= μ_α^T $(f\ (x))$ ∀ x∈X.

If μ is a fuzzy CI-ideal of Y, then f^{-1} (μ_α^T) is a fuzzy CI-ideal of X.

Proof :

Let μ be a fuzzy CI-ideal of Y. Let x, y ,z ∈X.

(1) $f^{-1}\left(\mu_\alpha^T(0)\right) = \mu_\alpha^T\left(f(0)\right) = \mu\left(f(0)\right) + \alpha \geq \mu\left(f(x)\right) + \alpha = \mu_\alpha^T\left(f(x)\right)$
$= f^{-1}(\mu_\alpha^T(x)) \Rightarrow f^{-1}(\mu_\alpha^T(0)) \geq f^{-1}(\mu_\alpha^T(0))$.

(2) $f^{-1}\left(\mu_\alpha^T(x*z)\right) = \mu_\alpha^T\left(f(x*z)\right) = \mu\left(f(x*z)\right) + \alpha$
$\geq min\{\mu(f(x*(y*z))), \mu(f(y)\} + \alpha = min\{\mu(f(x*(y*z))) + \alpha, \mu(f(y) + \alpha\}$
$= min\{\mu_\alpha^T(f(x*(y*z))), \mu_\alpha^T(f(y)\} = min\{f^{-1}(\mu_\alpha^T(x*(y*z))), f^{-1}(\mu_\alpha^T(y)\}$
$\Rightarrow f^{-1}(\mu_\alpha^T(x*z)) \geq min\{f^{-1}(\mu_\alpha^T(x*(y*z))), f^{-1}(\mu_\alpha^T(y)\}$.Hence f^{-1} (μ_α^T) is a fuzzy CI-ideal of X. ⌂

4- Fuzzy multiplications of fuzzy CI-ideals

We study the notion of fuzzy multiplication of CI-ideal on CI-algebra X and we give some properties of it.

Definition 4.1 [9]:

Let μ be a fuzzy subset of X and β∈ [0, 1]. **A fuzzy multiplication** of μ, denoted by μ_β^M is defined to be a mapping μ_β^M: X→[0,1] define by $\mu_\beta^M(x) = \beta\,.\mu(x)$, for all x ∈ X.

Theorem 4.2 :

Let μ be a fuzzy subset of a CI-algebra X and β ∈ (0,1].Then μ is a fuzzy CI-ideal of X if and only if the fuzzy multiplication μ_β^M is fuzzy CI-ideal of X .

Proof: (⟹)

Assume μ is a fuzzy CI-ideal of X and let β ∈ (0, 1] .Then

(1) $\mu_\beta^M(0) = \beta.\mu(0) \geq \beta.\mu(x) = \mu_\beta^M(x)$,

(2) $\mu_\beta^M(x*z) = \beta.\mu(x*z) \geq \beta.min\{\mu(x*(y*z)), \mu(y)\}$
$= min\{\beta.\mu(x*(y*z)), \beta.\mu(y)\} = min\{\mu_\beta^M(x*(y*z)), \mu_\beta^M(y)\}$ for all x, y, z ∈ X. Hence μ_β^M is a fuzzy CI-ideal of X. ⌂

(⟹) Let β ∈ (0, 1] be such that μ_β^M is a fuzzy CI-ideal multiplication of X. Then for all x, y , z ∈ X

(1) $\beta\,.\mu(0) = \mu_\beta^M(0) \geq \mu_\beta^M(x) = \beta\,.\mu(x) \Rightarrow \mu(0) \geq \mu(x)$

(2) $\beta\,.\mu(x*z) = \mu_\beta^M(x*z) \geq min\{\mu_\beta^M(x*(y*z)), \mu_\beta^M(y)\}$
$= min\{\beta.\mu(x*(y*z)), \beta.\mu(y)\} = \beta.min\{\mu(x*(y*z)), \mu(y)\}$
$\Rightarrow \mu(x*z) \geq min\{\mu(x*(y*z)), \mu(y)\}$. Hence μ is a fuzzy CI-ideal of X. ⌂

Proposition 4.3 :

Let μ be a fuzzy subset of a CI-algebra X, $\alpha \in [0,T]$ and $\beta \in (0, 1]$. Then every fuzzy CI-ideal translation μ_α^T of μ is a fuzzy extension CI-ideal of the fuzzy CI-ideal multiplication μ_β^M of μ.

Proof:

For every $x \in X$,we have

(1) Assume that μ_β^M is a fuzzy CI-ideal of X. Then μ is a fuzzy CI-ideal of X by Theorem (4.2). It follows from Theorem (3.2) that μ_α^T is a fuzzy CI-ideal of X for all $\alpha \in [0,T]$.

(2) $\mu_\alpha^T(x) = \mu(x) + \alpha \geq \mu(x) \geq \beta . \mu(x) = \mu_\beta^M(x) \Rightarrow \mu_\alpha^T$ is a fuzzy extension of μ_β^M. Hence every fuzzy translation μ_α^T is a fuzzy extension CI-ideal of the fuzzy CI-ideal multiplication μ_β^M. ⌂

The following example illustrates Proposition (4.3).

Example 4.4 :

Let X = {0,a,b,c,d} be a CI-algebra which is given in example (3.13). Define a fuzzy subset μ of X by :

X	0	a	b	c	d
μ	0.8	0.7	0.6	0.7	0.6

Then μ is a fuzzy CI-ideal of X. If we take β= 0.3 then the fuzzy multiplication $\mu_{0.3}^M$ of μ is given by :

X	0	a	b	c	d
$\mu_{0.3}^M$	0.24	0.21	0.18	0.21	0.18

Clearly $\mu_{0.3}^M$ is a fuzzy CI-ideal multiplication of X. Also, for any $\alpha \in [0, 0.2]$, the fuzzy translation μ_α^T of μ is given by:

X	0	a	b	c	d
μ_α^T	0.8+ α	0.7+ α	0.6+ α	0.7+ α	0.6 +α

Then μ_α^T is a fuzzy extension of $\mu_{0.3}^M$ and $\mu_\alpha^T$is always a fuzzy CI-ideal translation of X for all $\alpha \in [0, 0.2]$. Hence μ_α^T is a fuzzy extension CI-ideal of $\mu_{0.3}^M$ for all $\alpha \in [0, 0.2]$.

Proposition 4.5 :

Let $\gamma \in [0, 1]$ and μ be a fuzzy CI-ideal of X. Then The fuzzy multiplication $\mu_\gamma^M$of μ is a fuzzy subalgebra of X.

Proof:

Since μ fuzzy CI-ideal of a CI-algebra X, then by Proposition (2.10) μ is a fuzzy subalgebra of CI-algebra. Let $\gamma \in (0, 1]$ and $x, y \in X$. Then $\mu_\gamma^M(x * y) = \gamma . \mu(x * y)$ $\geq \gamma . min\{\mu(x), \mu(y)\} = min\{\gamma . \mu(x), \gamma . \mu(y)\} = min\{\mu_\gamma^M(x), \mu_\gamma^M(y)\}$. Hence $\mu_\gamma^M$of μ is a fuzzy subalgebra multiplication of X. ⌂

In general, the converse of the Proposition (4.5) is not true. As the following example shows;

Example 4.6 :

Let X={0,a,b,c}be a CI-algebra which is given in Example (3.18). Define a fuzzy subset μ of X by

X	0	a	b	c
μ	0.8	0.7	0.5	0.6

Then μ is not fuzzy CI-ideal of X. Since μ(0∗c)= μ(c)=0.6 < 0.7=min{ μ (0∗(a∗c), μ (a)} = min{ μ (a),μ (a)}= min{0.7, 0.7} ,and T=0.2. But if we take γ=0.1 the fuzzy multiplication μ_γ^{M} of μ is given as follows:

X	0	a	b	c
μ_γ^{M}	0.08	0.07	0.05	0.06

Then $\mu_\gamma^{M}$is a fuzzy subalgebra of X.

Proposition 4.7 :

If the fuzzy multiplication μ_γ^{M} of μ is a fuzzy CI-ideal of X, γ∈ (0,1] then μ is a fuzzy subalgebra.

proof:

Since μ_γ^{M} be a fuzzy CI-ideal of a CI-algebra X , then by Proposition (2.10) μ_γ^{M} be a fuzzy subalgebra of a CI-algebra X. Let γ∈ (0,1] and x, y ∈ X. Then $\gamma.\mu(x * y) = \mu_\gamma^{M}(x * y)$ $\geq min\{\mu_\gamma^{M}(x),\ \mu_\gamma^{M}(y)\} = min\{\gamma\ .\mu(x), \gamma\ .\mu(y)\} = \gamma\ .min\{\mu(x), \mu(y)\}$since γ ≠ 0 it follows that $\mu(x * y)\ \geq\ min\{\mu(x), \mu(y)\}$. Hence μ is a fuzzy subalgebra of X. ⌂

In general, the converse of the Proposition (4.7) is not true. As the following example shows;

Example 4.8 :

Let X={0,a,b,c}be a CI-algebra which is given in Example(3.20). Define a fuzzy subset μ of X by :

X	0	a	b	c
μ	0.7	0.6	0.4	0.5

Then μ is a fuzzy subalgera of X. But if we take γ=0.02 the fuzzy multiplication μ_γ^{M} of μ is given as follows:

X	0	a	b	c
μ_γ^{M}	0.014	0.012	0.008	0.01

Then μ_γ^{M} is not a fuzzy CI-ideal multiplication of X. sinceμ_γ^{M} (0∗2)= μ_γ^{M} (2) = 0.008 < 0.012 =min{μ_γ^{M} (0∗(1∗2)), μ_γ^{M} (1)}=min{μ_γ^{M} (0), μ_γ^{M} (1)}=min{0.014, 0.012} .

Proposition 4.9 :

Let f:X→Y be a homomorphism of CI-algebra X in to a CI-algebra Y and μ_γ^{M} be a fuzzy multiplication of μ ,then the pre-image of μ_γ^{M} denoted by $f^{-1}\left(\mu_\gamma^{\mathrm{M}}\right)$ is defined as $\{f^{-1}\left(\mu_\gamma^{\mathrm{M}}\right)\}$ = $\mu_\gamma^{\mathrm{M}}\,(f\,(x))$ ∀ x∈X. If μ is a fuzzy CI-ideal of Y, then $f^{-1}\left(\mu_\gamma^{\mathrm{M}}\right)$ is a fuzzy CI-ideal of X.

Proof :

Let μ be a fuzzy CI-ideal of Y. Let x, y ,z ∈X.

(1) $f^{-1}(\mu_\gamma^{\mathrm{M}}\,(0))=\mu_\gamma^{\mathrm{M}}\left(f\,(0)\right)=\gamma.\mu\big(f\,(0)\big)\geq\ \gamma.\mu\big(f\,(x)\big)=\mu_\gamma^{\mathrm{M}}\left(f\,(x)\right)=f^{-1}\left(\mu_\gamma^{M}\,(x)\right)$

$\Rightarrow\ f^{-1}(\mu_\gamma^{M}\,(0))\geq f^{-1}(\mu_\gamma^{M}\,(0))$.

(2) $f^{-1}\left(\mu_\gamma^{\mathrm{M}}\,(x*z)\right)=\mu_\gamma^{M}\big(f\,(x*z)\big)=\gamma.\mu\big(f\,(x*z)\big)$

$\geq\gamma.min\{\,\mu(f\,(x*(y*z))),\mu(f\,(y)\}=min\{\,\gamma.\mu(f(x*(y*z))),\gamma.\mu(f(y)\}$

$=min\{\mu_\gamma^{M}(f(x*(y*z))),\mu_\gamma^{M}(f\,(y)\}=min\{f^{-1}(\mu_\gamma^{\mathrm{M}}\,(x*(y*z))),(f^{-1}(\mu_\gamma^{\mathrm{M}}\,(y)\}$

$\Rightarrow\ f^{-1}(\mu_\gamma^{\mathrm{M}}\,(x*z))\geq\ min\{f^{-1}(\mu_\gamma^{\mathrm{M}}\,(x*(y*z))),f^{-1}(\mu_\gamma^{\mathrm{M}}\,(y)\}$. Hence $f^{-1}(\mu_\gamma^{\mathrm{M}})$ is a fuzzy CI-ideal of X .⌂

5- Homomorphism of fuzzy translation and fuzzy multiplication of CI-algebra:

We define of a homeomorphic of fuzzy translation and fuzzy multiplication of CI-algebra and we study some properties about it.

Definition 5.1 :

Let $f\colon(\,X;*\,,0\,)\to(Y;\acute{*},\acute{0})$ be an endomorphism and μ_α^{T} be a fuzzy translation of μ in CI-algebra X. We define a new fuzzy set in CI-algebra X by f in X as $(\mu_\alpha^{\mathrm{T}})_f$ in X as $(\mu_\alpha^{\mathrm{T}})_f(x)=\ (\mu_\alpha^{\mathrm{T}})\big(f(x)\big)=\mu\big(f(x)\big)+\alpha$ for all x∈X.

Proposition 5.2 :

Let $f\colon(\,X;*\,,0\,)\to(Y;\acute{*},\acute{0})$ be an endomorphism of CI-algebra X. If μ is a fuzzy CI-ideal of X then $(\mu_\alpha^{\mathrm{T}})_f$ is also fuzzy CI-ideal of X.

Proof :

Let μ be a fuzzy CI-ideal of X and let x, y ,z ∈X.

(1) $(\mu_\alpha^{\mathrm{T}})_f(0)=\ (\mu_\alpha^{\mathrm{T}})f(x_1)=y_1\big(f(0)\big)=\mu\big(f(0)\big)+\alpha\geq\mu\big(f(x)\big)+\alpha=\ (\mu_\alpha^{\mathrm{T}})_f(x)$

$\Rightarrow\ (\mu_\alpha^{\mathrm{T}})_f(0)\geq\ (\mu_\alpha^{\mathrm{T}})_f(x)$.

(2) $(\mu_\alpha^{\mathrm{T}})_f(x*z)=\ (\mu_\alpha^{\mathrm{T}})\big(f(x*z)\big)=\mu\big(f(x*z)\big)+\alpha\geq\min\{\mu\big(f(x*(y*z)\big),\mu(y)\}+\alpha$

$=\min\{\mu\big(f\big(x*(y*z)\big)+\alpha,\mu\big(f(y)\big)+\alpha\}=\min\{(\mu_\alpha^{\mathrm{T}})_f\big(x*(y*z)\big),\ (\mu_\alpha^{\mathrm{T}})_f(y)\}$

$\Rightarrow\ (\mu_\alpha^{\mathrm{T}})_f(x*z)\geq\min\{(\mu_\alpha^{\mathrm{T}})_f\big(x*(y*z)\big),\ (\mu_\alpha^{\mathrm{T}})_f(y)\}$. Hence $(\mu_\alpha^{\mathrm{T}})_f$ is a fuzzy CI-ideal of X. ⌂

Proposition 5.3 :

Let $f\colon(\,X;*\,,0\,)\to(Y;\acute{*},\acute{0})$ be an epimorphism of CI-algebra X. If $(\mu_\alpha^{\mathrm{T}})_f$ is a fuzzy CI-ideal of X then μ is also fuzzy CI-ideal of Y.

Proof :

Let $(\mu_\alpha^{\mathrm{T}})_f$ be a fuzzy CI-ideal of X and y_1, y_2 ,y_3 ∈Y. Then there exists x_1, x_2, x_3 ∈ X such that $f(x_1)=y_1$, $f(x_2)=y_2$ and $f(x_3)=y_3$

(1) $\mu(\acute{0})+\alpha=\ \mu_\alpha^{\mathrm{T}}(\acute{0})=\ \mu_\alpha^{\mathrm{T}}\big(f(0)\big)=\ (\mu_\alpha^{\mathrm{T}})_f(0)\geq\ (\mu_\alpha^{\mathrm{T}})_f(x_1)=\ \mu_\alpha^{\mathrm{T}}(f(x_1))$

$=\mu(f(x_1))+\alpha=\mu(y_1)+\alpha\Rightarrow\mu(\acute{0})\geq\mu(y_1)$

(2) $\mu(y_1\,\acute{*}\,y_3)+\alpha=\mu\big(f(x_1)\,\acute{*}\,f(x_3)\big)+\alpha=\mu\big(f(x_1*x_3)\big)+\alpha=\ \mu_\alpha^{\mathrm{T}}(f(x_1*x_3))$

$=\ (\mu_\alpha^{\mathrm{T}})_f(x_1*x_3)\geq\min\left\{\ (\mu_\alpha^{\mathrm{T}})_f\left((x_1*(x_2*x_3)),\ (\mu_\alpha^{\mathrm{T}})_f(x_2)\right)\right\}$

$=\min\{\ (\mu_\alpha^{\mathrm{T}})(f(x_1*(x_2*x_3))),\ (\mu_\alpha^{\mathrm{T}})(f(x_2))$

$=\min\{\ (\mu_\alpha^{\mathrm{T}})(f(x_1)\,\acute{*}\,\big(f(x_2)\,\acute{*}\,f(x_3)\big),\ (\mu_\alpha^{\mathrm{T}})(f(x_2))\}$

$= \min\{\mu(f(x_1) \acute{*} (f(x_2) \acute{*} f(x_3)) + \alpha, \mu(f(x_2)) + \alpha\}$
$= \min\{\mu(f(x_1) \acute{*} (f(x_2) \acute{*} f(x_3)), \mu(f(x_2))\} + \alpha$
$= \min\{\mu(y_1 \acute{*} (y_2 \acute{*} y_3), \mu(y_2)\} + \alpha$
$\Longrightarrow \mu(y_1 \acute{*} y_3) \geq \min\{\mu(y_1 \acute{*} (y_2 \acute{*} y_3), \mu(y_2)\}$. Hence μ is a fuzzy CI-ideal of Y. ⌂

Proposition 5.4 :

Let $f: (X;*,0) \to (Y;\acute{*},\acute{0})$ be a homomorphism of CI-algebra. If μ is a fuzzy CI-ideal of Y then $(\mu_\alpha^{\mathrm{T}})_f$ is also fuzzy CI-ideal of X.

Proof :

Let x, y, z ∈ X and let μ be a fuzzy CI-ideal of Y ,we get

(1) $(\mu_\alpha^{\mathrm{T}})_f(0) = \mu_\alpha^{\mathrm{T}}(f(0)) = \mu(f(0)) + \alpha \geq \mu(f(x)) + \alpha = \mu_\alpha^{\mathrm{T}}(f(x)) = (\mu_\alpha^{\mathrm{T}})_f(x)$

(2) $(\mu_\alpha^{\mathrm{T}})_f(x * z) = (\mu_\alpha^{\mathrm{T}})(f(x * z)) = \mu(f(x * z)) + \alpha$
$\geq \min\{\mu(f(x * (y * z)), \mu(f(y))\} + \alpha = \min\{\mu(f(x * (y * z)) + \alpha, \mu(f(y)) + \alpha\}$
$= \min\left\{\mu_\alpha^T\left(f(x * (y * z))\right), \mu_\alpha^T(f(y))\right\} = \min\{(\mu_\alpha^{\mathrm{T}})_f(x * (y * z)), (\mu_\alpha^{\mathrm{T}})_f(y)\}$
$\Longrightarrow (\mu_\alpha^{\mathrm{T}})_f(x * z) \geq \min\{(\mu_\alpha^{\mathrm{T}})_f(x * (y * z)), (\mu_\alpha^{\mathrm{T}})_f(y)\}$.

Hence $(\mu_\alpha^{\mathrm{T}})_f$ is a fuzzy CI-ideal of X. ⌂

Definition 5.5 :

Let $f: (X;*,0) \to (Y;\acute{*},\acute{0})$ be an endomorphism and μ_γ^{M} be a fuzzy multiplication of μ in CI-algebra X. We define a new fuzzy set in CI-algebra X by f in X as$(\mu_\gamma^{\mathrm{M}})_f$ in X as
$(\mu_\gamma^{\mathrm{M}})_f(x) = (\mu_\gamma^{\mathrm{M}})(f(x)) = \gamma.\mu(f(x))$ for all x∈X.

Proposition 5.6 :

Let $f: (X;*,0) \to (Y;\acute{*},\acute{0})$ be an endomorphism of CI-algebra X. If μ is a fuzzy CI-ideal of X then $(\mu_\gamma^{\mathrm{M}})_f$ is also fuzzy CI-ideal of X.

Proof :

Let μ be a fuzzy CI-ideal of X and let x, y ,z ∈X.

(1) $(\mu_\gamma^{\mathrm{M}})_f(0) = (\mu_\gamma^{\mathrm{M}})f(x_1) = \mu_\gamma^{\mathrm{M}}(f(0)) = \gamma.\mu(f(0)) \geq \gamma.\mu(f(x)) = (\mu_\gamma^{\mathrm{M}})_f(x)$
$\Longrightarrow (\mu_\gamma^{\mathrm{M}})_f(0) \geq (\mu_\gamma^{\mathrm{M}})_f(x)$.

(2) $(\mu_\gamma^{\mathrm{M}})_f(x * z) = (\mu_\gamma^{\mathrm{M}})(f(x * z)) = \gamma.\mu(f(x * z)) \geq \gamma.\min\{\mu(f(x * (y * z)), \mu(y)\}$
$= \min\{\mu(f(x * (y * z)), \gamma.\mu(f(y))\} = \min\{(\mu_\alpha^{\mathrm{T}})_f(x * (y * z)), (\mu_\alpha^{\mathrm{T}})_f(y)\}$
$\Longrightarrow (\mu_\gamma^{\mathrm{M}})_f(x * z) \geq \min\{(\mu_\gamma^{\mathrm{M}})_f(x * (y * z)), (\mu_\gamma^{\mathrm{M}})_f(y)\}$.

Hence $(\mu_\gamma^{\mathrm{M}})_f$ is a fuzzy CI-ideal of X. ⌂

Proposition 5.7 :

Let $f: (X;*,0) \to (Y;\acute{*},\acute{0})$ be an epimorphism of CI-algebra X. If $(\mu_\gamma^{\mathrm{M}})_f$ is a fuzzy CI-ideal of X then μ is also fuzzy CI-ideal of Y.

Proof :

Let $(\mu_\gamma^{\mathrm{M}})_f$ be a fuzzy CI-ideal of X and y_1, y_2 ,y_3 ∈Y. Then there exists x_1, x_2, x_3 ∈ X such that $f(x_1) = y_1$, $f(x_2) = y_2$ and $f(x_3) = y_3$

(1) $\gamma.\mu(\acute{0}) = \mu_\gamma^{\mathrm{M}}(\acute{0}) = \mu_\gamma^{\mathrm{M}}(f(0)) = (\mu_\gamma^{\mathrm{M}})_f(0) \geq (\mu_\gamma^{\mathrm{M}})_f(x_1) = \mu_\gamma^{\mathrm{M}}(f(x_1)) = \gamma.\mu(f(x_1))$
$= \gamma.\mu(y_1) \Longrightarrow \mu(\acute{0}) \geq \mu(y_1)$

(2) $\gamma.\mu(y_1 \acute{*} y_3) = \gamma.\mu(f(x_1) \acute{*} f(x_3)) = \gamma.\mu(f(x_1 * x_3)) = \mu_\gamma^{\mathrm{M}}(f(x_1 * x_3))$
$= (\mu_\gamma^{\mathrm{M}})_f(x_1 * x_3) \geq \min\left\{(\mu_\gamma^{\mathrm{M}})_f\left((x_1 * (x_2 * x_3)), (\mu_\gamma^{\mathrm{M}})_f(x_2)\right)\right\}$
$= \min\{(\mu_\gamma^{\mathrm{M}})(f(x_1 * (x_2 * x_3))), \mu_\gamma^{\mathrm{M}})(f(x_2))$
$= \min\{(\mu_\gamma^{\mathrm{M}})(f(x_1) \acute{*} (f(x_2) \acute{*} f(x_3)), (\mu_\gamma^{\mathrm{M}})(f(x_2))\}$

$= \min\{\gamma.\mu\big(f(x_1) \acute{*} \big(f(x_2) \acute{*} f(x_3)\big), \gamma.\mu\big(f(x_2)\big)\big\}$
$= \gamma.\min\{\mu\big(f(x_1) \acute{*} \big(f(x_2) \acute{*} f(x_3)\big), \mu\big(f(x_2)\big)\big\}$
$= \gamma.\min\{\mu(y_1 \acute{*} (y_2 \acute{*} y_3), \mu(y_2)\}$

$\Longrightarrow \mu(y_1 \acute{*} y_3) \geq \min\{\mu(y_1 \acute{*} (y_2 \acute{*} y_3), \mu(y_2)\}$. Hence μ is a fuzzy CI-ideal of Y. ⌂

Proposition 5.8 :

Let f:X → Y be a homomorphism of CI-algebra. If μ is a fuzzy CI-ideal of Y then $(\mu_\gamma^{\mathrm{M}})_f$ is also fuzzy CI-ideal of X.

Proof :

Let x, y, z ∈ X and let μ be a fuzzy CI-ideal of Y ,we get

(1) $(\mu_\gamma^{\mathrm{M}})_f(0) = \mu_\gamma^{\mathrm{M}}\big(f(0)\big) = \gamma.\mu\big(f(0)\big) \geq \gamma.\mu\big(f(x)\big) = \mu_\gamma^{\mathrm{M}}\big(f(x)\big) = (\mu_\gamma^{\mathrm{M}})_f(x)$

(2) $(\mu_\gamma^{\mathrm{M}})_f(x * z) = (\mu_\gamma^{\mathrm{M}})\big(f(x * z)\big) = \gamma.\mu\big(f(x * z)\big)$
$\geq \gamma.\min\{\mu(f\big(x * (y * z)\big), \mu(f(y))\} = \min\{\gamma.\mu(f\big(x * (y * z)\big), \gamma.\mu\big(f(y)\big)\}$
$= \min\left\{\mu_\gamma^{\mathrm{M}}\left(f\big(x * (y * z)\big)\right), \mu_\gamma^{\mathrm{M}}\big(f(y)\big)\right\} = \min\{(\mu_\gamma^{\mathrm{M}})_f\big(x * (y * z)\big), (\mu_\gamma^{\mathrm{M}})_f(y)\}$

$\Longrightarrow (\mu_\gamma^{\mathrm{M}})_f(x * z) \geq \min\{(\mu_\gamma^{\mathrm{M}})_f\big(x * (y * z)\big), (\mu_\gamma^{\mathrm{M}})_f(y)\}$.

Hence $(\mu_\gamma^{\mathrm{M}})_f$ is a fuzzy CI-ideal of X. ⌂

Conclusion:

In this paper we have introduced the notion of fuzzy translation and fuzzy multiplication on CI-algebras. Interestingly fuzzy extension CI-ideal of CI-algebras has been studied which adds an another dimension to the definition of CI-algebras, has been studying the a new fuzzy set to fuzzy translation and fuzzy multiplication to CI-algebra. It has been observed that the CI-algebra it's a generalization of BE-algebra by B.L. Meng [5].

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