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Compact In Intuitionistic Fuzzy Ideal Topological Space

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Abstract

In this paper ,we introduce the definition of intuitionistic fuzzy ideals , intuitionistic fuzzy ideals topological spaces and intuitionistic fuzzy open localfunction .Also we introduce the definition of (r,s)-intuitionistic fuzzy idealscompact ,and we define some types of (r,s)-compact in intuitionisticfuzzy ideals topological spaces ,and prove some results about them .

Keywords: Intuitionistic fuzzy ideals , Intuitionistic fuzzy ideals topologicalspaces , Compactness in intuitionistic fuzzy ideals topological spaces .

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التراص في الفضاء التبولوجي المثالي الضبابي الحدسي .

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الخلاصة
في هذا البند , قدمنا تعريف المثاليه الضبابيه الحدسيه , الفضاء التبولوجي المثالي الضبابي الحدسي والدوالالمحليه الضبابيه الحدسيه .كذالك قدمنا تعريف التراص في الفضاء التبولوجي المثالي الضبابي الحدسي وعرفنا بعض انواع التراص فيالفضاء التبولوجي المثالي الضبابي الحدسي وبرهنا بعض النتائج حول تلك الموضوعات .

1-Introdiction

The concept of fuzzy set was the firstly to introduced by L.A.Zadah in [1965] [14] as an extension of the classical notion of set . After three years C.L. Chang 1968 [5]axiomatizing a collection τ of fuzzy subset of a non-empty set X ,in order to introducea structure of fuzzy topology τ on X .

The "intuitionistic fuzzy set " was firstly defined by K.T.Atanassov in 1983[3] and many researches have followed the same author as [4],[9] .

On the other hand , in 1992 K.C. Chattopadhyay [7] had given a now definition of fuzzytopology by introducing a concept of gradation of openness of fuzzy set .D.Coker 1997 [6] gave the basic of definition of "intuitionistic fuzzy Topological space"and necessary example . The idea of fuzzy ideal and fuzzy local function in fuzzy set theory studied by D. Sarker in1997[12] and proved some results about them .

in 2002 T.K. Mondel and S.K. Smanta [8] ,introduced a concept, of intuitionistic gradation of openness of fuzzy subset of anon-empty set X and an intuitionistic fuzzy topological space with respect to gradation of openness T.K. Mondel and S.K. Smanta in 2002[8] .

Anew concept of mixed fuzzy ideal topological space introduced by Binod Chandra Tripathy ,Gautanm Ray in 2013[13] and investigated some properties of them .Y.M. Saber and M.A. Abdel-Sattar in 2014[11] introduced fuzzy ideal and r-fuzzy open local function .Also introduced r-fuzzy ideal compact, r-fuzzy ideal quasi H-closed andr-fuzzy compact modulo an fuzzy ideal in fuzzy ideal topological space .

In this paper we introduce the definition of intuitionistic fuzzy ideals , intuitionisticfuzzy ideals topological spaces , and intuitionistic fuzzy open local function .Also weintroduce the definition of (r,s)- intuitionistic fuzzy ideals compact (for short (r,s)- IF((I_1,I_2)compact) ,and we define some types of (r,s)-compact in intuitionistic fuzzy ideals topological spaces and prove some results about them.

2-PRELIMINARIES

We will give some preliminaries in this paper , X will denoted a non-empty set , $I = [o, 1]$, The closed nuit interval of real line ; $I_\circ = (0,1]$ and $I_1 = [0,1)$, I^X isthe set of all fuzzy subsets of X , By $\underline{0}$ and $\underline{1}$, we denote the constant fuzzy subset of Xtaking the values $\underline{0}$ and $\underline{1}$, respectively .

Definition:

A mapping $\tau: I^X \to I$, is a fuzzy topology satisfying the following conditions:

(1) $\tau(\underline{0}) = \tau(\underline{1})$.

(2) $\tau(A_1 \wedge A_2) \geq \tau(A_1) \wedge \tau(A_2)$.

(3) $\tau(\vee_{i\in\Gamma} A_i) \geq \wedge_{i\in\Gamma} \tau(A_i)$. for any $\{A_i\}_{i\in\Gamma} \subset I^X$, where$\Gamma$ is an index set the pair

(X, τ) is called fuzzy topological space .

Definition:2.2.[10]

A mapping $I: I^X \to I$ is called a fuzzy ideal on X if it satisfies the following condition:

1) $I(\underline{0}) = 1$, $I(\underline{1}) = 0$.

2)if $A \leq B$, then $I(B) \leq I(A)$, for each $A, B \in I^X$.

3)$I(A \vee B) \geq I(A) \wedge I(B)$, for $A, B \in I^X$.

The pair (X, τ, I) is called a fuzzy ideal topological space (FITS) .

Definition: 2.3.[5]

An intuitionistic fuzzy topology (for short , (IFT)) a mappings $\tau_1, \tau_2: I^X \to I$,

satisfies the following condition :

(1)$\tau_1(A) + \tau_2(A) \leq 1, \forall A \in I^X$,

$$(2)\tau_1(\underline{0}) = \tau_1(\underline{1}) = 1, \tau_2(\underline{0}) = \tau_2(\underline{1}) = 0,$$

(3)$\tau_1(A_1 \wedge A_2) \geq \tau_1(A_1) \wedge \tau_1(A_2)$ and $\tau_2(A_1 \wedge A_2) \leq \tau_2(A_1) \vee \tau_2(A_2)$

for each $A_i \in I^X, i = 1,2$,

(4) $\tau_1(\vee_{i\in\Gamma} A_i) \geq \wedge_{i\in\Gamma} \tau_1(A_i)$ and $\tau_2(\vee_{i\in\Gamma} A_i) \leq \vee_{i\in\Gamma} \tau_2(A_i)$

for each $A_i \in I^X, i \in \Gamma$.

The triple (X, τ_1, τ_2) is called an intuitionistic fuzzy topological spaces of openness (for short, (IFTS)).

Definition: 2.4.[2]

Let (X, τ_1, τ_2) be an intuitionistic fuzzy topological spaces , define an operator(X, τ_1, τ_2)be an intuitionistic fuzzy topological space. For $\mu, \lambda \in I^X, r \in I_\circ, s \in I_1$ the operator $I_{\tau_1,\tau_2}(\lambda, r, s)$ satisfies the following conditions:

$C_{\tau_1,\tau_2}: I^X \times I_\circ \times I_1 \to I^X$ by :

$$C_{\tau_1,\tau_2}(\lambda, r, s) = \wedge \{\mu \in I^X: \lambda \leq \mu, \tau_1(\underline{1} - \mu) \geq r, \tau_2(1 - \mu) \leq s\}.$$

Theorem: 2.1.[2]

Let (X, τ_1, τ_2) be an intuitionistic fuzzy topological spaces , for $\mu, \lambda \in I^X, r \in I_\circ$ $s \in I_1$ the operator C_{τ_1,τ_2} satisfies the following condition:

(C1) $C_{\tau_1,\tau_2}(\underline{0}, r, s) = \underline{0}$.

(C2) $\lambda \leq C_{\tau_1,\tau_2}(\lambda, r, s)$.

(C3) $C_{\tau_1,\tau_1}(\lambda, r, s) \vee C_{\tau_1,\tau_2}(\mu, r, s) = C_{\tau_1,\tau_2}(\lambda \vee \mu, r, s)$.

(C4) $C_{\tau_1,\tau_2}(\lambda, r, s) \leq C_{\tau_1,\tau_2}(\lambda, r_1, s_1)$ if $r \leq r_1$ and $s \geq s_1$

(C5) $C_{\tau_1,\tau_2}(C_{\tau_1,\tau_2}(\lambda, r, s), r, s) = C_{\tau_1,\tau_2}(\lambda, r, s)$.

Definition: 2.5.[1]

Let (X, τ_1, τ_2) be an intuitionistic fuzzy topological space define operator

$I_{\tau_1,\tau_2}: I^X \times I_\circ \times I_1 \to I^X$ by :

$I_{\tau_1,\tau_2}(\lambda, r, s) = \vee \{\mu \in I^X: \lambda \geq \mu, \tau_1(\mu) \geq r, \tau_2(\mu) \leq s\}$.

Theorem: 2.2.[1]

Let (X, τ_1, τ_2) be an intuitionistic fuzzy topological spaces , for $\mu, \lambda \in I^X, r \in I_\circ, s \in I_1$ the operator I_{τ_1,τ_2} satisfies the following condition:

(I1) $I_{\tau_1,\tau_2}(\underline{1}, r, s) = \underline{1} - C_{\tau_1,\tau_2}(\lambda, r, s)$ and $C_{\tau_1,\tau_2}(\underline{1}, r, s) = \underline{1} - I_{\tau_1,\tau_2}(\lambda, r, s)$.

(I2) For $\lambda, \mu \in I^X, r \in I_\circ, s \in I_1$, then

$I_{\tau_1,\tau_2}(\lambda, r_1, s_1) \leq I_{\tau_1,\tau_2}(\lambda, r, s)$ if $r \leq r_1$ and $s \geq s_1$.

(I3)$I_{\tau_1,\tau_2}(\underline{1}, r, s) = \underline{1}$.

(I4)$\lambda \geq I_{\tau_1,\tau_2}(\lambda, r, s)$.

(I5)$I_{\tau_1,\tau_2}(\lambda, r, s) \wedge I_{\tau_1,\tau_2}(\mu, r, s) = I_{\tau_1,\tau_2}(\lambda \wedge \mu, r, s)$

(I6)$I_{\tau_1,\tau_2}(I_{\tau_1,\tau_2}(\lambda, r, s), r, s) = I_{\tau_1,\tau_2}(\lambda, r, s)$.

Definition: 2.6

A Mappings $I_1, I_2: I^X \to I$, are called intuitionistic fuzzy ideals on X if it is satisfies the following condition:

$(I_1) I_1(A) + I_2(A) \leq 1, \forall A \in I^X$.

$(I_2) I_1(\underline{0}) = I_2(\underline{1}) = 1$ and $I_1(\underline{1}) = I_2(\underline{0}) = 0$.

(I_3) if $A \leq B$, then $I_1(B) \leq I_1(A)$ and $I_2(B) \geq I_2(A)$, for each $A, B \in I^X$.

$(I_4) I_1(A \vee B) \geq I_1(A) \wedge I(B)$ and $I_2(A \vee B) \leq I_2(A) \vee I_2(B)$, for each $A, B \in I^X$.

Then $(X, \tau_1, \tau_2, I_1, I_2)$ is called intuitionistic fuzzy ideals topological spaces .

Definition: 2.7

Let $(X, \tau_1, \tau_2, I_1, I_2)$ be an intuitionistic fuzzy ideals topological spaces , and $A \in I^X$, then the $(r, s)-$ fuzzy open local function $A^*_{(r,s)}(\tau_1, \tau_2, I_1, I_2)$ of A is the union of all fuzzy point x_t such that if $B \in Q_{\tau_1,\tau_2}(x_t, r, s)$ and $I_1(c) \geq r$,

$I_2(c) \leq s$ then there exists at least one $y \in X$ for which ,

$A(y) + B(y) - 1 > c(y)$.

3 −CompactnessinIntuitionisticFuzzyIdealsTopologicalSpaces

In this section , we study the definition some type of compact in intuitionistic fuzzy topological space and some type of compact in intuitionistic fuzzy ideals topological space and prove some results about them .

Definition: 3.1

Let (X, τ_1, τ_2) be a intuitionistic fuzzy topological spaces for each $A \in I^X$, $r \in I_\circ$, $s \in I_1$, then :

(1) A is called $(r, s)-$ intuitionistic fuzzy regular open (for short, $(r, s) - \text{FRO}$),

iff $A = I_{\tau_1,\tau_2}\left(C_{\tau_1,\tau_2}(A,r,s),r,s\right)$.

(2) A is called (r,s) − intuitionistic fuzzy pre − open (for short (r,s) − FPO) iff $A \leq I_{\tau_1,\tau_2}\left(C_{\tau_1,\tau_2}(A,r,s),r,s\right)$.

Definition: 3.2

An intuitionistic fuzzy topological space (X,τ_1,τ_2) is called (r,s) − intuitionistic fuzzy regular , iff for each $\tau_1(A) \geq r, \tau_2(A) \leq s$ and $r \in I_\circ, s \in I_1$,
$A = \vee\left\{B \in I^X, \tau_1(B) \geq r, \tau_2(B) \leq s, C_{\tau_1,\tau_2}(B,r,s) \leq A\right\}$.

Definition: 3.3

Let (X,τ_1,τ_2) be a intuitionitic fuzzy topological spaces , and $r \in I_\circ, \in I_1$, then X is called (r,s) − intuitionistic fuzzy compact (res , (r,s)-intuitionistic fuzzy almost compat) iff for each family $\gamma = \{A_i \in I^X: \tau_1(A_i) \geq r, \tau_2(A_i) \leq s\}$, such that $\vee_{i\in\Gamma} A_i = 1$, there exists a finite index $\Gamma_\circ \subseteq \Gamma$ such that $\vee_{i\in\Gamma\subseteq\Gamma_\circ} A_i = 1$,
$\left(\text{res.}\left(\vee_{i\subseteq\Gamma_\circ} C_{\tau_1,\tau_2}(A_i,r,s) = 1\right)\right)$.

Definition: 3.4

Let $(X,\tau_1,\tau_2,I_1,I_2)$ be an intuitionistic fuzzy ideals topological spaces and $r \in I_\circ, s \in I_1$, X is called (r,s) − intuitionistic fuzzy ideals compact(for short (r,s) − IF (I_1,I_2) compact) respectively ((r,s) − intuitionistic fuzzy ideals quasi H − closed (for short (r,s) − $IF(I_1,I_2)QHC$) , iff for each family $\{A_i \in I^X: \tau_1(A_i) \geq r, \tau_2(A_i) \leq s, i \in \Gamma\}$ such that $\vee_{i\in\Gamma} A_i = 1$, there exists a finite index set $\Gamma_\circ \subseteq \Gamma$, such that $I_1\left(1 - \vee_{i\in\Gamma_\circ} A_i\right) \geq r$ and $I_2\left(1 - \vee_{i\in\Gamma_\circ} A_i\right) \leq s$,
$((\text{res.}), I_1(1 - \vee_{i\in\Gamma_\circ}(C_{\tau_1,\tau_2}(A_i,r,s)) \geq r$ and $(I_2\left(1 - \vee_{i\in\Gamma_\circ}\left(C_{\tau_1,\tau_2}(A_i,r,s)\right) \leq s\right)$.

Definition: 3.5

Let $(X,\tau_1,\tau_2,I_1,I_2)$ be an intuitionistic fuzzy ideals topological spaces and $r \in I_\circ$, $s \in I_1$, X is called (r,s) − intuitionistic fuzzy ideals compact modulo an intuitioistic fuzzy ideals space (for short (r,s) − $IFC(I_1,I_2)$ compact) if for every$\tau_1(1 - A) \geq r$, $\tau_2(1 - A) \leq s$, and each family $\{B_i \in I^X: \tau_1(B_i) \geq r, \tau_2(B_i) \leq s, i \in \Gamma\}$ such that $A \leq \vee_{i\in\Gamma} B_i$, there exists a finite index set $\Gamma_\circ \subseteq \Gamma$ such that ,

$I_1\left(A \wedge \left[1 - \vee_{i\in\Gamma_\circ} C_{\tau_1,\tau_2}(B_i,r,s)\right]\right) \geq r$ and $I_2\left(A \wedge \left[1 - \vee_{i\in\Gamma_\circ} C_{\tau_1,\tau_2}(B_i,r,s)\right]\right) \leq s$.

Definition: 3.6

Let $(X,\tau_1,\tau_2,I_1,I_2)$ be an intuitionistic fuzzy ideals topological spaces and $r \in I_\circ$, $s \in I_1$, A is called $(r,s)-$ intuitionistic fuzzy ideals compact(for short $(r,s)-$ IF (I_1,I_2)compact and each family $\{B_i \in I^X : \tau_1(B_i) \geq r, \tau_2(B_i) \leq s , i \in \Gamma\}$ such that $A \leq \vee_{i\in\Gamma} B_i$ there exists a finite index set $\Gamma_\circ \subseteq \Gamma$ such that $I_1\left(1 - \vee_{i\in\Gamma_\circ} B_i\right) \geq r$ and

$$I_2\left(1 - \vee_{i\in\Gamma_\circ} B_i\right) \leq s \ .$$

Theorem: 3.1

Let $(X,\tau_1,\tau_2,I_1,I_2)$ be an intuitionistic fuzzy ideals topological spaces and $r \in I_\circ$, $s \in I_1$:

(1) If X is $(r,s)-$ IF compact then X is $(r,s)-$ IF(I_1,I_2)compact .

(2) If X is $(r,s)-$ IF(I_1,I_2)compact then X is $(r,s)-$ IFC(I_1,I_2)compact .

(3) If X is $(r,s)-$ IF(I_1,I_2)compact then X is $(r,s)-$ IF(I_1,I_2)QHC .

(4) If X is $(r,s)-$ IF almost compact then X is $(r,s)-$ IF(I_1,I_2)QHC .

Proof:

(1) Let $\{A_i \in I^X : \tau_1(A_i) \geq r , \tau_2(A_i) \leq s , i \in \Gamma\}$ be a family such that $\vee_{i\in\Gamma} A_i = 1$ since X is $(r,s)-$ intuitionistic fuzzy compact of X , there exists a finite subset $\Gamma_\circ \subseteq \Gamma$, such that $\vee_{i\in\Gamma_\circ} A_i = 1$, Now since $1 - \vee_{i\in\Gamma_\circ} A_i = 0$, then $I_1\left(1 - \vee_{i\in\Gamma_\circ} A_i\right) \geq r$ and $I_2\left(1 - \vee_{i\in\Gamma_\circ} A_i\right) \leq s$, therefore (X,τ_1,τ_2) is $(r,s)-$ IF(I_1,I_2)compact .

(2) Let $A \in I^X$, such that , $\tau_1(1 - A) \geq r$, $\tau_2(1 - A) \leq s$, and $\{B_i \in I^X : \tau_1(B_i) \geq r , \tau_2(B_i) \leq s, i \in \Gamma\}$ be a family such that $A < \vee_{i\in\Gamma} B_i$, since X is $(r,s)-$ IF(I_1,I_2)compact of A , there exists a finite index set $\Gamma_\circ \subseteq \Gamma$, such that $I_1\left(1 - \vee_{i\in\Gamma_\circ} B_i\right) \geq r$, and $I_2\left(1 - \vee_{i\in\Gamma_\circ} B_i\right) \leq s$,

since $B_i \leq C_{\tau_1,\tau_2}(B_i,r,s)$ (from theorem $(2.1.(C_2))$,

$A \wedge \left[1 - \vee_{i\in\Gamma_\circ} B_i\right] \geq A \wedge \left[1 - \vee_{i\in\Gamma_\circ} C_{\tau_1,\tau_2}(B_i,r,s)\right]$, hence $I_1\left(A \wedge \left[1 - \vee_{i\in\Gamma_\circ} B_i\right]\right) \geq r$, and $I_2\left(A \wedge \left[1 - \vee_{i\in\Gamma_\circ} B_i\right]\right) \leq s$, then $I_1\left(A \wedge \left[1 - \vee_{i\in\Gamma_\circ} C_{\tau_1,\tau_2}(B_I,r,s)\right]\right) \geq r$,

$I_2\left(A \wedge \left[1 - \vee_{i\in\Gamma_\circ} C_{\tau_1,\tau_2}(B_i,r,s)\right]\right) \leq s$, therefore $(X,\tau_1,\tau_2,I_1,I_2)$ is $(r,s)-$ IFC(I_1,I_2)

compact .

(3)Let $\{A_i \in I^X : \tau_1(A_i) \geq r, \tau_2(A_i) \leq s, i \in \Gamma\}$ be a family such that $\vee_{i\in\Gamma} A_i = 1$, since $(X, \tau_1, \tau_2, I_1, I_2)$ be $(r,s) - IF(I_1, I_2)$ compact there exists a finite index $\Gamma_\circ \subseteq \Gamma$ such that $I_1(1 - \vee_{i\in\Gamma_\circ} A_i) \geq r$, $I_2(1 - \vee_{i\in\Gamma_\circ} A_i) \leq s$, since $A_i \leq C_{\tau_1,\tau_2}(A_i, r, s)$ $(1 - \vee_{i\in\Gamma_\circ} A_i) \geq 1 - \vee_{i\in\Gamma_\circ} C_{\tau_1,\tau_2}(A_i, r, s)$, then $I_1(1 - \vee_{i\in\Gamma_\circ} C_{\tau_1,\tau_2}(A_i, r, s)) \geq r$, and $I_2(1 - \vee_{i\in\Gamma_\circ} C_{\tau_1,\tau_2}(A_i, r, s)) \leq s$. thus $(X, \tau_1, \tau_2, I_1, I_2)$ is $(r,s) - IF(I_1, I_2)QHC$.

(4) Let $\{A_i \in I^X : \tau_1(A_i) \geq r, \tau_2(A_i) \leq s, i \in \Gamma\}$ be a family, such that $\vee_{i\in\Gamma} A_i = 1$ since X is a $(r,s) -$ IF almost compact, there exists a finite index $\Gamma_\circ \subseteq \Gamma$, such that $\vee_{i\in\Gamma_\circ} C_{\tau_1,\tau_2}(A_i, r, s) = 1$, $1 - \vee_{i\in\Gamma_\circ} C_{\tau_1,\tau_2}(A_i, r, s) = 0$, then $I_1(1 - \vee_{i\in\Gamma_\circ} C_{\tau_1,\tau_2}(A_i, r, s)) \geq r$ and $I_2\left(1 - \vee_{i\in\Gamma_\circ} C_{\tau_1,\tau_2}(A_i, r, s)\right) \leq s$, thus $(X, \tau_1, \tau_2, I_1, I_2)$ is $(r,s) - IF(I_1, I_2)QHC$.

Theorem: 3.2

Let $(X, \tau_1, \tau_2, I_1, I_2)$ be a intuitionistic fuzzy ideals topological spaces, and $r \in I_\circ$, $s \in I_1$ then the following are equivalent:

(1)$(X, \tau_1, \tau_2, I_1, I_2)$ is $(r,s) - IF(I_1, I_2)$ compact.

(2) for any collection $\{A_i \in I^X : \tau_1(A_i) \geq r, \tau_2(A_i) \leq s, i \in \Gamma\}$, with $\wedge_{i\in\Gamma} A_i = 0$, there exists a finite subest $\Gamma_\circ \subseteq \Gamma$ with $I_1(\wedge_{i\in\Gamma} A_i) \geq r$, $I_2(\wedge_{i\in\Gamma} A_i) \leq s$.

Proof:

(1) → (2) Let $\{A_i \in I^X : \tau_1(A_i) \geq r, \tau_2(A_i) \leq s, i \in \Gamma\}$ be a family with $\wedge_{i\in\Gamma} A_i = 0$ then $\vee_{i\in\Gamma}(1 - A_i) = 1$, since $(X, \tau_1, \tau_2, I_1, I_2)$ is$(r,s) - IF(I_1, I_2)$ compact, there, there exists a finite subset $\Gamma_\circ \subseteq \Gamma$ such that $I_1(1 - \vee_{i\in\Gamma_\circ}(1 - A_i)) \geq r$ and $I_2(1 - \vee_{i\in\Gamma_\circ}(1 - A_i)) \leq s$, this implies that $I_1(\wedge_{i\in\Gamma_\circ} A_i) \geq r$, $I_2(\wedge_{i\in\Gamma_\circ} A_i) \leq s$.

(2) → (1)

Let $\{A_i \in I^X : \tau_1(A_i) \geq r, \tau_2(A_i) \leq s, i \in \Gamma\}$ be a family with $\vee_{i\in\Gamma} A_i = 1$, since $1 - \vee_{i\in\Gamma} A_i = 0$ then$\wedge_{i\in\Gamma}(1 - A_i) = 0$

by (2), then there exists a finite subset $\Gamma_\circ \subseteq \Gamma$ such that $I_1\left(\wedge_{i\in\Gamma_\circ}(1 - A_i)\right) \geq r$ and $I_2\left(\wedge_{i\in\Gamma_\circ}(1 - A_i)\right) \leq s$, this implies that $I_1(1 - \vee_{i\in\Gamma_\circ} A_i) \geq r$ and

$I_2(1 - \vee_{i\in\Gamma_\circ} A_i) \leq s$ therefore $(X, \tau_1, \tau_2, I_1, I_2)$ is $(r, s) - IF(I_1, I_2)$ compact .

Theorem: 3.3

Let $(X, \tau_1, \tau_2, I_1, I_2)$ be $(r, s) - IF(I_1, I_2)QHC$ and (r, s) − intuitionistic fuzzy regular then $(X, \tau_1, \tau_2, I_1, I_2)$ is $(r, s) - IF(I_1, I_2)$ compact .

Proof:

Let $\{A_i \in I^X : \tau_1(A_i) \geq r, \tau_2(A_i) \leq s, i \in \Gamma\}$ be a family with $\vee_{i\in\Gamma} A_i = 1$, since $(X, \tau_1, \tau_2, I_1, I_2)$ is (r, s) − intuitionistic fuzzy regular , for each $\tau_1(A_i) \geq r$, $\tau_2(A_i) \leq s$, $A_i = \vee \{B_j \in I^X : \tau_1(B_j) \geq r, \tau_2(B_j) \leq s, C_{\tau_1,\tau_2}(B_j, r, s) \leq A_i : j \in J\}$

hence $\vee_{i\in\Gamma} (\vee_{j\in J} B_j) = 1$,$1 - \vee_{i\in\Gamma} (\vee_{j\in J} B_j) \geq 1 - \vee_{i\in\Gamma} (\vee_{j\in J\subseteq\Gamma} C_{\tau_1,\tau_2}(B_j, r, s))$

since $(X, \tau_1, \tau_2, I_1, I_2)$ be $(r, s) - IF(I_1, I_2)QHC$, then there exists a finite subset $\Gamma_\circ \subseteq \Gamma$, $I_1\left(1 - \vee_{i\in\Gamma_\circ} \left(\vee_{j\in J\subseteq\Gamma_\circ} C_{\tau_1,\tau_2}(B_j, r, s)\right)\right) \geq r$, and

$I_2\left(1 - \vee_{i\in\Gamma_\circ} \left(\vee_{j\in J\subseteq\Gamma_\circ} C_{\tau_1,\tau_2}(B_j, r, s)\right)\right) \leq s$, for each $j \in J$

since $\vee_{j\in J\subseteq\Gamma_\circ} C_{\tau_1,\tau_2}(B_j, r, s) \leq\leq A_j$, it implies that

$\left(1 - \vee_{i\in\Gamma_\circ} \left(\vee_{j\in J\subseteq\Gamma_\circ} C_{\tau_1,\tau_2}(B_j, r, s)\right)\right) \geq 1 - \vee_{i\in\Gamma_\circ} A_j$, therefore

$I_1(1 - \vee_{j\in J\subseteq\Gamma_\circ} A_j) \geq r$ and $I_2(1 - \vee_{j\in J\subseteq\Gamma_\circ} A_j) \leq s$, thus $(X, \tau_1, \tau_2, I_1, I_2)$ is $(r, s) - IF$ (I_1, I_2) compact .

Definition: 3.8

A family $\{A_i\}_{i\in\Gamma}$ in X has the finite interscection property $((I_1, I_2) - FIP)$iff the intersection of no finite subfamily $\Gamma_\circ \subseteq \Gamma$ such that $I_1(\wedge_{i\in\Gamma_\circ} A_i) \geq r$ and $I_2(\wedge_{i\in\Gamma_\circ} A_i) \leq s$.

Theorem: 3.4

Let $(X, \tau_1, \tau_2, I_1, I_2)$ be intuitionistic fuzzy ideals topological spaces , is (r,s)-IF-(I_1, I_2) compact iff every collection $\{A_i \in I^X, \tau_1(1 - A_i) \geq r, \tau_2(1 - A_i) \leq s, i \in \Gamma\}$, A having the finite intersction proper $(I_1, I_2) - FIP$ has a non − empty intersction .

Proof:

Let an intuitionistic fuzzy ideals topological space $(X, \tau_1, \tau_2, I_1, I_2)$ be($(r, s) - IF$ (I_1, I_2)compact)and consider the family,

$\{A_i \in I^X, \tau_1(1-A_i) \geq r, \tau_2(1-A_i) \leq s, i \in \Gamma\}$ having the$(I_1, I_2) -$ FIP, and $\wedge_{i\in\Gamma} A_i = \underline{0}$, then $\vee_{i\in\Gamma} (1-A_i) = \underline{1}$, since $\{A_i \in I^X, \tau_1(1-A_i) \geq r , \tau_2(1-A_i) \leq s, i \in \Gamma\}$, is a collection of $(r,s) - IF(I_1, I_2)$ compact of X with $\vee_{i\in\Gamma} (1-A_i) = \underline{1}$, there exists a finite subset $\Gamma_\circ \subseteq \Gamma$ such that $I_1\left(1 - \vee_{i\in\Gamma_\circ} (1-A_i)\right) \geq r$ and $I_2\left(1 - \vee_{i\in\Gamma_\circ} (1-A_i)\right) \leq s$ this implies that then $I_1(\wedge_{i\in\Gamma_\circ} A_i) \geq r$ and $I_2(\wedge_{i\in\Gamma_\circ} A_i) \leq s$, this a contradiction $\wedge_{i\in\Gamma_\circ} A_i \neq \underline{0}$

$\Leftarrow$ Let $\{A_i \in I^X, \tau_1(A_i) \geq r, \tau_2(A_i) \leq s, i \in \Gamma\}$ be a family such that $\vee_{i\in\Gamma} (A_i) = \underline{1}$ $1 - \vee_{i\in\Gamma} (A_i) = \underline{0}$ then $\wedge_{i\in\Gamma_\circ} (1-A_i) = \underline{0}$ if $\vee_{i\in\Gamma} (A_i) \neq \underline{1}$, for every finite subset $\Gamma_\circ \subseteq \Gamma$ then $\wedge_{i\in\Gamma_\circ} (1-A_i) \neq \underline{0}$and the family $\{A_i \in I^X, \tau_1(1-A_i) \geq r, \tau_2(1-A_i) \leq s, i \in \Gamma\}$ has$(I_1, I_2) - FIP$, this contradiction there exists a finite subset $\Gamma_\circ \subseteq \Gamma$ such that $\{A_i \in I^X, \tau_1(A_i) \geq r, \tau_2(A_i) \leq s, i \in \Gamma\}$ is $(r,s) - IF(I_1, I_2)$ compact of X .

Theorem: 3.5

Let $(X, \tau_1, \tau_2, I_1, I_2)$ be intuitionistic fuzzy ideals topological spaces , and A is $(r,s) - IF(I_1, I_2)$ compact then for every collection $\{B_i \in I^X : B_i \leq I_{\tau_1,\tau_2}\left(C_{\tau_1,\tau_2}(B_i, r, s)\right) , i \in \Gamma\}$ with $A \leq \vee_{i\in\Gamma} B_i$, there exists a finite $\Gamma_\circ \subseteq \Gamma$, such that

$I_1\left(A \wedge \left[1 - \vee_{i\in\Gamma_\circ} I_{\tau_1,\tau_2}\left(C_{\tau_1,\tau_2}(B_i, r, s)\right)\right]\right) \geq r$ and

$I_2(A \wedge [1 - \vee_{i\in\Gamma_\circ} I_{\tau_1,\tau_2}(C_{\tau_1,\tau_2}(B_i, r, s), r, s)]) \leq s$.

Proof:

Let $\{B_i \in I^X : B_i \leq I_{\tau_1,\tau_2}\left(C_{\tau_1,\tau_2}(B_i, r, s)\right) , i \in \Gamma\}$ with $A \leq \vee_{i\in\Gamma} B_i$ then

$A \leq \vee_{i\in\Gamma} I_{\tau_1,\tau_2}(C_{\tau_1,\tau_2}(B_i, r, s), r, s)$, $\tau_1\left(I_{\tau_1,\tau_2}(C_{\tau_1,\tau_2}(B_i, r, s), r, s)\right) \geq r$,

$\tau_2\left(I_{\tau_1,\tau_2}(C_{\tau_1,\tau_2}(B_i, r, s), r, s)\right) \leq s$, since the $(r,s) - IF(I_1, I_2)$ compact of $A_i \in I^X$ there exists a finite $\Gamma_\circ \subseteq \Gamma$, $I_1(1 - \vee_{i\in\Gamma_\circ} B_i) \geq r$ and $I_2(1 - \vee_{i\in\Gamma_\circ} B_i) \leq s$, since $B_i \leq C_{\tau_1,\tau_2}(B_i, r, s)$ implies $B_i \leq I_{\tau_1,\tau_2}(C_{\tau_1,\tau_2}(B_i, r, s), r, s)$,

$1 - \vee_{i\in\Gamma_\circ} B_i \geq 1 - \vee_{i\in\Gamma_\circ} (I_{\tau_1,\tau_2}(C_{\tau_1,\tau_2}(B_i, r, s), r, s))$,

$A \wedge \left(1 - \vee_{i\in\Gamma_\circ} B_i\right) \geq A \wedge \left[1 - \vee_{i\in\Gamma_\circ} I_{\tau_1,\tau_2}\left(C_{\tau_1,\tau_2}(B_i, r, s)\right)\right]$ then ,

$I_1\left(A \wedge \left[1 - \vee_{i\in\Gamma_\circ} I_{\tau_1,\tau_2}\left(C_{\tau_1,\tau_2}(B_i, r, s)\right)\right]\right) \geq r$ and

$I_2(A \wedge [1 - \vee_{i\in\Gamma_\circ} I_{\tau_1,\tau_2}(C_{\tau_1,\tau_2}(B_i, r, s), r, s)]) \leq s$.

Theorem: 3.6

Let $(X, \tau_1, \tau_2, I_1, I_2)$ be a intuitionistic fuzzy ideals topological spaces and A_1, A_2 are $(r, s) - IF(I_1, I_2)$compact then $A_1 \vee A_2$ is $(r, s) - IF(I_1, I_2)$compact sub set relative to X .

Proof:

suppose that the family $\{B_i \in I^X : \tau_1(B_i) \geq r , \tau_2(B_i) \leq s , i \in \Gamma\}$, such that $A_1 \vee A_2 \leq \vee_{i\in\Gamma} B_i$, so $A_1 \leq \vee_{i\in\Gamma} B_i$ and $A_2 \leq \vee_{i\in\Gamma} B_i$, since A_1 and A_2 are $(r, s) - IF(I_1, I_2)$compact there exists a finite $\Gamma_\circ \subseteq \Gamma$ such that

$I_1(A_i \wedge [1 - \vee_{i\in\Gamma_\circ} B_i]) \geq r$, and $I_2\left(A_i \wedge \left[1 - \vee_{i\in\Gamma'_\circ} B_i\right]\right) \leq s$, $i = 1,2$

since $A_1 \wedge \left(1 - \vee_{i\in\Gamma_\circ} B_i\right) \vee A_2 \wedge \left(1 - \vee_{i\in\Gamma_\circ} B_i\right) = (A_1 \vee A_2) \wedge \left(1 - \vee_{i\in\Gamma_\circ} B_i\right)$

therefore $I_1\left[(A_1 \vee A_2) \wedge \left(1 - \vee_{i\in\Gamma_\circ} B_i\right)\right] \geq r$ and $I_2\left[(A_1 \vee A_2) \wedge \left(1 - \vee_{i\in\Gamma_\circ} B_i\right)\right] \leq s$.

Theorem: 3.7

Let $(X, \tau_1, \tau_2, I_1, I_1)$ be a intuitionistic fuzzy ideals topological spaces , and $r \in I_\circ$, $s \in I_1$ then the following are equivalent :

(1)$(X, \tau_1, \tau_2, I_1, I_1)$ is $(r, s) - IF(I_1, I_1)QHC$.

(2) For any collection $\{A_i \in I^X : \tau_1(1 - A_i) \geq r , \tau_2(1 - A_i) \leq s , i \in \Gamma\}$, with $\wedge_{i\in\Gamma} A_i = \underline{0}$, there exists a finite$\Gamma_\circ \subseteq \Gamma$ such that $I_1\left(\wedge_{i\in\Gamma_\circ} I_{\tau_1,\tau_2}(A_i, r, s)\right) \geq r$ and $I_2\left(\wedge_{i\in\Gamma_\circ} I_{\tau_1,\tau_2}(A_i, r, s)\right) \leq s$.

(3) $\wedge_{i\in\Gamma} A_i \neq \underline{0}$, holds for any collection $\{ A_i \in I^X : \tau_1(1 - A_i) \geq r$ $\tau_2(1 - A_i) \leq s : i \in \Gamma\}$such that $\left\{I_{\tau_1,\tau_2}(A_i, r, s) : \tau_1(1 - A_i) \geq r , \tau_2(1 - A_i), i \in \Gamma\right\}$ has the $(I_1, I_2) - FIP$.

(4) For any collection $\{A_i \in I^X : A_i$ is $(r, s) - FRO$ sets $i \in \Gamma\}$ such that $\vee_{i\in\Gamma} A_i = \underline{1}$, there exists a finite subsets $\Gamma_\circ \subseteq \Gamma$ such that

$I_1\left(1 - \vee_{i\in\Gamma_\circ} C_{\tau_1,\tau_2}(A_i, r, s)\right) \geq r$ and $I_1\left(1 - \vee_{i\in\Gamma_\circ} C_{\tau_1,\tau_2}(A_i, r, s)\right) \leq s$

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(5) For every collection $\{A_i \in I^X : A_i \text{ is } (r,s) - \text{FRC sets } i \in \Gamma\}$ such that $\wedge_{i\in\Gamma} A_i = \underline{0}$, there exists a finite subsets $\Gamma_\circ \subseteq \Gamma$ such that $I_1\left(\vee_{i\in\Gamma_\circ} I_{\tau_1,\tau_2}(A_i, r, s)\right) \geq r$ and $I_2\left(\vee_{i\in\Gamma_\circ} I_{\tau_1,\tau_2}(A_i, r, s)\right) \leq s$.

(6) $\wedge_{i\in\Gamma} A_i \neq \underline{0}$, for every collection $\{A_i \in I^X : A_i \text{ is } (r,s) - \text{FRC sets } i \in \Gamma\}$ such that $\{ I_{\tau_1,\tau_2}(A_i, r, s) : A_i \text{ is } (r,s) - \text{FRC sets } i \in \Gamma \}$ has the $(I_1, I_2) - \text{FIP}$.

Proof:

(1) ⇒ (2) Let $\{A_i \in I^X : \tau_1(1 - A_i) \geq r , \tau_2(1 - A_i) \leq s , i \in \Gamma\}$ be a family with $\wedge_{i\in\Gamma} A_i = \underline{0}$, then $\vee_{i\in\Gamma} (1 - A_i) = \underline{1}$,

Since $(X, \tau_1, \tau_2, I_1, I_2)$ is $(r,s) - \text{IF}(I_1, I_2)\text{QHC}$ there exists a finite $\Gamma_\circ \subseteq \Gamma$ such that $I_1\left(1 - \vee_{i\in\Gamma_\circ}\left(C_{\tau_1,\tau_2}(1 - A_i, r, s)\right)\right) \geq r$ and

$I_2\left(1 - \vee_{i\in\Gamma_\circ}\left(C_{\tau_1,\tau_2}(1 - A_i, r, s)\right)\right) \leq s$,

Since $1 - \vee_{i\in\Gamma_\circ}(C_{\tau_1,\tau_2}(1 - A_i, r, s)) = \wedge_{i\in\Gamma_\circ} I_{\tau_1,\tau_2}(A_i, r, s)$,

therefore $I_1(\wedge_{i\in\Gamma_\circ} I_{\tau_1,\tau_2}(A_i, r, s)) \geq r$, $I_2\left(\wedge_{i\in\Gamma_\circ} I_{\tau_1,\tau_2}(A_i, r, s)\right) \leq s$.

(2) ⇒ (1)

Let $\{A_i \in I^X : \tau_1(A_i) \geq r , \tau_2(A_i) \leq s , i \in \Gamma\}$ be a family such that $\vee_{i\in\Gamma} A_i = \underline{1}$ then $\vee_{i\in\Gamma} (1 - A_i) = \underline{0}$, and by (2) there exists a finite subset $\Gamma_\circ \subseteq \Gamma$ such

that $I_1\left(\wedge_{i\in\Gamma_\circ} I_{\tau_1,\tau_2}(1 - A_i, r, s)\right) \geq r$ and $I_2\left(\wedge_{i\in\Gamma_\circ} I_{\tau_1,\tau_2}(1 - A_i, r, s)\right) \leq s$,

Since $\wedge_{i\in\Gamma_\circ} I_{\tau_1,\tau_2}(1 - A_i, r, s) = 1 - \vee_{i\in\Gamma_\circ} C_{\tau_1,\tau_2}(A_i, r, s)$,

$I_1\left(1 - \vee_{i\in\Gamma_\circ} C_{\tau_1,\tau_2}(A_i, r, s)\right) \geq r$ and $I_2(1 - \vee_{i\in\Gamma_\circ} C_{\tau_1,\tau_2}(A_i, r, s)) \leq s$,

hence $(X, \tau_1, \tau_2, I_1, I_2)$ is $(r,s) - \text{IF}(I, I^*)\text{QHC}$.

(1) ⇒ (3)

Let $\{A_i \in I^X : \tau_1(1 - A_i) \geq r , \tau_2(1 - A_i) \leq s , i \in \Gamma\}$ be a family with

$\left\{I_{\tau_1,\tau_2}(A_i, r, s) : \tau_1(1 - A_i) \geq r , \tau_2(1 - A_i) \leq s, i \in \Gamma \right\}$ has the $(I_1, I_2) - \text{FIP}$, i

$\wedge_{i\in\Gamma} A_i = \underline{0}$ then $\vee_{i\in\Gamma} (1 - A_i) = \underline{1}$,

since $(X, \tau_1, \tau_2, I_1, I_2)$ is $(r, s) - IF(I_1, I_2)$ QHC, there exists a finite index $\Gamma_\circ \subseteq \Gamma$ such that $I_1\left(1 - \vee_{i\in\Gamma_\circ} C_{\tau_1,\tau_2}(1 - A_i, r, s)\right) \geq r$ and

$I_2\left(1 - \vee_{i\in\Gamma_\circ} C_{\tau_1,\tau_2}(1 - A_i, r, s)\right) \leq s$

since $1 - \vee_{i\in\Gamma_\circ} C_{\tau_1,\tau_2}(1 - A_i, r, s) = \wedge_{i\in\Gamma_\circ} I_{\tau_1,\tau_2}(A_i, r, s)$

then $I_1\left(\wedge_{i\in\Gamma_\circ} I_{\tau_1,\tau_2}(A_i, r, s)\right) \geq r$ and $I_2\left(\wedge_{i\in\Gamma_\circ} I_{\tau_1,\tau_2}(A_i, r, s)\right) \leq s$

it is contradiction $\wedge_{i\in\Gamma_\circ} A_i \neq \underline{0}$.

$(3) \Rightarrow (1)$

Let $\{A_i \in I^X : \tau_1(A_i) \geq r, \tau_2(A_i) \leq s, i \in \Gamma\}$ be a family , such that $\vee_{i\in\Gamma} A_i = \underline{1}$ there exists a index finite $\Gamma_\circ \subseteq \Gamma$ such that $I_1(1 - \vee_{i\in\Gamma_\circ} (C_{\tau_1,\tau_2}(A_i, r, s))) \geq r$ and $I_2(1 - \vee_{i\in\Gamma_\circ} (C_{\tau_1,\tau_2}(A_i, r, s))) \leq s$,

since $1 - \vee_{i\in\Gamma_\circ} C_{\tau_1,\tau_2}(A_i, r, s) = \wedge_{i\in\Gamma_\circ} I_{\tau_1,\tau_2}(1 - A_i, r, s)$, the family $\{ I_{\tau_1,\tau_2}(1 - A_i, r, s) : \tau_1(A_i) \geq r, \tau_2(A_i) \leq s : i \in \Gamma \}$ has the $(I_1, I_2) - FIP$ by (3) $\wedge_{i\in\Gamma_\circ} (1 - A_i) \neq \underline{0}$, this implies $\vee_{i\in\Gamma_\circ} (A_i) \neq \underline{1}$, it is a contradiction .

$(1) \Rightarrow (4)$

Let $\{A_i \in I^X : i \in \Gamma\}$ be a family of $(r, s) - IFRO$ sets with $\vee_{i\in\Gamma} A_i = \underline{1}$, then

$\vee_{i\in\Gamma} I_{\tau_1,\tau_2}\left(C_{\tau_1,\tau_2}(A_i, r, s)\right) = \underline{1}$, and $\tau_1(I_{\tau_1,\tau_2}\left(C_{\tau_1,\tau_2}(A_i, r, s)\right)) \geq r$,

$\tau_2\left(I_{\tau_1,\tau_2}\left(C_{\tau_1,\tau_2}(A_i, r, s)\right)\right) \leq s$,

Since $(X, \tau_1, \tau_2, I_1, I_2)$ is $(r, s) - IF(I_1, I_2)QHC$, then there exists a finite $\Gamma_\circ \subseteq \Gamma$ such that $I_1(1 - \vee_{i\in\Gamma_\circ} C_{\tau_1,\tau_2}(I_{\tau_1,\tau_2}(C_{\tau_1,\tau_2}(A_i, r, s), r, s))) \geq r$ and

$I_2(1 - \vee_{i\in\Gamma_\circ} C_{\tau_1,\tau_2}(I_{\tau_1,\tau_2}(C_{\tau_1,\tau_2}(A_i, r, s), r, s))) \leq s$, since for each $\tau_1(A_i) \geq r$, $\tau_2(A_i) \leq s$, $C_{\tau_1,\tau_2}(A_i, r, s) = C_{\tau_1,\tau_2}(I_{\tau_1,\tau_2}(C_{\tau_1,\tau_2}(A_i, r, s), r, s))$, therefore $I_1(1-\vee_{i\in\Gamma_\circ}\left(C_{\tau_1,\tau_2}(A_i,r,s)\right)) \geq r$ and $I_2(1-\vee_{i\in\Gamma_\circ}\left(C_{\tau_1,\tau_2}(A_i,r,s)\right)) \leq s$.

(4) ⇒ (5)

Let $\{A_i \in I^X : i \in \Gamma\}$ be a family of $(r,s)-$ IFRC sets with $\wedge_{i\in\Gamma} A_i = \underline{0}$, then $\vee_{i\in\Gamma}\ (1-A_i) = \underline{1}$ and $\{1-A_i \in I^X : i \in \Gamma\}$ be a family of $(r,s)-$ IFRO sets, by (4) there exists a finite subset $\Gamma_\circ \subseteq \Gamma$ such that

$I_1(1-\vee_{i\in\Gamma_\circ}\left(C_{\tau_1,\tau_2}(1-A_i,r,s)\right) \geq r$ and $I_2(1-\vee_{i\in\Gamma_\circ}\left(C_{\tau_1,\tau_2}(1-A_i,r,s)\right) \leq s$,

since $1-\vee_{i\in\Gamma_\circ}\left(C_{\tau_1,\tau_2}(1-A_i,r,s)\right) = \wedge_{i\in\Gamma_\circ} I_{\tau_1,\tau_2}(A_i,r,s)$, therefore

$I_1\left(\wedge_{i\in\Gamma_\circ} I_{\tau_1,\tau_2}(A_i,r,s)\right) \geq r$ and $I_2\left(\wedge_{i\in\Gamma_\circ} I_{\tau_1,\tau_2}(A_i,r,s)\right) \leq s$.

(5) ⇒ (1)

Let $\{A_i \in I^X : \tau_1(A_i) \geq r\ ,\ \tau_2(A_i) \leq s\ ,\ i \in \Gamma\}$ be a family , such that $\vee_{i\in\Gamma} A_i = \underline{1}$

then $\vee_{i\in\Gamma}\left(I_{\tau_1,\tau_2}(C_{\tau_1,\tau_2}(A_i,r,s),r,s)\right) = \underline{1}$,

thus $\wedge_{i\in\Gamma}\left(C_{\tau_1,\tau_2}(I_{\tau_1,\tau_2}(1-A_i,r,s),r,s)\right) = \underline{0}$

and $\left(C_{\tau_1,\tau_2}(I_{\tau_1,\tau_2}(A_i,r,s),r,s)\right)$ is $(r,s)-$ IFRC for the hypothesis , there exists a finite subset $\Gamma_\circ \subseteq \Gamma$ such that $I_1(\wedge_{i\in\Gamma_\circ} I_{\tau_1,\tau_2}\big(C_{\tau_1,\tau_2}\big(I_{\tau_1,\tau_2}(A_i,r,s),r,s\big),r,s\big) \geq r$ and $I_2\left(\wedge_{i\in\Gamma_\circ} I_{\tau_1,\tau_2}\big(C_{\tau_1,\tau_2}(I_{\tau_1,\tau_2}((1-A_i,r,s),r,s),r,s)\right) \leq s$, since $\tau_1(A_i) \geq r$ since $\tau_1(A_i) \geq r$ and $\tau_2(A_i) \leq s$,

$C_{\tau_1,\tau_2}(A_i,r,s) = C_{\tau_1,\tau_2}\big(I_{\tau_1,\tau_2}(C_{\tau_1,\tau_2}((A_i,r,s),r,s),r,s)\big)$

and hence $\wedge_{i\in\Gamma_\circ} I_{\tau_1,\tau_2}\big(C_{\tau_1,\tau_2}(I_{\tau_1,\tau_2}((1-A_i,r,s),r,s),r,s\big)$

$= 1-\vee_{i\in\Gamma_\circ} C_{\tau_1,\tau_2}\big(I_{\tau_1,\tau_2}(C_{\tau_1,\tau_2}((1-A_i,r,s),r,s),r,s\big) = 1-\vee_{i\in\Gamma_\circ} C_{\tau_1,\tau_2}(A_i,r,s)$

then $I_1\left(1-\vee_{i\in\Gamma_\circ} C_{\tau_1,\tau_2}(A_i,r,s)\right)\geq r$ and $I_2\left(1-\vee_{i\in\Gamma_\circ} C_{\tau_1,\tau_2}(A_i,r,s)\right)$

$\leq s$, therefore

$(X,\tau_1,\tau_2,I_1,I_2)$ is $(r,s)-IF(I_1,I_2)QHC$.

$(6)\Rightarrow(4)$

Let $\{A_i\in I^X : i\in\Gamma\}$ be a family of $(r,s)-$ IFRO sets with $\vee_{i\in\Gamma} A_i=\underline{1}$, there exists

a finite subset $\Gamma_\circ\subseteq\Gamma$ such that $I_1\left(1-\vee_{i\in\Gamma_\circ} C_{\tau_1,\tau_2}(A_i,r,s)\right)\geq r$ and

$I_2\left(1-\vee_{i\in\Gamma_\circ} C_{\tau_1,\tau_2}(A_i,r,s)\right)\leq s$,

Since $1-\vee_{i\in\Gamma_\circ} C_{\tau_1,\tau_2}(A_i,r,s)=\wedge_{i\in\Gamma_\circ} I_{\tau_1,\tau_2}(1-A_i,r,s)$ and by (6) the family $\{A_i\in I^X : (1-A_i)$ is $(r,s)-$ IFRC sets , $i\in\Gamma\}$ such that $\{I_{\tau_1,\tau_2}(1-A_i,r,s), 1-A_i$ is $(r,s)-$ IFRC sets , $i\in\Gamma\}$ has the $(I_1,I_1)-$ FIP , so $\wedge_{i\in\Gamma_\circ}(1-A_i)\neq\underline{0}$ this implies $\vee_{i\in\Gamma_\circ} A_i\neq\underline{1}$, this is a contradiction .

$(4)\Rightarrow(6)$

Let $\{A_i\in I^X : A_i$ is $(r,s)-$ IFRC sets, $i\in\Gamma\}$ be a family such that $\{I_{\tau_1,\tau_2}(A_i,r,s) : A_i$ is $(r,s)-$ IFRC sets , $i\in\Gamma\}$ has the $(I_1,I_2)-$ FIP,

If $\wedge_{i\in\Gamma}(A_i)=\underline{0}$, then $\vee_{i\in\Gamma}(1-A_i)=\underline{1}$ By (4) , there exists a finite subset $\Gamma_\circ\subseteq\Gamma$ such that , $I_1\left(1-\vee_{i\in\Gamma_\circ} C_{\tau_1,\tau_2}(1-A_i,r,s)\right)\geq r$

and $I_2\left(1-\vee_{i\in\Gamma_\circ} C_{\tau_1,\tau_2}(1-A_i,r,s)\right)\leq s$,

since $1-\vee_{i\in\Gamma_\circ} C_{\tau_1,\tau_2}(1-A_i,r,s)=\wedge_{i\in\Gamma_\circ} I_{\tau_1,\tau_2}(A_i,r,s)$ then $I_1\left(\wedge_{i\in\Gamma_\circ} I_{\tau_1,\tau_2}(A_i,r,s)\right)\geq r$ and $I_2\left(\wedge_{i\in\Gamma_\circ} I_{\tau_1,\tau_2}(A_i,r,s)\right)\leq s$, this is contradiction $\wedge_{i\in\Gamma_\circ}(A_i)\neq\underline{0}$.

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