Iraqi Journal of Statistical Sciences, Vol. 21, No. 2, 2024, pp (82-94)

Iraqi Journal of Statistical Sciences

www.stats.mosuljournals.com

Improved Ratio-Cum Regression Estinator Using Two Auxiliary Variables In Single Phase Sampling

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Article information

Article history:
Received June 1, 2024
Accepted August 20, 2024
Available online December 1, 2024

Keywords:
Ratio-Cum Regression, Regression, Single Phase Sampling , Auxiliary Variables

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Abstract

Auxiliary information has been confirmed to enhance precision in the estimators of ratio, regression and product respectively. Many cases of improved mixed estimators in single phase sampling have been advanced and recommendations made using more than one auxiliary variable and correct factors for extreme values. This study takes a look at case of extreme value in both study and auxiliary variable where the proposed mixed estimator is not corrected for extreme values in both study and auxiliary variable. However, it is interesting to know that the developed mixed estimator is efficient over developed single estimators of ratio and regression with correction factors for extreme value.

DOI: 10.33899/iqjoss.2024.185243 , ©Authors, 2024, College of Computer Science and Mathematics, University of Mosul.

1. Introduction

([Moha67]) has incorporated the use of more than one estimator. This is what is now known as mixed estimation method in single phase sampling. The use of mixed estimation over the years has been confirmed to enhance the efficiency of any estimator by ([OGUNS19]). Some notable authors that have use this method include ([SINNS67a]) and ([SINNS67b]), ([SSS78]) and ([TSS09]). This study is extending the work of ([KS13]) into mixed estimation (Ratio-cum-regression) by combining ([KS13]) improved ratio and regression estimators in the order of ([KC05]). The proposed estimator shall assume that both the study and the auxiliary variables have no extreme value in their distributions. This proposed estimator shall be called NEV. This study shall test the performance of NEV theoretically, empirically and using percentage relative efficiency against the improved ratio estimator of ([KS13]), the improved ratio estimator of ([AK14]) and the improved regression estimator of ([AK14])

1. METHODOLOGY

2.0. All symbol used in this article have been clearly defined in Appendix A.

2.1 Review on ([*S72*]) correction factor

([*S72*]) has advanced solution to extreme value by introducing a correction constant c such that if there exists extreme large value in a distribution and $\bar{y}_{max}$ is the sample mean using Simple Random Sampling without Replacement (SRSWOR), then c will be subtracted from $\bar{y}_{max}$ to obtain the corrected mean. This is stated as

$$\bar{y}_1 = \bar{y}_{max} - c \tag{1}$$

Likewise, if there exists extreme low value in a distribution and $\bar{y}_{min}$ is the sample mean with SRSWOR, then c will be added to $\bar{y}_{min}$ to obtain the corrected mean. This is stated as

$$\bar{y}_1 = \bar{y}_{min} + c \quad (2)$$

This can be written in a compressed form as

$$\bar{y}_1 = \begin{cases} \bar{y} + c \text{ if samples contains } y_{min} \text{ but not } y_{max} \\ \bar{y} - c \text{ if samples contains } y_{max} \text{ but not } y_{min} \\ \bar{y} \quad \text{for all other samples} \end{cases} \quad (3)$$

c is the correction constant. The minimum variance of $\bar{y}_1$ up to first order of approximation is given as

$$Var(\bar{y}_1)_{min} - \frac{\lambda \Delta^2 y}{2(N-1)} \quad (4)$$

where $\Delta_y = (y_{max-}y_{min})$ and the optimum value of c is given as

$$c_{opt} = \frac{\Delta y}{2(N-1)} \quad (5)$$

2.2 Review on ([*KC05*])

([*KC05*]) has advanced an estimator, which was derived from the combination of the regression estimate of $\bar{Y}$ and the estimator of ([*A-D03*]). **([*KC05*])** estimator is given as

$$\bar{y}_2 = \bar{y}\left(\frac{\bar{X}_1}{\bar{x}_1}\right)^{\alpha_1}\left(\frac{\bar{X}_2}{\bar{x}_2}\right)^{\alpha_1} + b_1(\bar{X}_1 - \bar{x}_1) + b_2(\bar{X}_2 - \bar{x}_2) \quad (6)$$

where α_1 and α_2 were real numbers and $b_1 = \frac{S_{yx_1}}{S^2_{x_1}}$, $b_2 = \frac{S_{yx_2}}{S^2_{x_2}}$. Here, $S^2_{x_1}$ and $S^2_{x_2}$ are the sample variance of x_1 *and* x_2 and S_{yx_1}, S_{yx_2} are the sample covariances between y and x_1 and between y and x_2 respectively. The MSE is given as:

$$MSE_{min}(\bar{y}_2) \cong \lambda s^2_y\left[1 + c^2_1 + c^2_2 + 2c_1c_2\rho_{\bar{x}_1\bar{x}_2} - 2c_1\rho_{y\bar{x}_1} - 2c_2\rho_{y\bar{x}_2}\right] \quad (7)$$

2.3 Review on ([*KS13*]) ratio estimator

([*KS13*]) has proposed an improved ratio estimator using one auxiliary variable with extreme value. The estimator is given as

$$\bar{y}_3 = \frac{\bar{y}_{c_{11}}}{\bar{x}_{c_{21}}}\bar{X}. \quad (8)$$

The corresponding MSE is given as:

$$MSE(\bar{y}_3)_{opt} \cong M(\bar{y}_R) - \frac{\lambda(\Delta y - R\Delta x)^2}{2(N-1)}, \quad (9)$$

where $M(\bar{y}_R) \cong \bar{Y}^2\lambda\left(C^2_yC^2_x - 2\rho_{yx}C_yC_x\right)$, is the mean square error of the conventional ratio estimator.

2.4 Review on ([*KS13*]), regression estimator

([*KS13*]) has proposed an improved regression estimator using one auxiliary variable with extreme value. The estimator is given as

$$\bar{y}_4 = \bar{y}_{c_{11}} + b\,(\bar{X} - \bar{x}_{c_{21}}), \quad (10)$$

with the corresponding MSE as

$$V(\bar{y}_4)_{opt} \cong M(\bar{y}_{lr}) - \frac{\lambda(\Delta y - \beta\Delta x)^2}{2(N-1)}, \quad (11)$$

where $M(\bar{y}_{lr}) = \lambda S^2_y\left(1 - \rho^2_{yx}\right)$ and b is the sample regression coefficient.

2.5 Review on ([*AK14*]) ratio estimators

([AK14]) has proposed an improved ratio estimator using two auxiliary variables with extreme value. The estimator is given as

$$\bar{y}_5 = \bar{y}_{c_{11}}\left(\frac{\bar{X}_1}{\bar{x}_{1_{c_{21}}}}\right)\left(\frac{\bar{X}_2}{\bar{x}_{2_{c_{31}}}}\right). \quad (12)$$

The corresponding MSE is presented as

$$MSE(\bar{y}_5)_{opt} \cong M(\bar{y}_{R2}) - \frac{\lambda(\Delta y - R_1\Delta x_1 - R_2\Delta x_2)^2}{2(N-1)}, \quad (13)$$

where $M(\bar{y}_{R2}) = \lambda\left(S^2_y + R^2_1S^2_{x_1} + R^2_2S^2_{x_2} + 2R_1R_2S_{x_1x_2} - 2R_2S_{yx_2} - 2R_1S_{yx_1}\right)$.

2.6 Review on ([*AK14*]) regression estimators

([AK14]) has proposed an improved regression estimator using two auxiliary variables with extreme value. The improved regression estimator of **([*AK14*])** is given as

$$\bar{y}_6 = \bar{y}_{c_{11}} + b_1\left(\bar{X} - \bar{x}_{1_{c_{21}}}\right) + b_2\left(\bar{X} - \bar{x}_{2_{c_{31}}}\right). \quad (14)$$

The corresponding MSE given as

$$MSE(\bar{y}_6)_{opt} \cong M(\bar{y}_{lr}) - \frac{\lambda(\Delta y-\beta_1\Delta x_1-\beta_2\Delta x_2)^2}{2(N-1)} \quad (15)$$

where $M(\bar{y}_{lr}) = \lambda S_y^2\left(1-\rho_{yx_1}^2-\rho_{yx_2}^2+2\rho_{yx_1}\rho_{yx_2}\rho_{x_1x_2}\right)$. Similarly,

$\beta_1 = \rho_{yx_1}\frac{S_y}{S_{x_1}}$ *and* $\beta_2 = \rho_{yx_2}\frac{S_y}{S_{x_1}}$ are the population regression coefficient between y and x_1 and between y and x_2.

2. Proposed Mixed Estimator (NEV)

This study has extended the ratio and regression estimators of ([*KS13*]) into mixed estimation without correction for extreme values. It has also extended the number of auxiliary variables from one to two. The proposed mixed estimator and the reviewed estimators were tested theoretically, empirically and with the use of percentage relative efficiency analysis under High maximum Extreme values and Low minimum Extreme values. The correction factor of ([*S72*]) is used only were necessary.

The proposed estimator (NEV) is presented as:

$$\bar{y}_{st} = \left(\frac{\bar{y}}{\bar{x}_1}\right)\bar{X}_1 + b(\bar{X}_2-\bar{x}_2) \quad (16)$$

The relative error terms are defined as

$$\left.\begin{aligned} \varepsilon_0 &= \frac{\bar{y}-\bar{Y}}{\bar{Y}} \Rightarrow \bar{y} = \bar{Y}(1+\varepsilon_0) \\ \varepsilon_1 &= \frac{\bar{x}_1-\bar{X}_1}{\bar{X}_1} \Rightarrow \bar{x}_1 = \bar{X}_1(1+\varepsilon_1) \\ \varepsilon_2 &= \frac{\bar{x}_2-\bar{X}_1}{\bar{X}_2} \Rightarrow \bar{x}_2 = \bar{X}_2(1+\varepsilon_2) \end{aligned}\right\}, \quad (17)$$

such that

$$E(\varepsilon_0)=E(\varepsilon_1)=E(\varepsilon_2)=0,\ E(\varepsilon_0^2)=E\left[\frac{\bar{y}-\bar{Y}}{\bar{Y}}\right]^2=\frac{(\bar{y}-\bar{Y})^2}{\bar{Y}^2}$$

$$\left.\begin{aligned} E(\varepsilon_0^2) &= \frac{Var(\bar{y})}{\bar{Y}^2} = \frac{\lambda}{\bar{Y}^2}S_y^2 \\ Similarly,\ E(\varepsilon_1^2) &= \frac{Var(\bar{x}_1)}{\bar{X}_1} = \frac{\lambda}{\bar{X}_1^2}S_{x_1}^2 \\ E(\varepsilon_2^2) &= \frac{Var(\bar{x}_2)}{\bar{X}_2} = \frac{\lambda}{\bar{X}_2^2}S_{x_2}^2 \end{aligned}\right\} \quad (18)$$

$$E(\varepsilon_0\varepsilon_1) = E\left[\frac{\bar{y}-\bar{Y}}{\bar{Y}}\right]\left[\frac{\bar{x}_1-\bar{X}_1}{\bar{X}_1}\right]$$

This implies that

$$\left.\begin{aligned} E(\varepsilon_0\varepsilon_1) &= \frac{E(\bar{y}-\bar{Y})(\bar{x}_1-\bar{X}_1)}{\bar{Y}\bar{X}_1} = \frac{\lambda}{\bar{Y}\bar{X}_1}S_{yx_1} \\ Similarly,\ E(\varepsilon_0\varepsilon_2) &= \frac{E(\bar{y}-\bar{Y})(\bar{x}_2-\bar{X}_2)}{\bar{Y}\bar{X}_2} = \frac{\lambda}{\bar{Y}\bar{X}_2}S_{yx_2} \\ E(\varepsilon_1\varepsilon_2) &= \frac{E(\bar{x}_1-\bar{X}_1)(\bar{x}_2-\bar{X}_2)}{\bar{X}_1\bar{X}_2} = \frac{\lambda}{\bar{X}_1\bar{X}_2}S_{x_1x_2} \end{aligned}\right\} \quad (19)$$

Substituting equation (17) into equation (16), gives

$$\bar{y}_{st} = \frac{\bar{Y}\,(1+\varepsilon_0\,)\bar{X}_1}{\bar{X}_1\,(1+\varepsilon_1\,)} + b\left[\bar{X}_2 - \left(\bar{X}_2\,(1+\varepsilon_2\,)\right)\right]$$

$$\bar{y}_{st} = \bar{Y}(1+\varepsilon_0\,)(1+\varepsilon_1)^{-1} - b\bar{X}_2\varepsilon_2$$

Applying Taylor series, and expanding $(1+\varepsilon_1)^{-1}$ up to 2nd order of degree

$$\bar{y}_{st} = \bar{Y}(1-\varepsilon_1+\varepsilon_1^2+\varepsilon_0-\varepsilon_0\varepsilon_1) - \mathrm{b}\varepsilon_2\bar{X}_2$$

$$Bias(\bar{y}_{st}) = E(\bar{y}_{st} - \bar{Y})$$

$$But \quad \bar{y}_{st} - \bar{Y} = \bar{Y}(1-\varepsilon_1+\varepsilon_1^2+\varepsilon_0-\varepsilon_0\varepsilon_1) - b\varepsilon_2\bar{X}_2 - \bar{Y}$$

$$\bar{y}_{st} - \bar{Y} = \bar{Y}(\varepsilon_0-\varepsilon_1+\varepsilon_1^2-\varepsilon_0\varepsilon_1) - b\varepsilon_2\bar{X}_2$$

$$E(\bar{y}_{st} - \bar{Y}) = E(\bar{Y}(\varepsilon_0-\varepsilon_1+\varepsilon_1^2-\varepsilon_0\varepsilon_1) - \mathrm{b}\varepsilon_2\bar{X}_2) \tag{20}$$

Substituting equation (19) into equation (20), gives

$$E(\bar{y}_{st} - \bar{\mathrm{Y}}) = \bar{Y}\left[\frac{\lambda}{\bar{X}_1^2}\,S_{x_1}^2 - \frac{\lambda}{\bar{Y}\bar{X}_1}S_{yx_1}\right]$$

since $E(\varepsilon_2) = 0$, this implies that

$$E(\bar{y}_{st} - \bar{\mathrm{Y}}\,) = \frac{\bar{Y}\lambda}{\bar{X}_1^2}\,S_{x_1}^2 - \frac{\bar{Y}\lambda}{\bar{Y}\bar{X}_1}S_{yx_1}$$

$$E(\bar{y}_{st} - \bar{\mathrm{Y}}\,) = \frac{\bar{Y}^2\lambda}{\bar{Y}\bar{X}_1^2}\,S_{x_1}^2 - \frac{R_1\lambda}{\bar{Y}}S_{yx_1}$$

$$E(\bar{y}_{st} - \bar{\mathrm{Y}}\,) = \frac{R_1^2\lambda}{\bar{Y}}\,S_{x_1}^2 - \frac{R_1\lambda}{\bar{Y}}S_{yx_1}$$

This implies that

$$Bias(\bar{y}_{st}) = \frac{R_1\lambda}{\bar{Y}}\left[R_1 S_{x_1}^2 - S_{yx_1}\right] \tag{21}$$

$$MSE(\bar{y}_{st}) = E(\bar{y}_{st} - \bar{\mathrm{Y}})^2$$

$$MSE(\bar{y}_{st}) = E(\bar{Y}(\varepsilon_0 - \varepsilon_1 + \varepsilon_1^2 - \varepsilon_0\varepsilon_1) - \mathrm{b}\varepsilon_2\bar{X}_2)^2$$

$$E(\bar{Y}^2\varepsilon_0^2 + \bar{Y}^2\varepsilon_1^2 - 2\bar{Y}^2\varepsilon_0\varepsilon_1 + b^2\varepsilon_2^2\bar{X}_2^2 - 2b\bar{X}_2\bar{\mathrm{Y}}\varepsilon_0\varepsilon_2 + 2b\bar{X}_2\bar{\mathrm{Y}}\varepsilon_1\varepsilon_2)$$

$$\Rightarrow E[\bar{Y}^2(\varepsilon_0^2 + \bar{Y}^2\varepsilon_1^2 - 2\bar{Y}^2\varepsilon_0\varepsilon_1) + b^2\varepsilon_2^2\bar{X}_2^2 - 2b\bar{X}_2\bar{\mathrm{Y}}(\varepsilon_0\varepsilon_2 + \varepsilon_1\varepsilon_2)]$$

Applying expectation,

$$MSE(\bar{y}_{st}) = \bar{Y}^2\left[\frac{\lambda}{\bar{Y}^2}S_y^2 + \frac{\lambda}{\bar{X}_1^2}\,S_{x_1}^2 - 2\frac{\lambda}{\bar{Y}\bar{X}_1}S_{yx_1}\right] + \left[-2b\bar{X}_2\bar{\mathrm{Y}}\left(\frac{\lambda}{\bar{Y}\bar{X}_2}S_{yx_2} - \frac{\lambda}{\bar{X}_1\bar{X}_2}S_{x_1x_2}\right)\right] + b^2\bar{X}_2^2\frac{\lambda}{\bar{X}_2^2}\,S_{x_2}^2$$

$$MSE(\bar{y}_{st}) = \lambda\left[S_y^2 + R_1^2S_{x_1}^2 - 2R_1S_{yx_1}\right] - 2b\lambda S_{yx_2} + 2bR_1\lambda S_{x_1x_2} + b^2\lambda S_{x_2}^2$$

$$MSE(\bar{y}_{st}) \cong \lambda\left[S_y^2 + R_1^2S_{x_1}^2 - 2R_1S_{yx_1} - 2bS_{yx_2} + 2bR_1S_{x_1x_2} + b^2S_{x_2}^2\right] \qquad (22)$$

To obtain the b_{opt} , differentiate equation (22) and equate to zero.

$$\frac{\partial[MSE(\bar{y}_{st})]}{\partial b} = 0$$

$$\hat{b}_{opt} = \frac{S_{yx_2} - R_1S_{x_1x_2}}{S_{x_2}^2} \qquad (23)$$

To obtain the $MSE(\bar{y}_{st})_{min}$, substitute equation (23) into equation (22)

$$MSE(\bar{y}_{st})_{min} = \lambda\left[S_y^2 + R_1^2S_{x_1}^2 - 2R_1S_{yx_1}\right] + \lambda S_{x_2}^2\left[\frac{S_{yx_2} - R_1S_{x_1x_2}}{S_{x_2}^2}\right]^2 + 2\lambda\left[R_1S_{x_1x_2} - S_{yx_2}\right]\left[\frac{S_{yx_2} - R_1S_{x_1x_2}}{S_{x_2}^2}\right]$$

$$MSE(\bar{y}_{st})_{min} = \lambda\left\{\left(S_y^2 + R_1^2S_{x_1}^2 - 2R_1S_{yx_1}\right) + \frac{\left(S_{yx_2} - R_1S_{x_1x_2}\right)^2}{S_{x_2}^2} + \frac{2\left(R_1S_{x_1x_2} - S_{yx_2}\right)\left(S_{yx_2} - R_1S_{x_1x_2}\right)}{S_{x_2}^2}\right\}$$

$$= \lambda\left\{\frac{\left(S_{x_2}^2\left(S_y^2 + R_1^2S_{x_1}^2 - 2R_1S_{yx_1}\right) + \left(S_{yx_2} - R_1S_{x_1x_2}\right)^2 + 2\left(R_1S_{x_1x_2} - S_{yx_2}\right)\left(S_{yx_2} - R_1S_{x_1x_2}\right)\right)}{S_{x_2}^2}\right\}$$

$$MSE(\bar{y}_{st})_{min} = \lambda\left\{\frac{S_{x_2}^2\left(S_y^2 + R_1^2S_{x_1}^2 - 2R_1S_{yx_1}\right) - \left(S_{yx_2}^2 + R_1^2S_{x_1x_2}^2 - 2R_1S_{yx_2}S_{x_1x_2}\right)}{S_{x_2}^2}\right\}$$

$$MSE(\bar{y}_{st})_{min} \cong \lambda\left\{S_y^2 + R_1^2S_{x_1}^2 - 2R_1S_{yx_1}\right\} - \lambda\frac{\left(S_{yx_2} - R_1S_{x_1x_2}\right)^2}{S_{x_2}^2}$$

Or

$$MSE(\bar{y}_{st})_{min} \cong \lambda\left\{S_y^2 + R_1^2 S_{x_1}^2 - 2R_1 S_{yx_1} - \frac{\left(S_{yx_2} - R_1 S_{x_1x_2}\right)^2}{S_{x_2}^2}\right\} \quad (24)$$

3. RESULTS AND DISCUSSIONS

4.1 Theoretical Analysis

The theoretical comparison of the proposed estimators with the reviewed estimators is followed by empirical analysis and percentage relative efficiency analysis.

The condition for the theoretical analysis is if $MSE(\bar{y}_{st})_{min} - MSE(\bar{y}_3)_{min} < 0$. $\bar{y}_{st}$ is more efficient than $\bar{y}_3$; otherwise reverse the decisions in favour of $\bar{y}_3$.

4.11 Comparing MSE of NEV with the MSE of ([*KS13*]) improved ratio estimator

$$MSE(\bar{y}_{st})_{min} - MSE(\bar{y}_3)_{min} < 0$$

$$\left\{\lambda\left[S_y^2 + R_1^2 S_{x_1}^2 - 2R_1 S_{yx_1} - \frac{\left(S_{yx_2} - R_1 S_{x_1x_2}\right)^2}{S_{x_2}^2}\right]\right\} - \lambda\left[S_y^2 + R^2 S_y^2 - 2\frac{RS_{xy}}{\bar{Y}^2}\right] - \frac{\lambda(\Delta y - R\Delta x)^2}{2(N-1)} < 0$$

$$\lambda\left[R_1^2 S_{x_1}^2 - 2R_1 S_{yx_1} - \frac{\left(S_{yx_2} - R_1 S_{x_1x_2}\right)^2}{S_{x_2}^2}\right]$$

$$-\lambda R^2 S_y^2 + 2\frac{RS_{xy}}{\bar{Y}^2}\lambda - \frac{\lambda(\Delta y - R\Delta x)^2}{2(N-1)} < 0 \quad (25)$$

This implies that $\bar{y}_{st}$ is more efficient than $\bar{y}_3$.

4.12 Comparing the MSE of NEV with the MSE of ([*AK14*]) ratio estimator

$$MSE(\bar{y}_{st})_{min} - MSE(\bar{y}_5)_{min} < 0$$

$$\left\{\lambda\left[S_y^2 + R_1^2 S_{x_1}^2 - 2R_1 S_{yx_1} - \frac{\left(S_{yx_2} - R_1 S_{x_1x_2}\right)^2}{S_{x_2}^2}\right]\right\}$$

$$-\left\{\lambda\left(S_y^2 + R_1^2 S_{x_1}^2 + R_2^2 S_{x_2}^2 + 2R_1R_2 S_{x_1x_2} - 2R_2 S_{yx_2} - 2R_1 S_{yx_1}\right) - \frac{\lambda(\Delta y - R_1\Delta x_1 - R_2\Delta x_2)^2}{2(N-1)}\right\}$$

$$\Rightarrow -\lambda\frac{\left(s_{yx_2} - R_1 s_{x_1x_2}\right)^2}{s_{x_2}^2} -$$

$$\left\{\lambda\left[R_2^2 S_{x_2}^2 + 2R_1R_2 S_{x_1x_2} - 2R_2 S_{yx_2}\right] - \frac{\lambda(\Delta y - R_1\Delta x_1 - R_2\Delta x_2)^2}{2(N-1)}\right\} < 0 \quad (26)$$

This implies that $\bar{y}_{st}$ is more efficient that $\bar{y}_5$

4.13 Comparing the MSE of NEV with the MSE of ([*AK14*]) regression estimator

$$MSE(\bar{y}_{st})_{min} - MSE(\bar{y}_6)_{min} < 0$$

This implies that

$$\left\{\lambda\left[S_y^2 + R_1^2 S_{x_1}^2 - 2R_1 S_{yx_1} - \frac{\left(S_{yx_2} - R_1 S_{x_1x_2}\right)^2}{S_{x_2}^2}\right]\right\} - \lambda S_y^2\left[1 - b_1^2\frac{S_{x_1}^2}{S_y^2} - b_2^2\frac{S_{x_2}^2}{S_y^2}\right] + \lambda S_y^2\left[2b_1\frac{S_{x_1}}{S_y}\, b_2\frac{S_{x_2}}{S_y}\frac{S_{x_1x_2}}{S_{x_1}S_{x_2}}\right]$$

$$-\frac{\lambda(\Delta y - b_1\Delta x_1 - b_2\Delta x_2)^2}{2(N-1)} < 0 \quad (27)$$

The efficiency of $\bar{y}_{st}$ over $\bar{y}_6$ will be determined empirically using equation (27)

4.2 Empirical Analysis

In the empirical comparison, R statistical software was used to write and compile 728-line code to stimulate and following the normal population of a pre-defined mean and standard deviation of a twenty population. The essence of twenty stimulated population is to test the efficiency of the estimators asymptotically (that is with different populations and sample sizes). Each population has one study variable Y and two auxiliary variables (x_1, x_2) with the exception of *([KS13]*) with one auxiliary variable. The code was developed to compare the estimators under two conditions. The conditions are High Maximum Extreme Value (HMaEV) and Low Minimum Extreme Value (LMiEV).

Table1: Rank and Comparison of the proposed estimator with the reviewed estimators for the twenty stimulated populations for HMaEV cases

S/N Populations	1	2	3	4	5	6	7
1. MSE ($\bar{y}_{st}$)	9969.802	3844.868	10257.3	8945.47	7904.129	9118.818	10692.95
2. MSE ($\bar{y}_3$)	253180.2	92865.49	271747.6	226558.7	193342.2	254878.2	313252.8
3. MSE ($\bar{y}_5$)	13928.04	7307.471	16462.41	14393.79	12197.14	16976.95	22577.38
4. MSE ($\bar{y}_6$)	9572.582	4243.1	10553.71	8893.569	7672.536	10508.48	12693.74
Rank MSE($\bar{y}_{st}$)	2	1	1	2	2	1	1
Rank MSE($\bar{y}_3$)	4	4	4	4	4	4	4
Rank MSE($\bar{y}_5$)	3	3	3	3	3	3	3
Rank MSE($\bar{y}_6$)	1	2	2	1	1	2	2

Table2: Rank and Comparison of the proposed estimator with the reviewed estimators for the twenty stimulated populations for HMaEV cases continues

S/N Populations	8	9	10	11	12	13	14
1. MSE ($\bar{y}_{st}$)	14054.15	24199.44	27390.01	32016.56	41112.32	15676.78	17689.88
2. MSE ($\bar{y}_3$)	419045.3	754112	923106.7	1177701	1612307	573734.1	667053.4
3. MSE ($\bar{y}_5$)	27112.14	34497.47	50521.47	62759.85	88084.77	45497.15	50967.38
4. MSE ($\bar{y}_6$)	15921.55	23275.36	29689.32	38058.76	48689.05	23201.94	26482.01
Rank MSE($\bar{y}_{st}$)	1	2	1	1	1	1	1
Rank MSE($\bar{y}_3$)	4	4	4	4	4	4	4
Rank MSE($\bar{y}_5$)	3	3	3	3	3	3	3
Rank MSE($\bar{y}_6$)	2	1	2	2	2	2	2

Table3: Rank and Comparison of the proposed estimator with the reviewed estimators for the twenty stimulated populations for HMaEV cases continues

S/N Populations	15	16	17	18	19	20	Overall Ranking
1. MSE ($\bar{y}_{st}$)	115012.2	40820.85	34655.54	114867.1	317787.5	666269.9	
2. MSE ($\bar{y}_3$)	6258080	1956279	2026384	7303558	31577741	90611571	
3. MSE ($\bar{y}_5$)	174441.8	139203.4	184225.9	452657.2	1432352	3191902	
4. MSE ($\bar{y}_6$)	111874.1	68002.77	79395.13	192497	629051	1290081	
Rank MSE($\bar{y}_{st}$)	2	1	1	1	1	1	**1**
Rank MSE($\bar{y}_3$)	4	4	4	4	4	4	**4**
Rank MSE($\bar{y}_5$)	3	3	3	3	3	3	**3**

Rank MSE($\bar{y}_6$)	1	2	2	2	2	2	**2**

4.3 Low and Minimum Value

Table4: Comparison of the proposed estimators with the reviewed estimators for the twenty stimulated populations for LMiEV

S/N Populations	1	2	3	4	5	6	7
1. MSE($\bar{y}_{st}$)	5055.9794	11212.1637	15542.0748	10960.398	15646.7526	15604.1283	10184.2256
2. MSE ($\bar{y}_3$)	62526.0429	80547.27837	95128.42097	91465.53736	108745.356	94488.16596	109258.909
3. MSE ($\bar{y}_5$)	7530.87287	11565.93007	15222.01492	12934.9066	17750.6313	15748.2352	14597.4654
4. MSE ($\bar{y}_6$)	5095.52272	9645.987777	13355.4278	9921.678177	13977.5592	13385.63772	10373.735
Rank MSE($\bar{y}_{st}$)	1	2	3	2	2	2	1
Rank MSE($\bar{y}_3$)	4	3	4	4	4	4	4
Rank MSE($\bar{y}_5$)	3	2	2	3	3	3	3
Rank MSE($\bar{y}_6$)	2	1	1	1	1	1	2

Table5: Comparison of the proposed estimators with the reviewed estimators for the twenty stimulated populations for LMiEV cases (continue)

S/N Populations	8	9	10	11	12	13	14
1. MSE($\bar{y}_{st}$)	33993.5011	32557.2075	40727.7258	36032.6038	29158.3298	86656.3293	74320.0911
2. MSE ($\bar{y}_3$)	105446.317	117322.785	86960.1988	91591.5829	144668.6619	99446.11086	62115.4812
3. MSE ($\bar{y}_5$)	31630.5049	32116.9394	36588.7598	33135.9329	33323.84253	70175.05245	61616.8991
4. MSE ($\bar{y}_6$)	28836.7787	27993.4493	34131.6335	30228.739	26280.09414	72475.27807	61926.4792
Rank MSE($\bar{y}_{st}$)	3	3	3	3	2	3	4
Rank MSE($\bar{y}_3$)	4	4	4	4	4	4	3
Rank MSE($\bar{y}_5$)	2	2	2	2	3	1	1
Rank MSE($\bar{y}_6$)	1	1	1	1	1	2	2

Table6: Comparison of the proposed estimators with the reviewed estimators for the twenty stimulated populations for LMiEV

S/N Populations	15	16	17	18	19	20	Overall Ranking
1. MSE($\bar{y}_{st}$)	89598.9762	91014.7522	110285.8414	160817.7648	20948231.13	1860028.551	
2. MSE ($\bar{y}_3$)	111018.1903	120989.8886	500293.3261	223833578.3	3130175596	367888798.1	
3. MSE ($\bar{y}_5$)	72291.75637	71855.23721	86482.73769	1150713.922	32801061.58	22311430.98	
4. MSE ($\bar{y}_6$)	74927.23977	76224.78422	95954.58546	515671.6613	920584.0752	2969628.188	
Rank MSE($\bar{y}_{st}$)	3	3	3	1	2	1	**2**
Rank MSE($\bar{y}_3$)	4	4	4	4	4	4	**4**
Rank MSE($\bar{y}_5$)	1	1	1	3	3	3	**2**
Rank MSE($\bar{y}_6$)	2	2	2	2	1	2	**1**

4.4 High and Maximum Value

*Table 7 :*The Relative Efficiency (RE) of estimators developed by ([*KS13*]) ratio, ([*AK14*]) regression and ([*AK14*]) ratio with the proposed estimator for the twenty simulated populations (measured in percentages)

S/N	Populations	1	2	3	4	5	6	7
1.	$RE(\bar{y}_{st}/\ \bar{y}_3)$	2539.471	2415.31	2649.311	2532.664	2446.091	2795.08	2929.527
2.	$RE(\bar{y}_{st}/\ \bar{y}_5)$	139.7022	190.0578	160.4947	160.9059	154.3135	186.1749	211.1427
3.	$RE(\bar{y}_{st}/\ \bar{y}_6)$	96.01577	110.3575	102.8898	99.4198	97.06998	115.2395	118.7113
4.	$RE(\bar{y}_5/\ \bar{y}_3)$	1817.774	1270.829	1650.716	1574.003	1585.144	1501.32	1387.463
5.	$RE(\bar{y}_6/\ \bar{y}_3)$	2644.848	2188.624	2574.9	2547.444	2519.925	2425.453	2467.773
6.	$RE(\bar{y}_6/\ \bar{y}_5)$	145.4993	172.2201	155.9869	161.8449	158.9714	161.5547	177.8623

Table 8: The Relative Efficiency (RE) of estimators developed by *([KS13]*) ratio, ([*AK14*]) regression and ([*AK14*]) ratio with the proposed estimator for the twenty simulated populations (measured in percentages)

S/N Populations	8	9	10	11	12	13	14
1. $RE(\bar{y}_{st}/\ \bar{y}_3)$	2981.649	3116.237	3370.232	3678.412	3921.712	3659.771	3770.82
2. $RE(\bar{y}_{st}/\ \bar{y}_5)$	192.9121	142.5548	184.4522	196.0231	214.2539	290.22	288.116
3. $RE(\bar{y}_{st}/\ \bar{y}_6)$	113.2872	96.18138	108.3947	118.8721	118.4293	148.002	149.7015
4. $RE(\bar{y}_5/\ \bar{y}_3)$	1545.6	2185.992	1827.157	1876.52	1830.404	1261.033	1308.785
5. $RE(\bar{y}_6/\ \bar{y}_3)$	2631.938	3239.958	3109.221	3094.428	3311.437	2472.785	2518.892
6. $RE(\bar{y}_6/\ \bar{y}_5)$	170.2858	148.2145	170.1672	164.9025	180.9129	196.092	192.4603

Table 9 : The Relative Efficiency (RE) of estimators developed by (*[KS13]*) ratio, ([*AK14*]) regression and ([*AK14*]) ratio with the proposed estimator for the twenty simulated populations (measured in percentages)

S/N Populations	15	16	17	18	19	20	Average
1. $RE(\bar{y}_{st}/\ \bar{y}_3)$	5441.231	4792.354	5847.216	6358.266	9936.746	13599.83	4439.096
2. $RE(\bar{y}_{st}/\ \bar{y}_5)$	151.6724	341.0105	531.5915	394.0703	450.7264	479.0704	252.9733
3. $RE(\bar{y}_{st}/\ \bar{y}_6)$	97.27153	166.5883	229.0979	167.5823	197.947	193.6275	132.2343
4. $RE(\bar{y}_5/\ \bar{y}_3)$	3587.488	1405.339	1099.945	1613.485	2204.607	2838.796	1768.62
5. $RE(\bar{y}_6/\ \bar{y}_3)$	5593.858	2876.764	2552.278	3794.116	5019.901	7023.709	3230.413
6. $RE(\bar{y}_6/\ \bar{y}_5)$	155.9269	204.7025	232.0368	235.1503	227.7005	247.4186	182.9955

4.5 Low and Minimum Value

Table10: The Relative Efficiency (RE) of estimators developed by *([KS13]*) ratio, ([*AK14*]) regression and ([*AK14*]) ratio with the proposed estimator for the twenty simulated populations (measured in percentages)

S/N Populations	1	2	3	4	5	6	7
1. $RE(\bar{y}_{st}/\ \bar{y}_3)$	1236.675	718.3919	612.0703	834.5093	695.0027	605.5331	1072.825
2. $RE(\bar{y}_{st}/\ \bar{y}_5)$	148.9498	103.1552	97.94069	118.0149	113.4461	100.9235	143.3341
3. $RE(\bar{y}_{st}/\ \bar{y}_6)$	100.7821	86.03146	85.93079	90.52297	89.33201	85.78267	101.8608
4. $RE(\bar{y}_5/\ \bar{y}_3)$	830.2629	696.4185	624.9397	707.1217	612.6281	599.9921	748.4786
5. $RE(\bar{y}_6/\ \bar{y}_3)$	1227.078	835.034	712.2828	921.8757	777.9996	705.8922	1053.226
6. $RE(\bar{y}_6/\ \bar{y}_5)$	147.7939	119.9041	113.9762	130.3701	126.9938	117.6502	140.7156

Table11: The Relative Efficiency (RE) of estimators developed by ([*KS13*]) ratio, ([*AK14*]) regression and ([*AK14*]) ratio with the proposed estimator for the twenty simulated populations (measured in percentages)

S/N Populations	8	9	10	11	12	13	14
1. $\boldsymbol{RE}(\bar{\boldsymbol{y}}_{st}/\bar{\boldsymbol{y}}_3)$	310.1955	360.3589	213.516	254.1909	496.1487	114.7592	83.57832
2. $\boldsymbol{RE}(\bar{\boldsymbol{y}}_{st}/\bar{\boldsymbol{y}}_5)$	93.04868	98.64771	89.83747	91.96097	114.2858	80.98087	82.90746
3. $\boldsymbol{RE}(\bar{\boldsymbol{y}}_{st}/\bar{\boldsymbol{y}}_6)$	84.83027	85.98234	83.80442	83.89274	90.12894	83.6353	83.32401
4. $\boldsymbol{RE}(\bar{\boldsymbol{y}}_5/\bar{\boldsymbol{y}}_3)$	333.3691	365.2988	237.6692	276.4117	434.1296	141.7115	100.8092
5. $\boldsymbol{RE}(\bar{\boldsymbol{y}}_6/\bar{\boldsymbol{y}}_3)$	365.6661	419.108	254.7789	302.995	550.4876	137.2138	100.3052
6. $\boldsymbol{RE}(\bar{\boldsymbol{y}}_6/\bar{\boldsymbol{y}}_5)$	109.6881	114.7302	107.199	109.6173	126.8026	96.82619	99.50008

DISCUSSIONS

Theoretical Analysis

The comparing the proposed estimator NEV ($\bar{y}_{st}$) with the ratio estimator of ([*KS13*]), ($\bar{y}_3$). It is obvious from equation 25 that NEV (st) is superior to ($\bar{y}_3$). In addition, comparing NEV ($\bar{y}_{st}$) with the ratio estimator of ([*AK14]*), ($\bar{y}_5$). it could be seen from equation 26 that NEV ($\bar{y}_{st}$) is efficient over this estimator. Finally, comparing NEV ($\bar{y}_{st}$) with the regression estimator of ([*AK14*]), ($\bar{y}_6$) using equation 27, the result here could not be determined theoretically therefore an empirical analysis has been used.

Empirical Analysis

High Maximum Extreme Value (HMaEV) Case

It is revealed that ($\bar{y}_{st}$) has a smaller MSE when compared to that of

([KS13]) ratio estimator ($\bar{y}_3$)

([AK14]) ratio estimator ($\bar{y}_5$) and

([AK14]) regression estimator ($\bar{y}_6$) .

Hence, it's ranked first among the estimators. This means that for HMaEV case. NEV ($\bar{y}_{st}$) is asymptotically efficient over all the reviewed estimators.

LMiEV CASE

Comparing NEV ($\bar{y}_{st}$) with the reviewed estimators using their MSE for the LMiEV case

It is revealed that ($\bar{y}_{st}$) has a smaller MSE when compared to that of

([*KS13*]) ratio estimator ($\bar{y}_3$) and ([*AK14*]) ratio estimator ($\bar{y}_5$) but a bigger MSE when compared to ([*AK14*]) regression estimator ($\bar{y}_6$). Hence, it is ranked second. This means that for LMiEV case, NEV ($\bar{y}_{st}$) is efficient over ([*KS13*]) ratio estimator ($\bar{y}_3$) and over ([*AK14*]) ratio estimator ($\bar{y}_5$) but not over ([*AK14]*) regression estimator ($\bar{y}_6$). The observed difference here between the HMaEV case and the LMiEV case could be due to shift in the line of best fit. Hence, there is the need to know by what percentage is one estimator efficient over the other. This necessitates the use of percentage relative efficiency analysis.

Percentage Relative Efficiency

Using the percentage relative efficiency, table 9 reveals that $\bar{y}_{st}$(NEV) is 4439.096%, 152.9733%, and 32.2342% relatively efficient over $\bar{y}_3$, $\bar{y}_5$ and $\bar{y}_6$ respectively for the HMaEV cases. Likewise, table 12 reveals that $\bar{y}_{st}$ (NEV) is 9011.197% and 89.3562% relatively efficient over $\bar{y}_3$ and $\bar{y}_5$ respectively but less efficient by 1.25367% to $\bar{y}_6$.for the case of LMiEV. This implies that the proposed estimator is partially efficient over $\bar{y}_6$ but asymptotically efficient over the rest of the reviewed estimators.

5.1 Summary

This research work had extended the work of ([*KS13*]) into a mixed estimation (Ratio-cum-regression) in single phase sampling without replacement. The proposed estimator was a combination of the improved ratio and regression estimators of ([*KS13*]) without extreme values (NEV) correction of ([*S72*]). The mixed estimator was combined in the order of (*KC05*) while following the procedure of ([*AK14*]). This proposed estimator used one study variable and two auxiliary variables without the presence of extreme values in its distribution and assumed that the population information of both the study and auxiliary variables were available.

This study has made theoretical and empirical comparison of the proposed estimator with the reviewed estimators. The efficiency of the proposed estimator had been established using the Mean Square Errors (MSE). Similarly, the biases of the proposed estimator were ascertained in the empirical analysis. Finally, this study also made use of percentage relative efficiency analysis in other to ascertain by what percentage was the proposed estimator efficient over the reviewed.

5.2 Conclusion

The proposed estimator No Extreme Value (NEV) was asymptotically efficient over the reviewed estimators except in comparison with the regression estimator of *([AK14]*) where it was partially efficient.

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نسبة محسنة–مقدر انحدار نائب الرئيس باستخدام متغيرين مساعدين في أخذ العينات أحادية الطور

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الخلاصة: تم تأكيد المعلومات المساعدة لتعزيز الدقة في مقدرات النسبة والانحدار والمنتج على التوالي .تم تقديم العديد من حالات المقدرات المختلطة المحسنة في أخذ العينات أحادية الطور وقدمت توصيات باستخدام أكثر من متغير مساعد وعوامل صحيحة للقيم القصوى .تلقي هذه الدراسة نظرة على حالة القيمة القصوى في كل من الدراسة والمتغير المساعد حيث لا يتم تصحيح المقدر المختلط المقترح للقيم القصوى في كل من الدراسة والمتغير المساعد .ومع ذلك ، من المثير للاهتمام معرفة أن المقدر المختلط المطور هو مقدرات فردية عالية الكفاءة للنسبة والانحدار مع عوامل تصحيح للقيمة القصوى.

الكلمات المفتاحية: نسبة– الرئيس الانحدار ، الانحدار ، أخذ العينات مرحلة واحدة ، المتغيرات المساعدة.