Certain properties of contra-T^*_{12}-continuous functions

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Abstract—*The concept of contra function was introduced by Dontchev [2], in this work, we use the notion of* T^*_{12}*-open to study a new class of function called a contra-*T^*_{12}*-continuous function as generalization of contra- continuous.*

Keywords: T^*_{12}-open sets; contra-T^*_{12}-continuous function; operator topological space; contra-T^*_{12}-closed graph.

I. INTRODUCTION

In 1996, Dontchev [2] introduced contra-continuous functions. In [10], the authors introduced the concept of almost contra-T*-continuous function. In this paper, we introduce a new class of function called contra-T^*_{12}-continuous function where T_1, T_2 are operators associated with the topology τ on X. Throughout the paper, the space X and Y or (X, Y) and (Y, δ) stand for topological space, let A be a subset of X. the closure of A and the interior of A will be denoted by Cl(A) and Int(A), respectively.

II. PRELIMINARIES

In this section, we recall the basic facts and definitions needed in this work.

2.1 Definition: A subset A of a space X is said to be:

i) Semi-open [6] if $A \subseteq Cl(Int(A))$,

ii) Pre-open [7] if $A \subseteq Int(Cl(A))$,

iii) b-open [1] if $A \subseteq Cl(Int(A)) \cup Int(Cl(A))$.

The complement of semi-open (pre-open, b-open) is said to be semi-closed (pre-closed, b-closed). The family of all semi-open (pre-open, b-open, semi-closed, pre-closed, b-closed) subset of a space X is denoted by SO(X)(PO(X), BO(X), SC(X), PC(X), BC(X), respectively).

2.2 Definition [4]: A function $f : X \to Y$ is called semi-continuous (pre-continuous, b-continuous) if for each $x \in X$ and each open set V of Y containing f(x), there exists $U \in SO(X)$ ($U \in PO(X)$, $U \in BO(X)$) such that $f(U) \subseteq V$.

2.3Definition: A function $f : X \to Y$ is called contra-continuous [2] (contra-semi continuous [4], contra-pre-continuous [3], contra-b-continuous [5]) if $f^{-1}(V)$ is closed (semi-closed, pre-closed, b-closed, resp.) in X for each open set V of Y.

III. OPERATOR TOPOLOGICAL SPACES

3.1 Definition [8]: Let (X, τ) be a topological space and let $T: p(X) \to p(X)$ be a function (where p(X) is the power set of X) we say that T is an operator associated with the topology τ on X if $W \subseteq T(W)$ $(W \in \tau)$ and the triple (X, τ, T) is called an operator topological space.

3.2 Definition [9]: Let (X, τ, T) be an operator topological space, let $A \subseteq X$

i) A is called T-open if given $x \in A$, then there exists $V \in \tau$ there exists $x \in V \subseteq T(V) \subseteq A$.

ii) A is called T*-open if $A \subseteq T(A)$ (A is not necessarily open).

3.3 Remarks:

i) Every T-open set is open.

ii) Every open set is T*-open, so we have the following implications:

T-open $\to$ open $\to$ T*-open

iii) Let (X, τ) be a topological space define $T: p(X) \to p(X)$ as follows: $T(A) = \text{Int}\, Cl(A)$ then T is an operator associated with the topology τ on X and the triple (X, τ, T) is an operator topological space.

As an example, we can suppose $X = R$, $\tau = t_u$ = the usual topology on R, if

$T(A) = \text{Int}\, Cl(A)$,

then the triple (R, t_u, T) is an operator topological space,

notice that $Q \subset R$ satisfies $Q \subseteq \text{Int}(Cl(Q))$, so Q is a T*-open (pre-open) which is not open.

3.3 Definition: Let (X, τ) be a topological space and let T_1, T_2 be two operators associated with the topology τ on X then (X, τ, T_1, T_2) is called a bi operator topological space.

3.4 Definition: Let (X, τ, T_1, T_2) be an operator topological space and let $A \subseteq X$, we say that A is a T^*_{12}-open if $A \subseteq T_1(A) \cup T_2(A)$, the complement of T^*_{12}-open is called T^*_{12}-closed for example if:

$T_1(A) = Cl(\text{Int}(A))$,

$T_2(A) = \text{Int}(Cl(A))$, Then

$A \subseteq Cl(\text{Int}(A)) \cup \text{Int}(A)$,

this is the definition of b-open set.

Notice that every T^*_1-open (T^*_2-open) is T^*_{12}-open because if A is T^*_1-open then $A \subseteq T_1(A) \subseteq T_1(A) \cup T_2(A)$, so A will be T^*_{12}-open.

IV. CONTRA-T^*_{12}-CONTINUOUS FUNCTIONS

In this section, we obtain some properties of contra-T^*_{12}-continuous functions.

4.1 Lemma [1]: Let (X, τ) be a topological space then:

1) The intersection of an open set and a b-open set is a b-open set.

2) The union of any family of b-open sets is a b-open set.

Now, we generalize Lemma 4.1 as follows:

4.2 Lemma: Let (X, τ, T_1, T_2) be a bi operator topological space assume that

$T_1(W \cap B) = T_1(W) \cap T_1(B)$, $W \in \tau$, $B \subseteq X$,

$T_2(W \cap B) = T_2(W) \cap T_2(B)$, $W \in \tau$, $B \subseteq X$, therefore:

1) The intersection of an open set and a T^*_{12}-open set is T^*_{12}-open.

2) The union of any family T^*_{12}-open sets is a T^*_{12}-open set.

Proof:

1) Let $W \in X$ be an open set and let V be a T^*_{12}–open set we have to prove that $W \cap V$ is also a T^*_{12}–open set. Since W is open then:

$W \subseteq T_1(W)$ … (1)

$W \subseteq T_2(W)$ … (2)

Since V is a T^*_{12}–open then

$V \subseteq T_1(V) \cap T_2(V)$ … (3)

$W \cap V \subseteq W \cap (T_1(V) \cap T_2(V))$

$= (W \cap T_1(V)) \cup (W \cap T_2(V))$

$\subseteq (T_1(W) \cap T_1(V)) \cup (T_2(W) \cap T_2(V))$

$= (T_1(W \cap V)) \cup (T_2(W \cap V))$

Then $W \cap V$ is T^*_{12}-open set.

2) Let $\mathcal{L} = \{w_\alpha \mid \alpha \in I\}$ be any family of T^*_{12}-open sets we must prove that $\bigcup_\alpha w\alpha$ is also a T^*_{12}-open

$w_\alpha \subseteq T_1(w_\alpha) \cup T_2(w_\alpha)$ for each $\alpha \in I$

$\bigcup_\alpha w\alpha \subseteq \bigcup_\alpha (T_1(w_\alpha) \cup T_2(w_\alpha))$

$= \bigcup_\alpha T_1(w_\alpha) \cup \bigcup_\alpha T_2(w_\alpha)$

Now $\bigcup_\alpha T_1(w_\alpha) = T_1(\bigcup_\alpha w_\alpha)$

Also $\bigcup_\alpha T_2(w_\alpha) = T_2(\bigcup_\alpha w_\alpha)$

Then $\bigcup_\alpha w_\alpha \subseteq T_1(\bigcup_\alpha w_\alpha) \cup T_2(\bigcup_\alpha w_\alpha)$ and $\bigcup_\alpha w_\alpha$ is a T^*_{12}-open.

4.3 Remarks:

i) The intersection of two T^*_{12}-open is not necessarily T^*_{12}-open, so the collection of all T^*_{12}-open sets is not necessarily a topology on X.

Let $\tau^*_{(12)}$ be the topology generated by the collection of all T^*_{12}-open sets.

ii) The intersection of any collection of T^*_{12}-closed sets is T^*_{12}-closed. Let T^*_{12}-Cl(B)-intersection of all T^*_{12}-closed sets containing B.

Recall that for a function $f: X \to Y$, the subset $\{(x, f(x)) \mid x \in X\} \subseteq X \times Y$ is called the graph of f and denoted by G (f).

4.4 Definition: Let $f:(X, \tau, T_1, T_2) \to (Y, \delta)$be a function the graph G(f) of f is said to be contra-T^*_{12}-closed graph if for each $(x, y) \in (X \times Y)$-G(f) there exists U which is T^*_{12}-open containing x and a closed set V of Y containing y such that $(U \times V) \cap G(f) = \emptyset$. The implies that $f(U) \cap V = \emptyset$.

4.5 Definition: A space X is said to be contra-compact if every closed cover of X has a finite sub cover.

4.6 Theorem : Let (X, τ, T_1, T_2) be a bi operator topological space and suppose $f :(X, \tau, T_1, T_2) \to (Y, \delta)$ has a contra-T^*_{12}-closed graph, then the inverse image of a contra –compact set A of Y is T^*_{12}–closed in X.

Proof: Assume that A is contra-compact set of A and $x \notin f^{-1}(A)$ for each $a \in A$, $(x, a) \notin G(f)$. Then there exists U_a which is T^*_{12}-closed containing x and V_a which is closed in Y containing a such that

$f(U_a) \cap V_a = \emptyset$.

Consider $\mathcal{L} = \{A \cap V_a \mid a \in A\}$ and $\mathcal{L}$ is a closed cover of the subspace A, but A is contra-compact then there exists $a_1, a_2, a_3 ... a_n$ such that

$A \subseteq \bigcup_{i=1}^{n} V_{ai}$.

Let $U = \bigcap_{i=1}^{n} U_{ai}$,

then U is T^*_{12}-closed containing x and $f(U) \cap A = \emptyset$, therefore

$U \cap f^{-1}(A) = \emptyset$ and hence $x \notin T^*_{12}$ -Cl ($f^{-1}(A)$), this show that $f^{-1}(A)$ is T^*_{12}-closed.

4.7 Theorem : Let Y be contra –compact space and let (X, $\tau^*_{(12)}$,T_1, T_2) be operator topological space ,suppose f : (X, $\tau^*_{(12)}$,T_1, T_2) → (Y, δ) has a contra-T^*_{12} -closed graph then f is contra T^*_{12}-continuous.

Proof: First we show that an open set U of Y is contra –compact and let $\mathcal{L}$ ={ V_α | α∈ ∧ } be a cover of U by closed sets V_α of U for each α∈ ∧ , then there exists a closed set K_α of Y such that $V_\alpha = K_\alpha \cap U$,then the family { K_α | α∈ ∧ } U { U^c} is closed cover of Y. But Y is contra-compact then there exists $\alpha_1, \alpha_2 \ldots \alpha_n$ such that

Y= ($\bigcup_{i=1}^{n} K_{\alpha i}$) U ($U^c$), hence

U = $\bigcup_{i=1}^{n} V_{\alpha i}$.

This show that U is contra-compact by (theorem 4.6) $f^{-1}(U)$ is a T^*_{12}-closed in X then for f is contra T^*_{12} -continuous.

4.8 Theorem: Let f : (X, τ,T_1, T_2) → (Y, δ) be a function and g : X → X × Y the graph function of f defined by g(x) = (x, f(x)) for every x ∈X, if g is contra-T^*_{12}-continuous then f is contra-T^*_{12}-continuous.

Proof: Since g is contra-T^*_{12}-continuous then $f^{-1}(U) = g^{-1}(X \times U)$ is a T^*_{12}-closed in X. Then f is contra-T^*_{12}-continuous.

4.9 Theorem : If f : (X, τ,T_1, T_2) → (Y, δ) is contra-T^*_{12}-continuous and g : (X, τ,T_1, T_2) → (Y, δ) is contra-continuous and Y is Urysohn space then E={ x ∈ X | f(x) = g(x) } is T^*_{12}-closed in X.

Proof: Let $x \in E^c$, then f(x)≠g(x), since Y is a Urysohn then there exists open sets V and W such that f(x) ∈ V, g(x) ∈ W, and

Cl(V) ∩ Cl(W)=∅.

Since f is contra-T^*_{12} –continuous then f^{-1}(Cl(V)) is T^*_{12}-open in X and g is contra-continuous then g^{-1}(Cl(W)) is open in X, let U = f^{-1}(Cl(V)), G = g^{-1}(Cl(W)).

Then x ∈ U ∩ G = A, where A is T^*_{12}-open in X and

f(A) ∩ g(A) ⊆ f(U) ∩ g(G) ⊆ Cl(V) ∩ Cl(W) = ∅, hence

f(A) ∩ g(A) = ∅ and A ∩ E = ∅, A ⊆ E^c,

where A is T^*_{12}-open there for $x \notin T^*_{12}$-Cl(E), then E is T^*_{12}-closed in X.

4.10 Definition: A subset A of operator topological space (X, τ, T_1, T_2) is said to be T^*_{12}-dense in X if T^*_{12}-Cl (A) = X.

4.11 Remarks: Let (X, τ) be a topological space define:

T_1: p(X) → p(X)

T_2: p(X) → p(X) as follows

T_1 (A) = Int (Cl (A))

T_2 (A) = Cl (Int(A)), then T^*_{12}-dense subset will be b-dense and T^*_{12}-Cl(A) will be b-Cl(A) so b-dense in X mean that b-Cl(A) = X.

4.12 Corollary: Let f: (X, τ,T_1, T_2) → (Y, δ) is contra-T^*_{12}-continuous and g : (X, τ,T_1, T_2) → (Y, δ) is contra continuous if Y is Urysohn and f = g on T^*_{12}-dense set A ⊆ X then f = g on X.

Proof: since f is contra -T^*_{12}-continuous and is contra continuous and Y is Urysohn by previous Theorem E = {x ∈ X: f (x) = g(x)} is a T^*_{12}-closed in X. We have f = g on T^*_{12}-dense set A ⊆ E, then X = T^*_{12}-Cl (A) ⊆T^*_{12}-Cl (E) = E. Hence f = g on X.

4.13 Definition: A bi operator topological space (X, τ, T_1, T_2) is called T^*_{12}-connected if X is not the Union of two non-empty T^*_{12}-open sets.

4.14 Theorem: If f: (X, τ, T_1, T_2) → (Y, δ) is contra-T^*_{12}-continuous from a T^*_{12}-connected space onto Y, then Y is not a discrete space.

Proof: Suppose that *Y* is discrete. Let $\emptyset \neq A \subset Y$ then A is proper nonempty open and closed subset of Y. Then $f^{-1}(A)$ is a proper nonempty T^*_{12}-clopen (T^*_{12}-open and T^*_{12}-closed) subset of X such that $X = f^{-1}(A) \cup (f^{-1}(A))^c$ which means that X is T^*_{12}-disconnected which is a contradiction. Hence Y is not discrete.

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