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Sometimes-pool Shrinkage Estimation of the Entropy Function for the Exponential Distribution under Different Loss Functions Using Progressive Type II Censored Sample

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Abstract

In this paper, sometimes-pool shrinkage estimation procedure is suggested to estimate the entropy function of the exponential distribution with Type II progressive sampling scheme under symmetric and asymmetric loss functions. Two types of loss functions are considered; squared error and LINEX loss function. We obtain that the proposed estimators have good performance comparing with a classical estimator in terms of relative risk.

Keywords and phrases: entropy function, exponential distribution, shrinkage estimation, progressive censoring type II sample.

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مقدرات Sometimes-pool المقلصة الى دالة لانتروبي للتوزيع الاسي بأستخدام العينات المراقبة التتابعية من النوع الثاني تحت نوعين مختلفين من دوال الخسارة

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الخلاصة

تناول هذا البحث اقتراح مقدرات sometimes-pool المقلصة الى دالة الانتروبي للتوزيع الاسي باستخدام العينات المراقبة التتابعية من النوع الثاني ودراسة خواصها لنوعين من دوال الخسارة ؛ دالة متوسط مربعات الخطا (SELF) و دالة الخطية الاسية LINEX . من خلال الدراسة العددية لمعادلات الخطرالنسبية للمقدرات المقترحة يالنسبة الى المقدر الكلاسيكي لكلا النوعين من دوال الخسارة (SELE & LINEX) أتضح افضلية المقدرات المقترحة عن المقدر الكلاسيكي .

1. Introduction

Entropy is one of the essential elements in statistical mechanics. It was defined originally in physics particularly in the second law inthermodynamics. However, entropy was defined in information theorem by Shannon (1948) based on probability and statistics and his definition is given below as follows

If X is random variable with probability density function f and distribution function F, then the entropy function is given by

$$H(f) = E[-\ln(f(x))] \qquad (1)$$

In fact, the entropy measures of the uniformity of distribution , in other words uniformity of distribution has a positive relationship with entropy function this means that the distribution has a small entropy becomes more difficult to predict an outcome of drawn from it . Many authors interested to estimation entropy for different life distribution one may refer to Lazo and Rathie (1978), Misra et al. (2005), Jeevanand and Abdul-Sathar (2009) and Kayal and Kumar (2011).

Now, a prior information of about the unknown parameter θ as an initial guess value θ_0, based on the past experience, must be used while estimating θ. Thompson (1968) suggested shrinkage estimation to modify the usual estimator (like MLE or MVUE) of the unknown parameter θ by moving it closer to θ_0. The shrinkage estimator performs batter than the usual estimator when the initial value is close to the true value of the parameter θ.The shrinkage estimators have been discussed for different parameters or parametric functions under different life of distributions by number of authors.

It is of importance in estimation theory how to get a good estimators using minimum number of sample units and economize the cost of experimentation. Thus, the sometimes-pool estimation is a suitable procedure to obtain this target. Katti (1962) proposed sometimes-pool estimation for estimating of the normal distribution mean when the variance is given and the initial value μ_0 of μ is provided. Katti applied the initial value just to build a region for of the first sample mean, in other words, when the first sample mean involved to this region, then it will be applied like estimate of μ else, second sample will be used and pooled mean will be built as estimator of μ. Arnold and Al-Bayyati (1970, 1972) suggested sometimes-pool shrinkage estimator for the normal distribution mean and the regression model parameters, by applying the first stage sample for testing the initial value accuracy and when it is excat then apply shrinkage of the usual estimator across an initial value, else apply pooled estimator like Katti (1962).On the same direction many researchers study sometimes-pool shrinkage estimations for example Waikar et al (1984), Adke et al (1987), Handa et al (1988) and Kambo et al (1991). In recent years, Srivastava and Tanna (2007), Prakash and Singh (2009) and Al-Hemyari (2009) studied sometimes-pool shrinkage estimation for various life distributions under different sampling systems.

Therefore, the aim of this article is to propose and investigate the properties of sometimes-pool shrinkage estimators of the entropy function of exponential distribution, given below under symmetric and asymmetric loss functions using progressive type II censored sample.

$$f(x;\theta)=\frac{1}{\theta}e^{-\frac{x}{\theta}} \qquad , \quad x\geq 0 \ ;\theta>0 \qquad (2)$$

In progressive censored type II sample the experimenter put n identical units, with life time distribution having cumulative function F(x) and probability density function f(x), for life testing. After observing the first failure, R_1 surviving units are removed from the test at random; next, immediately following the second failure, R_2 surviving units are removed from the test at random, and et cetera. Lastly, at the time of the m^{th} failure all the remaining $R_m = n - m - \sum_{i=1}^{m-1} R_i$ surviving units are removed from the test. In this progressive censoring scheme $R_1, R_2, \ldots, R_m$ are assumed to be pre-fixed . The complete sample and usual censored sample are special cases of type II progressive censored scheme. If $R_1=R_2= \ldots = R_m=0$ we get complete sample and if $R_1=R_2= \ldots = R_{m-1}=0$, $R_m=$ n-m we get type II censored sample. Balakrishnan (2007) gave a complete idea and recent developments on progressive censored sample.

2. Sometimes-pool Shrinkage Estimators of H (f)

The expression of entropy function for exponential distribution, defined in (2) one can be obtained as follows

$$H(f) = 1 + \ln(\theta) \qquad (3)$$

Cleary estimating H (f) is equivalent to estimating ln (θ). Let I (θ) =ln (θ).

Let $X_{1:m:n}, X_{2:m:n}, \ldots, X_{m:m:n}$ be Type II progressive censored sample from the exponential distribution (2). Then the joint density of the progressive censored sample (see Balakrishnan and Aggarwala 2000) is

$$f(x_{1:m::n}, x_{2:m:n}, \ldots, x_{m:m:n}) = C \prod_{i=1}^{m} f(x_{i:m:n})(1 - F(x_{i:m:n}))^{R_i}, \ 0 \le x_{1:m::n} \le x_{2:m:n} \le \ldots \le x_{m:m:n},$$

(4)

where C= n (n-R_1-1)(n-R_1-R_2-2) . . . (n-R_1-R_2- . . . – R_{m-1}-m+1) .

$$f(x_{1:m::n}, x_{2:m:n}, \ldots, x_{m:m:n}) = C\left(\frac{1}{\theta}\right)^m \exp\left(-\frac{\sum_{i=1}^{m}(R_i+1)\, x_{i:m:n}}{\theta}\right), \ 0 \le x_{1:m::n} \le x_{2:m:n} \le \ldots \le x_{m:m:n}$$

(5)

Then the MLE of θ is

$$\hat{\theta} = \frac{\sum_{i=1}^{m}(R_i+1)\, x_{i:m:n}}{m} \qquad (6)$$

Now, by replacing θ by its MLE $\hat{\theta}$ in ln (θ) we get the MLE of the entropy function of exponential distribution as follows

$$\hat{I}(\theta) = \ln(\hat{\theta}) \qquad (7)$$

We can show that the distribution of $\hat{\theta}$ has distribution as (see Balakrishnan and Aggarwala 2000)

$$f(\hat{\theta};\theta) = \left(\frac{m}{\theta}\right)^m \frac{\hat{\theta}^{m-1}\exp(-\frac{m\hat{\theta}}{\theta})}{\Gamma(m)}, \hat{\theta} > 0 \qquad (8)$$

Now, let ${}_1X_{i:m_1:n_1}, (i = 1,2,...,m_1)$ with schemes $R_{1i}, (i = 1,2,...,m_1)$ and ${}_2X_{j:m_2:n_2}, (j = 1,2,...,m_2)$ with schemes $R_{2j}, (j = 1,2,...,m_2)$ be the two independent progressive type II censored samples of sizes m_1 and m_2 respectively, drawn from exponential distribution defined in (2). Then by (6) the MLE estimators for the scale parameter θ based on the first sample and the second sample are given, respectively, by

$$\hat{\theta}_1 = \frac{\sum_{i=1}^{m_1}(R_i + 1)\, {}_1x_{i:m:n}}{m_1} \qquad (9)$$

$$\hat{\theta}_2 = \frac{\sum_{j=1}^{m_2}(R_i + 1)\, {}_2x_{j:m:n}}{m_2}, \qquad (10)$$

and the pooled estimator of θ is given by

$$\hat{\theta}_p = \frac{m_1\hat{\theta}_1 + m_2\hat{\theta}_2}{m_1 + m_2}. \qquad (11)$$

Now, by replacing θ by its MLE $\hat{\theta}$ of the first sample, second sample and pooled estimator in $I(\theta)$ we get the MLE of $I(\theta)$ first, second and pooled estimator respectively as

$\hat{I}_1(\theta) = ln(\hat{\theta}_1)$, (12) $\hat{I}_2(\theta) = ln(\hat{\theta}_2)$
(13)

and $\hat{I}_P(\theta) = ln(\hat{\theta}_P)$. (14)

Based on the testing of hypothesis H_0: $\theta=\theta_0$ against H_1:$\theta\neq\theta_0$ at level α, if H_0 accepted we take the shrinkage estimator $k\, ln(\hat{\theta}_1) + (1-k)\, ln(\theta_0)$, otherwise we take the estimator defined in (7). Therefore, we proposed the following sometimes-pool shrinkage estimators,

$$\tilde{I}_1(\theta) = \begin{cases} k_2\, Ln(\hat{\theta}_1) + (1-k_2)\, Ln(\theta_0) & H_0: \theta = \theta_0 \text{ Accepted} \\ Ln(\hat{\theta}_P) & \text{otherwise} \end{cases} \quad (15)$$

where k_1 is a constant such that $0 \le k_1 \le 1$. An alternative way to choose for shrinkage factor is to minimize mean square error function for estimator above with respected k_1, in this case we get k_2 a variable and a second proposed shrinkage estimator is

$$\tilde{I}_2(\theta) = \begin{cases} k_2\, Ln(\hat{\theta}_1) + (1-k_2)\, Ln(\theta_0) & H_0: \theta = \theta_0 \text{ Accepted} \\ Ln(\hat{\theta}_P) & \text{otherwise} \end{cases} \quad (16)$$

In the next section, we derive risk functions of the estimators with asymmetric loss function (LINEX loss function (LLF)) and also with symmetric loss function (square error loss function (SELF)). As many real life situations overestimation or underestimation are not of equal consequences and hence for this cases, a beneficial asymmetric loss function is LINEX loss function was defined by Varian (1975). It is given by

$$L(\Delta) = b(e^{a\Delta} - a\,\Delta - 1)$$

(17)

Where $\Delta = \hat{\theta} - \theta$ and a, b are the shape parameter and scale parameter respectively. The sign and values of a respectively represent the direction and degree of asymmetry, respectively. The positive value of a is applied when overestimation is more substantial than underestimation and negative value is used for the other states.

2.Risk of the Estimators

2.1 The Risk of the MLE estimator, $\hat{I}_P(\theta)$

The risk of the estimator $\hat{I}(\theta)$ under LLF is defined as follows.

$R_{LLF}(\hat{I}_p(\theta)) = E(\hat{I}_P(\theta) \mid LLF)$

$$R_{LLF}(\hat{I}_P(\theta)) = \frac{\Gamma\left(m_1\left(\frac{a}{(m_1+m_2)}+m_1\right)\right)}{m_1^{\frac{am_1}{m_1+m_2}}\,\Gamma(m_1)}\,\frac{\Gamma\left(m_2\left(\frac{a}{(m_1+m_2)}+m_2\right)\right)}{m_2^{\frac{am_2}{m_1+m_2}}\,\Gamma(m_2)} - a\left[\frac{m_1}{m_1+m_2}\left(\psi(m_1)-\ln(m_1)\right)+\frac{m_2}{m_1+m_2}\left(\psi(m_2)-\ln(m_2)\right)\right]-1 \qquad (18)$$

where $\psi(n) = \dfrac{\frac{d}{dn}\Gamma(n)}{\Gamma(n)}$

Moreover, it is possible to defined the risk of estimator $\hat{R}(t)$ under SELF as

$R_{SELF}(\hat{I}_p(\theta)) = E(\ln(\hat{\theta}_P) - \ln(\theta))^2$

$$\begin{aligned} R_{SELF}(\hat{I}_P(\theta)) = {} & \frac{m_1^2}{m_1+m_2}\left[\left[G(0,\infty,(\ln(x))^2,m_1)\right]^2 - 2\ln(x)\,\psi(m_1) + \left(\ln(m_1)\right)^2\right] \\ & + \frac{m_2^2}{m_1+m_2}\left[\left[G(0,\infty,(\ln(x))^2,m_2)\right]^2 - 2\ln(x)\,\psi(m_2) + \left(\ln(m_2)\right)^2\right] \\ & + \frac{2m_1m_2}{m_1+m_2}\left[\left(\psi(m_1)-\ln(m_1)\right)\left(\psi(m_2)-\ln(m_2)\right)\right] , \end{aligned}$$

(19) where $G(t_1,t_2,W,n) = \dfrac{\int_{t_1}^{t_2} W\, x^{n-1} e^{-x}\, dx}{\Gamma(n)}$ and W is function of x .

2.2 The Risk of the sometimes-pool shrinkage estimator $\tilde{I}_1(\theta)$

The risk of the estimator $\tilde{I}_1(\theta)$ under LLF is defined as follow:

$R_{GELF}(\tilde{I}_1(t)) = E(\tilde{I}_1(\theta) \mid LLF)$

$$R_{LLF}(\tilde{I}_1(\theta)) = \frac{\lambda^a \Gamma(ak_1+m_1)}{(\lambda_1)^{ak_1}\Gamma(m_1)}\left[I(r_2',ak_1+m_1)-I(r_1',ak_1+m_1)\right]-a\ln(\lambda)\left[I(r_2',m_1)-I(r_1',m_1)\right]$$

$$-ak\left[G(r_1',r_2',\ln(x),m_1)-\ln(\lambda m_1)\left(I(r_2',m_1)-I(r_1',m_1)\right)\right]$$

$$+\frac{\Gamma(m_1(\frac{a}{(m_1+m_2)}+m_1)}{m_1^{\frac{am_1}{m_1+m_2}}\Gamma(m_1)}\frac{\Gamma(m_2(\frac{a}{(m_1+m_2)}+m_2)}{m_2^{\frac{am_2}{m_1+m_2}}\Gamma(m_2)}$$

$$-a\left[\frac{m_1}{m_1+m_2}\left(\psi(m_1)-\ln(m_1)\right)+\frac{m_2}{m_1+m_2}\left(\psi(m_2)-\ln(m_2)\right)\right]-1$$

$$-\frac{\Gamma(m_1(\frac{a}{(m_1+m_2)}+m_1)}{m_1^{\frac{am_1}{m_1+m_2}}\Gamma(m_1)}\frac{\Gamma(m_2(\frac{a}{(m_1+m_2)}+m_2)}{m_2^{\frac{am_2}{m_1+m_2}}\Gamma(m_2)}\left[I(r_2',m_1(\frac{a}{(m_1+m_2)}+m_1)-I(r_1',m_1(\frac{a}{(m_1+m_2)}+m_1)\right]$$

$$+a\left[\frac{m_1}{m_1+m_2}\left(G(r_1',r_2',\ln(x),m_1)-\ln(m_1)\left[I(r_2',m_1)-I(r_1',m_1)\right]\right)+\frac{m_2}{m_1+m_2}\left(\psi(m_2)-\ln(m_2)\right)\left[I(r_2',m_1)-I(r_1',m_1)\right]\right], \quad (20)$$

where $r_1' = \frac{2\chi_1^2}{\lambda}$, $r_2' = \frac{2\chi_2^2}{\lambda}$, $\lambda = \frac{\theta_0}{\theta}$ and I(x, n) is incomplete gamma distribution given by

$$I(x,n) = \frac{\int_0^x t^{n-1} e^{-t} dt}{\Gamma(n)}.$$

The risk of the estimator $\tilde{I}_1(\theta)$ under SELF is given by

$R_{SELF}(\tilde{I}_1(\theta)) = E(\tilde{I}_1(\theta) - I(\theta))^2$

$$\mathbf{R_{SELF}(\tilde{I}_1(\theta)) = k_1^2\left[G(r_1',r_2',(\ln(x))^2) - 2\ln(\lambda m_1)G(r_1',r_2',\ln(x)) + (\ln(\lambda m_1))^2\left(I(r_2',m_1) - I(r_1',m_1)\right)\right]}$$

$$+2k\ln(\lambda)\Big(G(r_1',r_2',\ln(x))-\ln(\lambda m_1)\big(I(r_2',m_1)-I(r_1',m_1)\big)\Big)$$
$$+\frac{m_1^2}{m_1+m_2}\Big[\big[G(0,\infty,(\ln(x))^2,m_1)\big]^2-2\ln(m_1)\psi(m_1)+(\ln(m_1))^2\Big]$$
$$+\frac{m_2^2}{m_1+m_2}\Big[\big[G(0,\infty,(\ln(y))^2,m_2)\big]^2-2\ln(m_2)\psi(m_2)+(\ln(m_2))^2\Big]$$
$$+\frac{2m_1m_2}{m_1+m_2}\big[(\psi(m_1)-\ln(m_1))(\psi(m_2)-\ln(m_2))\big]$$
$$-\frac{m_1^2}{m_1+m_2}\Big[\big[G(r_1',r_2',(\ln(x))^2,m_1)\big]^2-2\ln(m_1)G(r_1',r_2',(\ln(x)),m_1)+(\ln(m_1))^2\big(I(r_2',m)-I(r_1',m)\big)\Big]$$
$$-\frac{m_2^2}{m_1+m_2}\Big[\big[G(0,\infty,(\ln(y))^2,m_2)\big]^2-2\ln(m_2)\psi(m_2)+(\ln(m_2))^2\Big]\big(I(r_2',m)-I(r_1',m)\big)$$
$$-\frac{2m_1m_2}{m_1+m_2}\Big[(\psi(m_2)-\ln(m_2))\Big(G(r_1',r_2',(\ln(x)),m_1)+(\ln(m_1))\big(I(r_2',m)-I(r_1',m)\big)\Big)\Big]$$

(21)

2.3 The Risk of the sometimes-pool shrinkage estimator $\tilde{I}_2(\theta)$

Minimizing the risk function under SELF with respect k_1 defined in (21), gives us

$$k^*=\frac{\ln(\lambda)\Big[\ln(\lambda m_1)\big(I(r_2',m_1)-I(r_1',m_1)\big)-G(r_1',r_2',\ln(x))\Big]}{G(r_1',r_2',(\ln(x))^2)-2\ln(\lambda m_1)G(r_1',r_2',\ln(x))+(\ln(\lambda m_1))^2\big(I(r_2',m_1)-I(r_1',m_1)\big)} \quad (22)$$

However, we are not able to prove that k^* lies between 0 and 1 theoretically, thus, we can restrict the value of 'k' by

$$k_2=\begin{cases}0 & k^*<0\\ k^* & 0\le k^*\le 1\\ 1 & k^*>1\end{cases} \qquad (23)$$

Now , by replacing k_1 by k_2 we get the estimator $\tilde{I}_2(\theta)$, defined in (16) , also by replacing k_1 by k_2 in (21) we get the risk function under SELF for the estimator $\tilde{I}_2(\theta)$.

3. Relative Risk

To study the properties of estimators $\tilde{I}_1(\theta)$ and $\tilde{I}_2(\theta)$ under SELF and LLF, we compare the relative risks of the estimators given above. The relative risk of $\tilde{I}_1(\theta)$ with respect to $\hat{I}_p(\theta)$ under LLF is

$$RR_{LLF}(\tilde{I}_1(\theta)) = \frac{R_{LLF}(\hat{I}_p(\theta))}{R_{LLF}(\tilde{I}_1(\theta))} \quad (24)$$

Also, the relative risk of $\tilde{I}_1(\theta)$ with respect to $\hat{I}_p(\theta)$ under SELF is

$$RR_{SELF}(\tilde{I}_1(\theta)) = \frac{R_{SELF}(\hat{I}_p(\theta))}{R_{SELF}(\tilde{I}_1(\theta))} \quad (25)$$

Finally, the relative risk of $\tilde{I}_2(\theta)$ with respect to $\hat{I}_p(\theta)$ under SELF is

$$RR_{SELF}(\tilde{I}_2(\theta)) = \frac{R_{SELF}(\hat{I}_p(\theta))}{R_{SELF}(\tilde{I}_2(\theta))} \quad (26)$$

We observe that equations $RR_{LLF}(\tilde{I}_1(\theta))$, $RR_{SELF}(\tilde{I}_1(\theta))$ and $RR_{SELF}(\tilde{I}_2(\theta))$ mainly depend on a, α , k_1, m_1,m_2 and λ. To assess the performance of the suggested estimators under LLF and SELF, we assume the some following sets of constants

$a=1, 2, 3, \alpha=0.01, 0.05$ $k_1=0.2, 0.4, 0.6$ $m_1= 6, 9, 12$, $m_2=3, 6, 9$, $\lambda= 0.25(0.25)1.75$

The computed values of the relative risks of the proposed estimators are presented and tabled in Tables 1 – 3. From Table 1-3, we obtain the following the conclusions.

4. Conclusion

i. The relative risk of the both estimators we proposed under SELF and LLF are high in around $\lambda=1$. i.e. if the true value of θ is close to θ_0.Further the rang of the relative risk is greater than one and become small as m increases.

ii. The relative risk of the estimator $\tilde{I}_1(\theta)$ under LLF is higher than the relative risk of the estimator $\tilde{I}_1(\theta)$ under SELF.

iii. The relative risk of the estimator $\tilde{I}_2(\theta)$ under SELF is higher than the relative risk of the estimator $\tilde{I}_1(\theta)$ under SELF.

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iv. The relative risk of the estimator $\tilde{I}_1(\theta)$ under LLF is increasing function of a for $0.25 \le \lambda \le 0.50$ and decreases for $0.0.75 \le \lambda \le 1.75$ except for $k_1=0.2$, $m_1=6$.

v. The relative risk of the estimator $\tilde{I}_1(\theta)$ under SELF and LLF are increasing function of m_1. Moreover, it decreases with m_2. It is a increasing function of k_1, for $0.25 \le \lambda \le 0.50$. But, it is a decreasing function of other value of λ.

vi. The relative risk of the estimator $\tilde{I}_2(\theta)$ under SELF is an increasing function of $m_1$for $0.25 \le \lambda \le 1$ and decreasing function of m_2.

vii. The relative risk of the estimator $\tilde{I}_1(\theta)$ under SELF and LLF and the relative risk of the estimator $\tilde{I}_2(\theta)$ under SELF are decreasing functions of α.

Thus, in general, we observe that our proposed estimators perform better than the classical estimator and single stage shrinkage estimator underboth SELF and LLF.

Appendix

Table (1) show the relative risk of the estimator $\tilde{I}_1(t)$ under LLF when α=0.01

	λ									
m1	m2	k	a	0.25	0.50	0.75	1.00	1.25	1.50	1.75
6	3	0.2	1	0.5181	0.4847	1.6299	9.0387	2.6745	0.9309	0.4924
			2	0.5898	0.5157	1.5840	8.7808	2.4702	0.8027	0.4026
			3	0.6551	0.5569	1.5706	8.4024	2.3405	0.7128	0.3376
		0.4	1	0.5767	0.5901	1.6196	3.7012	2.4695	1.2532	0.7424
			2	0.6414	0.6163	1.6040	3.6034	2.2477	1.0891	0.6240
			3	0.7003	0.6522	1.6108	3.5653	2.0900	0.9666	0.5342
		0.6	1	0.6448	0.7101	1.3950	1.8886	1.7061	1.3069	0.9755
			2	0.7007	0.7336	1.4162	1.8428	1.5539	1.1395	0.8307
			3	0.7513	0.7642	1.4511	1.8285	1.4368	1.0051	0.7131
	6	0.2	1	0.4054	0.3666	1.2783	8.2334	2.1371	0.7171	0.3753
			2	0.4776	0.4022	1.2904	8.1123	1.9545	0.6171	0.3071
			3	0.5454	0.4454	1.3283	8.0193	1.8307	0.5436	0.2559
		0.4	1	0.4548	0.4497	1.2696	2.9450	1.9595	0.9835	0.5782

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			2	0.5235	0.4854	1.3083	2.9107	1.7681	0.8478	0.4830
			3	0.5878	0.5283	1.3671	2.9284	1.6261	0.7435	0.4091
			1	0.5133	0.5457	1.0828	1.4409	1.3192	1.0289	0.7755
		0.6	2	0.5772	0.5845	1.1420	1.4276	1.2005	0.8890	0.6521
			3	0.6365	0.6284	1.2147	1.4303	1.1025	0.7742	0.5512
			1	0.3350	0.2952	1.0462	7.3907	1.7677	0.5811	0.3024
		0.2	2	0.4037	0.3299	1.0781	7.3273	1.6106	0.5003	0.2479
			3	0.4694	0.3706	1.1307	7.3360	1.4986	0.4389	0.2058
			1	0.3781	0.3639	1.0389	2.428	1.6142	0.8051	0.4713
	9	0.4	2	0.4452	0.4007	1.0938	2.4196	1.452	0.6922	0.3931
			3	0.509	0.4432	1.1661	2.4489	1.327	0.6032	0.331
			1	0.4297	0.4441	0.8811	1.161	1.0712	0.8438	0.6395
		0.6	2	0.4943	0.4862	0.9487	1.1603	0.9758	0.7267	0.5349
			3	0.5552	0.5326	1.028	1.1666	0.8928	0.6286	0.4483
			1	0.7238	0.4294	1.2805	9.0813	2.007	0.6754	0.3634
		0.2	2	0.783	0.464	1.2576	8.8499	1.879	0.5922	0.3022
			3	0.8284	0.5048	1.2542	8.4615	1.7926	0.5295	0.255
			1	0.7686	0.5279	1.4012	3.9669	2.1386	0.9552	0.5492
	3	0.4	2	0.8181	0.5578	1.3836	3.8723	1.9875	0.853	0.4742
			3	0.8564	0.5938	1.3805	3.8124	1.8735	0.7724	0.414
			1	0.8164	0.6521	1.3485	2.063	1.709	1.1484	0.7854
		0.6	2	0.8555	0.676	1.353	2.017	1.581	1.0325	0.6949
			3	0.8859	0.705	1.3656	1.9966	1.478	0.9344	0.6171
9			1	0.6387	0.346	1.0694	8.7407	1.6964	0.5565	0.2971
		0.2	2	0.7078	0.3816	1.0791	8.6095	1.5738	0.4859	0.2465
			3	0.7627	0.4221	1.1049	8.439	1.4867	0.4314	0.2067
			1	0.6834	0.4277	1.177	3.3925	1.8164	0.8025	0.458
	6	0.4	2	0.7442	0.4619	1.1967	3.3456	1.6699	0.7101	0.3925
			3	0.7926	0.5008	1.2292	3.3394	1.5567	0.636	0.3394
			1	0.7318	0.5321	1.1298	1.6939	1.4289	0.9782	0.6722
		0.6	2	0.7833	0.5648	1.1681	1.6729	1.313	0.8682	0.5866
			3	0.8246	0.6013	1.2145	1.6695	1.217	0.775	0.5132
	9	0.2	1	0.5741	0.2906	0.9149	8.2559	1.462	0.4718	0.2507
			2	0.6481	0.3248	0.9387	8.174	1.3499	0.4113	0.2078

			3	0.7083	0.3633	0.9762	8.1102	1.2672	0.3636	0.1736
		0.4	1	0.6181	0.3608	1.0107	2.9436	1.5703	0.6889	0.3913
			2	0.685	0.3954	1.0465	2.9201	1.4352	0.6066	0.3341
			3	0.7395	0.434	1.0938	2.9322	1.3284	0.5397	0.2871
		0.6	1	0.6665	0.4513	0.9686	1.4328	1.2229	0.8471	0.584
			2	0.7252	0.4868	1.0201	1.4237	1.1200	0.7464	0.5055
			3	0.7732	0.5255	1.0798	1.4256	1.0325	0.6606	0.4382
12	3	0.2	1	0.8814	0.4143	1.0568	9.0125	1.6053	0.5346	0.2975
			2	0.9126	0.452	1.0483	8.8096	1.5133	0.4728	0.2496
			3	0.9338	0.4937	1.0531	8.4488	1.448	0.4241	0.211
		0.4	1	0.9041	0.5074	1.2195	4.0905	1.8505	0.7688	0.4447
			2	0.9288	0.5407	1.2074	4.0045	1.7423	0.6965	0.3886
			3	0.9459	0.5781	1.2059	3.9394	1.6575	0.6374	0.3421
		0.6	1	0.9267	0.6281	1.2636	2.1543	1.6416	0.9962	0.6536
			2	0.9452	0.6548	1.2624	2.1119	1.5376	0.9139	0.5898
			3	0.9581	0.685	1.2671	2.0899	1.4509	0.8417	0.5335
	6	0.2	1	0.839	0.3472	0.915	8.8852	1.4033	0.4585	0.2534
			2	0.879	0.3843	0.926	8.7568	1.3125	0.4038	0.2119
			3	0.9067	0.425	0.9481	8.5589	1.2454	0.3599	0.1783
		0.4	1	0.864	0.4269	1.0643	3.6413	1.6329	0.6714	0.3853
			2	0.8971	0.4621	1.0771	3.5915	1.5215	0.6032	0.3345
			3	0.9205	0.5007	1.0991	3.5722	1.4329	0.5468	0.292
		0.6	1	0.8892	0.5311	1.1051	1.8472	1.4369	0.8857	0.5804
			2	0.9156	0.5632	1.1302	1.8239	1.3345	0.8025	0.5178
			3	0.9345	0.598	1.1606	1.8169	1.248	0.7299	0.4626
	9	0.2	1	0.802	0.2999	0.805	8.615	1.2419	0.4004	0.2204
			2	0.8489	0.3354	0.8257	8.5279	1.156	0.3518	0.184
			3	0.8819	0.3741	0.856	8.4241	1.0906	0.3123	0.1542
		0.4	1	0.829	0.3699	0.9415	3.262	1.4546	0.5937	0.3389
			2	0.8687	0.405	0.9672	3.232	1.3467	0.5307	0.293
			3	0.8972	0.4431	1.0012	3.2323	1.2594	0.478	0.2544
		0.6	1	0.8564	0.4623	0.9791	1.6128	1.2729	0.7931	0.5193
			2	0.8891	0.4964	1.0173	1.5998	1.176	0.7129	0.4597
			3	0.9128	0.5327	1.061	1.5986	1.093	0.6428	0.4072

Table (2) show the relative risk of the estimator $\tilde{I}_1(t)$ under SELF

α=0.01			λ						
m_1	m_2	k	0.25	0.50	0.75	1.0	1.25	1.50	1.75
6	3	0.2	0.4394	0.4645	1.7243	9.1115	2.9342	1.1029	0.6138
		0.4	0.5055	0.5752	1.6705	3.8583	2.7411	1.4586	0.8929
		0.6	0.5829	0.695	1.3944	1.9676	1.8909	1.5046	1.1454
	6	0.2	0.329	0.3378	1.297	8.3637	2.3799	0.8504	0.4661
		0.4	0.3815	0.4206	1.2543	3.0323	2.2028	1.1563	0.6997
		0.6	0.4441	0.5113	1.0373	1.4703	1.4605	1.1971	0.9244
	9	0.2	0.2643	0.2657	1.036	7.5195	1.9757	0.6867	0.3734
		0.4	0.3083	0.3321	1.0009	2.4743	1.8191	0.9478	0.5702
		0.6	0.3614	0.4053	0.8236	1.1681	1.1801	0.9833	0.7658
9	3	0.2	0.646	0.401	1.3307	9.1058	2.1674	0.7817	0.4415
		0.4	0.7037	0.5047	1.4411	4.094	2.315	1.0776	0.6402
		0.6	0.7654	0.6341	1.3573	2.1353	1.857	1.2778	0.8865
	6	0.2	0.5519	0.3151	1.0789	8.8134	1.8539	0.6462	0.3612
		0.4	0.6069	0.3982	1.1731	3.4803	1.9946	0.9149	0.5378
		0.6	0.6668	0.5029	1.1015	1.7328	1.5647	1.1049	0.7703
	9	0.2	0.4841	0.2602	0.9057	8.347	1.6057	0.548	0.3044
		0.4	0.5365	0.33	0.9873	3.0031	1.7356	0.7888	0.4608
		0.6	0.5944	0.4185	0.9252	1.4529	1.3422	0.9641	0.6746
12	3	0.2	0.8338	0.3809	1.0826	9.0195	1.7191	0.6109	0.3567
		0.4	0.867	0.4784	1.2469	4.1947	1.9743	0.8536	0.511
		0.6	0.8996	0.6053	1.2742	2.2176	1.7575	1.0857	0.7239
	6	0.2	0.7797	0.3135	0.917	8.9268	1.517	0.5258	0.3044
		0.4	0.8155	0.3949	1.0628	3.7212	1.7644	0.752	0.4454
		0.6	0.8512	0.5016	1.0873	1.8871	1.5538	0.9784	0.6503
	9	0.2	0.7342	0.2673	0.7948	8.6765	1.3492	0.4599	0.2648
		0.4	0.7721	0.3377	0.9251	3.3225	1.5831	0.6682	0.3931
		0.6	0.8102	0.4304	0.9471	1.6379	1.3837	0.8835	0.5863
α=0.05									
6	3	0.2	0.6153	0.5459	1.4979	4.2172	1.9318	0.899	0.5637
		0.4	0.6613	0.6248	1.4587	2.8234	1.9255	1.1018	0.7347
		0.6	0.7097	0.7033	1.2965	1.8206	1.6027	1.1968	0.9056

	6	0.2	0.4868	0.3993	1.1825	4.5043	1.7601	0.7443	0.4502
		0.4	0.5277	0.4589	1.1482	2.5861	1.7528	0.9475	0.6095
		0.6	0.5715	0.5188	1.0085	1.5126	1.3934	1.048	0.7817
	9	0.2	0.4054	0.3161	0.9729	4.5982	1.5755	0.6265	0.3712
		0.4	0.4422	0.3645	0.943	2.3262	1.5679	0.8166	0.5142
		0.6	0.482	0.4133	0.8223	1.2757	1.2083	0.9141	0.6763
9	3	0.2	0.8262	0.5156	1.2109	3.9969	1.5125	0.6845	0.4514
		0.4	0.8541	0.5966	1.2717	2.8115	1.6142	0.8558	0.5827
		0.6	0.8816	0.687	1.2332	1.8814	1.4824	0.9976	0.7369
	6	0.2	0.7517	0.4067	1.0092	4.3102	1.4027	0.5924	0.3805
		0.4	0.7818	0.4721	1.0642	2.7141	1.5174	0.7633	0.5044
		0.6	0.8119	0.5455	1.0293	1.6783	1.3694	0.913	0.6588
	9	0.2	0.6925	0.3376	0.864	4.5046	1.2862	0.5177	0.3272
		0.4	0.7239	0.3931	0.9135	2.5688	1.4056	0.6811	0.4415
		0.6	0.7557	0.4557	0.8821	1.4968	1.2519	0.8301	0.5898
12	3	0.2	0.9416	0.524	1.0264	3.8531	1.2609	0.5725	0.4054
		0.4	0.9522	0.6035	1.125	2.7802	1.3988	0.7203	0.517
		0.6	0.9624	0.6946	1.1536	1.8989	1.3665	0.8679	0.6553
	6	0.2	0.9146	0.4342	0.8839	4.1444	1.1809	0.5081	0.3525
		0.4	0.927	0.5015	0.9741	2.7445	1.332	0.6545	0.458
		0.6	0.9389	0.579	1.0005	1.7559	1.2961	0.8079	0.5947
	9	0.2	0.8906	0.3727	0.776	4.3583	1.0979	0.4542	0.3109
		0.4	0.9045	0.4317	0.8589	2.6644	1.2547	0.5953	0.4095
		0.6	0.918	0.5	0.8832	1.617	1.2171	0.7486	0.5413

Table (3)show the relative risk of the estimator $\tilde{I}_2(t)$ under SELF

α=0.01		λ						
m_1	m_2	0.25	0.50	0.75	1.0	1.25	1.50	1.75
6	3	0.3834	0.3729	1.5124	16.6832	2.1891	0.7531	0.4166
	6	0.2851	0.27	1.1295	20.2052	1.7141	0.5655	0.3095
	9	0.228	0.2117	0.8988	23.476	1.3954	0.4503	0.2452
9	3	0.5925	0.3212	1.0995	15.3829	1.5956	0.5444	0.3089
	6	0.5018	0.2515	0.884	18.0152	1.3276	0.4409	0.2484
	9	0.4373	0.2072	0.7382	20.5155	1.13	0.3692	0.2072

12	3	0.8005	0.307	0.8696	14.628	1.2585	0.4315	0.255
	6	0.7441	0.2521	0.7307	16.7263	1.086	0.3652	0.2146
	9	0.697	0.2145	0.6298	18.7458	0.9513	0.3159	0.185
α=0.05								
6	3	0.5719	0.4725	1.3938	5.0477	1.6159	0.6898	0.4268
	6	0.449	0.3443	1.092	5.9837	1.4074	0.55	0.3309
	9	0.3717	0.2718	0.8942	6.818	1.2219	0.4528	0.2684
9	3	0.7982	0.4452	1.0758	4.6506	1.2507	0.5321	0.3503
	6	0.722	0.3502	0.8889	5.3611	1.1212	0.4486	0.2895
	9	0.6617	0.29	0.7566	6.0156	1.003	0.3853	0.2457
12	3	0.9305	0.4557	0.8906	4.4219	1.0349	0.449	0.3202
	6	0.9018	0.3767	0.761	4.9935	0.9437	0.3909	0.2745
	9	0.8762	0.3226	0.6644	5.5303	0.8598	0.3447	0.2397

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