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Comparative Between Backstepping and Adaptive Backstepping Control for Controlling Prosthetic Knee

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Abstract— *The lower limb amputees are increasing day by day. This has led to an increase in research in the field of prosthetic knee. In this work, a prosthetic knee was designed and developed to assist human movements and more quality of life for millions of individuals who have lost lower limbs. The dynamic model and parameter identification of a two degree of freedom (2-DOF) joint prosthetic knee is derived according to the Lagrangian dynamic approach. The two controllers Backstepping and Adaptive Backstepping are adopted to control the system. Stability analysis and controller design dependent on Lyapunov theory are assessed to prove a tracking of a desired trajectory. From the results, found that the quantitative comparison between the two controllers, showed significant improvement in results in position tracking. To comparison between Backstepping control and Adaptive Backstepping control, at the control action consumptions. It was found that the position error of the prosthetic knee in Backstepping control is by 9% at link 1 (thigh) and 7.4% at link 2 (shank) compared with desired trajectory, while in Adaptive Backstepping control is by 1.16% at link 1 and 1.65% at link 2 compared with desired trajectory. When comparing between Backstepping control and Adaptive Backstepping control, the improvement rate was 7.84 at link 1 and 5.75 at link 2 , the proposed Adaptive Backstepping control, it may be concluded, is more robust against this perturbation and to deal with uncertainty. Therefore, the controller is built in a MATLAB environment, and its performance and robustness are assessed.*

Index Terms— *Prosthetic knee, Backstepping control, Adaptive Backstepping control, Lyapunov theory.*

I. INTRODUCTION

Millions of people have had difficulty using their lower limbs in recent. As a result, some of them have lost their ability to work and are unable to participate in normal social activities [1]. Accidents, cancer, diabetes, vascular disease, congenital deformities, and paralysis are among reasons for amputation. Transtibial (below the knee), transfemoral (above the knee), and foot amputations, as well as hip and knee disarticulations, are all examples of amputation (amputation through the joint). Amputees can try to reclaim their normal walking gait by wearing prosthetic legs [2, 3]. The tools available to people who lost their lower limbs were walkers, wheelchairs, wooden braces, and crutches. Nowadays, advances in medical science and technology can be used to help people with amputations using motorized lower limbs [4]. There are many difficult problems such as system uncertainty, high nonlinearity, and external perturbations, which can occur during movement, problems with imbalance, falls and sudden bending of the knee while standing. Since there are many control strategies used to control the movement of prosthetic limbs, including Backstepping Control (BC) and Adaptive Backstepping Control (ABC). Many researchers have also proposed some ways for controlling prosthetic limbs.

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In 2008, Scandaroli et al. [5] presented a design a prosthetic limb above the knee. Proportional–Integral–Derivative (PID) and model references Adaptive controllers are used in their models. They found that the results revealed difficulty in controlling such a nonlinear plant.

Chen et al. in 2015 [6], used Backstepping Adaptive Robust Control (ARC) algorithm for 1-DOF knee joint exoskeleton. A Backstepping control method it is proposed to bypass the bandwidth restriction of the commonly utilized cascade control by using the entire dynamics. The coupling effect between different layers of dynamics can be used to achieve a larger bandwidth. The proposed ARC algorithm delivers guaranteed force tracking performance in both transient and steady-state conditions. They should note, that the suggested adaptive robust Backstepping force controller not only provides a high level of robustness in the face of model uncertainty, but also provides faster closed loop responses and lower contact forces.

Mefoued et al. (2015) [7] a Second order Sliding Mode Control (SoSMC) was created to aid in the relocation of dependents. The wearer's desired movement was calculated in real time using the RBF neural network and electromyography data of the quadriceps muscle. This controller (SoSMC) was chosen because of its robustness in the face of parameter uncertainty, atypical dynamics, and external disturbances.

Wen et al. in 2016, presented Adaptive Dynamic Programming (ADP) based controller performance testing that automatically configures prosthetic control parameters. The system was evaluated on a physically healthy person, walking with an electrical prosthesis on a treadmill. The goal was for the user to be able to approximate conventional knee kinematics using ADP to alter the Finite State Impedance Control (FSIC) resistance values. They tested the practicality of ADP for adaptive control of a powered prosthesis and discovered that in about 10 minutes, the prosthetic controller could be tuned to provide modular kinematics of the knee [8].

In 2017, Yousefi et al. [9] developed a knee rehabilitation robot and control it. The system is controlled by two Sliding-Backstepping controllers in their models. The Sliding Backstepping controller is used to monitor the prescribed trajectory dictated by the expert physiotherapist, while the admittance control is used to create a delicate and smooth interface between the robot and the human leg in this hybrid control scheme. The results demonstrating the effectiveness of the proposed controller in terms of compensating for an involuntary leg movement and noise rejection.

In 2018, Khamar et al. [10] presented a Backstepping Sliding Control (BSC) approach in combination with a nonlinear observer for designing a knee exoskeleton to support human movements in knee flexion and extension. Based on the Lyapunov theory, the asymptotic stability of the given controller and the convergence of the nonlinear turbulence observer were mathematically validated. The benefits of NDO-based BSC were proven by simulation results. The durability and stability of the NDO-based SBC approach have been confirmed, improving tracking accuracy and reducing the time required to remove disturbance.

Zhang et al. (2020) [11] introduced an Electro Hydraulic Actuator (EHA) system with a robust adaptive backstepping sliding mode control technique. An adaptive backstepping sliding mode control (BSMC) strategy is used to handle the nonlinearity problem of changes in the dynamic system. The Lyapunov function verifies the stability of the control system. SMC and PID control schemes are also used in computer simulation to evaluate the performance and resilience of BSMC. When compared to PID and SMC, the simulation shows that this suggested control system has a high robust tracking.

In the present work the Backstepping and Adaptive Backstepping control strategy for a mathematical model of a system of 2-DoF that includes the thigh-leg. The BC is based on a control strategy for a

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certain sort of nonlinear systems. Due to its ability to handle the nonlinearity and uncertainty with high efficiency. The Lyapunov theory combined with BC improves the closed-loop system's dynamic performance while also ensuring its stability.

Backstepping and Adaptive Backstepping are a control strategy that can be applied to a certain type of nonlinear system. The Lyapunov theory in combination with the controllers ensures the closed-loop system's stability while also improving its dynamic performance. [12, 13].

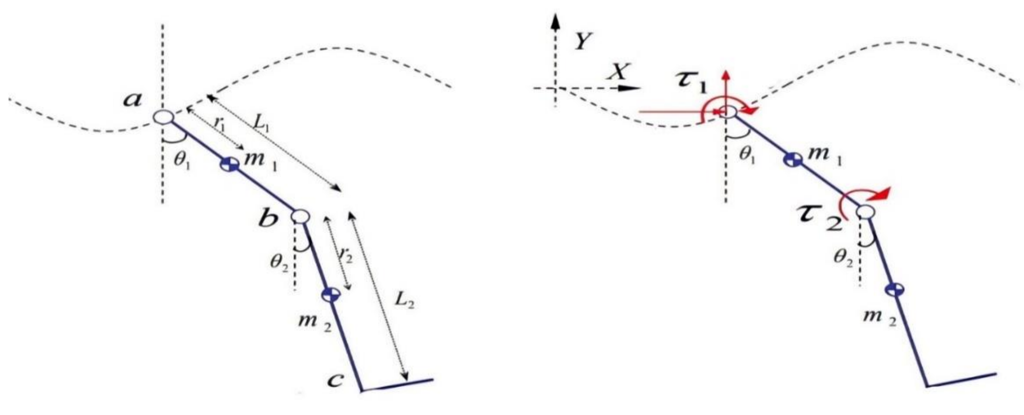
The aim of this study is to know how to design Backstepping and Adaptive Backstepping control so that it can arrange and control the tracking of intended walking patterns while limiting the effects of unknown disturbances, non-linear uncertainties in the system, and ensuring the prosthetic knee's stability

The Contribution for this research will be stated in this points:

1. This research showed other possibilities of imbalance, falling, and sudden bending of the knee (due to uncertainty in the system and high non-linearity) which were not discovered by previous researchers.
2. Design of Backstepping and Adaptive Backstepping controllers in order to stabilize the prosthesis knee, performed to analyze trajectory tracking and estimate the position and velocity states.

II. DYNAMIC MODEL OF PROSTHETIC KNEE.

Fig. (1a and 1b) shows the free body diagram of the prosthetic knee and the location of force effect on the prosthetic knee. The main objective is to derive the second order ordinary differential equations system. The motion of the prosthetic knee is controlled as a serial manipulator with rigid link, prosthetic knee can be modeled. In this case, it is easy to readily obtain the equations of motion. The method of Lagrangian can be used to obtain motion equation for a serial kinematic chain system [4].



(a) Schematic diagram to the artificial   (b) Free body diagram to the artificial knee

FIG. 1. SCHEMATIC DIAGRAM AND FREE BODY DIAGRAM OF THE PROSTHETIC KNEE.

Axes involving the displacement of a prosthetic knee about a fixed axis should be established in a cartesian coordinate system and specify the sign and direction of the x-axis and y-axis, as shown in *Fig. 1b.*

$$X_1 = r_1 sin\theta_1 \; Y_1 = r_1 cos\theta_1 \; X_2 = L_1 sin\theta_1 + r_2 sin\theta_2 \; Y_2 = L_1 cos\theta_1 + r_2 cos\theta_2 \quad (1)$$

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, where X_1 and Y_1 are the displacements for the x- and y-axes for joint1. In addition, X_2 and Y_2 are the displacements of the x-axis and y-axis for joint2.

To derive the displacements in equation (1) with respect to time, the components of velocity are obtained as equations (2).

$$\frac{d}{dt}X_1 = r_1\,\dot{\theta}_1 cos\theta_1 \;\; \frac{d}{dt}Y_1 = -r_1\,\dot{\theta}_1 sin\theta_1 \;\; \frac{d}{dt}X_2 = L_1\,\dot{\theta}_1 cos\theta_1 + r_2\,\dot{\theta}_2 cos\theta_1 \;\; \frac{d}{dt}Y_2 = -L_1\,\dot{\theta}_1 sin\theta_1 - r_2\,\dot{\theta}_2 sin\theta_2 \;\} \quad (2)$$

, where r_1 and r_2 are the distance between the center of mass of each link (thigh and shank), L_1 is the length of link 1, θ_1 and θ_2 are the rotation angle of link 1 and link 2, respectively.

Langragian's equation is used in this analysis to determine the equation of motion, the mathematical formula to Langragian's equation can be written as follows [14]:

$$L = KE - PE \quad (3)$$

$$KE = \frac{1}{2}\,m\,v^2 \quad (4)$$

, where L is defined as the difference between the kinetic energy (*KE)* and potential energy (*PE)* of the mechanical system,

The *KE* equation is the summation of kinetic energy for individual links, and can be expressed by the following formula:

$$KE = \frac{1}{2}\,m_1\,(\dot{X}_1^2 + \dot{Y}_1^2) + \frac{1}{2}\,I_1\dot{\theta}_1^2 + \frac{1}{2}\,m_2\,(\dot{X}_2^2 + \dot{Y}_2^2) + \frac{1}{2}\,I_2\dot{\theta}_2^2 \quad (5)$$

By substituting equation (2) into equation (5) to determine the total *KE* for two links

$$KE = \frac{1}{2}\,m_1((r_1\dot{\theta}_1\cos\theta_1)^2 + (\,r_1\,\dot{\theta}_1 sin\theta_1)^2) + \frac{1}{2}(\frac{mL^2}{12} * \dot{\theta}_1^2) + \frac{1}{2}\,m_2((L_1\dot{\theta}_1\cos\theta_1 + r_2\,\dot{\theta}_2\cos\theta_2)^2 + (L_1\dot{\theta}_1\sin\theta_1 + r_2\,\dot{\theta}_2\sin\theta_2)^2) + \frac{1}{2}(\frac{mL^2}{12} * \dot{\theta}_2^2) \quad (6)$$

In addition, PE is the potential energy of system can be written as:

$$PE = mgh \quad (7)$$

$$PE = m_1\,y_1\,g + m_2\,y_2\,g \quad (8)$$

$$PE = m_1 r_1 g\,\cos\theta_1 + m_1\,g\,(L_1\cos\theta_1 + r_2\cos\theta_2) \quad (9)$$

Substitute equation (6) and equation (9) into equation (3), to get the following equation:

$$L = \frac{1}{2}\,m_1 r_1^2\,\dot{\theta}_1^2 + \frac{1}{2}I_1\dot{\theta}_1^2 + \frac{1}{2}m_2(L_1^2\dot{\theta}_1^2 + r_2^2\dot{\theta}_2^2 + 2L_1r_2\dot{\theta}_1\dot{\theta}_2\,cos\,cos\,(\,\theta_1 - \theta_1)) + \frac{1}{2}\,I_2\dot{\theta}_2^2 - m_1 r_1 g\,\cos\theta_1 - m_1\,gL_1\cos\theta_1 + \;m_1\,g\,r_2\cos\theta_2) \quad (10)$$

The equations of motion for the manipulator are derived using the Lagrangian in equation (3) as the following:

$$\tau_{Total} = \frac{d}{dt}(\frac{\partial\,L}{\partial\dot{\theta}}) - \frac{\partial L}{\partial\theta} \quad (11)$$

, where τ is torque acting on the system to each joint. The hip τ_1 and knee τ_2 torque expressions can be written as:

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$$\tau_1 = (I_1 + m_1 r_1^2 + m_2 L_1^2 - m_2 L_1 r_2 \cos\cos(\theta_1 - \theta_2))\ddot{\theta_1} + (m_2 r_2^2 + I_2 + m_2 L_1 r_2 \cos\cos(\theta_1 - \theta_2))\ddot{\theta}_2 - (m_2 L_1 r_2 \sin\sin(\theta_1 - \theta_2))\dot{\theta}_1^2 + (m_2 L_1 r_2 \sin\sin(\theta_1 - \theta_2))\dot{\theta}_2^2 - m_1 g\, r_1 \sin\sin\theta_1 - m_2 g\, L_1 \sin\sin\theta_1 - m_2 g\, r_2 \sin\sin\theta_2 \quad (12)$$

$$\tau_2 = ((m_2 r_2^2 + I_2)\ddot{\theta}_2 + m_2 L_1 r_2 \ddot{\theta_1} \cos\cos(\theta_1 - \theta_2) - m_2 L_1 r_2 \dot{\theta}_1^2 \sin\sin(\theta_1 - \theta_2) - m_2 g\, r_2 \sin\sin\theta_2 - L_1 \sin\sin\theta_1 F_1 - L_2 \sin\sin\theta_2 F_2 \quad (13)$$

Assuming that there is no friction force, the dynamics model of the system can be expressed as general form below is [15].

$$\mathrm{M}(\theta)\,\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + G(\theta) = \tau \quad (14)$$

, where (θ) is an angular position vector, which is expected to be usable by measurement. $\mathrm{M}(\theta)$ represents the inertia matrix of the links, while τ is the control torque, $C(\theta, \dot{\theta})\dot{\theta}$ represents the vector of the Coriolis and centripetal torques, and $G(\theta)$ represents gravitational torque.

Equation (14) shows the nonlinear dynamics of the prosthetic knee system. The following can be represented using a state variable in the state equation:

$$x_1 = \theta_1, x_2 = \dot{\theta}_1\ x_3 = \theta_2, x_4 = \dot{\theta}_2\ \dot{x}_1 = \dot{\theta}_1, \dot{x}_2 = \ddot{\theta}_1, \dot{x}_3 = \dot{\theta}_2, \dot{x}_4 = \ddot{\theta}_2 \} \quad (15)$$

, where, $[\theta_1, \theta_2]$ is angular position of upper and lower link. $[\dot{\theta}_1, \dot{\theta}_2]$ which represent angular velocity of upper and lower link respectively [16].

Equation (15) can be substituted into Equation (14), which is a nonlinear dynamics equation, so Equation (14) can be written as:

$$\dot{x}_1 = x_2 \quad (16)$$

$$\dot{x}_2 = \frac{1}{M_{11}}[\tau_1 - M_{12}\,\dot{x}_4 - C_1\,x_2 - G_1] \quad (17)$$

$$\dot{x}_3 = x_4 \quad (18)$$

$$\dot{x}_4 = \frac{1}{M_{22}}[\tau_2 - M_{21}\,\dot{x}_2 - C_2\,x_4 - G_2] \quad (19)$$

Fig. 2 shows the MATLAB/SIMULINK of the prosthetic knee model. To simulate the prosthetic knee model representation by using Equations (16, 17, 18, and 19). Table I exhibits the model of prosthetic knee parameters values that utilized in the simulation.

TABLE I. PHYSICAL PARAMETERS VALUES OF THE PROSTHETIC KNEE [17]

Prosthetic knee parameters	Parameter value
m_1	5.28 kg
m_2	2.23 kg
I_1	0.033 $kg.m^2$
I_2	0.033 $kg.m^2$
L_1	0.302 m
L_2	0.332 m

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r_1	0.236 m
r_2	0.189 m
g	$9.81\ m/s^2$

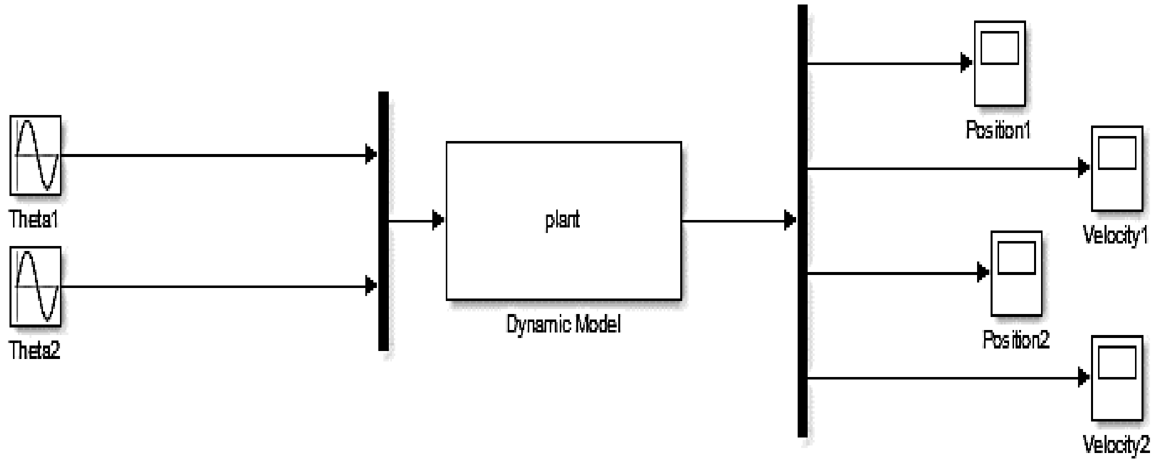

FIG. 2. OPEN LOOP PROSTHESIS KNEE SYSTEM REPRESENTED BY MATLAB SIMULINK.

Fig. 3 represents the results of the open loop trajectory and the speed with which the prosthesis moves within the initial conditions ($\theta = 10°$) [18]. The main problem is the instability and controllability of the movement of the prosthesis resulting from the lack of control over the location and speed which will certainly lead to undesirable movement of the limb which in turn must be controlled. Clearly from *Fig. 3*, the open loop system is instable. Consequently, the Backstepping controller is utilized to stabilized the prosthetic knee and make its states reach the asymptotically stable region with maximal angle.

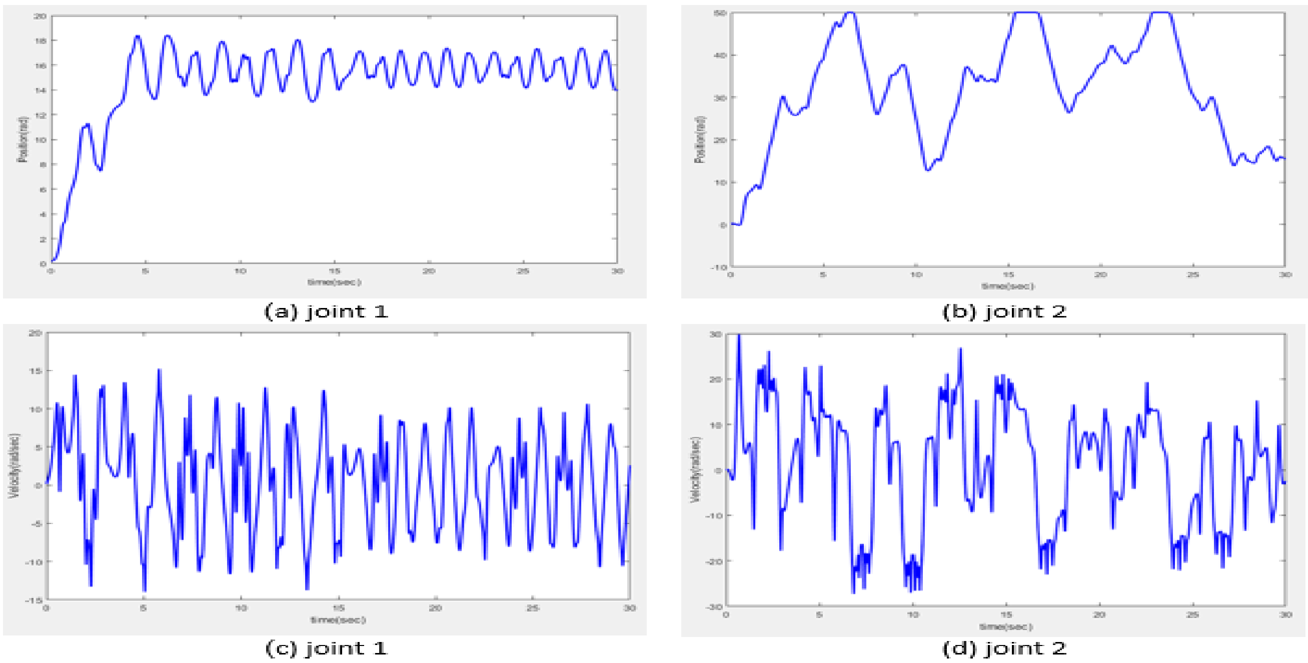

FIG. 3. OPEN LOOP RESPONSE OF PROSTHESIS KNEE, (A, AND B) REPRESENTS POSITION OF LINK 1AND 2, IN ADDITION TO (C, AND D) IS THE VELOCITY OF LINK 1 AND LINK 2.

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III. BACKSTEPPING CONTROLLER (BC) DESIGN

The backstepping approach provides a systematic method for designing a control structure to monitor a reference signal. Suggested BC to control the lower prosthesis as the dynamic model is based on this approach [19]. The controller is introduced which ensures convergent stability in tracking the desired position and speed trajectories. The control laws are derived from Lyapunov theory-based stability assessments of the Backstepping controller to control the prosthesis knee [20, 21].

In order to create the BC algorithm for a prosthetic knee system, follow the procedures listed [22]

Steps 1: Suppose that the error e_1, represents the actual stat x_1 and intended trajectory x_{d1} described by the

$$e_1 = x_1 - x_{d1} \tag{20}$$

The time derivative of the error in equation (20), the tracking velocity, can be written as follows:

$$\dot{e}_1 = \dot{x}_1 - \dot{x}_{d1} \tag{21}$$

Defining the first virtual control $\alpha_1 = x_2$ and sub in equation (21) to get

$$\dot{e}_1 = \alpha_1 - \dot{x}_{d1} \tag{22}$$

The positive Lyapunov function:

$$V_1 = \frac{1}{2} e^2{}_1 \tag{23}$$

The Lyapunov functions derivative during time is called

$$\dot{V}_1 = e_1 \dot{e}_1 \tag{24}$$

As follows: by substituting equation (22) into equation (24) to gate a new derivative of the Lyapunov function, which can be written as follows:

$$\dot{V}_1 = e_1(\alpha_1 - \dot{x}_{d1}) \tag{25}$$

A virtual control is created ($\alpha_1 = -c_1 e_1 + \dot{x}_{d1}$) , and sub it into equation (25) then:

$$\dot{V}_1 = -c_1 e^2{}_1 \tag{26}$$

This means that $\dot{V}_1 < 0$

Let the error e_2, between actual state x_2 and the first virtual control α_1 described by

$$e_2 = x_2 - \alpha_1 \tag{27}$$

Taking the time derivative of equation (27) and using equation (17) to get:

$$\dot{e}_2 = \frac{1}{M_{11}} [\tau_1 - M_{12}\, \dot{x}_4 - C_1\, x_2 - G_1] + \alpha_1 \tag{28}$$

The second Lyapunov function is $V_2 = \frac{1}{2} e^2{}_1 + \frac{1}{2} e^2{}_2$

Using the time derivative of Lyapunov function and the presumption that ($\alpha_1 = -c_1 e_1 + \dot{x}_{d1}$) to get

$$\dot{V}_2 = -c_1 e^2{}_1 + e_2(e_1 + (\frac{1}{M_{11}} [\tau_1 - M_{12}\, \dot{x}_4 - C_1\, x_2 - G_1]) + c_1 \dot{e}_1 - \ddot{x}_{d1}) \tag{29}$$

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Choosing the first control law

$$\tau_1 = M_{11}[-e_1 - c_1\dot{e}_1 - c_2 e_2 + \ddot{x}_{d1}] + M_{12}\,\dot{x}_4 + \,C_1\,x_2 + \,G_1 \tag{30}$$

The derivative of Lyapunov function leads to:

$$\dot{V}_2 = -c_1 e^2{}_1 - c_2 e^2{}_2 \tag{31}$$

, where c_1 and c_2 are a positive constants to be determined using Bat algorithm, and $\dot{V}_2 < 0$ are negative definite

Step 2: Let e_3, represent the actual state x_3 of the desired trajectory x_{d3} as defined by:

$$e_3 = x_3 - x_{d3} \tag{32}$$

The time derivative of equation (32), and in addition to assigning the second virtual control ($\dot{x}_3 = x_4$) in order to get the error of the tracking velocity, it is written as follows:

$$\dot{e}_3 = \alpha_2 - \dot{x}_{d3} \tag{33}$$

By using third Lyapunov function

$$V_3 = V_2 + \frac{1}{2}e_3^2 \tag{34}$$

Since the time derivative of V_3 is given by

$$\dot{V}_3 = \dot{V}_2 + e_3\dot{e}_3 \tag{35}$$

Using equation (33) and equation (35) to get

$$\dot{V}_3 = -c_1 e^2{}_1 - c_2 e^2{}_2 + e_3(\alpha_2 - \dot{x}_{d3}) \tag{36}$$

By substitution, the virtual control ($\alpha_2 = -c_3 e_3 + \dot{x}_{d3}$) in to $\dot{V}_3$ to equation becomes:

$$\dot{V}_3 = -c_1 e^2{}_1 - c_2 e^2{}_2 - c_3 e^2{}_3 \tag{37}$$

Step 3: Consider the error e_4 as a representation of x_4 and the second virtual control α_2 as:

$$e_4 = x_4 - \alpha_2 \tag{38}$$

The time derivative of the error e_4, and Sub $\dot{x}_4$ from equation (19) to get

$$\dot{e}_4 = \frac{1}{M_{22}}[\tau_2 - \,M_{21}\,\dot{x}_2 - \,C_2\,x_4 - \,G_2\,] - \,\alpha_2 \tag{39}$$

Using fourth Lyapunov function

$$V_4 = V_3 + \frac{1}{2}e_4^2 \tag{40}$$

Taking the time derivative of Lyapunov function, and compensation equation (37) and (39)

$$\dot{V}_4 = -c_1 e^2{}_1 - c_2 e^2{}_2 - c_3 e^2{}_3 + e_4(e_3 + [\frac{1}{M_{22}}[\tau_2 - \,M_{21}\,\dot{x}_2 - \,C_2\,x_4 - \,G_2\,]c_3\dot{e}_3 - \ddot{x}_{d3}\,) \tag{41}$$

Choosing the second control law

$$\tau_2 = M_{22}[-e_3 - c_3\dot{e}_3 - c_4 e_4 + \ddot{x}_{d3}] + M_{21}\,\dot{x}_2 + \,C_2\,x_4 + \,G_2 \tag{42}$$

The derivative of Lyapunov function leads to

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$$\dot{V}_4 = -c_1{e^2}_1 - c_2{e^2}_2 - c_3{e^2}_3 - c_4{e^2}_4 \tag{43}$$

As a result, the selection of control law τ_2 assures that $\dot{V}_4$ to be negative definite, ensuring the whole system's asymptotic stability characteristics. *Fig. 4* shows a schematic design of BC for lower limb prosthesis.

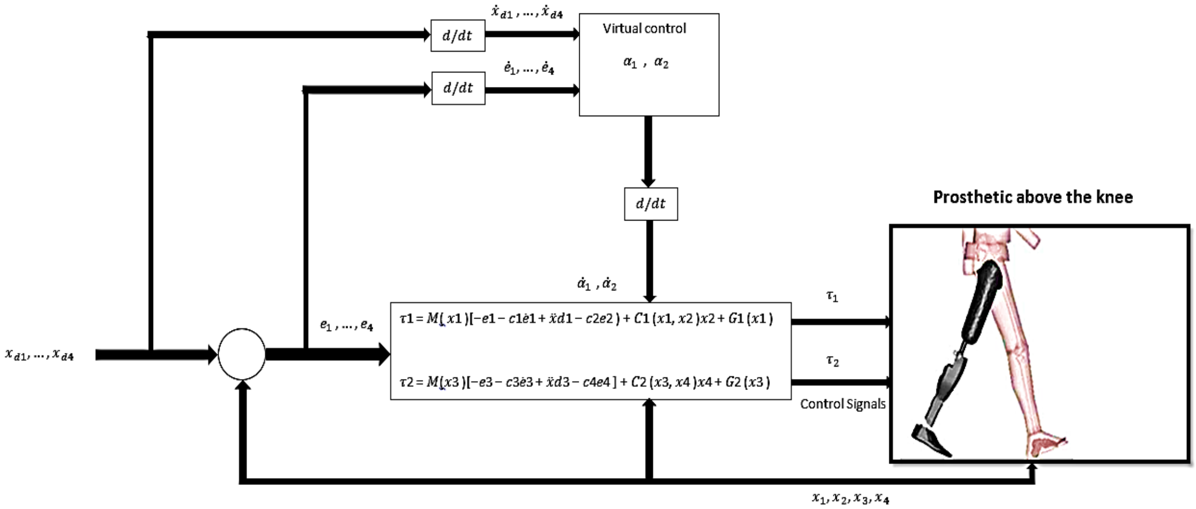

FIG. 4. BACKSTEPPING CONTROL FOR A PROSTHETIC KNEE.

IV. ADAPTIVE BACKSTEPPING CONTROLLER (ABC) DESIGN

Adaptive control is a good way to deal with uncertainty. Adaptive control based on backstepping technology is a nonlinear recursive design methodology for tracking that is based on the systematic building of Lyapunov functions, and it gives you the option of dealing with unknown parameters and nonlinear effects [23, 24].

ABC was design in this part to stabilize position and angular position of prosthetic knee, and estimates the disturbance. An adaptive controller is designed by combining a parameter estimator, which provides estimates of unknown parameters, with a control law. Which is able to ensure the boundedness of the closed-loop states and asymptotic tracking [25, 26].

In order to create the ABC algorithm for a prosthetic knee system, follow the procedures listed

Steps 1:

e_1 is selected as the trajectory tracking error, which represents the actual state x_1 and intended trajectory x_{d1} defined as

$$e_1 = x_1 - x_{d1} \tag{44}$$

Derivation of the error in equation (44) with respect to time, the tracking velocity, can be written as follows:

$$\dot{e}_1 = \dot{x}_1 - \dot{x}_{d1} \tag{45}$$

$$\dot{e}_1 = x_2 - \dot{x}_{d1} \tag{46}$$

Defining the virtual controller variable for the first error subsystem $\alpha_1 = x_2$ and sub in equation (46) to get

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$$\dot{e}_1 = \alpha_1 - \dot{x}_{d1} \tag{47}$$

The positive Lyapunov function:

$$V_1 = \frac{1}{2} {e^2}_1 \tag{48}$$

The Lyapunov functions derivative during time is called

$$\dot{V}_1 = e_1 \dot{e}_1 \tag{49}$$

By substituting equation (47) into equation (49) to gate a new derivative of the Lyapunov function, which can be written as follows:

$$\dot{V}_1 = e_1(\alpha_1 - \dot{x}_{d1}) \tag{50}$$

A virtual control is created

$$\alpha_1 = -c_1 e_1 + \dot{x}_{d1} \tag{51}$$

Sub the virtual control into equation (50) then:

$$\dot{V}_1 = -c_1 {e^2}_1 \tag{52}$$

Tacking the time derivative of virtual control in equation (51)

$$\dot{\alpha}_1 = -c_1 \dot{e}_1 + \ddot{x}_{d1} \tag{53}$$

Sub Equation (46) into Equation

$$\dot{\alpha}_1 = -c_1 x_2 + c_1 \dot{x}_{d1} + \ddot{x}_{d1} \tag{54}$$

Selected e_2 as the trajectory tracking error, between the actual state x_2 and the virtual controller variable for the first error subsystem α_1 defined as

$$e_2 = x_2 - \alpha_1 \tag{55}$$

The following equation can be found using the equation

$$x_2 = e_2 + \alpha_1 \tag{56}$$

Taking the time derivative of equation (55)

$$\dot{e}_2 = \dot{x}_2 - \dot{\alpha}_1 \tag{57}$$

Sub the Equation and from Equation of system

$$\dot{e}_2 = \frac{1}{M_{11}} [\tau_1 - M_{12}\, \dot{x}_4 - C_1\, x_2 - G_1 - F_1] + c_1 x_2 - c_1 \dot{x}_{d1} - \ddot{x}_{d1} \tag{58}$$

The second Lyapunov function is

$$V_2 = \frac{1}{2} {e^2}_1 + \frac{1}{2} {e^2}_2 + \frac{1}{2} \gamma_1^{-1} {\tilde{F}_1}^2 \tag{59}$$

, where the $\tilde{F}_1$ represents the estimation error disturbance

$$\tilde{F}_1 = F_1 - \hat{F}_1 \tag{60}$$

F_1 is supposed to be unknown external disturbance and $\hat{F}_1$ the estimation of disturbance F_1 denoted.

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Tacking the time derivative of estimation error disturbance

$$\dot{\tilde{F}}_1 = -\dot{\hat{F}}_1 \quad (61)$$

Taking the derivation of equation (V_2) with respect of time

$$\dot{V}_2 = e_1\dot{e}_1 - e_2\dot{e}_2 + \gamma_1^{-1}\,\tilde{F}_1\,\dot{\tilde{F}}_1 \quad (62)$$

$$\dot{V}_2 = -c_1{e^2}_1 + e_2\,(e_1 + \frac{1}{M_{11}}[\tau_1 - M_{12}\,\dot{x}_4 - C_1\,x_2 - G_1 - F_1] + c_1x_2 - c_1\dot{x}_{d1} - \ddot{x}_{d1}) - \gamma_1^{-1}\,\tilde{F}_1\,\dot{\tilde{F}}_1 \quad (63)$$

Choosing the first control law

$$\tau_1 = M_{11}\,[-e_1 - c_1x_2 + c_1\dot{x}_{d1} + \ddot{x}_{d1} - c_2e_2\,] + M_{12}\,\dot{x}_4 + C_1\,x_2 + G_1 + \hat{F}_1 \quad (64)$$

From Equation (60), and sub Equation (64) into (63), can beget:

$$\dot{V}_2 = -c_1{e^2}_1 - c_2{e^2}_2 + \tilde{F}_1\left(e_2 - \gamma_1^{-1}\dot{\hat{F}}_1\right) \quad (65)$$

Choosing the first update adaptive control law provided by for Equation (65)

$$\dot{\hat{F}}_1 = \gamma_1 e_2 \quad (66)$$

Using the first update adaptive law into Equation (63) gives

$$\dot{V}_2 = -c_1{e^2}_1 - c_2{e^2}_2 \quad (67)$$

Step 2:

e_3 is selected as the trajectory tracking error, which represents the actual state x_3 and intended trajectory x_{d3} defined as

$$e_3 = x_3 - x_{d3} \quad (68)$$

Time derivative of equation (68)

$$\dot{e}_3 = \dot{x}_3 - \dot{x}_{d3} \quad (69)$$

Replace ($\dot{x}_3 = x_4$)from Equation of model (18)

$$\dot{e}_3 = x_4 - \dot{x}_{d3} \quad (70)$$

, and in addition to assigning the virtual controller variable for the second error subsystem ($x_4 = \alpha_2$) in order to get the error of the tracking velocity, it is written as follows:

$$\dot{e}_3 = \alpha_2 - \dot{x}_{d3} \quad (71)$$

By using third Lyapunov function

$$V_3 = V_2 + \frac{1}{2}e_3^2 \quad (72)$$

Since the time derivative of V_3 is given by

$$\dot{V}_3 = \dot{V}_2 + e_3\dot{e}_3 \quad (73)$$

Using equation (67) and equation (71) to get

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$$\dot{V}_3 = -c_1 {e^2}_1 - c_2 {e^2}_2 + e_3(\alpha_2 - \dot{x}_{d3}) \tag{74}$$

Let's selected the second virtual control.

$$\alpha_2 = -c_3 e_3 + \dot{x}_{d3} \tag{75}$$

By substitution, the equation (75) in $\dot{V}_3$ to equation becomes:

$$\dot{V}_3 = -c_1 {e^2}_1 - c_2 {e^2}_2 - c_3 {e^2}_3 \tag{76}$$

The time derivative of second virtual control

$$\dot{\alpha}_2 = -c_3 \dot{e}_3 + \ddot{x}_{d3} \tag{77}$$

Sub Equation (70) into Equation

$$\dot{\alpha}_2 = -c_3 x_4 + c_3 \dot{x}_{d3} + \ddot{x}_{d3} \tag{78}$$

Step3:

Selected e_4 as the trajectory tracking error, between the actual state x_4 and the virtual controller variable for the second error subsystem α_2 defined as

$$e_4 = x_4 - \alpha_2 \tag{79}$$

The following equation can be found using the equation

$$x_4 = e_4 + \alpha_2 \tag{80}$$

Taking the time derivative of equation (79)

$$\dot{e}_4 = \dot{x}_4 - \dot{\alpha}_2 \tag{81}$$

Sub the Equation and from Equation of system

$$\dot{e}_4 = \frac{1}{M_{22}}[\tau_2 - M_{21}\,\dot{x}_2 - C_2\,x_4 - G_2 - F_2] + c_3 x_4 - c_3 \dot{x}_{d3} - \ddot{x}_{d3} \tag{82}$$

The second Lyapunov function is

$$V_4 = V_2 + \frac{1}{2} {e^2}_3 + \frac{1}{2} {e^2}_4 + \frac{1}{2} \gamma_2^{-1} {\tilde{F}_2}^2 \tag{83}$$

, where the $\tilde{F}_2$ represents the estimation error disturbance

$$\tilde{F}_2 = F_2 - \hat{F}_2 \tag{84}$$

F_2 is supposed to be unknown external disturbance and $\hat{F}_2$ the estimation of disturbance F_1 denoted.

The derivative of estimation error disturbance

$$\dot{\tilde{F}}_2 = -\dot{\hat{F}}_2 \tag{85}$$

Taking the time derivative of equation (V_2)

$$\dot{V}_4 = \dot{V}_2 + e_3 \dot{e}_3 + e_4 \dot{e}_4 + \gamma_2^{-1} \tilde{F}_2 \dot{\tilde{F}}_2 \tag{86}$$

$$\dot{V}_2 = -c_1 {e^2}_1 - c_2 {e^2}_2 - c_3 {e^2}_3 + e_4 \left(e_3 + \frac{1}{M_{22}}[\tau_2 - M_{21}\,\dot{x}_2 - C_2\,x_4 - G_2 - F_2] + c_3 x_4 - c_3 \dot{x}_{d3} - \ddot{x}_{d3}\right) - \gamma_2^{-1} \tilde{F}_2 \dot{\tilde{F}}_2 \tag{87}$$

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Choosing the second control law

$$\tau_2 = M_{22}\left[-e_3 - c_3 x_4 + c_3 \dot{x}_{d3} + \ddot{x}_{d3} - c_4 e_4\right] + M_{21}\,\dot{x}_2 + C_2\,x_4 + G_2 + \widehat{F_2} \tag{88}$$

From Equation (84), and sub Equation (88) into (87) we will get:

$$\dot{V}_4 = -c_1 {e^2}_1 - c_2 {e^2}_2 - c_3 {e^2}_3 - c_4 {e^2}_4 + \tilde{F}_2 \left(e_4 - \gamma_2^{-1}\dot{\hat{F}}_2\right) \tag{89}$$

Choosing the second update adaptive control law provided by for Equation (89)

$$\dot{\hat{F}}_2 = \gamma_2 e_4 \tag{90}$$

Using the second update adaptive law into Equation (87) gives

$$\dot{V}_4 = -c_1 {e^2}_1 - c_2 {e^2}_2 - c_3 {e^2}_3 - c_4 {e^2}_4 \tag{91}$$

Using the control laws stated in Equations (64) and (88), utilizing the most updated adaptive control rules, which are used to estimate the applied disturbances, given by Equation (66), and (90) could guarantee the asymptotic stability of adaptive backstepping controlled prosthetic knee. *Fig. 5* shows the schematic diagram of ABC for prosthetic knee.

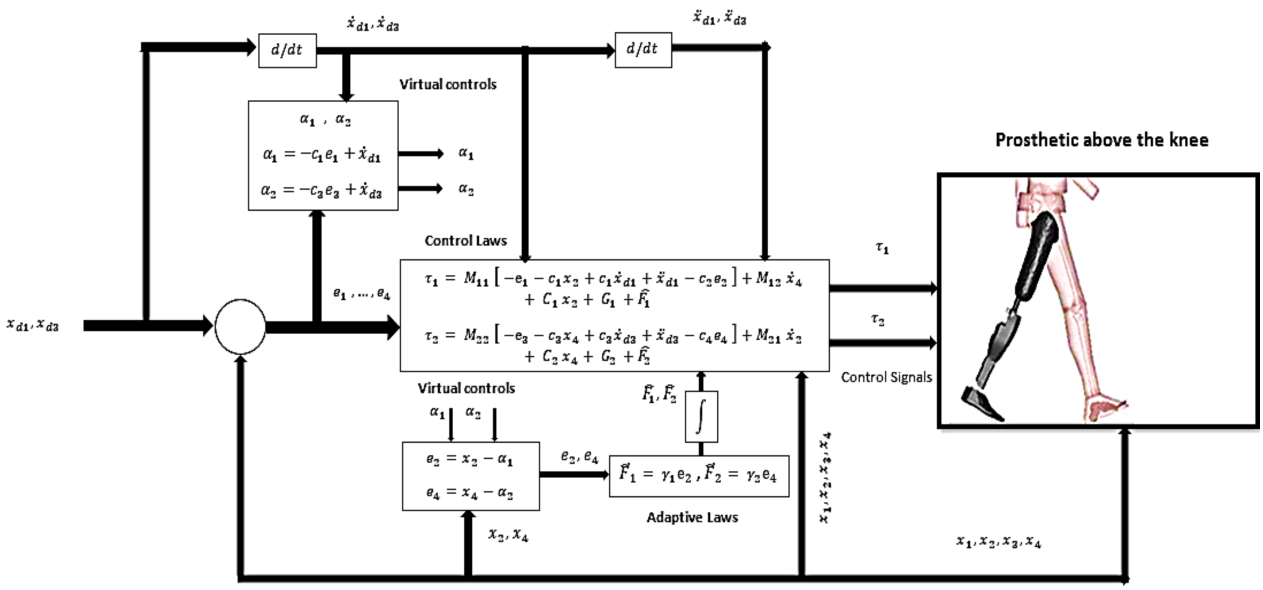

FIG. 5. ADAPTIVE BACKSTEPPING CONTROL FOR A PROSTHETIC KNEE.

V. SIMULATION RESULTS

MATLAB/SIMULINK simulation is used to evaluate the controller and analyze the performance of the Backstepping and Adaptive Backstepping controlled system. As shown in Table I. the values of the system parameters for a prosthetic knee with a 2-DOF joint are provided. The Trial-and-Error values of the controllers design parameters are listed in Table II.

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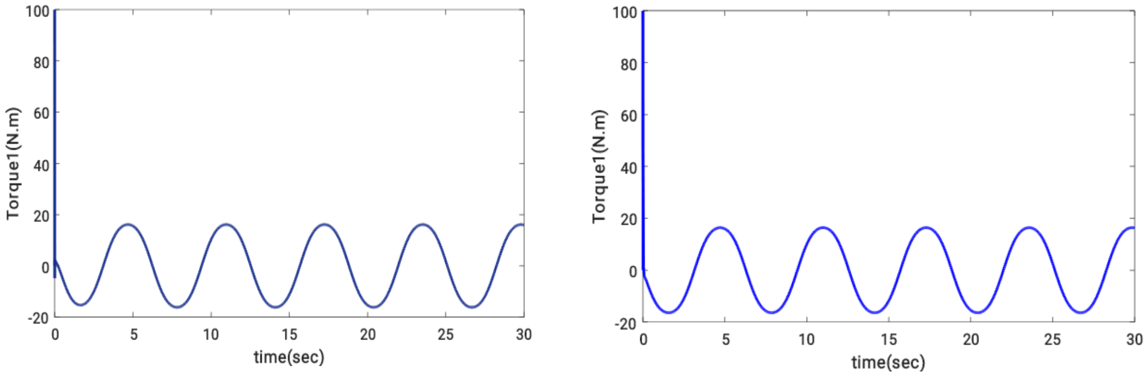
TABLE II. TRIAL-AND-ERROR VALUES OF THE CONTROLLER BS AND ABS DESIGN PARAMETERS

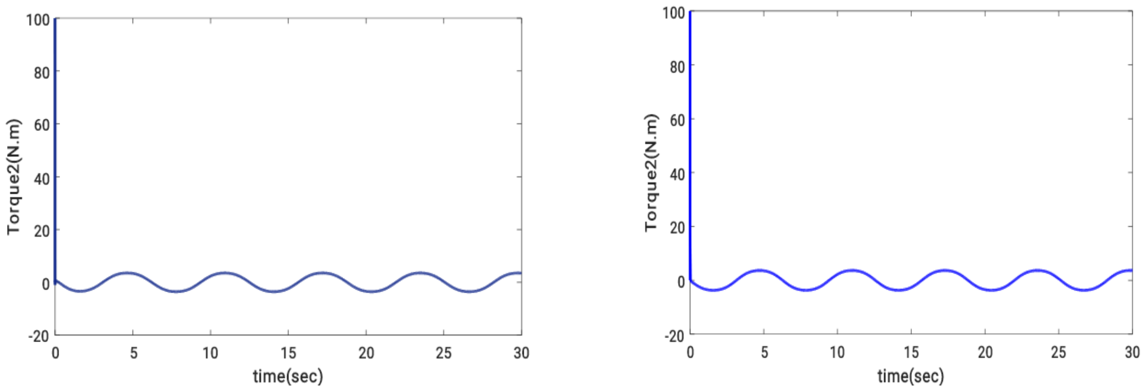
Design parameters	BS	ABS
c_1	9	11
c_2	9	12
c_3	9	15
c_4	9	20

Fig. 6 and 7 show the drive torque of each joint. Controlling torques τ_1 and τ_2 are restricted within the range [100, -100] N.m. The results show that with the Backstepping and Adaptive Backstepping control the system can tracks the desired trajectory when there are disturbances in the system. RMS error value in Adaptive Backstepping control is 1.403× 10^{-4} for link1 and 5.072× 10^{-5} for link2 is reasonably small to the extent to verify a good convergence. This proves that ABS controller is much better for enhances the tracking performance and could guarantee the asymptotic stability.



(a) Backstepping control (b) Adaptive Backstepping control

FIG. 6. CONTROL SIGNAL APPLIED TO THE FIRST LINK.



(a) Backstepping control (b) Adaptive Backstepping control

FIG. 7. CONTROL SIGNAL APPLIED TO THE SECOND LINK.

Fig. 8 and 9 depict the actual the actual position converges to the desired trajectory. The comparison was made between the two controllers Backstepping and Adaptive Backstepping. The proposed ABC greatly improves the position tracking compared to the BC where the tracking position error has been greatly reduced. To comparison between BC and ABC, at the control action consumptions. It was found that the position error of the prosthetic knee in Adaptive Backstepping control is by 1.16% at link 1 and 1.65% at link 2 to compression with BC at peak, respectively.

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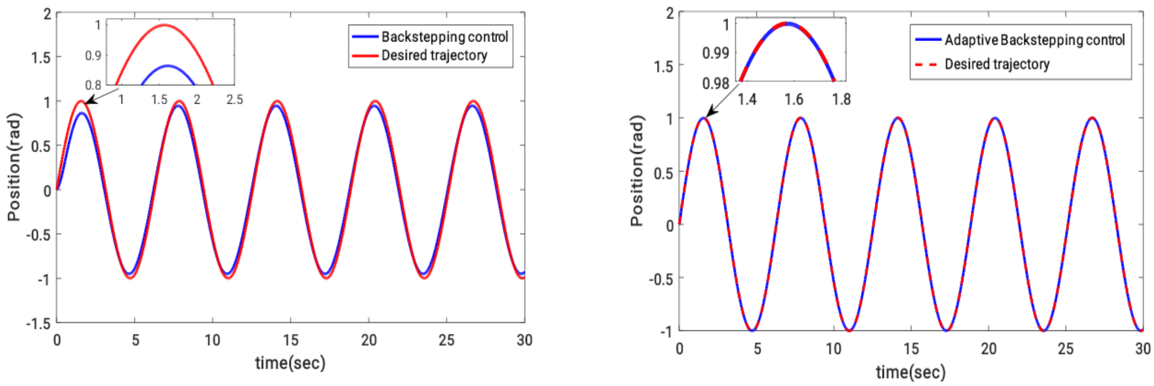
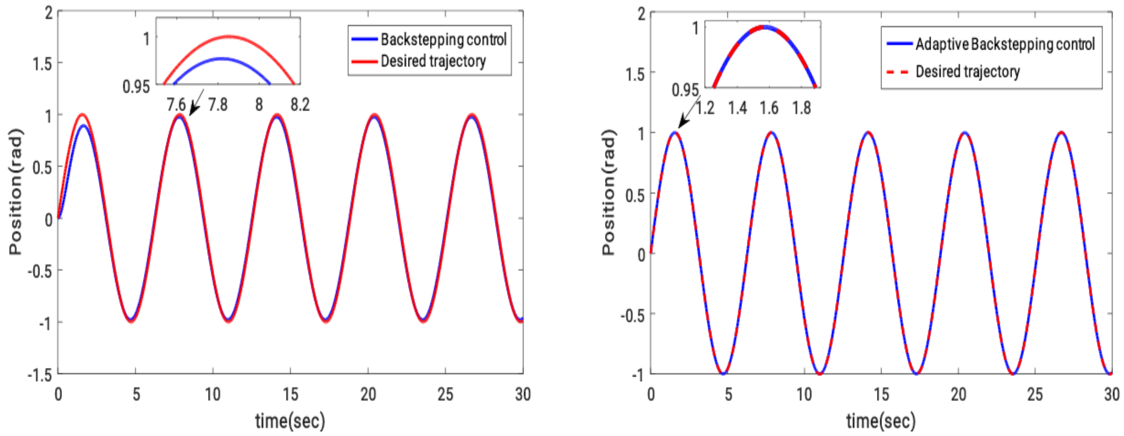


(a) Backstepping control (b) Adaptive Backstepping control

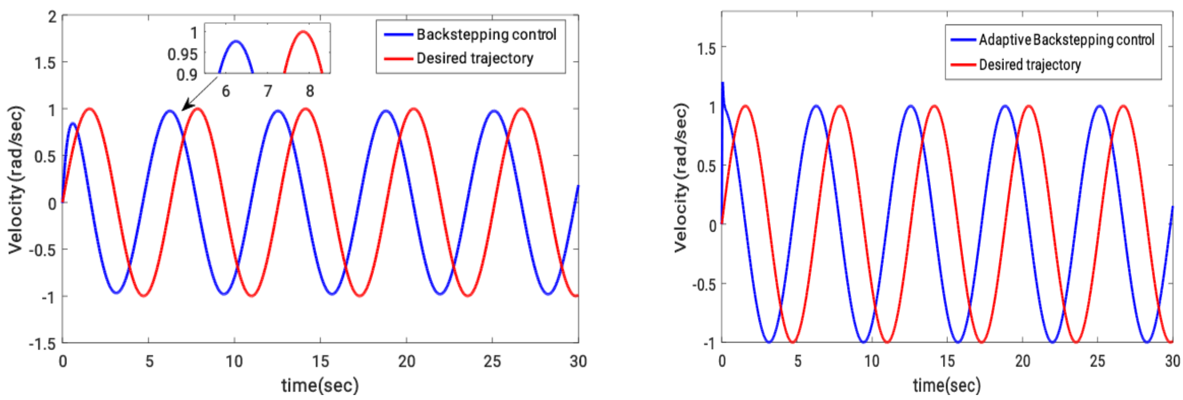
FIG. 8. POSITION TRAJECTORY FOR LINK 1.



(a) Backstepping control (b) Adaptive Backstepping control

FIG. 9. POSITION TRAJECTORY FOR LINK 2.

Fig. 10 and 11 represent velocity tracking for the both joints with the controllers. This proves that Adaptive Backstepping controller is much better for enhances the tracking performance and could guarantee the asymptotic stability.



(a) Backstepping control (b) Adaptive Backstepping control

FIG. 10. VELOCITY TRAJECTORY FOR LINK 1.

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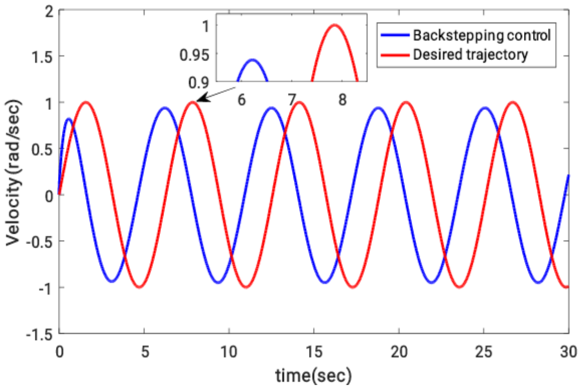
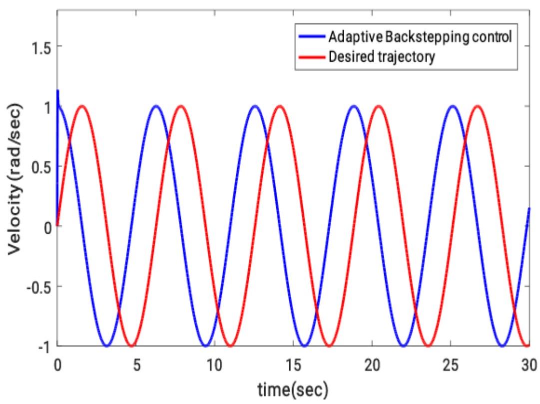


(a) Backstepping control



(b) Adaptive Backstepping control

FIG. 11. VELOCITY TRAJECTORY FOR LINK 2.

Table III show the value of Root Mean Square (RMS) error and Enhancement rate of position for BSC and ABSC.

TABLE III. VALUE OF RMS AND ENHANCEMENT RATE FOR BSC AND ABSC IN POSITION

RMS	Trial-and-Error BSC	Trial-and-Error ABSC	Enhancement rate
Link 1	0.02744	1.403×10^{-4}	94.887%
Link 2	0.06229	5.072×10^{-5}	99.1857%
Both links	0.08974	1.911×10^{-4}	99.7871%

VI. CONCLUSIONS

In this paper presented to design and developed for a 2-DOF prosthetic knee using the Backstepping and Adaptive Backstepping control approach. According to the Lagrangian dynamic principle, the dynamic model of the prosthesis knee was derived. Backstepping and Adaptive Backstepping control scheme was proposed for the prosthetic knee to solve its nonlinearity, uncertainty and external disturbance problems. The stability of the control scheme is proved by Lyapunov stability theorem. The simulation found that the quantitative comparison between the two controllers, showed significant improvement in results in position tracking. The comparison between Backstepping control and Adaptive Backstepping control, at the control action consumptions. It was found that the position error of the prosthetic knee in Backstepping control is by 9% at link 1 and 7.4% at link 2 compared with desired trajectory, while in Adaptive Backstepping control is by 1.16% at link 1 and 1.65% at link 2 compared with desired trajectory. When comparing between Backstepping control and Adaptive Backstepping control, the improvement rate was 7.84 at link 1 and 5.75 at link 2, the proposed Adaptive Backstepping control, it may be concluded, is more robust against this perturbation and to deal with uncertainty. In future work using optimization algorithms to tune the controller parameters such as BAT Algorithms, Gray Wolf optimization techniques, and implementing the developed controllers on the real system hardware to obtain accurate results and good performance.

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