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**Original Research**

# EXPERIMENTAL INVESTIGATION OF SCOUR DOWNSTREAM OF A C-TYPE TRAPEZOIDAL PIANO KEY WEIR WITH STILLING BASIN

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**Abstract:** In recent years, engineers have focused on finding a solution to reduce scouring downstream of piano key weirs. Piano key weirs have high efficiency in flood flow and a higher discharge coefficient. In this research, a type C trapezoidal piano key weir with a type 1 stilling basin was used. Three discharges and three water depths were also used. The results showed that the existence of the stilling basin reduces scour. In the weir with the stilling basin, the maximum scour depth is reduced and the scouring hole becomes more elongated. The maximum distance of the scouring depth increases compared to the toe of the weir. The maximum scour depth and the maximum scour depth distance in the weirs with the stilling basin are about 63.4% less and 20.4% more, respectively than in the weirs without the stilling basin. Additionally, by increasing the flow rate and decreasing the depth of the downstream flow, the amount of scour increases.

**Keywords:** *piano key weir (PKW); scour; scour index; stilling basin; type C*

## 1. Introduction

Piano key weirs are nonlinear weir structures and an advanced version of labyrinth weirs. These weirs have longer crest lengths within a limited width, resulting in increased water passage coefficients [1]. Piano key weirs can have rectangular, trapezoidal, or triangular shapes in plan view and are classified into types A, B, C, and D. Type A features hanging edges both upstream and downstream, while types B and C have hanging edges upstream and downstream, respectively. Type D does not have hanging edges. Given the high efficiency of these weirs, it is crucial to address the issue of downstream scouring and explore solutions to minimize it. By reducing energy loss and mitigating scouring downstream of piano key weirs, financial and safety risks can be reduced. Previous studies by researchers such as Sajadi [2], Al-Shukur and Al-Khafaji [3], Naghibzadeh et al. [4], Eslinger and Crookston [5], Singh and Kumar [6], Fathi et al. [7], Elyass [8] and Hassan [9] have provided valuable insights into strategies for increasing energy loss and discharge coefficient. Jüstrich et al. [10] conducted a laboratory study on rectangular type A piano key weirs and found that reducing discharge, increasing abutment depth, using coarser materials, and lowering the flow drop height can decrease scouring. Gohari and Ahmadi [11] investigated rectangular type A piano key weirs and observed that reducing the

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*Corresponding Author: khoshfetrat@khuisf.ac.ir number of weir cycles reduces scouring. Ghafouri et al. [12] studied trapezoidal type A piano key weirs and noted that weir depth and flow rate directly influence scouring rates. Kumar and Ahmad [13] introduced the concept of an apron in front of the toe of rectangular type A piano key weirs, which was found to reduce scouring. Yazdi et al. [14] conducted laboratory investigations on rectangular and trapezoidal type A piano key weirs and discovered that trapezoidal weirs experience significantly lower scouring at a greater distance from the toe of the weir compared to rectangular weirs Ghodsian et al. [15] studied trapezoidal and triangular piano weirs of type A and found that triangular weirs experience higher scouring closer to the toe of the weir compared to trapezoidal weirs. Jamal et al. [16] investigated type C rectangular piano key weirs and found that increasing the weir height and the input key width-to-output width ratio leads to higher scouring rates. Lantz et al. [17] introduced the concept of an apron with lengths ranging from 0 to 2 times the height of the weir in front of rectangular type A piano key weirs. Their study revealed that the presence of an apron reduces scouring, with an optimal length of 1.5 times the weir height. Abdi Chooplou et al. [18] conducted laboratory investigations on trapezoidal type A piano key weirs and observed that an increase in the densimetric Froude number leads to an increase in the maximum scouring depth. In a separate study, under high weir conditions, they found that the amount of scouring is influenced by the scouring depth, and reducing it increases scouring [19]. Bodaghi et al. [20] conducted laboratory investigations on submerged and free flow conditions for trapezoidal type A piano key weirs and discovered that despite being submerged, the scour rate is much lower than under free flow conditions. Abdi Chooplou et al. [21] conducted laboratory and numerical studies on the effect of the lateral wall crest shape on the flow field and downstream scouring of rectangular type A piano key weirs. Their findings showed that modifying the shape of the lateral wall crest reduces scouring rates. Considering the high efficiency of piano key weirs, it becomes crucial to investigate and address the issue of downstream scouring in order to reduce its occurrence It is also understood that the implementation of a stilling basin can lead to lower energy losses and ultimately mitigate scouring. Therefore, this research aims to incorporate a type 1 stilling basin in front of the C-type piano key weir, considering three tailwater depths, three discharge rates, and utilizing a specific sand material in the downstream bed .This study has also been investigated for the stability of hydraulic structures, especially type C piano spillways, and a solution has been proposed for this important structure to reduce the amount of erosion in it.

## 2. Dimensional Analysis

The parameters that influence the scouring of the C-type trapezoidal piano key weir are shown in Fig. 1 and Eq. 1 describes the factors associated with the scour by the PKW:

$$\boldsymbol{X_s, Z_s = f(q, H_u, H_d, g, \mu, \sigma, \rho_w, \rho_s, d_{50})} \qquad (1)$$

In which; $Xs$ is the distance from the toe of the weir to the maximum scour depth, and $Zs$ is the height of the maximum scour depth. In this case, the two parameters are assigned the symbol $\xi$ for convenience. This makes it easier to refer to the parameters throughout the equation.

Additionally, $q$ represents the flow rate per unit width, $H_u$ represents the flow depth upstream of the weir, $H_d$ represents the flow depth downstream of the weir (tailwater), $g$ represents the gravitational force, $\mu$ represents the dynamic viscosity, $\sigma$ represents the surface tension coefficient, $\rho_w$ represents the density of water, $\rho_s$ represents the density of bed materials, and $d_{50}$ represents the average particle diameter of the bed material .By Applying the π theorem and considering three independent parameters, therefore, in this study; $q$, $H_u$, and $\rho_w$, the variables were chosen in such a way that they could be combined to form dimensionless groups, which is the form given in Eq. 2:

$$\frac{\xi}{H_u} = f\left(\frac{H_d}{H_u}, Fr = \frac{q}{H_u\sqrt{gH_u}}, Re = \frac{q}{\vartheta}, We = \frac{\rho_w q^2}{H_u \sigma}, \frac{\rho_s - \rho_w}{\rho_w} = S - 1, \frac{d_{50}}{H_u}\right) \quad (2)$$

Due to the high turbulence of the flow, the Reynolds number ($Re > 4000$) is omitted and the Weber number is also omitted because the depth on the weir crest is greater than 0.03 m [22, 23]. By combining the Froude number ($Fr_d$), the parameter $d_{50}/H_u$, the parameter $S$-1, and the parameter $H_d/H_u$, we obtain the dimensionless number $Fr_d = \frac{q}{H_d\sqrt{gd_{50}(s-1)}}$, which is referred to as the densimetric Froude number. The following are present in the densimetric Froude number: discharge per unit width, tailwater depth, and sediment characteristics, including $d_{50}$ and sediment density .Thus, scouring can be considered as a function of the parameters outlined in Eq. 3.

$$\frac{\xi}{H_u} = f(Fr_d) \quad (3)$$

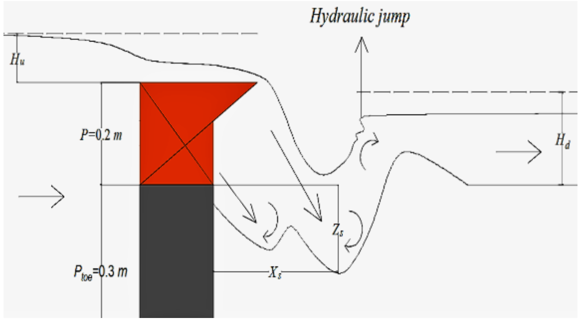

**Figure 1**. Flow and parameters affecting the scouring of the C-type trapezoidal PKW

## 3. Materials and Methods

Experiments were conducted in the hydraulic laboratory of the Civil Engineering Department using a laboratory flume with dimensions of 10 meters in length, 0.6 meters in width, and 0.8 meters in height. The flow, controlled by a pump with an error of ±0.01%, is directed through two ground tanks and then into the laboratory flume before reaching the weir section located 5.5 meters downstream. A valve and a monitor were employed to adjust and measure the flow rate. The flow would then return to the first tank after entering the experimental flume, passing through the flow straighteners, passing over the weir, and then passing through the end gate of the flume. The first and second tanks are connected by a pipe with a diameter of 130 millimeters (approximately 5 inches). After the pump (of the centrifugal type) was turned on, the flow was drawn into the beginning of the experimental flume by a pipe with the same specifications. The flume tilt system was adjusted by an electric motor and gearbox. The slope of the experimental flume was constant at 0.04. The flow depth upstream of the weir was measured using a needle depth gauge with an error margin of ±0.001 meters. To prevent initial scour, a galvanized plate was initially placed in front of the weir. Once the flow rate and weir depth were adjusted, the plate was removed .Three different flow rates of 0.03, 0.035, and 0.04 cubic meters per second were tested, along with three tailwater depths of 0.05, 0.1, and 0.15 meters. Downstream of the weir, materials with an average particle diameter of 0.0075 meters were utilized. The materials were poured into the channel with a width of 0.6 meters, a height of 0.3 meters, and a length of 3 meters. The volume of the materials was 0.54 cubic meters. The uniformity coefficient of the materials was also less than 1.5, which indicates the uniformity of the materials . The equilibrium time for material scouring was set at 150 minutes, following Chiew's criteria [22]. According to this criterion, in 150 minutes, the changes in erosion were less than 1 millimeter .The weir has a width (*W*) of 0.6 meters, outlet key width ($W_o$) of 0.075 meters, inlet key width ($W_i$) of 0.215 meters, side wall length (*B*) of 0.5 meters, overhanging edge of the outlet ($B_o$) with a length of 0.13 meters, thickness (*Ts*) of 0.01 meters, height (*P*) of 0.2 meters, and a crest length (*L*) of 2.6 meters. The ratio of Wi/Wo is also equal to 2.8. Fig. 2 illustrates the weir configuration used in the research. The stilling basin employed in this study is a type 1 USBR (United States Bureau of Reclamation) stilling basin with a length equal to the height of the weir [15].

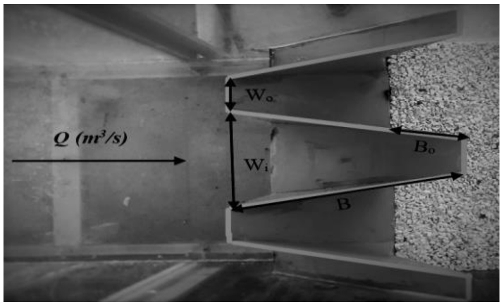

**Figure 2**. Specifications of the Weir

This type of stilling basin design is suitable for Froude numbers ranging between 1 and 2.5. For the flow rates of 0.03 and 0.04 meters and the minimum weir depth of 0.05 meters, producing Froude numbers of 1.43 and 1.9, respectively, accordingly. Additionally, a cut-off wall was implemented to prevent material erosion underneath the basin. Also, the cut-off wall is an anti-seepage control device to prevent seepage and develop the piping phenomena beneath hydraulic structures. The wall was also used at the end of the apron and inside the sand materials. Table (1) presents the hydraulic characteristics of the conducted tests, where *Q* represents the flow rate.

The velocity will be calculated given the discharge and depth of flow upstream of the weir. The Reynolds number is also calculated given the velocity. Also, the velocity of the flow downstream is calculated by adjusting the tailwater depth by the end gate and using the continuity equation.

## 4. Results and Discussion

The flow behavior over the C-type piano key weir involves the transfer of flow as a free-falling jet from the inlet keys and as an inclined jet downstream from the outlet keys. In this type of weir, the absence of a hanging edge upstream of the weir results in steeper slopes in the outlet keys and a smaller submerged area or local protrusion of flow. The flow indentation occurs at the entrance of the outlet keys due to the lack of a hanging edge upstream of the weir. Additionally, the increased slope in the outlet keys leads to scouring near the toe of the weir. The overhanging edge downstream of the weir causes the flow to hit the bed in a jet-like manner, creating flow disturbances in that region.

Table 1. Test specifications

| *Row* | model | $Q$ (m$^3$/s) | $H_u$ (m) | $H_d$ (m) | $Fr_d$ | $Z_s/H_u$ | $X_s/H_u$ |
|---|---|---|---|---|---|---|---|
| 1 | Without Apron | 0.03 | 0.039 | 0.101 | 1.421 | 2.419 | 3.616 |
| 2 | Without Apron | 0.03 | 0.039 | 0.113 | 1.273 | 2.292 | 3.438 |
| 3 | Without Apron | 0.03 | 0.039 | 0.156 | 0.922 | 2.165 | 2.954 |
| 4 | Without Apron | 0.035 | 0.048 | 0.119 | 1.402 | 2.506 | 3.738 |
| 5 | Without Apron | 0.035 | 0.048 | 0.117 | 1.427 | 2.380 | 3.571 |
| 6 | Without Apron | 0.035 | 0.048 | 0.158 | 1.062 | 2.193 | 3.237 |
| 7 | Without Apron | 0.04 | 0.057 | 0.141 | 1.361 | 2.441 | 3.644 |
| 8 | Without Apron | 0.04 | 0.057 | 0.123 | 1.560 | 2.388 | 3.537 |
| 9 | Without Apron | 0.04 | 0.057 | 0.160 | 1.195 | 2.264 | 3.484 |
| 10 | With Apron | 0.03 | 0.039 | 0.101 | 1.421 | 1.273 | 4.737 |
| 11 | With Apron | 0.03 | 0.039 | 0.113 | 1.273 | 1.070 | 4.355 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 12 | With Apron | 0.03 | 0.039 | 0.156 | 0.922 | 0.764 | 3.565 |
| 13 | With Apron | 0.035 | 0.048 | 0.119 | 1.402 | 1.232 | 4.657 |
| 14 | With Apron | 0.035 | 0.048 | 0.117 | 1.427 | 1.148 | 4.469 |
| 15 | With Apron | 0.035 | 0.048 | 0.158 | 1.062 | 0.898 | 3.988 |
| 16 | With Apron | 0.04 | 0.057 | 0.141 | 1.361 | 1.256 | 4.775 |
| 17 | With Apron | 0.04 | 0.057 | 0.123 | 1.560 | 1.185 | 4.687 |
| 18 | With Apron | 0.04 | 0.057 | 0.160 | 1.195 | 1.026 | 4.245 |

This flow disturbance causes the materials to be deposited along the side of the weir, resulting in a more inclined scour slope downstream of the weir. The gravel materials slightly protrude where the flow falls from the hanging edge of the inlet key to the bed (see Fig. 3).

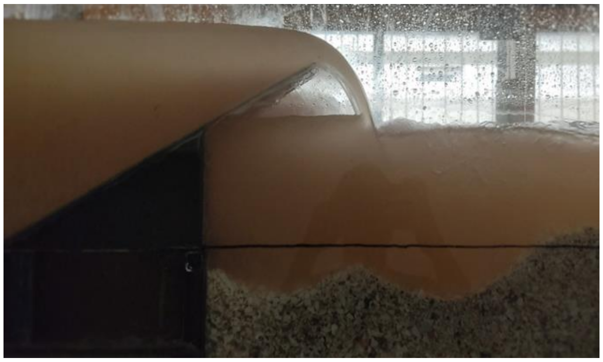

**Figure 3.** The erosion bed slope in the C-type piano key weir

The presence of a type 1 stilling basin, acting as an apron, reduces the depth and speed of eddies, thereby decreasing flow velocity. The flow rate and the depth of the reservoir influence the scour rate. Higher flow rates correspond to increased flow velocities and consequently more significant scouring. The higher velocity intensifies the strength of eddies and further disrupts the flow. An increased flow rate also expands the volume and length of the scour hole. Conversely, a greater depth of the downstream flow reduces flow velocity downstream and hampers additional scouring. The maximum scour depth in the weir, combined with the presence of the stilling basin, is significantly less than the maximum scour depth in the main weir.

Fig. 4 shows the effect of densimetric Froude number on maximum scour depth.

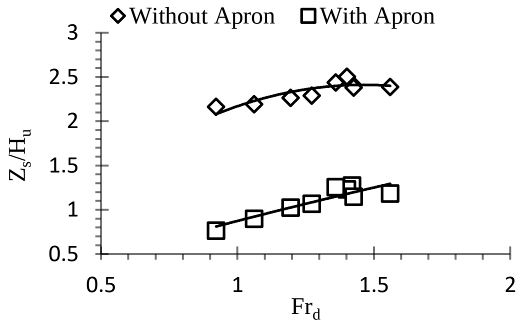

**Figure 4.** Effect of densimetric Froude number on dimensionless maximum scour depth

As depicted in the figure, an increase in the densimetric Froud number corresponds to an increase in the maximum scour depth. The depth of the downstream flow has a significant impact on the densimetric Froud number, with a decrease in downstream flow depth and an increase in flow rate per unit width resulting in higher Froude numbers. The densimetric Froude number strongly affects the maximum scour depth. The average maximum scour depth in the main Weir is 0.112 meters, while in the weir with the stilling basin, it is 0.041 meters. Consequently, the presence of the stilling basin reduces the maximum scour depth by 63.4% compared to the main weir. Eq. 4 presents the calculation of the maximum scour depth, with K1 and $K_2$ depending on the presence or absence of the stilling basin, as indicated in Table (2). This equation is accepted with a correlation coefficient of 98.84% and an acceptable error.

$$\frac{Z_s}{H_u} = -k_1(Fr_d)^2 + k_2 Fr_d \tag{4}$$

Table 2. Calculation of coefficient $K_1$ and $K_2$

| Row | Name weir | $K_1$ | $K_2$ |
|---|---|---|---|
| 1 | With Apron | 0.0796 | 0.9534 |
| 2 | Without Apron | 1.1264 | 3.2964 |

Fig. 5shows the values observed and calculated by Eq. 4. As can be seen from the figure, it is accepted with an acceptable error .

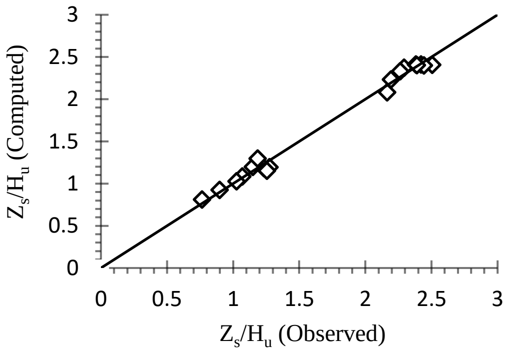

**Figure 5.** Observed and calculated values of the maximum scour depth

Fig. 6, Eq. 5, and Table (3) depict the relationship between the distance of the maximum scour depth from the toe of the weir and the maximum scour depth.

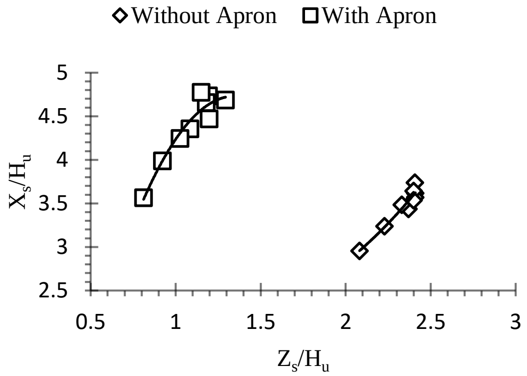

**Figure 6.** The distance between the maximum scour depth and the maximum scour depth

Table 3. Calculation of coefficient $K_3$ and $K_4$

| Row | Name weir | $K_3$ | $K_4$ |
|---|---|---|---|
| 1 | With Apron | -1.6122 | 5.801 |
| 2 | Without Apron | 0.2599 | 0.8747 |

As observed from the figure, as the maximum scour depth increases, its distance from the toe weir toe also increases. The average maximum scour depth from the toe weir toe in the main weir is 0.168 meters, while in the weir with the stilling basin, it measures 0.211 meters. Furthermore, the maximum scour depth in the weir with a stilling basin is approximately 20.4% higher than that in the weir without a stilling basin.

In Eqs. 4 and 5, the coefficients $K_1$, $K_2$, $K_3$, and $K_4$ are coefficients that show the difference between the maximum scour depth and its distance from the toe of the weir in the cases with and without an apron.

It is also necessary to have a wider range of $Z_s/H_u$. In this study, the $Z_s/H_u$ range does not exist between the numbers 1.3 and 2.2, and therefore it is necessary to use other aprons lengths as well and design the optimal state. This action can be like Lantz et al. [17]. To better illustrate the above sentence, the laboratory data of Jüstrich et al. [10], were also used for comparison. They conducted their experiments on a rectangular weir type A with sand materials [10]. The data of Abdi Chooplou et al. [18] were also used. Abdi Chooplou et al. [19] conducted their experiments on a trapezoidal piano key weir type A with sand materials. The average maximum scour depth in the research of Abdi Chooplou et al. [19] and Jüstrich et al. [10] is equal to 0.39 and 0.17 meters, respectively. In the current research, the average maximum scouring depth with and

$$\frac{X_s}{H_u} = K_3 \left(\frac{Z_s}{H_u}\right)^2 + K_4 \frac{Z_s}{H_u} \tag{5}$$

without aprons was observed to be 0.05 and 0.11 meters. In the present study, with the presence of the apron, the maximum scouring depth is about 87.2% and 70.6% less than Abdi Chooplou et al. and Jüstrich et al. respectively [10, 19]. The reason for that could be the existence of an apron reduced speed and coarser materials (gravel). As mentioned, the maximum scour depth is higher in sand materials, which can be attributed to the finer grain size of the materials. The presence of an apron also significantly reduces the maximum scour depth, which is beneficial for the weir. Scour is also less in the piano key weir type C than in type A, which can be attributed to the smaller cycle, the ratio of inlet width to outlet width of the keys, and the effective length of the weir.

## 5. Conclusion

Experimental investigations were conducted to examine the impact of a type 1 stilling basin on the performance of the piano key weir, and Three different flow rates (0.03, 0.035, and 0.04 cubic meters per second), three tailwater depths (0.05, 0.1, and 0.15 meters), and gravel materials were utilized in the experiments, The findings demonstrate that the presence of the stilling basin leads to a reduction in the maximum scour depth and a displacement of the scour away from the weir toe of the weir, and specifically, the maximum scour depth in the piano key weir with the stilling basin was approximately 63.4% less compared to the weir without the stilling basin. Furthermore, the distance of the maximum scour depth and its relation to the weir with the stilling basin was about 20.4% greater, and The scour index, calculated as twice the maximum scour depth divided by its distance ($=2Z_s/X_s$), was found to be 0.496 in the weir with the stilling basin and 1.35 in the main weir, indicating a reduction of approximately 63.2% compared to the main weir, By considering the distance of the maximum scour depth to the toe of the weir and including the length of the stilling basin, the scour index in the weir with the stilling basin, compared to the toe of the weir, was 0.253, reflecting an 81.3% reduction compared to the main weir. A lower scour index significantly reduces the risk of the weir overturning, thus enhancing the overall stability and performance of the weir .Also, factors such as tailwater depth, flow rate, and the presence or absence of an apron affect the amount of scour .It is suggested to use blocks with steps at the end of the apron to dissipate energy. This action can reduce scour.

## Conflict of interest

The authors confirm that the publication of this article causes no conflict of interest.

## Contribution of Author

Ali Qasim Rdhaiwi, Ali Khoshfetrat, and Amirhossein Fathi: proposed the research problem. Authors Ali Qasim Rdhaiwi and Ali Khoshfetrat.: developed the theory and performed the computations. Authors Ali Khoshfetrat and Amirhossein Fathi.: checked the theoretical analysis methods and supervised the results of this research. The authors checked and discussed the results and contributed to this work to present the final manuscript.

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## Nomenclature

| | | |
|---|---|---|
| B | lateral wall length | m |
| $B_i$ | upstream overhanging length | m |
| $B_o$ | downstream overhanging length | m |
| $d_{50}$ | The average particle diameter of the bed material | m |

| | | |
|---|---|---|
| Fr | Froude number | - |
| Frd | densimetric Froude number | - |
| g | acceleration of gravity | $m/s^2$ |
| $H_d$ | total tailwater head downstream of the weir | m |
| $H_u$ | total flow head upstream of the weir | m |
| $K_1$, $K_2$, $K_3$ and $K_4$ | Fixed coefficient | - |
| P | weir height | m |
| q | flow rate per unit width | $m^2/s$ |
| Q | discharge | $m^3/s$ |
| Re | Reynolds number | - |
| Ts | wall thickness | m |
| W | weir width | m |
| $W_i$ | inlet key width | m |
| $W_o$ | outlet key width | m |
| We | Weber number | - |
| $X_s$ | distance of maximum scour depth to the weir toe | m |
| $Z_s$ | maximum scour depth | m |
| $\rho_s$ | sediment density | $Kg/m^3$ |
| $\rho_w$ | water density | $Kg/m^3$ |
| $\mu$ | dynamic viscosity | Kg/m.s |
| $\sigma$ | Surface tension | $Kg/s^2$ |

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