Data Dependence of Modified $\mathcal{S}-$Iteration for Asymptotically Quasi pseudocontractive operator.

البيانات المعتمدة لتكرار $\mathcal{S}$ للتطبيق الشبه الكاذب المحاذي

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Abstract:

In this paper , we prove a convergence result and a data dependence result of modified $\mathcal{S}-$Iteration for uniformly $L-$lipschitzion and Quasi asymptotically pseudocontractive mapping in Banach space .

Keywords : modified $\mathcal{S}-$ Iteration , uniformly $L-$lipschitzian , Asymptotically Quasi pseudocontractive operator , Data dependence.

المستخلص :

في هذا البحث نبرهن نتيجة التقارب ونتيجة البيانات المعتمدة للتكرار من نمط $\mathcal{S}$ للتطبيق لبشيز المنتظم والتطبيق شبه الكاذب المحاذي في فضاء بناخ .

1. Introduction

Chang [1] introduced one step iterative sequence (Mann iteration) of self mapping in uniformly smooth Banach space E as follows , for any $x_1 \in E$

$$x_{n+1} = (1-\varpi_n)x_n + H^n x_n \ , \forall\, n \geq 1 \qquad (i)$$

Where $\{\varpi_n\}$ is a sequence in [0,1] and then ,he established the convergence result of modified iteration (i) for asymptotically pseudocontractive self mapping . Zhiqun and Guiwen [2] introduced two step iterative sequences (Ishikawa iteration) with error of self mapping in Banach space E as follows , for any $x_1 \in E$

$$x_{n+1} = (1-\varpi_n - ƀ_n)\, x_n + \varpi_n H^n y_n + ƀ_n \vartheta_n$$
$$y_n = (1-\xi_n - \varepsilon_n)\, x_n + \xi_n H^n x_n + \varepsilon_n \sigma_n \ , \forall\, n \geq 1 \qquad (ii)$$

Where $\{\varpi_n\},\{\xi_n\},\{ƀ_n\},\{\varepsilon_n\},\{\vartheta_n\} and\ \{\sigma_n\}$ under some conditions and then ,they proved the convergence result of modified iteration (ii) for asymptotically pseudocontractive self mapping .

On the other hand, some authors had studied data dependence for several iterative sequences and diffrent mappings . In [3] Solutz established the data dependence result of Ishikawa method for contraction mappings. In[4] Soltuz and Grosan proved data dependence result for the same iteration when dealing with contractive like operators.

2. Preliminaries

Definition(2.1): [5]

Let G is a nonempty subset of Banach space E . A mapping $H: G \to G$ is said to be
(i)Uniformly L − Lipschitzian if $\exists\, L > 0$ such that $\|H^n x - H^n y\| \leq L \,\|x-y\|$ (1)
$\forall\, x, y \in G$ and $\forall\, n \in N$.

Definition(2.2): $[2]$

Let E is a real Banach space with the dula space E^* and the mapping $J: E \to 2^{E^*}$ is defined by
$J(x) = \{f \in E : \langle x, f\rangle = \|f\|\|x\|, \|f\| = \|x\|\}$, $\forall x \in G$ is said to be normalized duality mapping.

Definition(2.3): $[1]$

Let G is a nonempty subset of Banach space E . A mapping $H: G \to G$ is said to be
(i) Asymptotically pseudocontractive if $\exists$ a sequence $\{\hbar_n\} \subset [1,\infty[$ *and* $\hbar_n \to 1$ *as* $n \to \infty$ Such that

$$\langle H^n x - H^n y, j(x-y)\rangle \leq \hbar_n \|x-y\|^2 \quad (2)$$

$\forall x, y \in G$ *and* $\forall n \in N$.

(ii) Asymptotically quasi pseudocontractive if $\mathrm{F}(H) = \{x : x \in G ; Hx = x\} \neq \emptyset$ and sequence $\{\hbar_n\} \subset [1,\infty[$ *with* $\hbar_n \to 1$ *as* $n \to \infty$ such that

$$\langle H^n x - \rho, j(x-\rho)\rangle \leq \hbar_n \|x-\rho\|^2 \quad (3)$$

$\forall x \in G, \rho \in \mathrm{F}$ *and* $\forall n \in N$.

Definition(2.4): $[6]$

Let G is a nonempty and convex subset of Banach space E and $H: G \to G$ *is* a mapping for any $x_1 \in E$, the sequence $\{x_n\}$ define by

$$x_{n+1} = (1 - \varpi_n) H^n x_n + \varpi_n H^n y_n$$
$$y_n = (1 - \xi_n) x_n + \xi_n H^n x_n, \forall n \geq 1 \quad (4)$$

Is said to be $\mathcal{S}$ $-$iteration sequence ,where $\{\varpi_n\}, \{\xi_n\}$ are sequences in [0,1].

Definition(2.5): $[7]$

Let G is a nonempty subset of Banach space E and $H, \widetilde{H}: G \to G$ are two mappings. A mapping $\widetilde{H}$ is an approximate mapping of E , if $\forall x \in G$ and for $\Upsilon > 0$,then

$$\|Hx - \widetilde{H}x\| \leq \Upsilon \quad (5)$$

Lemma (2.6) : $[8]$

Let E is a real Banach space and $J: E \to 2^{E^*}$ is the normalized duality mapping , then $\langle x, j(y)\rangle \leq \|x\|\|y\|$ $\forall x, y \in G$ *and* $\forall j(x) \in J(x)$.

Lemma (2.7): $[9]$

Let $\{\tau_n\}$ is a bounded sequence in $[0,\infty[$, $\boldsymbol{\mu} : [0,\infty[\to [0,\infty[$ continuous strictly increasing map with $\varphi(0) = 0$ and $\exists\, n_0 \in N$ such that

$$\tau_{n+1} \leq (1-\omega)\tau_n + \omega\, \tau_{n+1} - \omega \frac{\Upsilon(\tau_{n+1})}{\tau_{n+1}} + \omega\, \beta_n, \ \forall n \geq n_0$$

Where $\omega \in (0,1)$, $\beta_n \geq 0$, for all $n \in N$ and $\lim_{n\to\infty} \beta_n = 0$. Then $lim_{n\to\infty} \tau_n = 0$.

***Lemma*(2.8) : $[4]$**

Let $\{\eta_n\}$ is a bounded sequence in $[0,\infty[$, $\exists\, n_0 \in N$, for all $n \geq n_0$, we have

$$\eta_{n+1} \leq (1 - đ_n)\eta_n + đ_n\, e_n ,$$

Where $đ_n \in (0,1)$, $\forall n \in N$, $\sum_{n=0}^{\infty} đ_n = \infty$ and $e_n \geq 0$. Then

$$0 \leq lim_{n\to\infty} sup\ \eta_n \leq lim_{n\to\infty} sup\ e_n .$$

3. Main Theorem

Theorem (3.1):

Let G is a nonempty convex and bounded subset of Banach space E and $H: G \to G$ *is* uniformly L $-$lipschitzian with $L \geq 1$ and asymptotically quasi pseudocontractive mapping with a sequence $\hbar_n \subset [1,\infty[$ *and* $\hbar_n \to 1$ *as* $n \to \infty$. Let $\{x_n\}$ is a sequence in (4) satisfying
(i) $\varpi_n \to 0$ *as* $n \to \infty$ *and* $\xi_n \to 0$ *as* $n \to \infty$ (ii) $\varpi_n + \xi_n = 1$, $\forall n \in N$.
(iii) $\sum_{n=1}^{\infty} \varpi_n = \infty$ and for a fixed $\omega \in (0,1)$ such that $(1 - \varpi_n) < \omega$, $\forall n \in N$.

If $F(H) \neq \emptyset$, $\exists$ continuous strictly increasing map $\boldsymbol{\mu} : [0,\infty[\to [0,\infty[$ with $\boldsymbol{\mu}(0) = 0$ such that

$$\langle H^n x - \rho , j(x-\rho) \rangle \leq \hbar_n \|x-y\|^2 - \boldsymbol{\mu}\| x-\rho \|$$
$$\forall\, j(x-\rho) \in J(x-\rho) , \rho \in F \text{ . Then } \|x_n - \rho\| = 0$$

Proof:

Let $\rho \in F(H)$, from condition (4) and condition (1), we get

$$\|H^n y_n - \rho\| \leq L\, \|y_n - \rho\|$$
$$= L\,\{ (1-\xi_n)\|x_n - \rho\| + \xi_n\|(H^n x_n - \rho)\|\}$$
$$= L\,\{ (1-\xi_n) + \xi_n L\}\|(x_n - \rho)\|$$
$$\leq \{(1-\xi_n)\, L + \xi_n L^2\}\, \|(x_n - \rho)\| \qquad (6)$$

By using condition (4), Lemma(2.6), Lemma (2.7) condition (3) and condition (1) , we obtain

$$\|x_{n+1} - \rho\|^2 = \langle x_{n+1} - \rho \ , j(x_{n+1} - \rho)\rangle$$
$$= \langle (1-\varpi_n)\,(H^n x_n - p) + \varpi_n H^n y_n - \rho, j(x_{n+1} - \rho)\rangle$$
$$= (1-\varpi_n)\,\langle H^n x_n - H^n x_{n+1} + H^n x_{n+1} - \rho, j(x_{n+1} - \rho)\rangle$$
$$+\varpi_n \langle H^n y_n - \rho, j(x-\rho) \rangle$$
$$\leq (1-\varpi_n)\langle H^n x_n - H^n x_{n+1} , j(x_{n+1} - \rho)\rangle$$
$$+(1-\varpi_n)\langle H^n x_{n+1} - \rho, j(x_{n+1} - \rho)\rangle$$
$$+\varpi_n \langle H^n y_n - \rho, j(x_{n+1} - \rho)\rangle$$
$$\leq (1-\varpi_n)\,\|H^n x_n - H^n x_{n+1}\|\,\|x_{n+1} - \rho\|$$
$$+(1-\varpi_n)\,\{\hbar_n \|x_{n+1} - \rho\| - \boldsymbol{\mu}\, \|x_{n+1} - \rho\|$$
$$+\varpi_n L\,\|y_n - \rho\|\,\|x_{n+1} - \rho\| \qquad (7)$$

Putting (6) into (7), imply that

$$\|x_{n+1} - \rho\|^2 \leq (1-\varpi_n)\,\|H^n x_n - H^n x_{n+1}\|\,\|x_{n+1} - \rho\|$$
$$+(1-\varpi_n)\hbar_n\,\|x_{n+1} - \rho\|^2 - (1-\varpi_n)\,\boldsymbol{\mu}\,\|x_{n+1} - \rho\|$$
$$+\varpi_n\,\{(1-\xi_n)\, L + b_n L^2\}\|x_n - p\|\,\|x_{n+1} - \rho\|$$
$$\|x_{n+1} - \rho\|^2 \leq \|x_{n+1} - \rho\|\,\{(1-\varpi_n)\|H^n x_n - H^n x_{n+1}\|$$
$$+(1-\varpi_n)\hbar_n\|x_{n+1} - p\| - (1-\varpi_n)\,\frac{\boldsymbol{\mu}\|x_{n+1}-\rho\|}{\|x_{n+1}-\rho\|}$$
$$+\varpi_n\,\{(1-\xi_n)\, L + \xi_n L^2\,\}\,\|x_n - \rho\|$$

Since $\varpi_n \to 0$ *as* $n \to \infty$ *and* $\xi_n \to 0$ *as* $n \to \infty$, then $\varpi_n\{(1-b_n)L + b_n L^2\} = \varpi_n L - \varpi_n\xi_n L + \varpi_n\xi_n L^2 < (1-\omega)$, $\omega \in (0,1)$ *and* $(1-\varpi_n) < \omega$, *we get*

$$\|x_{n+1} - \rho\| \leq (1-\omega) \quad \|x_n - \rho\| \quad + \quad \omega \quad \hbar_n\|x_{n+1} - \rho\| - \omega\,\frac{\boldsymbol{\mu}\|x_{n+1}-\rho\|}{\|x_{n+1}-\rho\|}$$
$$+\ \omega\,\|H^n x_n - H^n x_{n+1}\|$$

Since G is bounded set in E and $\|H^n x_n\|$, $\|H^n y_n\|$ *and* $\|x_n\|$ *in* G ,then $\|H^n x_n\|$, $\|H^n y_n\|$ and $\|x_n\|$ are bounded sequences.

From condition (4) , $(\varpi_n + \xi_n) = 1$, $\varpi_n \to 0$ *as* $n \to \infty$ *and* $\xi_n \to 0$ *as* $n \to \infty$, then

$$\|x_{n+1} - x_n\| = (1-\varpi_n)\,\|H^n x_n\| + \varpi_n\|H^n y_n\| + (1-\varpi_n)\,\|x_n\| + \varpi_n\|x_n\|$$
$$= \xi_n\|H^n x_n\| + \varpi_n\|H^n y_n\| + \xi_n\|x_n\| + \varpi_n\|x_n\| \to 0 \ \text{as}\ n \to \infty \qquad (8)$$

Since H is uniformly $L-$lipschitzian ,then it is uniformly equi$-$continuous [10] and condition (8) lead to

$$lim_{n\to\infty}\|H^n x_n - H^n x_{n+1}\| = 0 \qquad (9)$$

We set $\tau_n = \|x_n - \rho\|$

$\beta_n = \|H^n x_n - H^n x_{n+1}\|$

Since $\omega \in (0,1)$ *and* $\hbar_n \to 1$ *as* $n \to \infty$, *then* $\exists\, n_0 \in N$ *such that* $\omega\hbar_n \in (0,1)$ and use lemma (2.7) we obtain $lim_{n\to\infty}\, \tau_n = 0$

By using theorem (1) , we establish the data dependence theorem.

Theorem (3.2) :

Let E , G and H as in the theorem (1) and $\widetilde{H}$ is an approximate operator of H satisfying $\|Hx - \widetilde{H}x\| \leq \Upsilon$, $for\ all\ x \in G\ \ and\ \ for\ \ \Upsilon > 0$. Let $\{x_n\}$ is a sequence in (4) and define an iteration sequence $\{\tilde{x}_n\}$ as follows , for any $\tilde{x}_1 \in E$

$$\tilde{x}_{n+1} = (1 - \varpi_n)\tilde{x}_n + \varpi_n \widetilde{H}^n \tilde{y}_n$$
$$\tilde{y}_n = (1 - \xi_n)\tilde{x}_n + \xi_n \widetilde{H}^n \tilde{x}_n \qquad (10)$$

where $\{\varpi_n\}$, $\{\xi_n\}$ be sequences in (0,1) satisfying : $(i)\ \varpi_n + \xi_n = 1\ ,\forall\ n \in N$. $(ii)\ \varpi_n \to 0\ \ as\ \ n \to \infty\ and\ \xi_n \to 0\ \ as\ n \to \infty.$ $(iii)\ \sum_{n=1}^{\infty} \varpi_n = \infty$.
If $H\rho = \rho$ and $\widetilde{H}\,\tilde{\rho}=\tilde{\rho}$ such that $\tilde{x}_n \to \tilde{\rho}$ as $n \to \infty$. Then $\|\rho - \tilde{\rho}\| \leq \frac{L\Upsilon + 2\Upsilon}{1-k}$.
where $\Upsilon > 0$ is a fixed number $and\ \kappa \in (0,1)$.

Proof:

From condition (10), lemma(2.6) and condition (3), we get

$$\begin{aligned}\|x_{n+1} - \tilde{x}_{n+1}\|^2 &= \langle x_{n+1} - \tilde{x}_{n+1}, j(x_{n+1} - \tilde{x}_{n+1})\rangle \\ &= \langle (1 - \varpi_n)(H^n x_n - \widetilde{H}^n \tilde{x}_n) + \varpi_n (H^n y_n - \widetilde{H}^n \tilde{y}_n), j(x_{n+1} - \tilde{x}_{n+1})\rangle \\ &= (1 - \varpi_n)\langle (H^n x_{n+1} - H^n \tilde{x}_{n+1}), j(x_{n+1} - \tilde{x}_{n+1})\rangle \\ &\quad + (1 - \varpi_n)\langle (H^n x_n - H^n x_{n+1}), j(x_{n+1} - \tilde{x}_{n+1})\rangle \\ &\quad + (1 - \varpi_n)\langle H^n \tilde{x}_{n+1} - \widetilde{H}^n \tilde{x}_n), j(x_{n+1} - \tilde{x}_{n+1})\rangle \\ &\quad + (1 - \varpi_n)\langle H^n y_n - \widetilde{H}^n \tilde{y}_n, j(x_{n+1} - \tilde{x}_{n+1})\rangle \\ &\leq (1 - \varpi_n)\{\hbar_n \|x_{n+1} - \tilde{x}_{n+1}\|^2 - \boldsymbol{\mu}\|x_{n+1} - \tilde{x}_{n+1}\|\} \\ &\quad + (1 - \varpi_n)\|H^n x_n - H^n x_{n+1}\|\|x_{n+1} - \tilde{x}_{n+1}\| \\ &\quad + (1 - \varpi_n)\left\|H^n\ \tilde{x}_{n+1} - \widetilde{H}^n \tilde{x}_n\right\|\|x_{n+1} - \tilde{x}_{n+1}\| \\ &\quad + \varpi_n \left\|H^n y_n - \widetilde{H}^n \tilde{y}_n\right\|\|x_{n+1} - \tilde{x}_{n+1}\|\end{aligned}$$

Hence

$$\begin{aligned}\|x_{n+1} - \tilde{x}_{n+1}\| &\leq (1 - \varpi_n)\,\hbar_n \|x_{n+1} - \tilde{x}_{n+1}\| - (1 - \varpi_n)\frac{\mu\|x_{n+1} - \tilde{x}_{n+1}\|}{\|x_{n+1} - \tilde{x}_{n+1}\|} \\ &\quad + (1 - \varpi_n)\|H^n x_n - H^n x_{n+1}\| + (1 - \varpi_n)\left\|H^n \tilde{x}_{n+1} - \widetilde{H}^n \tilde{x}_n\right\| \\ &\quad + \varpi_n \left\|H^n y_n - \widetilde{H}^n \tilde{y}_n\right\| \qquad (11)\end{aligned}$$

From conditions (1) and (5) , we get

$$\begin{aligned}\left\|H^n y_n - \widetilde{H}^n \tilde{y}_n\right\| &= \|H^n y_n - H^n \tilde{y}_n\| + \left\|H^n\ \tilde{y}_n - \widetilde{H}^n \tilde{y}_n\right\| \\ &\leq L\,\|y_n - \tilde{y}_n\| + \Upsilon \qquad (12)\end{aligned}$$

From conditions (4) , (10) and (1), then

$$\begin{aligned}\|y_n - \tilde{y}_n\| &\leq \left\|(1 - \varpi_n)(x_n - \tilde{x}_n) + \varpi_n(\ H^n x_n - \widetilde{H}^n \tilde{x}_n\ )\right\| \\ &\leq (1 - \varpi_n)\|x_n - \tilde{x}_n\| + \varpi_n \left\|H^n x_n - \widetilde{H}^n \tilde{x}_n\right\| \\ &\leq (1 - \varpi_n)\|x_n - \tilde{x}_n\| + \varpi_n \|H^n x_n - H^n \tilde{x}_n\| \\ &\quad + \varpi_n \left\|H^n\ \tilde{x}_n - \widetilde{H}^n \tilde{x}_n\right\| \\ &\leq (1 - \varpi_n)\|x_n - \tilde{x}_n\| + \varpi_n\ L\,\|\ x_n - \tilde{x}_n\| + \varpi_n \Upsilon \qquad (13)\end{aligned}$$

Putting (13) into (12) , then

$$\begin{aligned}\left\|H^n y_n - \widetilde{H}^n \tilde{y}_n\right\| &\leq (1 - \varpi_n)\ L\|x_n - \tilde{x}_n\| + \varpi_n\ L^2\|x_n - x_n\| \\ &\quad + L\,\varpi_n\,\Upsilon + \Upsilon\end{aligned}$$

Hence

$$\begin{aligned}\varpi_n\left\|H^n y_n - \widetilde{H}^n \tilde{y}_n\right\| &\leq (1 - \varpi_n)\,L\,\|x_n - x_n\| + \varpi_n{}^2\,L^2\,\|x_n - \tilde{x}_n\| \\ &\quad + L\,\varpi_n{}^2\ \Upsilon + \varpi_n \Upsilon \\ &\leq \{\varpi_n\,(1 - \varpi_n)\,L + \varpi_n{}^2\ L^2\}\,\|x_n - \tilde{x}_n\| + L\,\varpi_n{}^2 \Upsilon + \varpi_n \Upsilon \qquad (14)\end{aligned}$$

By using conditions (1) and (5), we get

$$\begin{aligned}\left\|H^n\ \tilde{x}_{n+1} - \widetilde{H}^n \tilde{x}_n\right\| &= \left\|H^n\ \tilde{x}_{n+1} - H^n \tilde{x}_n + H^n\ \tilde{x}_n - \widetilde{H}^n \tilde{x}_n\right\| \\ &\leq \|H^n\ \tilde{x}_{n+1} - H^n \tilde{x}_n\| + \left\|H^n\ \tilde{x}_n - \widetilde{H}^n \tilde{x}_n\right\| \\ &\leq L\ \|\ \tilde{x}_{n+1} - \tilde{x}_n\| + \Upsilon \qquad (15)\end{aligned}$$

$$\| \tilde{x}_{n+1} - \tilde{x}_n \| = \|\tilde{x}_{n+1} - x_{n+1} + x_{n+1} - x_n\|$$
$$\leq \| \tilde{x}_{n+1} - x_{n+1}\| + \|x_{n+1} - \tilde{x}_n\| \quad (16)$$

Putting (16) into (15), then

$$(1-\varpi_n)\left\|H^n \tilde{x}_{n+1} - \widetilde{H}^n \tilde{x}_n\right\| \leq (1-\varpi_n)\, L\, \|\tilde{x}_{n+1} - x_{n+1}\|$$
$$+(1-\varpi_n)\, L\, \|x_{n+1} - \tilde{x}_n\| + (1-\varpi_n)\, \Upsilon \quad (17)$$

Putting conditions (14) and (17) in condition (11), we get

$$\|x_{n+1} - \tilde{x}_{n+1}\| \leq \{(1-\varpi_n)h_n + (1-\varpi_n)\, L\}\ \|x_{n+1} - \tilde{x}_{n+1}\|$$
$$+\{\varpi_n(1-\varpi_n)\, L + \varpi_n^{\,2}\, L^2\}\ \|x_n - \tilde{x}_n\| + L\, \varpi_n^{\,2}\Upsilon + \Upsilon$$
$$- \quad (1-\varpi_n)\frac{\boldsymbol{\mu}\|x_{n+1} - \tilde{x}_{n+1}\|}{\|x_{n+1} - \tilde{x}_{n+1}\|} + (1-\varpi_n)\ \|H^n\, x_n - H^n\, x_{n+1}\|$$
$$+(1-\varpi_n)\hbar_n\ \|x_{n+1} - \tilde{x}_n\|$$

Using the same of the proof in conditions (8) and (9) in the theorem (1), then

$lim_{n\to\infty}\|x_{n+1} - \tilde{x}_n\| = 0$ and $\lim_{n\to\infty}\|H^n\, x_n - H^n\, x_{n+1}\| = 0$

$$\|x_{n+1} - \tilde{x}_{n+1}\| \leq \{(1-\varpi_n)\hbar_n + (1-\varpi_n)\, L\ \}\ \|x_{n+1} - \tilde{x}_{n+1}\|$$
$$+\{\varpi_n\, (1-\varpi_n)\mathrm{L} + \varpi_n^{\,2}\mathrm{L}^2\ \}\ \|x_n - \tilde{x}_n\| + L\, \varpi_n^{\,2}\Upsilon + \Upsilon$$
$$\{1 - \hbar_n + \varpi_n\, \hbar_n - L + \varpi_n\, L\}\ \|x_{n+1} - \tilde{x}_{n+1}\| \leq$$
$$\{\varpi_n\, (1-\varpi_n)\, L + \varpi_n^{\,2}\ L^2\}\|x_n - \tilde{x}_n\| + L\varpi_n^{\,2}\Upsilon + \Upsilon$$

$$\|x_{n+1} - \tilde{x}_{n+1}\| \leq \frac{\varpi_n\,(1-\varpi_n)\mathrm{L} + \varpi_n^{\,2}\, L^2}{1-\hbar_n+\varpi_n\,\hbar_n - L + \varpi_n L}\ \|x_n - \tilde{x}_n\| + \frac{L\varpi_n^{\,2}\,\Upsilon + \Upsilon}{1-\hbar_n+\varpi_n\,\hbar_n - L + \varpi_n L}$$
$$\leq 1 - \frac{\varpi_n\,(\varpi_n\, L - \varpi_n L - L + \hbar_n\,)}{1-\hbar_n+\varpi_n\,\hbar_n - L + \varpi_n L}\ \|x_n - \tilde{x}_n\| + \frac{L\varpi_n^{\,2}\,\Upsilon + \Upsilon}{1-\hbar_n+\varpi_n\,\hbar_n - L + \varpi_n L}$$

Since $\varpi_n \to 0$, $\hbar_n \to 0\ \ as\ \ n \to \infty$, then $\exists\ n_0 \in N$ such that

$(\varpi_n\, \mathrm{L} + \varpi_n\, \mathrm{L} - \mathrm{L} + \hbar_n\,) \leq\ (1-\kappa), \kappa \in (0,1)$

$$\|x_{n+1} - \tilde{x}_{n+1}\| \leq 1 - \varpi_n(1-\kappa)\ \|x_n - \tilde{x}_n\| + L\varpi_n^{\,2}\Upsilon + \Upsilon$$
$$\leq 1 - \varpi_n(1-\kappa)\|x_n - \tilde{x}_n\| + L\varpi_n\Upsilon + 2\varpi_n\Upsilon$$
$$\leq 1 - \varpi_n(1-\kappa)\ \|x_n - \tilde{x}_n\|\ + \varpi_n(1-k)\frac{(L\Upsilon+2\Upsilon)}{(1-\kappa)}$$

Denote that

$\eta_n = \|x_n - \tilde{x}_n\|$, $đ_n = \ \varpi_n(1-\kappa) \in (0,1)$

$e_n = \frac{(L\Upsilon\kappa+2\Upsilon)}{(1-\kappa)}$

It follows from lemma (2.8) that

$$0 \leq \lim_{n\to\infty} sup\ \|x_n - \tilde{x}_n\|\ \leq \lim_{n\to\infty} sup\,\frac{(L\Upsilon+2\Upsilon)}{(1-\kappa)}$$

From theorem(1) we have that $\lim_{n\to\infty} x_n = \rho$, thus , using this fact together with the assumption $\lim_{n\to\infty} \tilde{x}_n = \tilde{\rho}$, we obtain

$$\|\rho - \tilde{\rho}\|\ \ \leq \frac{(L\Upsilon+2\Upsilon)}{(1-\kappa)} \quad , \ \Upsilon > 0\ \ and\ \ \kappa\ \in\ (0,1)$$

4. Conclusions

The convergence result for modified $\mathcal{S}-$Iteration has been proved when applied to the mapping which that $H: G \to G$ is quasi asymptotically pseudocontractive in a real Banach space . Also the data Dependence result of modified $\mathcal{S}-$Iteration has been established when applied to the mapping which that $H: G \to G$ is quasi asymptotically pseudocontractive and the mapping $\widetilde{\mathrm{H}}$: G → G is an approximate operator of H satisfying $\|Hx - \widetilde{H}x\| \leq\ \Upsilon$, $\forall x \in\ G$ and for $\Upsilon > 0$.

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