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Solving Three Objectives Single-Machine Scheduling Problem Using Fuzzy Multi-Objective Linear Programming

Dara A. Hassan[1], **Nezam M. Amiri**[2], **Ayad M. Ramadan**[3]

[1] *College of Basic Education, Mathematical Sciences, University of Sulaimani, KRG, Iraq*
[2] *Faculty of Mathematical Sciences, Sharif University of Technology, Tehran, Iran*
[3] *College of Science, Mathematics Department, University of Sulaimani, KRG, Iraq*

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Corresponding Author:

Name: Dara A. Hassan

E-mail: dara.hassan@univsul.edu.iq

Tel:

ABSTRACT

In this paper, three criteria scheduling problem of n jobs on a single machine is considered. Each of these n jobs is to be processed without interruption and becomes available for processing at time zero. The problem is to minimize three objectives simultaneously, which are the completion time, maximum tardiness, and maximum earliness. Here, we develop a fuzzy multi-objective linear programming (FMOLP) model for solving multi-objective scheduling problem in a fuzzy environment by using piecewise linear membership function (PLMF). A numerical example demonstrates the feasibility of applying the proposed model to scheduling problem, and yields a compromised solution to help the decision maker's overall levels of satisfaction. The algorithm is tested to show the ability of applying this model to three criteria.

1. Introduction

In today's manufacturing industry multi-criteria optimization simultaneously is considered rather than only individual criteria [1]. In job scheduling problems, to meet customer's requirements usually, jobs must be arranged in an orderly sequence [2]. More often, there are one or more objectives to meet in assigning a finite number of resources to a number of jobs over a period of time. This assignment is called scheduling. According to [3], the most significant elements in any modern manufacturing systems is scheduling of jobs and controlling their flow via a production process. There are different type of scheduling problems, the single-machine is one of the simplest and basis for other complex types. Usually, measures of performance such as completion time, earliness, tardiness, sum of maximum earliness and tardiness are account for optimization of single-machine scheduling problems. In this type of problem there is one machine that is responsible of processing all the jobs in order to optimize one or more objectives. It is obvious that a model with only single-criterion can be solved in a totally different way than a model with multiple-criteria, especially the conflicted ones.

Quite often, decision maker DM is faced with minimizing one of the above mentioned measures. However, some times DM wants to optimize multiple-objectives at the same time, in another word simultaneously, which might conflict each other. In these instances, a solution may perform well to optimize one of the objectives and perform poor to optimize the other objective. An example of such situation is when there is a single-machine with no constraints. The optimum sequences of the jobs is found by SPT-rule to minimize mean flow time, and to find optimum sequence to minimize maximum tardiness the EDD-rule is used.

In reality, all decision maker's desire is to minimize a given criterion. For example, satisfying customers and then minimizing tardiness is of interest the commercial manager of a company. While, minimizing the makespan or the work in process by minimizing the maximum flow time are the goals the production manager wants to achieve by optimizing the use of machines. Each of these objectives is valid from a general perspective. That is why scheduling problem by its nature have a multiple-objective structure rather than single objective [4]. While in most researches in single-machine scheduling, minimizing single-criterion is their main concern, multiple-criteria is more realistically practical [4- 6]. Lee et al. [7] used linguistic values to solve a multi-criteria single machine scheduling problem. They evaluated each of the criterion as "very poor, poor, fair, good, and very good". Also they represented their relative weights as "very unimportant, unimportant, moderately important, important, and very important".

Another approach is used by Adamopoulos and Pappis [8], they have proposed a fuzzy-linguistic to solve a multi-criteria sequencing problem. In their perspective, each job was characterized by fuzzy processing times in a single-machine environment. They sequenced the jobs on the machine by associating penalty values with due dates assigned, tardiness, and earliness and processing time of the jobs as the objective. Another single-machine scheduling problem was considered by Ishi and Tada [9]. The objective was minimizing the maximum lateness of the jobs with fuzzy precedence relations. In their model the crisp precedence relation was relaxed by a fuzzy precedence and the satisfaction level with respect to the precedence between two jobs was presented. In order to maximize the minimum satisfaction level that is gained through the fuzzy precedence relations an additional objective was introduced. Though for determining non-dominated solutions an algorithm was determined based on the precedence relations graph representations.

According to Chanas and Kasperski [10], the difference between fuzzy completion time and fuzzy due date of a job or fuzzy maximum of zero is the fuzzy tardiness of that job in a given sequence. Knowing that the problem was a single-machine scheduling problem with fuzzy processing times and fuzzy due dates. Another example of single-machine problem which gives parameters in the form of fuzzy numbers. This type of problem was considered in [11] by assuming that optimal schedule is not easily determined to be precise. This is showing how to calculate the degrees of necessary of optimality of a given schedule in one of the special cases of single-machine scheduling problems.

The two-machine flow shop scheduling was considered by Toktas et al. [12]. The objective was minimizing makespan and maximum earliness simultaneously. They proposed a heuristic procedure to generate approximate efficient solutions and they developed a branch-and-bound procedure that generates all efficient solutions with respect to the two mentioned criteria. Also, a bi-criteria single machine scheduling problem with maximum weighted tardiness and number of tardy jobs as objective was considered by [13]. They kept one of these two criteria as the primary and the other one as secondary criterion, and gave Np-hardness proofs for the scheduling problem.

A fuzzy goal programming approach was presented by [14]. The approach was to solve a mixed integer model of a single-machine scheduling problem in order to minimize the total weighted flow time and total weighted tardiness, since these objectives conflict each other as they stated, they introduced a fuzzy goal programming approach to solve the extended mathematical model of a single machine scheduling problem. They argued that they constructed their approach based on the desirability of the DM and tolerances considered on goal values.

Zimmermann [15] first extended his fuzzy linear programming FLP approach to a conventional multi-objective linear programming (MOLP) problem [16]. For each of the objective functions of this problem, assume that the decision maker DM has a fuzzy goal such as 'the objective functions should be essentially less than or equal to some value'. Then, the corresponding linear membership function is defined and the minimum operator proposed by Bellman and Zadeh [17] is applied to combine all the objective functions. This problem can be transformed into an equivalent conventional linear programming LP problem by introducing auxiliary variables, and can be solved easily by the simplex method. [19- 21] are the subsequent works on fuzzy goal programming FGP.

The three criteria have been considered before by many authors individually or composite in different environments. Here we will present some recent works regarding these criteria. Jawad et al. [22] presented heuristic approaches to the problem of total completion time and the total earliness, where Ali and Jawad [23] applied local search methods for the same problem. In the fuzzy environment, Cheachan and Kadhim [24] introduced a branch and bound method to the total completion time and the maximum earliness. By composing four criteria, Chachan and Hameed [25] used branch and bound method to find exact solution for the problem namely, completion time, the tardiness, the earliness, and the late work. Aneed [26] with unequal release date used a branch and bound method to the sum of completion times, maximum earliness and maximum tardiness.

The aim of this paper is to develop a fuzzy multi-objective linear programming (FMOLP) model that finds a solution for a multi-objective single-machine scheduling problem in an environment that is fuzzy by it is nature (minimizing completion time, maximum earliness, and maximum tardiness) simultaneously. So, initially, a (MOLP) model for a multi-objective single-machine scheduling problem is constructed. The model attempts to minimize the total completion time, maximum earliness, and maximum tardiness. Since these objectives are conflicting each, the initially constructed model is converted into a (FMOLP) model by integrating fuzzy sets and multi-objective programming approaches.

2. Notation and Basic Concepts

The following notations are used in order to describe the multi-objective single-machine scheduling model [27].

N = number of jobs,

p_j = processing time for job j $\forall j = (1, \dots, N)$,

d_j = due date for job j,

M = a large positive integer value,

MST: (minimum slack times) here, the jobs are sequenced in non-decreasing order of minimum slack time s_j, where $s_j = d_j - p_j$,

SPT: (shortest processing time) jobs are sequenced in non-decreasing order of p_j,

EDD: (Early due date) jobs are sequenced in non-decreasing order of d_j.

For the decision variables we have:

$$X_{ij} = \begin{cases} 1 & \text{if job j is scheduled after job i,} \\ 0 & \text{otherwise.} \end{cases}$$

C_j = Completion time for job j,

$\sum_{j=1}^{n} C_j$ = Total completion time for job j,

$E_j = \text{Max}\,\{d_j - C_j\,, 0\}$; the earliness of job j,

$E_{max} = \text{Max}\,\{E_j\}$; the maximum earliness,

$T_j = \text{Max}\,\{C_j - d_j, 0\}$; the tardiness of job j.

$T_{max} = \text{Max}\,\{T_j\}$; the maximum tardiness.

3. Mathematical Model

The following model has three criteria namely, Z_1: total completion time, Z_2: maximum earliness, Z_3: maximum tardiness [27], the aim is finding the best possible (optimal) schedule that minimizes these criteria. We should note that at least two of these objectives are in conflict with each other [4].

The problem can be stated as follows [3]:

$\text{Min } Z_1 = \sum_{j=1}^{n} C_j \quad (1)$

$\text{Min } Z_2 = E_{max} \quad (2)$

$\text{Min } Z_3 = T_{max} \quad (3)$

s.t.

$C_i \geq p_i \qquad \forall i, \quad (4)$

$X_{ij} \in \{0,1\} \qquad \forall i, j; i \neq j, \quad (5)$

$X_{ij} + X_{ji} = 1 \qquad \forall i, j; i \neq j, \quad (6)$

$C_i - C_j + MX_{ij} \geq p_i \qquad \forall i, j; i \neq j, \quad (7)$

$E_i = \text{Max}\{d_i - C_i, 0\} \qquad \forall i, j; i \neq j, \quad (8)$

$T_i = \text{Max}\{C_i - d_i, 0\} \qquad \forall i, j; i \neq j, \quad (9)$

$C_i,\ E_i,\ T_i,\ p_i, d_i \geq 0 \qquad i = 1, 2, \dots, n. \quad (10)$

The (4)[th] constraint ensures the completion time must be greater or equal to its processing time. Constraint (5) ensures that each job is assigned to only one position in the sequence. Constraint (6) specifies the order relation between two jobs scheduled. The (7)[th] constraint stipulates relative completion times of any two jobs, and M should be large enough [3]. Constraints (8, 9) specify the earliness and tardiness of each job, respectively, and the non-negativity constraint is in (10).

Now, piecewise linear membership function (PLMF) given in [18] can be used in order to convert the original (MOLP) into a (FMOLP) model [3]. This enables us to represent the fuzzy goals of the DM that is given in the [8] as (MOLP) model. Generally, the problem is converted to a solvable ordinary LP problem using the (PLMF) that is given in [17].

3.1. The Algorithm:

The proposed algorithm has the following steps:

Step 1: Use some values for each objectives Z_i, then specify a membership function for all Z_i individually (see Table 1).

Step 2: Graph the (PLMF).

Step 3: For each (PLMF) formulate a linear equation $f_i(Z_i)$ specify the intervals for each Z_i.

The intervals for possible values of each objective function Z_i was specified by the user as $[A_{i,u_i+1}\,, A_{i0}]$, implicating a piecewise membership function (PMF) (see Table 1). In general, (PMF) divided into two intervals. $[\,0, A_{i,u_i+1}]$ which represents the values with realistic solution. $[A_{i,u_i+1}\,, A_{i0}\,]$, represents the values that are unrealistic.

Table 1: Membership function $f_i(Z_i)$

Z_1	$> A_{10}$	A_{10}	A_{11}	A_{12}	…	A_{1u1}	A_{1,u_1+1}	$< A_{1,u_1+1}$
$f_1(Z_1)$	0	0	q_{11}	q_{12}	…	q_{1u1}	1	1
Z_2	$> A_{20}$		A_{21}	A_{22}	…	A_{2u2}	A_{2,u_2+1}	$< A_{2,u_2+1}$
$f_2(Z_2)$	0	0	q_{21}	q_{22}	…	q_{2u2}	1	1
Z_3	$> A_{30}$		A_{31}	A_{32}	…	A_{3u3}	$A_{3,u3+1}$	$< A_{3,u_3+1}$
$f_3(Z_3)$	0	0	q_{31}	q_{32}	…	q_{3u3}	1	1

$\mathbf{(0 \leq q_{ib} \leq 1, q_{ib} \leq q_{ib+1}, i = 1, 2, 3\,, b = 1, 2, \dots, u_i)}$

Step 3.1: Convert $f_i(Z_i)$ to new form.

$f_i(Z_i) = \sum_{b=1}^{P_i} \alpha_{ib} |Z_i - A_{ib}| + \beta_i Z_i + \theta_i, \qquad i = 1,2,3, \quad (11)$

where

$\alpha_{ib} = -\frac{\gamma_{i,b+1} - \gamma_i}{2}, \quad \beta_i = \frac{\gamma_{i,u_i+1} + \gamma_{i1}}{2}, \quad \theta_i = \frac{S_{i,u_i+1} + S_{i1}}{2}. \quad (12)$

Assume that $f_i(Z_i) = \gamma_{ir} Z_i + S_{ir}$, where $A_{i,r-1} \leq Z_i \leq A_{ir}$, γ_{ir} is the slope and S_{ir} is the y-intercept of the line on $[A_{i,r-1}, A_{ir}]$ in the (PLMF), get the following:

$$f_i(Z_i) = -\left(\frac{\gamma_{i2} - \gamma_{i1}}{2}\right) |Z_i - A_{i1}| - \left(\frac{\gamma_{i3} - \gamma_{i2}}{2}\right) |Z_i - A_{i2}| - \cdots - \left(\frac{\gamma_{I,u_i+1} - \gamma_{iu_i}}{2}\right) |Z_i - A_{ip_i}| + \left(\frac{\gamma_{I,u_i+1} + \gamma_{i1}}{2}\right) Z_i + \frac{S_{I,u_i+1} + S_{i1}}{2} \left(\frac{\gamma_{I,b+1} - \gamma_{ib}}{2}\right) \neq 0,$$

$i = 1,2,3\,, b = 1,2,\dots, u_i,$ (13)

where, $\gamma_{i1} = \left(\frac{q_{i_1} - 0}{A_{i_1} - A_{i_0}}\right)$, $\gamma_{i2} = \left(\frac{q_{i_2} - q_{i_1}}{A_{i_2} - A_{i_1}}\right)$, $\dots, \gamma_{I,u_i+1} = \left(\frac{1 - q_{iu_i}}{A_{I,u_i+1} - A_{iu_I}}\right)$, (14)

The i^{th} objective function has u_i number of broken points and S_{i,u_i+1} is the y-intercept on $[A_{i,u_i}, A_{i,u_i+1}]$.

Step 3.2: Introduce the following variables:

$Z_i + w_{ib}^- - w_{ib}^+ = A_{ib}, \quad i = 1,2,3, \quad b = 1,2,\dots,u_i,$ (15)

where, w_{ib}^- and w_{ib}^+ are the deviational variables in both directions of i^{th} point, and A_{ib} is the values of objective function of the i^{th} point.

Step 3.3: Put Eq. (15) in Eq. (13), we get the equation:

$$f_i(Z_i) - \left(\frac{\gamma_{i_2} - \gamma_{i_1}}{2}\right)(w_{i_1}^- - w_{i_1}^+) - \left(\frac{\gamma_{i_3} - \gamma_{i_2}}{2}\right)(w_{i_2}^- - w_{i_2}^+) - \dots - \left(\frac{\gamma_{i,u_i+1} - \gamma_{i,u_i}}{2}\right)(w_{iu_i}^- - w_{iu_i}^+) + \left(\frac{\gamma_{i,u_i+1} - \gamma_{i_1}}{2}\right) Z_i + \frac{S_{i,u_i+1} - S_{i_1}}{2}, \quad i = 1,2,3. \quad (16)$$

Step 4: To transform the problem into conventional LP problem, introduce a two-phase approach for the variable φ . This variable represents the overall degree of satisfaction with the DM's goals.

Step 4.1: Introduced "max-min" operator to solve a single-objective problem:

Max φ_0 (17)

s.t.

$$\varphi_0 - \leq \left(\frac{\gamma_{i_2} - \gamma_{i_1}}{2}\right)(w_{i_1}^- - w_{i_1}^+) - \left(\frac{\gamma_{i_3} - \gamma_{i_2}}{2}\right)(w_{i_2}^- - w_{i_2}^+) - \dots - \left(\frac{\gamma_{i,u_i+1} - \gamma_{i,u_i}}{2}\right)(w_{iu_i}^- - w_{iu_i}^+) + \left(\frac{\gamma_{i,u_i+1} - \gamma_{i_1}}{2}\right) Z_i + \frac{S_{i,u_i+1} - S_{i_1}}{2}, \quad i = 1,2,3. \quad (18)$$

$Z_i + w_{ib}^- - w_{ib}^+ = A_{ib}, \quad i = 1,2,3, \quad b = 1,2,\dots,u_i,$ (19)

along with constraints (4) - (10).

Step 4.2: We use the results obtained from this step in order to correct any disadvantages of step 4.1 might have. Here, the solution is forced to improve further, modify, and dominate the results obtained from step 4.1. We also add new auxiliary objective function with new constraints to obtain at least the satisfactory degree obtained from step 4.1. Thus, we have the final model as:

$$\text{Max}\ \varphi = \varphi_0 + \frac{1}{3}\sum_{i=1}^{3}(\varphi_i - \varphi_0) \quad (20)$$

s.t.

$$\varphi_0 \leq \varphi_i \leq -\left(\frac{\gamma_{i_2} - \gamma_{i_1}}{2}\right)(w_{i_1}^- - w_{i_1}^+) - \left(\frac{\gamma_{i_3} - \gamma_{i_2}}{2}\right)(w_{i_2}^- - w_{i_2}^+) - \dots - \left(\frac{\gamma_{i,u_i+1} - \gamma_{i,u_i}}{2}\right)(w_{iu_i}^- - w_{iu_i}^+) + \left(\frac{\gamma_{i,u_i+1} - \gamma_{i_1}}{2}\right) Z_i + \frac{S_{i,u_i+1} - S_{i_1}}{2}, i = 1,2,3. \quad (21)$$

$Z_i + w_{ib}^- - w_{ib}^+ = Y_{ib}, \quad i = 1,2,3\,, b = 1,2,\dots,u_i\,.$ (22)

and constraints (4) - (10).

Step 5: We implement and modify the model until a satisfactory solution is chosen by the DM.

3.2. Numerical Illustrations

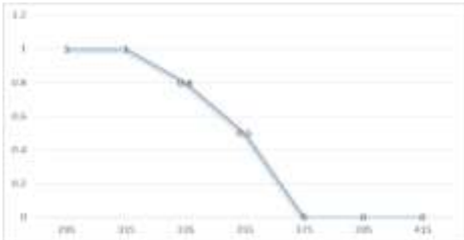
Consider the following 5-jobs example

Jobs	1	2	3	4	5
p_i	21	28	23	20	20
d_i	43	79	48	72	39

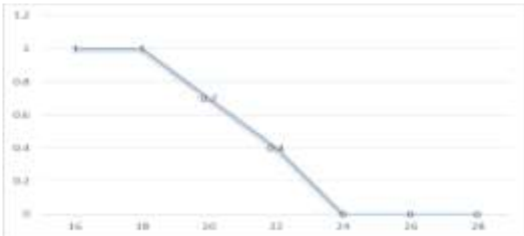
Now to formulate the (FMOLP) model, Firstly, find the initial solutions for each objective function by conventional LP individually. We get Z_1=317, Z_2=19, Z_3=33. After the initial solutions are obtained, the (FMOLP) is formulated using these findings. Also, we formulate the (MOLP) model which previously presented in Section 4. The results of the (PLMF) functions is presented in Table 2.Then, the shapes of the (PLMF) are presented in the Figs.1, 2 and 3.

Table 2: The (PLMF) for 5-jobs example

Z_1	>= 395	375	355	335	315	< 315
$f_1(Z_1)$	0	0	0.50	0.80	1	1
Z_2	> = 26	24	22	20	18	< 18
$f_2(Z_2)$	0	0	0.40	0.70	1	1
Z_3	>= 41	39	37	35	33	< 33
$f_3(Z_3)$	0	0	0.40	0.70	1	1



Fig, 1: $(Z_1, f_1(Z_1))$ membership function



Fig, 2: $(Z_2, f_2(Z_2))$ membership function

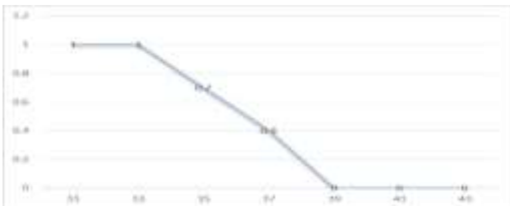


Fig. 3:$(Z_3, f_3(Z_3))$ membership function

For the 5-jobs problem, the complete (FMOLP) model is :

$$\text{Max}\ \varphi = \varphi_0 + \frac{1}{3}\sum_{i=1}^{3}((\lambda_1 - \varphi_0) + (\lambda_2 - \varphi_0) + (\lambda_3 - \varphi_0)) \quad (23)$$

s.t.

$$\varphi_0 \leq \lambda_1 \leq -0.005(w_{11}^- - w_{11}^+) - 0.0025(w_{12}^- - w_{12}^+) - 0.0175\{\textstyle\sum_{i=1}^{N} C_i\} + 6.85 \quad (24)$$

$$\varphi_0 \leq \lambda_2 \leq -0.025(w_{21}^- - w_{21}^+) - 0.175\{E_{Max}\} + 4.25 \quad (25)$$

$\varphi_0 \leq \lambda_3 \leq -0.025(w_{31}^- - w_{31}^+) - 0.175\{T_{Max}\} + 6.875$ (26)

$\{\sum_{i=1}^{N} C_i\} + w_{11}^- - w_{11}^+ = 355$ (27)

$\{\sum_{i=1}^{N} C_i\} + w_{12}^- - w_{12}^+ = 335$ (28)

$\{E_{Max}\} + w_{21}^- - w_{21}^+ = 22$ (29)

$\{T_{Max}\} + w_{31}^- - w_{31}^+ = 37$ (30)

$C_i \geq p_i \quad \forall i$ (31)

$X_{ij} \in \{0,1\} \quad \forall i,j; i \neq j,$ (32)

$X_{ij} + X_{ji} = 1 \quad \forall i,j; i \neq j$ (33)

$C_i - C_j + MX_{ij} \geq p_i \quad \forall i,j; i \neq j$ (34)

$E_i = Max\{d_i - C_i, 0\} \quad \forall i,j; i \neq j$ (35)

$T_i = Max\{C_i - d_i, 0\} \quad \forall i,j; i \neq j$ (36)

$C_i,\ E_i,\ T_i,\ w_{11}^-, w_{11}^+, w_{12}^-, w_{12}^+, w_{21}^-, w_{21}^+, w_{31}^-, w_{31}^+ \geq 0 \quad i = 1, 2, \ldots, n.$ (37)

The results of the formulated problem are $\varphi = 0.79$, Z1= 321, Z_2= 19, and Z_3= 33 with the schedule (5, 1, 3, 4, 2).

3.3. Computational Results:

Here, we use Lingo18 program to implement the model on Intel (R) Core (TM) i5- 2450 M CPU@ 2.50 GHz, with RAM 4.00 GB personal computer. The input data are the processing times p_i which generated from a uniform distribution on [20, 30], the due dates d_i are generated from a uniform distribution on [35, 80]. The obtaining results are Z_1=321, Z_2=19, Z_3=33 regarding with the objectives: total completion time, maximum earliness, and maximum tardiness respectively. The initial results obtained from solving each objective function individually using linear programing. The overall degree of satisfaction is 0.79 with the best sequence (5, 1, 3, 4, 2).

Table 3: Comparison between the individual LP sequences and the proposed method

	LP1	**LP2**	**LP3**	**Proposed Method**
Objective Function	Min Z_1	Min Z_2	Min Z_3	Max φ
φ	100	100	100	0.79
Z_1	317	321	321	321
Z_2	52	19	19	19
Z_3	36	33	33	33
Optimal Sequence	4,5,1,3,2	5,1,3,4,2	5,1,3,4,2	5,1,3,4,2

Conclusion

Solving conflicting objectives in scheduling is a task to find the best solution. To judge about tri-criteria scheduling problem we developed in this paper a (FMOLP) method for solving single-machine scheduling problems with multiple fuzzy objective. The tri-objectives function were the completion time, maximum tardiness, and maximum earliness. This method is presented for the first time to three objectives, so the overall level of satisfaction is acceptable comparing with two objectives. A numerical example was implemented to show the feasibility of applying this method to scheduling problems.

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حل مشكلة جدولة الماكنة الواحدة ثلاثية الاهداف باستخدام البرمجة الخطية الغامضة متعددة الأهداف

دارا علي حسن [1] ، نيزام مهداوي اميري [2]، اياد محمد رمضان [3]

[1] كلية التربية الأساسية ، العلوم الرياضية ، جامعة السليمانية ، إقليم كردستان العراق

[2] كلية العلوم الرياضية ، جامعة الشريف للتكنولوجيا ، طهران ، إيران

[3] كلية العلوم ، قسم الرياضيات ، جامعة السليمانية ، إقليم كردستان العراق

الملخص

في هذا البحث، تم دراسة جدولة مشكلة n من الوظائف لدالة ثلاثية المعاييروعلى ماكنة واحدة. تتم معالجة n من الاعمال دون انقطاع وتتطلب معالجة من الوقت صفر. تكمن المشكلة في تقليل ثلاثة أهداف في وقت واحد ، وهي وقت الاتمام ، اكبر تأخير، واكبر تبكير. هنا ، نقوم بتطوير نموذج برمجة خطية متعددة الأهداف (FMOLP) لحل مشكلة الجدولة متعددة الأهداف في بيئة ضبابية باستخدام دالة خطية متعددة الاجزاء (PLMF). يوضح المثال العددي جدوى تطبيق النموذج المقترح على جدولة المشكلة ، ويؤدي إلى حل وسط لمساعدة مستويات رضا صانع القرار بشكل عام. تم اختبار الخوارزمية لإظهار القدرة على تطبيق هذا النموذج على ثلاثة معايير.