New Properties of T_D – SPACES

خصائص جديده للفضاءات - T_D

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Abstract:

The main purpose of this paper is to study T_D – space and we will given the relation between T_D – space and T0 – spaces, T0 - Alexandroff spaces Also ,som other properties of T_D – spaces are proved.

المستخلص:

الهدف الرئيسي في هذا البحث هو دراسة الفضاء T_D وسوف نقوم بتقديم العلاقة بين الفضاءات T_D و الفضاءات -T0 وفضاءات الكيساندروف - T0 كذلك بعض خصائص الفضاءات T_D– وستبرهن .

1- Introduction:

The concept of T_D - spaces were introduced and studied by [1] . in order to extend many of the properties of locally closed sets introduced and investigated by [2] . in this work , we study the relationships between the T_D - spaces, T0-spaces and T0- Alexandoffspaces . further we prove that the property of being T_D - spaces is atopological property , but the containuous image of a T_D - spaces is not necessarily a T_D - spaces . Also several other properties of T_D - spaces are proved.

2- Basic Definitions, *Remarks and Examples* :

In this section, we recall the basic definitions,remarks and examples needed in this work.

Definition (2.1):[1]

Let (X,T) be a topological space,Then X is called a T_D – space if every point (singleton set) is locally closed .

Definition (2.2):[2]

Let (X, τ) be a topological spaces X is called a T½ - space if every point is open.

Definition (2.3):[2]

Let (X, τ) be a topological spaces let $S \subseteq X$ we say that S is locally closed if S = A1 $\cap$ A2 where A1 is open and A2 is closed .

Remarks and Examples (2.4):

(i) Every open set is locally closed also every closed set is locally closed .

(ii) Every T½ - spaces is a T_D - space let X be a T½ - space let $x \in X$ now $\{x\}$is either open or closed so $\{x\}$ is locally closed which means that X is a T_D - space

(iii) (IR , Tu) is a T_D - space (Tu is the usual topology on IR) because (IR,Tu) is a T1- space hence it is a T½ - space therefore (IR,,Tu) is a T_D - space

3. Main Results:

In this section, we state and prove several new properties of T_D - spaces .

proposition (3.1) :

Every T_D - space is a T0 - space.

Proof:

Let (X, τ) be a T_D - space let $x \in X$, $\{x\}$ is locally closed now every locally closed set is λ - closed [2] this means that every singleton is λ - closed in [3] , it was proved that X is a T0 – space iff every singleton is λ - closed hence X is a T0 – space before we state the next proposition we recall the following definition

Definition (3.2)[5]

Let (X, τ) be a topological spaces X is called Alexandroff space if every intersection of open sets is again open .

proposition (3.3):

Every T0 - Alexandroff space is a T_D - space .

Proof:

Let X be a T0 - Alexandroff space , let $x \in X$ now $x \in \{\bar{x}\}$ = closure of $\{x\}$ and $x \in ker\{x\}$ = the intersection of all open sets containing $\{x\}$ see [3] .
so $x \in ker\{x\} \cap \{\bar{x}\}$ we will prove that $\{x\} = ker\{x\} \cap \{\bar{x}\}$ suppose that $y \in ker\{x\} \cap \{\bar{x}\}$,$y \neq x$ since $y \in ker\{x\}$, every neighborhood of x contains y now $y \in \{\bar{x}\}$, so every neighborhood of y contains x now this is impossible for two distinct points in a T0 – space (recall that x is aT0 – space if given $x \neq y$, then there exists a neighborhood containing one of them but not the other) hence $\{x\} = ker\{x\} \cap \{\bar{x}\}$ which means that every singleton is locally closed therefore X is a T_D - space .

Remark (3.4):

Both T½ and T0 – alexandroff imply T_D, But there is no relation between T½ and T0 – alexandroff spaces because

(i) (IR,Tu) is T½ but is not alexandroff consider $\bigcap_{N}^{\infty}$ (-1/n , 1/n) this intersection = {0} which is not open .

(ii) X = {a ,b ,c} ,T = { ∅, X, {a}, {a,b} } now (X,T) is a T0 - alexandroff space but (X,T) is not T½ (the point {b} is neither open nor closed)

proposition (3.5):

the continuous image of a T_D - space , is not necessarily a T_D - space .

Proof:

Consider $I_{\Re}$: (IR , Tu) $\rightarrow$ (IR , Ti) (Ti is the indiscrete topology on IR, and $I_{\Re}$ is the identity on IR now $I_{\Re}$ *is* continuous and onto (IR,Tu) is a T_D - space but (IR ,Ti) is not a T0 – space hence it is not a T_D - space .

proposition (3.6):
the property of being T_D is a topological property

Proof:
Let X be a T_D - space and let f :X → Y be a homomorphism let y∈Y and x ∈ X such that y = f (x) now X is a T_D - space hence { x } is locally closed hence { x } = U ∩ V , where U is open in X and V is closed in X now y = f(x) ,then = f (U ∩V) = f (U) ∩ f (V) now f(U) is open and f(V) is closed in Y hence{ y } is locally closed which means that Y is T_D - space ■

4. References :

[1] C .E. Aull and W. J .Thorn , “separation axioms between T0 and T1” , Indagationesmath . 24 (1962) , 26 – 37 .

[2] N. Boubaki,"General Topology " , part 1 , Addision- Wesley, Reading . Mass, 1966 .

[3] -, H . J . Mustafa , “ T½ - spaces ” , Journal of the college of Education , AL – Mustansiriyah University ,(2002) .

[4] Hadi J Mustafa, and Awaad , B . K ., "Certain properties of λ- closed spaces ", accepted in the Journal of the University of karbala 2016.

[5] Hadi J Mustafa , and A . AL . Badeery " Al exandroff Spaces ", ,M.sc Thesis , Kufa University , 2011.