Using Δ - Discriminate Method to Determine the Stability and Bifurcation of Chen Chaotic System

Saad Fawzi AL-Azzawi
saad_fawzi78@yahoo.com
Department of Mathematics, College of Computer Sciences and Mathematics, University of Mosul, Mosul, Iraq

Karam A. Abed
Dar Ibn Al-Atheer for Printing and Publisher, University of Mosul, Mosul, Iraq

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ABSTRACT

The aim of this paper to found that the stability and bifurcation of Chen System by using a new method which called Δ- discriminate method as will as we found the stability at the second critical point $P_+(\sqrt{\beta(2r-\sigma)}\,,\sqrt{\beta(2r-\sigma)}\,,2\,r-\sigma\,)$ by another method, and we showed that the new method depended on the roots to determent the stability and bifurcation of this system while the previous methods depended on the parameters $\sigma,\, 2r$, also we showed the method which used to find the stability at the second critical point depended on the critical value β_c and parameter β. Finally, we get the same previous results but easily method.

Keywords: Chen system, stability, bifurcation, Δ- discriminate, Routh-Hurwitz.

استخدام طريقة Δ – المميزة لتحديد الاستقرارية والتشعب لنظام Chen المضطرب

سعد فوزي العزاوي
كلية علوم الحاسوب والرياضيات، جامعة الموصل

كرم عادل عبد
دار ابن الاثير للطباعة والنشر، جامعة الموصل

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الملخص

الهدف من هذا البحث هو إيجاد الاستقرارية والتشعب لنظام تشين بطريقة جديدة هي طريقة Δ – المميزة بالإضافة لذلك فقد تم إيجاد الاستقرارية عند النقطة الحرجة الثانية $P_+(\sqrt{\beta(2r-\sigma)}\,,\sqrt{\beta(2r-\sigma)}\,,2\,r-\sigma\,)$ بطريقة أخرى، وتبين بان الطريقة الجديدة تعتمد على الجذور في تحديد الاستقرارية والتشعب لهذا النظام بينما كانت الطرق السابقة تعتمد على المتغيرات $\sigma,\, 2r$. كما تبين بان الطريقة المستخدمة لإيجاد استقرارية النقطة الحرجة الثانية تعتمد على القيمة الحرجة β_c وعلى المتغير β. وأخيرا حصلنا على نفس النتائج السابقة لكن بطرق أكثر سهولة.

الكلمات المفتاحية: نظام تشين, الاستقرارية , التشعب, Δ – المميز , معايير Routh-Hurwitz .

1. Introduction

Chaos as a very interesting complex nonlinear phenomenon has been intensively studied in the last three decades within the science, mathematics and engineering communities [5].

In 1963, Lorenz discovered chaos in a simple system of three autonomous ordinary differential equations [7]. In 1999, Chen found a similar but not topological equivalent chaotic attractor to Lorenz's, which is now known to be the dual of the Lorenz system [4]. It is noticed that these systems can be classified into two types by the definition of Vanece and Celikovsky [3]: the Lorenz system satisfies the condition $a_{12}a_{21} > 0$, the Chen system satisfies $a_{12}a_{21} < 0$, where a_{12} and a_{21} are the corresponding elements in the linear part matrix $A = [a_{ij}]_{3\times3}$ of the system.

The mathematical model of **Lorenz system** is a system of nonlinear ordinary differential equations which has the following form:

$$\left.\begin{aligned}\dot{x} &= \sigma(y-x)\\ \dot{y} &= rx - xz - y\\ \dot{z} &= xy - \beta z\end{aligned}\right\} \qquad \ldots(1)$$

where $\sigma, r, \beta > o$, and Lorenz studied this system when $\sigma = 10$, $r = 28, \beta = 8/3$. [3]

Chen system it also has the form of nonlinear ordinary differential equations:

$$\left.\begin{aligned}\dot{x} &= \sigma(y-x)\\ \dot{y} &= (r-\sigma)x - xz + ry\\ \dot{z} &= xy - \beta z\end{aligned}\right\} \qquad \ldots(2)$$

where $\sigma, r, \beta > 0$ and $r < \sigma < 2r$, Chen take $\sigma = 35$, $r = 28, \beta = 3$. [6]

In the following we briefly describe some basic properties of the system (2).

1- Symmetry and Invariance:

Chen system has a symmetry S because the transformation:
$S:(x, y, z) \rightarrow (-x, -y, z)$

Which permits system invariant for all values of the system parameters $\sigma,\ r\ and\ \beta$. Obviously, the z-axis itself is an orbit [7].

2- Dissipative:

The system (2) can be a dissipative, because the divergenence of the vector field, also called the trace of the Jacobian matrix is negative if and only if the sum of the parameters $\sigma,\ r\ and\ \beta$ is positive

$$div\vec{V} = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} = Tr(J) = -(\sigma - r + \beta)\ , \qquad \ldots(3)$$

$$V(t) = V(0)e^{-(\sigma - r + \beta)t}$$

So, the system will always be dissipative if and only if when $\sigma - r + \beta > 0$ with an exponential rate: $\dfrac{d\vec{V}}{dt} = e^{-(\sigma - r + \beta)}$ [8] .

3- Critical points:

The critical points of system (2) can be easily found by solving the three equations $\dot{x} = \dot{y} = \dot{z} = 0$, which lead to

$\sigma(y - x) = 0$,

$(r-\sigma)x - xz + ry = 0$,

and $xy - \beta z = 0$.

It can be easily verified that there are three critical points [3]:

$p_o(0, 0, 0)$

$P_+(\sqrt{\beta(2r-\sigma)}, \sqrt{\beta(2r-\sigma)}, 2r-\sigma)$

$P_-(-\sqrt{\beta(2r-\sigma)}, -\sqrt{\beta(2r-\sigma)}, 2r-\sigma)$

In [1, 4, 5, 7, 10, 11] studied the stability and bifurcation of Lorenz system, a new Lorenz-like system, a generalized Lorenz-like system, unified chaotic system, system derived from the Lorenz system, a new chaotic system respectively, by using Routh-Hurwitz method. Where we can conclusion that the real part of the roots λ are negative if conditions of Routh-Hurwitz are satisfied without found the value of roots itself.

[8] studied the stability and bifurcation of Chen system by using Routh – Hurwitz method where depended on the parameters σ and $2r$ to determine the stability at the point $P_o(0,0,0)$, while depened on the direct substitute for Routh-Hurwitz method to determine the stability at the point
$P_{\pm}(\pm\sqrt{\beta(2r-\sigma)}\ ,\pm\sqrt{\beta(2r-\sigma)}\ ,2\,r-\sigma\ )$.

In this paper, we study the stability and bifurcation for the Chen system by using a new method which is called Δ- discriminate method. by this method we can find the exact roots of this system as well as we found the stability at the point $p_{\pm}$ by easily second method which depended on the critical value β_c, and we justified the same previous results, finally we showed that the relation between critical cases and bifurcation.

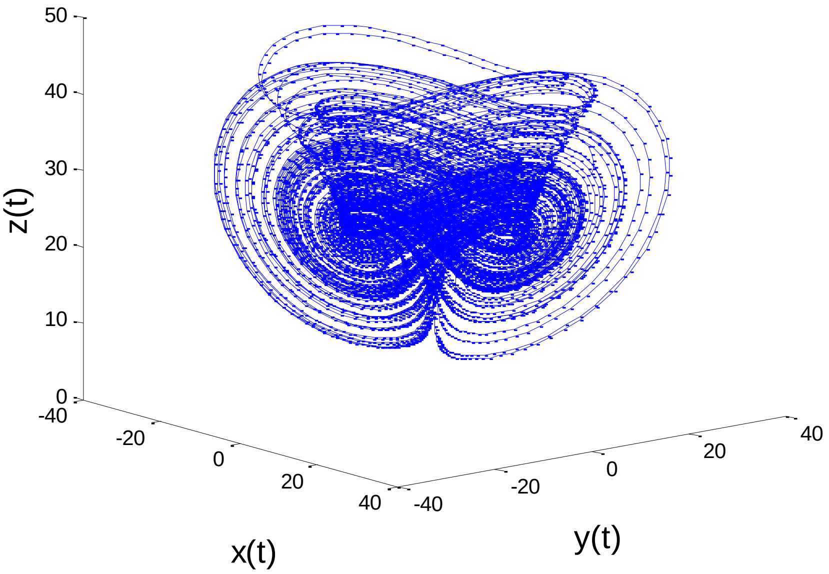

Figure 1: The attractor of Chen system

2- Helping Results:

In the context of ordinary differential equations ODEs the word "bifurcation" has come to mean any marked change in the structure of the orbits of a system (usually nonlinear) as a parameter passes through a critical value[2].

The theory of bifurcations of parameterized dynamical system is well known. One consider a vector field

$$\dot{x} = f_{\mu}(x) \qquad \mu \in R\,, \quad x \in R^n \qquad \ldots(4)$$

Depending on a parameter μ the critical point of the vector field are those x_0, μ_0 such that $f_{\mu 0}(x_0) = 0$.

Perhaps the most important property of critical point is its stability. In the first approximation, which is determined by stability of its liberalized system around x_0, μ_0

$$\dot{x} = D\, f_{\mu 0}(x_0) \qquad \mu_0 \in R\ , \quad x_0 \in R^n \qquad ...(5)$$

Where $D\, f_{\mu 0}(x_0)$ is the Jacobian matrix of f.

Theorem 1: (Hopf Bifurcation Theorem) [1]

Suppose that the system $\dot{x} = f_{\mu}(x)\ ,\mu \in R\ ,\ x \in R^n$ has critical point (x_0, μ_0), then this system has a Hopf bifurcation if the following properties are satisfied:

1- $D\, f_{\mu 0}(x_0)$ has a simple pair of pure imaginary eigenvalues and no other eigenvalues with zero real parts.

2- $\frac{d}{d\mu}(\mathrm{Re}(\lambda_{2/3}(\mu)))\big|_{\mu=\mu 0} = d \neq 0$

Remark 1: [1]

Let $\lambda^3 + a\lambda^2 + b\lambda + c = 0$ be the characteristic equation for a three-component system, where a , c indicate the trace and determinant rest, then a hopf bifurcation takes place of the transit through the surface

$$a\,b - c = 0 \quad \text{if} \quad a,\, b,\, c > 0 \qquad \ldots(6)$$

This condition is a necessary condition for a hopf bifurcation.

Remark 2: (Routh– Hurwitz Test) [2]

All the roots of the indicated polynomial have negative real parts precisely when the given conditions are met.

$$\lambda^3 + a\lambda^2 + b\lambda + c : \qquad a > 0, c > 0,\ ab - c > 0\ . \qquad \ldots(7)$$

Remark 3: (Critical Cases) [2]

Critical cases in the theory of stability for differential equation means, that cases when the real part of all roots of the characteristic equation are non positive with the real part of at least one root being zero, other express which is neither stable nor unstable.

Remark 4: [8]

(i) if $\sigma > 2r$, then system (2) has only one critical point, $p_o(\ 0\ ,0\ ,\ 0\ )$;

(ii) if $\sigma < 2r$, then system (2) has three critical point: $p_o(\ 0\ ,0\ ,\ 0\ )$,

$p_{\pm}(\pm\sqrt{\beta(2r-\sigma)}\ \ ,\ \ \pm\sqrt{\beta(2r-\sigma)}, 2\sigma - r)$

Theorem 2: [8]

(*i*) The solution of system (2) at critical point p_0 has the following cases:

1) Asymptotically stable if $\sigma > 2r$,

2) Unstable if $\sigma < 2r$.

(*ii*) The solution of system (2) at critical point p_+ or p_- has the following cases:

1) Asymptotically stable if $(\beta+\sigma-r)\beta r-2\sigma\beta(2r-\sigma)>0$,
2) Unstable if $(\beta+\sigma-r)\beta r-2\sigma\beta(2r-\sigma)<0$.

Remark 5: (Critical Value) [8]

Chen system has critical value which is $\sigma=2r$ at origin and $\beta_c=\dfrac{(3\sigma r-2\sigma^2+r^2)}{r}$ at the critical point p_+ .

Remark 6: (Pitchfork Bifurcation and Hopf Bifurcation) [8]

1-) Chen system has pitchfork bifurcation if $\sigma=2r$.
When $\sigma=2r$, this system has a one root being zero while the other two roots have a nonzero real part. In this case the solutions are $\lambda_1=-\beta$, $\lambda_2=0$, $\lambda_3<0$. and pitchfork bifurcation may appear only at the critical points p_0 .

2-) Chen system has hopf bifurcation if $\beta=\beta_c$ and $r>(\sqrt{17}-3)\sigma/2$.
When $\beta=\beta_c$, this system has is purely imaginary roots. In this case the solutions are $\lambda_1=2\sigma(\sigma-2r)/r$, $\lambda_{2,3}=\pm i\sqrt{\beta_c c}$ and hopf bifurcation may appear only at the critical points p_+ or p_-. Due to the symmetry of p_+ and p_-.

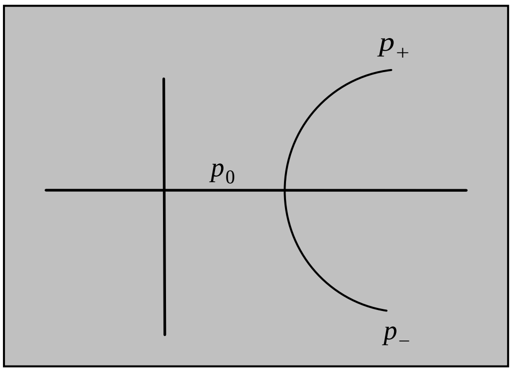

Figure 2: Sketch of the Pitchfork Bifurcation Diagram

Let us denote

$$q=c-\frac{1}{3}ab+\frac{2}{27}a^3, \qquad \ldots(8)$$

$$\Delta=c^2+\frac{4}{27}b^3-\frac{2}{3}abc-\frac{1}{27}a^2b^2+\frac{4}{27}a^3c \qquad \ldots(9)$$

We will use the following theorem, which enables us to find the exact roots for cubic equation (three degree).

Theorem 3: (The Δ-discriminate) [9]

1- If $\Delta=0$, then has three real roots , but one is multiple:

$$\lambda_1=-2\sqrt[3]{\frac{q}{2}}-\frac{a}{3}, \qquad \lambda_2=\lambda_3=\sqrt[3]{\frac{q}{2}}-\frac{a}{3} \qquad \ldots(10)$$

2- If $\Delta<0$, then has three different real roots:

$$\lambda_{k+1} = \sqrt[6]{16(q^2-\Delta)} \quad \cos\frac{\cos^{-1}\dfrac{-q}{\sqrt{q^2-\Delta}} + 2\pi k}{3} - \frac{a}{3} \quad , \; k = 0, 1, 2 \; . \qquad \ldots(11)$$

3- If $\Delta > 0$, then has one real root λ_1 and two complexes conjugate roots λ_2, λ_3 with non-vanishing imaginary parts:

$$\left.\begin{aligned}
\lambda_1 &= \sqrt[3]{\frac{-q-\sqrt{\Delta}}{2}} + \sqrt[3]{\frac{-q+\sqrt{\Delta}}{2}} - \frac{a}{3}, \\
\lambda_2 &= -\frac{1}{2}\left(\sqrt[3]{\frac{-q-\sqrt{\Delta}}{2}} + \sqrt[3]{\frac{-q+\sqrt{\Delta}}{2}}\right) - \frac{a}{3} + i\frac{\sqrt{3}}{2}\left(\sqrt[3]{\frac{-q-\sqrt{\Delta}}{2}} - \sqrt[3]{\frac{-q+\sqrt{\Delta}}{2}}\right), \\
\lambda_3 &= -\frac{1}{2}\left(\sqrt[3]{\frac{-q-\sqrt{\Delta}}{2}} + \sqrt[3]{\frac{-q+\sqrt{\Delta}}{2}}\right) - \frac{a}{3} - i\frac{\sqrt{3}}{2}\left(\sqrt[3]{\frac{-q-\sqrt{\Delta}}{2}} - \sqrt[3]{\frac{-q+\sqrt{\Delta}}{2}}\right)
\end{aligned}\right\} \qquad \ldots(12)$$

3- Main Results:

Corollary 1:

System (2) has one negative real root $\lambda_1 = -\beta$ at the point p_o and also one negative real root at the point p_+.

Theorem 4:

(i) The solution of system (2) at critical point p_0 has the following cases:

1) Asymptotically stable if $\Delta > 0$ and $\operatorname{Re}\lambda_2 < 0$,

2) Unstable if $\Delta < 0$ and $\lambda_2 > 0$ *or* $\lambda_3 > 0$,

3) Critical case if $\Delta < 0$ and $\lambda_2 = 0$ *or* $\lambda_3 = 0$.

(ii) The solution of system (2) at critical point p_+ when $\Delta > 0$ has the following case:

1) Asymptotically stable if $\operatorname{Re}\lambda_2 < 0$,

2) Unstable if $\operatorname{Re}\lambda_2 > 0$,

3) Critical case if $\operatorname{Re}\lambda_2 = 0$.

Proof:

(i) When $\Delta > 0$ we obtain: one real root and two complex conjugate roots (Theorem 3), and $\lambda_1 = -\beta$, $\beta > 0$ by corollary 1 and $\operatorname{Re}\lambda_2 < 0$ (given) then $\operatorname{Re}\lambda_3 < 0$ Since $\operatorname{Re}\lambda_2 = \operatorname{Re}\lambda_3$, consequently, all roots are negative real part, hence the system (2) is asymptotically stable, while when $\Delta < 0$, we get three different real roots $\lambda_1 = -\beta$ and $\lambda_2 > 0$ *or* $\lambda_3 > 0$ (given) then at least one of roots is positive therefore system (2) is unstable, finally when $\lambda_2 = 0$ *or* $\lambda_3 = 0$ then satisfied Remark 3 therefore the system (2) is critical case the proof is completed.

(ii) since $\Delta > 0$ we have one real root and two complex conjugate roots (Theorem 3), and the real root is negative at the point p_+ by corollary 1 and

1- if $\operatorname{Re}\lambda_2 < 0$ then $\operatorname{Re}\lambda_3 < 0$ also, therefore all roots are negative real part, hence the system (2) is asymptotically stable,
2- if $\operatorname{Re}\lambda_2 > 0$ then $\operatorname{Re}\lambda_3 > 0$, consequently, real parts of two roots are positive therefore system (2) is unstable,
3- if $\lambda_2 = 0$ then $\lambda_3 = 0$ satisfied Remark 3 therefore the system (2) is critical case the proof is completed.

Corollary 2:

All three critical point p_0, p_+ and p_- are coincide when $\sigma = 2r$.

Corollary 3:

In the critical point p_0 if:

(1) $\sigma = 2r = \beta$ then $\lambda_1 = 0$, $\lambda_2 = -\beta$, $\lambda_3 = -r$

(2) $\sigma = r = \beta$ then $\lambda_1 = -\beta$, $\lambda_{2,3} = \pm r = \pm\sigma = \mp\beta$

(3) $\sigma = r$ then $\lambda_1 = -\beta$, $\lambda_{2,3} = \pm r = \pm\sigma$.

Theorem 5:

The solution of system (2) at critical point p_+ has the following cases

1) Asymptotically stable if $\beta > \beta_c$,
2) Unstable if $\beta < \beta_c$,
3) Critical case if $\beta = \beta_c$.

Proof:

Linearizing system (2) about the critical points p_+ or p_- yields the following characteristic equation :

$$f(\lambda) = \lambda^3 + (\beta + \sigma - r)\lambda^2 + \beta r\lambda + 2\sigma\beta(2r - \sigma) = 0 \qquad \ldots(13)$$

Obviously, the two points $p_\pm$ have the same stability. Let

$a = \beta + \sigma - r$

$b = \beta r$

$c = 2\sigma\beta(2r - \sigma)$

Then the Routh-Hurwitz criterion, (Remark 2) lead to the conclusion that the real part of the roots λ are negative if and only if

$$\beta + \sigma - r > 0,\ 2\sigma\beta(2r - \sigma) > 0 \ \ and \ \ (\beta + \sigma - r)\beta r - 2\sigma\beta(2r - \sigma) > 0$$

Since $a = \beta + \sigma - r$ and β , σ and r are a positive parameters and $r < \sigma$, consequently, $a > 0$ always and $c > 0$ also since $\sigma \in (r, 2r)$, when $ab > c$ we get:

$$(\beta + \sigma - r)\beta r > 2\sigma\beta(2r - \sigma) \quad \Rightarrow \quad \beta > \beta_c$$

Consequently, when $\beta > \beta_c$ the system (2) is asymptotically stable, the proof of first condition is completed, while when $\beta < \beta_c$ we get $a > 0$ and $c > 0$ always, analogously as in proof of first case, and $ab < c$ hence one of Routh-Hurwitz conditions not satisfied, consequently the system (2) is unstable, finally when $\beta = \beta_c$ $a, c > 0$ and $ab = c$ critical cases, the proof is completed.

We explain our results by the following table

Relation between σ and $2r$	p_o	$p_{\pm}$	
$\sigma > 2r$	asymptotically stable	not exist	
$\sigma < 2r$	unstable	exist	asymptotically stable if $\beta > \beta_c$
			unstable if $\beta < \beta_c$
			critical case if $\beta = \beta_c$
$\sigma = 2r$	critical case	not exist	

Corollary 4:

Every hopf bifurcation and pitchfork bifurcation is critical cases but the conversely is not true.

Theorem 6:

When system (2) has critical case then this system is:

1 - Hopf Bifurcation if $\Delta > 0$,

2 - Pitchfork Bifurcation if $\Delta < 0$ or $\Delta = 0$.

Proof:

When $\Delta > 0$ we obtain: one real root and two complex conjugate roots (Theorem 3), since system (2) has critical case then

either $\lambda_1 < 0$ and $\operatorname{Re}\lambda_2 = 0$ ………. 1

or $\lambda_1 = 0$ and $\operatorname{Re}\lambda_2 < 0$ ……….. 2

but equation 1 satisfied theorem 1 and remark 6, hence the system (2) has hopf Bifurcation when $\Delta > 0$, the proof is complete .

When $\Delta = 0$ or $\Delta < 0$ we get either three real roots, but one is multiple or three different real roots (without complexes conjugate roots) and at last one of this root begin zero, therefore satisfied remark 6, hence the system (2) has Pitchfork Bifurcation when $\Delta < 0$ or $\Delta = 0$, the proof is complete .

4- Illustrative examples:

Example 1:

Investigates the stability and bifurcation of the following Chen System

$$\dot{x} = -4x + 4y$$
$$\dot{y} = -3x - xz + y$$
$$\dot{z} = xy - 5z$$

<u>Solution:</u>

At critical point p_o:

Equation parameters this system with system (2) resulting: $\sigma = 4$, r = 1, By theorem (2) the Chen System is **asymptotically stable** since $(\sigma > 2r)$,

To compare this result with Δ-discriminate theorem the characteristic equation of Chen system of the form: $\lambda^3 + 8\lambda^2 + 23\lambda + 40 = 0$

and $q = \frac{448}{27}$, $\Delta = 276$, $\lambda_1 = -5$, $\lambda_{2,3} = -\frac{3}{2} \pm \frac{1151}{480} i$

The Chen system is **asymptotically stable** at the point p_0 and also not has pitchfork Bifurcation.

At critical point p_+:

When $\sigma > 2r$, the critical point $P_+(\sqrt{\beta(2r-\sigma)}, \sqrt{\beta(2r-\sigma)}, 2r-\sigma)$ does not exist, consequently, the Chen not has hopf bifurcation.

Example 2:

Investigates the stability and bifurcation of the following Chen System

$$\dot{x} = -6x + 6y$$
$$\dot{y} = -2x - xz + 4y$$
$$\dot{z} = xy - 4z$$

<u>**Solution:**</u>

At critical point p_o:

Equation parameters this system with system (2) resulting: $\sigma = 6$, r = 4, By theorem (2) the Chen System is **unstable** since $(\sigma < 2r)$.

To compare this result with Δ-discriminate theorem the characteristic equation of Chen system of the form: $\lambda^3 + 6\lambda^2 - 4\lambda - 48 = 0$

and $q = -24$, $\Delta = -\frac{832}{27}$, $\lambda_1 = -4$, $\lambda_2 = -\frac{829}{180}$, $\lambda_3 = \frac{3567}{1369}$

The Chen system **unstable** is at the point p_0 and also not has pitchfork Bifurcation.

At critical point p_+:

By theorem (5) the Chen System is **critical case** since $\beta_c = 4$, $\beta = 4$ and $\beta = \beta_c$, to compare this result with Δ-discriminate theorem the characteristic equation of Chen system of the form: $\lambda^3 + 6\lambda^2 + 16\lambda + 96 = 0$

and $q = 80, \Delta = \frac{173056}{27}$, $\lambda_1 = -6$, $\lambda_{2,3} = \pm 4i$

The Chen system is **critical case** at the point p_+, consequently, the Chen has hopf bifurcation.

If $\beta = 5$ and $\beta_c = 4$ then by theorem (5) we get that the system (2) **asymptotically stable**, to compare this result with Δ-discriminate theorem the characteristic equation of Chen system of the form: $\lambda^3 + 7\lambda^2 + 20\lambda + 120 = 0$

and $q=\frac{2666}{27}$, $\Delta=\frac{263440}{27}$, $\lambda_1=-\frac{1907}{285}$, $\lambda_{2,3}=-\frac{1077}{6976}\pm\frac{2061}{487}i$

The Chen system is **asymptotically stable** at the point p_+, consequently, the Chen has not hopf bifurcation.

We are used MATLAB to simplify computation our results at critical point p_+, and we will denote the symbols σ, r, β, λ, β_c, Δ, λ_1, λ_2, λ_3 in the MATLAB program by $r1$, $r2$, $r3$, P, d, dalta, root 1, root 2, root 3, respectively.

5- Program:

```
% Stability of Chen System at Second Critical Point
clc
clear
format rat
r1=input('r1=');
r2=input('r2=');
r3=input('r3=');
a=r1+r3-r2
b=r2*r3
c=2*r1*r3*(2*r2-r1)
d=(r2^2+3*r1*r2-2*r1^2)/r2
f='p^3+a*p^2+b*p+c';
q=c-(1/3)*a*b+(2/27)*(a^3)
dalta=c^2+(4/27)*b^3-(2/3)*(a*b*c)-
(1/27)*(a^2)*(b^2)+(4/27)*(a^3)*c;
dalta=ss1(dalta)

if dalta<0
    root1=(16*(q^2-dalta))^(1/6)*cos((acos(-q/(q^2-
dalta)^(1/2)))/3)-(a/3);
    root2=(16*(q^2-dalta))^(1/6)*cos((acos(-q/(q^2-
dalta)^(1/2))+(2*pi))/3)-(a/3);
    root3=(16*(q^2-dalta))^(1/6)*cos((acos(-q/(q^2-
dalta)^(1/2))+(4*pi))/3)-(a/3);
    root1=ss1(root1)
    root2=ss1(root2)
    root3=ss1(root3)
    if (root1<0)&( root2<0)&(root3<0)
        disp ('asymptotically stable')
        break
  end
        elseif dalta>0
    root1=sign(-q)*norm(((-q-(sqrt(dalta)))/2)^(1/3))+norm(((-
q+(sqrt(dalta)))/2)^(1/3))-(a/3);
    root2=(-1/2)*(sign(-q)*norm(((-q-
(sqrt(dalta)))/2)^(1/3))+norm(((-q+(sqrt(dalta)))/2)^(1/3)))-
(a/3)...
        +i*(sqrt(3)/2)*(sign(-q)*norm(((-q-
(sqrt(dalta)))/2)^(1/3))-norm(((-q+(sqrt(dalta)))/2)^(1/3)));
```

```
    root3=(-1/2)*(sign(-q)*norm(((-q-
(sqrt(dalta)))/2)^(1/3))+norm(((-q+(sqrt(dalta)))/2)^(1/3)))-
(a/3)...
        -i*(sqrt(3)/2)*(sign(-q)*norm(((-q-
(sqrt(dalta)))/2)^(1/3))-norm(((-q+(sqrt(dalta)))/2)^(1/3)));
    root1=ss1(root1)
    root2=ss1(root2)
    root3=ss1(root3)
    if (root1<0)&( real(root2<0))&(real(root3<0))
        disp ('asymptotically stable')
        break
       elseif (root1<0)&(real(root2>0))
        disp( 'unstable')
        break
        elseif (root1<0)&(real(root2==0))
        disp( 'critical case')
        break
        end
 end
%  This function to used in main program and saved as
%  ss1.m  in a single  m-file
function r=ss1 (r)
if (abs(real(r))<1.e-5)
    r=0+imag(r)*i;
end
```

6- Conclusion:

In this paper, we study the stability and bifurcation for the Chen system by using a new method which is called Δ- discriminate method. and we concluded that the difference between the previous methods (Routh-Hurwitz method)and this method is depended as the previous methods on the estimate the singe of roots without found the value of this roots while in the a new method we found the exact value of this roots.

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