NUMERICAL STUDY ON THE NONLINEAR BEHAVIOR OF FULLY PRESTRESSED HIGH STRENGTH CONCRETE BEAMS

دراسة عددية للسلوك اللاخطي للعتبات الخرسانية ذات مقاومة انضغاط عالية ومسبقة الاجهاد

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ABSTRACT :

This paper involves the numerical study of nonlinear behavior of fully prestressed high strength concrete beams. A three dimensional nonlinear finite element model was developed using a computer program (ANSYS V.12).The material and geometrical nonlinearity were adopted in the current analysis.This study investigates the effects of some factors on the flexural behavior of prestressed concrete beams related to the compressive strength of concrete, prestressing reinforcement ratio, prestressing force level, and ratio of shear span to the effective depth. The obtained results showed that the ultimate load capacity are affected slightly with increase the compressive strength of concrete. Increasing of the prestressing reinforcement ratio leads to improvement the overall performance of the prestressed concrete beam , moreover increasing this ratio makes prestressed concrete beam more ductile and has the ability to pass high deflections before reaching failure load. Also increase the level of force of prestressing led to increase the load which causes emergence of the first crack ,but the beam stiffness after cracking and the ultimate load was not remarkably sensitive to the effect of the level of force of prestressing. Finally, it is revealed that the ultimate load capacity of those beams increased greatly with decreasing ratio of shear span to the effective depth .

الخلاصة:

هذا البحث يتضمن دراسة نظرية للسلوك اللاخطي العتبات الخرسانية المسبقة الاجهاد كليا وذات مقاومة الانضغاط العالية .تم إجراء التحليل اللاخطي الثلاثي الأبعاد باستخدام طريقة العناصر المحددة وبالاعتماد على برنامج الحاسوب ANSYS V.12).تم اخذ الخصائص اللاخطية المادية والهندسية بنظر الاعتبار في هذه الدراسة.تم التحري عن تاثير بعض العوامل على سلوك الانثناء للعتبات الخرسانية مسبقة الاجهاد ومنها مقاومة الانضغاط للخرسانة ، نسبة حديد التسليح ، مستوى قوة السحب للحديد ، نسبة فضاء القص الى العمق الفعال.

النتائج المستحصلة من هذه الدراسة بينت انه قابلية التحمل للعتبات الخرسانية مسبقة الاجهاد تتاثر بشكل طفيف بزيادة مقاومة الانضغاط للخرسانة.ان زيادة نسبة حديد التسليح يؤدي الى تحسين اداء العتبات الخرسانية مسبقة الاجهاد ، علاوة على ذلك فان زيادة نسبة التسليح يجعل تلك العتبات اكثر مطالية(ductility) ولها قابلية حدوث الانحرافات عالية (deflections) قبل الوصول الى التحمل الاقصى.كذلك فان زيادة مستوى قوة السحب للحديد يؤدي الى زيادة الحمل الذي يحدث عنده التشقق،لكن ما بعد حدوث الشقوق فان صلابة وتحمل العتبات لم تتاثر بزيادة قوة السحب للحديد.اخيرا ، تبين ان قابلية التحمل الاقصى لتلك العتبات ازدادت كثيرا بتناقص نسبة فضاء القص الى العمق الفعال.

Keywords: prestressed beams, finite element , reinforced concrete,high strength

1. INTRODUCTION :

Recently, spread the use of prestressed concrete dramatically in many areas of civil engineering and especially in the field of bridges engineering due to extensive development in the construction techniques, as well as the growing need for bridges with large span. These days, most of the bridges that were built in all parts of the world relied on the use of prestressing techniques [1].

Fully prestressing is defined as a complete elimination of tensile stresses in the concrete structural elements under full service load or allow a few tensile stresses, that can be resisted by concrete only, while partially prestressing allow for larger tensile stresses in concrete , as well as cracking under full service loads [2,3]. The main advantage of the fully prestressed concrete system is the absence of cracks in the concrete at the nominal service load and therefore better durability will be achieved.

Branson ,et al., in 1970 [4] presented a systematic procedure for predicting the material behavior of different weight concretes and the time-dependent structural deformation of non-composite and composite prestressed concrete structure. In their study,continuous time functions were provided for all needed parameters, so that the general equations for predicting loss of prestress,camber and deflection readily lend themselves to computer solution. The results computed by the material parameter equations were compared with representative data in the literature for normal weight, sand-lightweight,and all-lightweight concrete. Their results were reasonably good, but not precise,and probably indicate the nature of the correlation that might be expected,at best, for this type of behavior.

In 1972 , Faherty[5] searched in study a reinforced and prestressed concrete beam using the finite element method. His study included concrete charestrestics , a bond slip relation between the steel and concrete, and steel properties. His results showed that the deflections occurring in the prestressed beam and calculated theoretically were very close to those obtained by Branson, et al. (1970)[4]. Nevertheless, the load-deflection relationship beyond the point of cracking has not established because he used only three distinct patterns of cracks in this analysis. He recommended that additional analysis should be taken for the prestressed concrete beam after placing a technique to represent the tensile rupture of the concrete.

Fanning, in 2001[6] studied the experimental load-deflection response of ordinary reinforced concrete beams and post-tensioned concrete T-beams and used it to assess the suitability of numerical modelling implemented in the FE software ANSYS, in predicting the ultimate response of RC beams. The correlation of test and numerical data was found to based on the values of linear and nonlinear material charestrestics assigned to the materials, most importantly the Young’s modulus of elasticity of concrete and the yield strengths of the reinforcing bars and the post-tensioning tendons.

Ibrahim and Mahmood,in 2009 [7] created an FE model in ANSYS to simulate experimental testing of shear capacity for six RC beams strengthened with fiber reinforced polymer laminates. Comparisons between emprical results and the ANSYS FE analysis were then carried out with regard to load deflection curves, crack patterns and failure loads. All these comparisons showed good agreement. One of their main conclusions was that their finite element model could be used in additional studies to develop design rules for the use of FRP laminates as strengthening material for RC members.

Also in these days, there is a big spread for the use of high strength concrete as a result of the broad development of concrete technology, as well as the growing demand for this type of concrete which results a best control on the quality of concrete.In 2012, Hussien , et al.,[8] presented the emprical study on the conduct of bonded and unbounded prestressed normal strength (NSC) and high strength concrete (HSC) beams. Their study consisted of a total of nine beams; two specimens were reinforced with non-prestressed reinforcement, four specimens were reinforced with bonded tendons, and the remaining three specimens were reinforced with unbonded tendons.Their results showed that increasing the nominal compressive strength from 72 to 97 MPa for bonded prestressed beams led to a slight increase in the failure and cracking loads by 4% and 18%, respectively.

In 2013,Gunnarsson [9] studied the behavior of the prestressed concrete beams reinforced with basalt FRP or steel tendons. In his study, experimental results regarding prestressed BRFP reinforced concrete beams were simulated with the FEM and then compared with some of the shear resistance equations provided by codes and guidelines.Also,he could estimate the long term relaxation of basalt FRP tendons through experimental tests. He concluded that in order to keep the behavior of the bond between the concrete and reinforcement should not exceed the amount of strain (4-5 %) in BFRP longitudinal tension reinforcement. Finally, he confirmed on existence of a large reservation in the equations in the codes and guidelines for FRP reinforced concrete, and therefore, this reservation makes the design not be efficiently.

2. PROBLEM CONSIDERED FOR THE STUDY:

The experimental analysis provides the actual behavior of the structure. But it is time consuming and expensive. Finite element analysis is also used to analyze these structural components. There are three reasons why the finite element method is an effective way: First, thrift in design time and secondly cost-effective in the construction and finally an increase in construction safety.In the present study , nonlinear finite element analysis was used to study the factors affecting the nonlinear behavior of fully prestressed high strength concrete beams.

3. FINITE ELEMENT ANALYSIS:

Through the use of ANSYS software (version 12) can be made non-linear finite element analysis. The numerical models adopted brick elements (SOLID65) depicted in Figure(1). There is in this element eight nodes each of which has three degrees of freedom ,i.e,displacements in the three directions X, Y, and Z. SOLID65 is used to model the concrete beam. This component has the ability to represent cracks in the tension zone, as well as crushing in the compression zone [10].The steel reinforcement was idealized by using three dimensional spar elements (LINK8).This element is a uniaxial tension-compression element with two nodes having three degrees of freedom at each node.The geometry of the element (LINK8)showed in Figure(2). SOLID 45 was used to represent the supports and loading points, and this element has eight nodes in each one of them three degrees of freedom similar to those found in SOLID 65, as shown in Figure(3).The element has plastisty, creep,swelling, stress stiffening ,large deflection and large strain capabilities [10].

The objective of the present analysis is to determine first cracking load ,ultimate load and the maximum deflection under the effect of various parameters on the flexural behavior of prestressed concrete beams related to the compressive strength for the concrete, percentage of prestressing reinforcement, level of prestressing force, and shear span to effective depth ratio.

In order to represent the phenomena of collapse realistically,material and geometrical nonlinearities were included in the analysis by the finite element method . To find a non-linear solution has been adopted Newton Raphson algorithm.The displacement convergence criterion is used to monitor equilibrium.

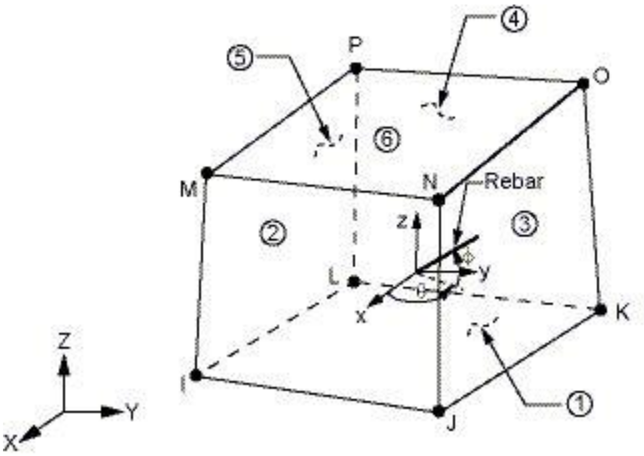


Fig. (1): SOLID65 Geometry [10]

Fig. (2): LINK8 Geometry [10]

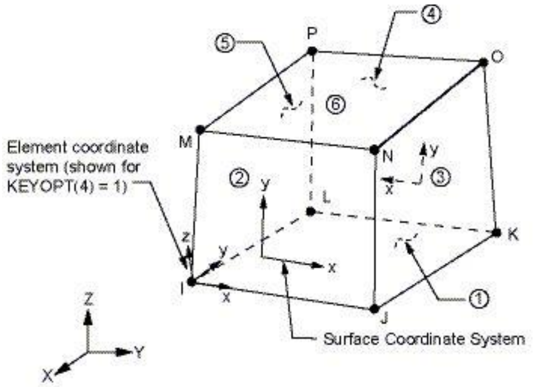


Fig. (3): SOLID45 Geometry [10]

3.1. Modeling of Material Properties:

3.1.1. Concrete Beam

In the finite element analysis, SOLID65 element was used to represent the concrete beams. Two categories: linear isotropic and multi-linear isotropic were adopted in order to represent the characteristics of the concrete material accurately. For definition of the failure in the concrete , the Von- Mises failure criterion along with the Willam and Warnke model were used. ANSYS software requires the uniaxial stress-strain relation for concrete in compression. The following equations(MacGregor 1992 [11]), were used to construct the multilinear isotropic stress-strain curve for concrete in this study.

$$f=\frac{E_c\,\varepsilon}{1+\left(\frac{\varepsilon}{\varepsilon_o}\right)^2} \quad ; \quad \varepsilon_o=\frac{2f_c'}{E_c} \quad ; \quad E_c=\frac{f}{\varepsilon}$$

Stress-strain relationship which had been adopted in the current study showed in Figure(4**).**

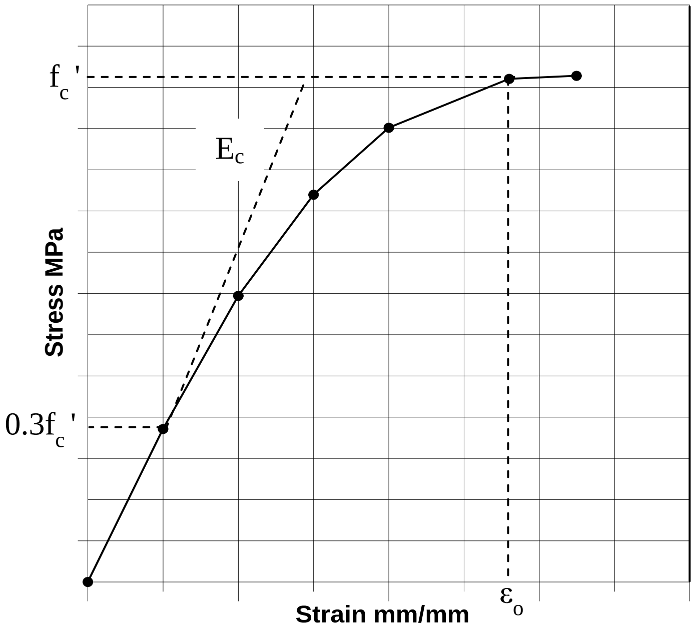


Fig.(4): Stress-strain relationship for concrete

3.1.2 Prestressed Steel Tendon:

In ANSYS 12.0 program, initial conditions were taken to represent the appling of the prestressing force in the tendon. This is done through given of the value of the initial strain in LINK8 properties. The behavior of prestressed steel tendon used in the present study was assumed to have bilinear elastic-plastic with hardening stress-strain relation as illustrated in Figure(5).

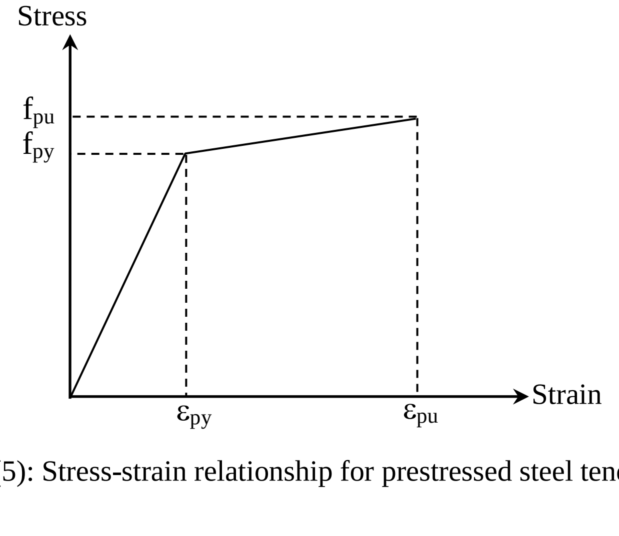


Fig.(5): Stress-strain relationship for prestressed steel tendon

3.1.3 Steel Plates :

To simulate the practical reality and to avoid the stress concentration problems, steel plates used in the loading and supports locations. The representation of these plates was through using of solid element (SOLID45). Lineary elastic stress-strain relationship was imposed to represent the behavior of these steel plates, as illustrated in Figure(6).

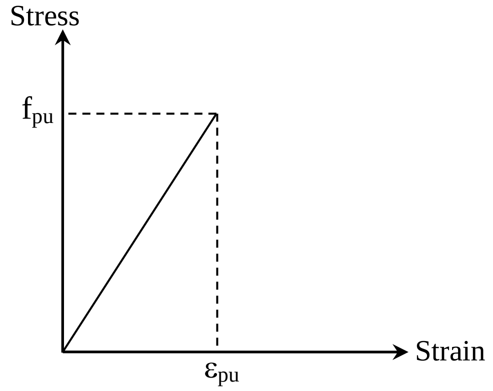


Fig.(6): Stress-strain relationship for Steel plates

3.2. Verification Study :

To check the validity and accuracy level of the adopted numerical analysis in prediction the structural behavior of the fully prestressed high strength concrete beams , an analysis process was perform for some of the experimental previous work that carried out by other researchers as follows:

Prestressed Concrete Beam:

The finite element model adopted in the presnt study was verified against the test data of prestressed concrete beam (Model 6) ,as reported by Gunnarsson in 2013 [9].The detailes of the test sample are explained in Figure (7) and Table (1).

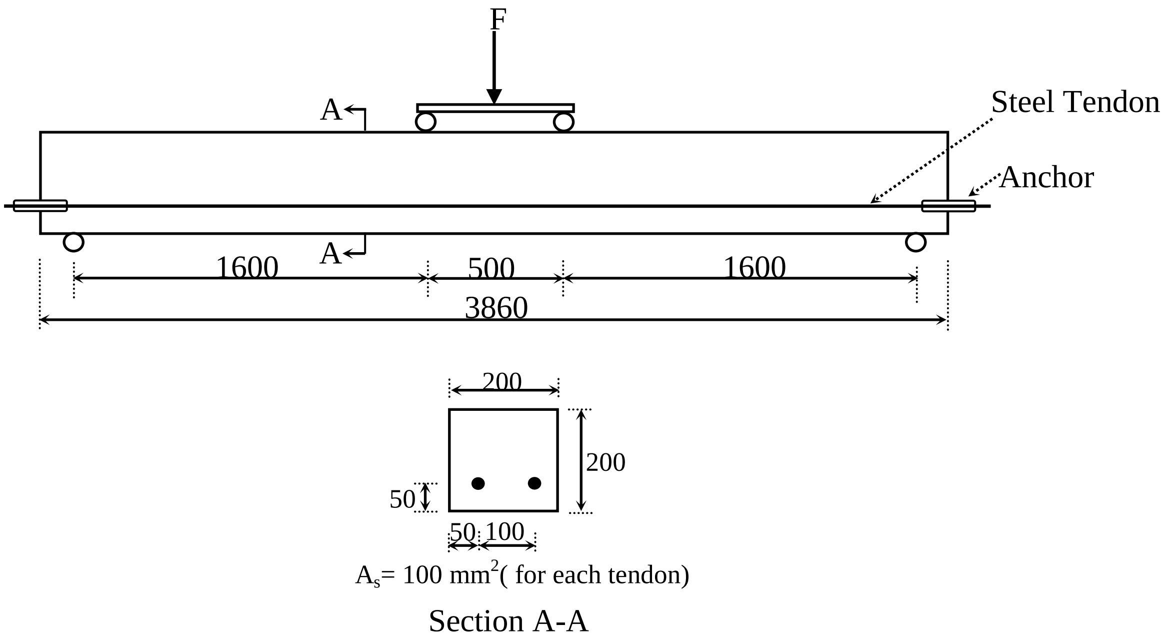
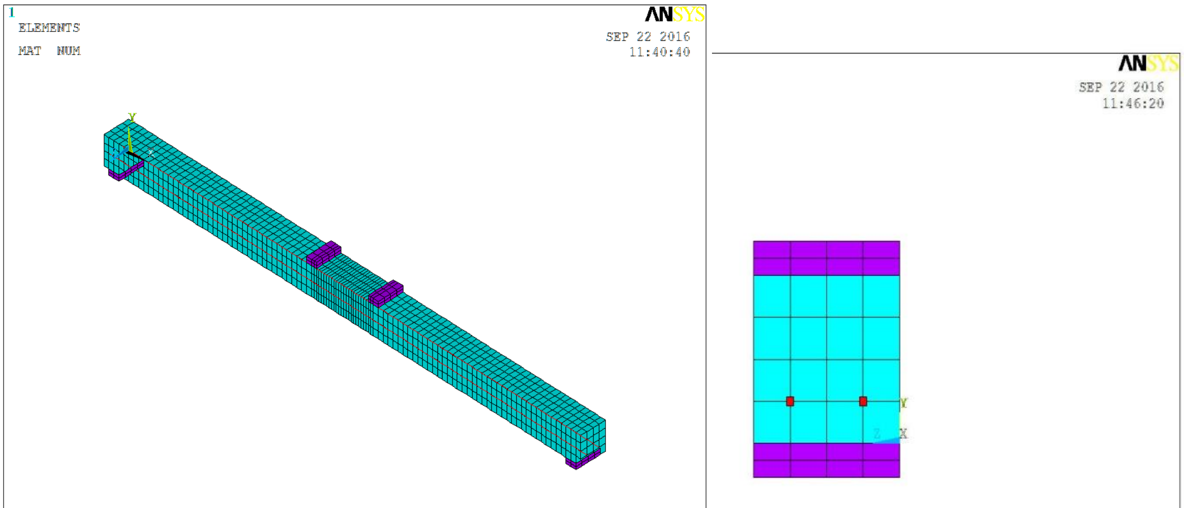


Fig.(7): Geometrical properties of the test specimen (all dimensions in mm)[9]

Table (1) : Material Properties of Prestressed Concrte Beam (Model 6) [9]

Property	Length (mm)	fc' (MPa)	E_c (MPa)	Effective prestress force in each tendon(kN)	f_y (MPa)	f_u (MPa)	E_s (MPa)
Model 6	3860	61.7	38000	24	1786	1975	197800

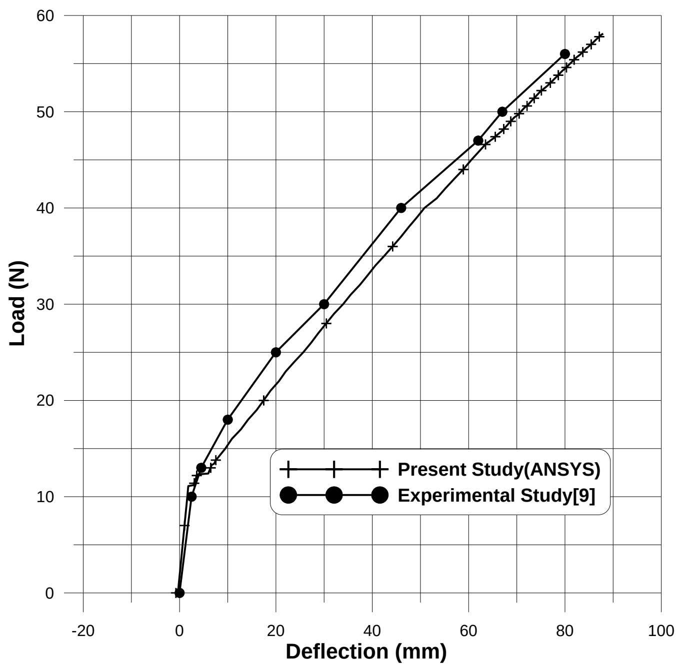
In the present finite element analysis obtained from ANSYS software,the same prestressed concrete beam was reanalyzed , SOLID65 element was chosen to represent concrete beam and LINK8 to model steel tendon, the prestressing force was entered to the ANSYS program as the initial strain in the LINK8 equal to (0.0012133).The finite element idealization which is adopted in the current study explained in Figure(8).



(a) 3-D idealization (b) Side view

Fig.(8) : Finite element idealization of prestressed concrete beam

Figure(9) presents the load-deflection relation at midspan of the beam of both Gunnarsson experimental results and the numerical finite element results that obtained from the present study .It was observed that the overall nonlinear behavior of the prestresssed concrete beam as predicted by the ANSYS software is close to the actual behavior obtained from experimental test with a slight difference especialy in plastic range. Such a difference can be related to the model used in the ANSYS in which the failure criteria and the related factors as anticipated by the software are so sensitive that a slight change in values of these factors could result in either unstable solution (no convergence) or non-real values of the element stiffness. However , changing the failure criteria with the related coefficients might result in closer solutions at some cases .



Figure(9): Load-deflection relationship of prestressed concrete beam

4. Parametric Study:

The effects of several parameters on the flexural behavior of prestressed concrete beams were studied in the present study .These parameters are:-

1-Effect of the compressive strength of concrete

2- Effect of the prestressing reinforcement ratio

3- Effect of the prestressing force level

4- Effect of ratio of the shear span to effective depth

The same prestressed beam tested by Gunnarsson [9] was used in the parametric study with the different parameters and the numerical results for each parameter are disscussed below:

4.1 Effect of the Compressive Strength of Concrete:

Three different values of the compressive strength of concrete (40 , 70 and 100 Mpa) were used.Figure(10) shows the effect of the compressive strength of concrete (fc') on the behavior of prestressed beam. It can be clearly seen from this figure that when increasing the compressive strength of concrete from 40 to100 MPa, the first cracking load and ultimate load capacity were increased by 52% and 10 % respectively.Also, for the same increase in the compressive strength of concrete the maximum deflection at midspan of the beam decreases by 17.6%.This behavior can be interpreted to the increasing in the mouduls of elasticity due to increasing of compressive strength ,and this leads to decreasing in the deflection and increasing in the stiffness of the beam.

Figure(11) shows deflected shape of the prestressed concrete beam at prestress force only and at the failure load.The stresses in the steel tendons and concrete were drawn in the figures(12 and 13).

The cracking behavior was explained in Figure(14). First, vertical flexural cracks started to develop at midspan , then they were followed with a formation of diagonal tension cracks and multiple sloping cracking beside the loading points.

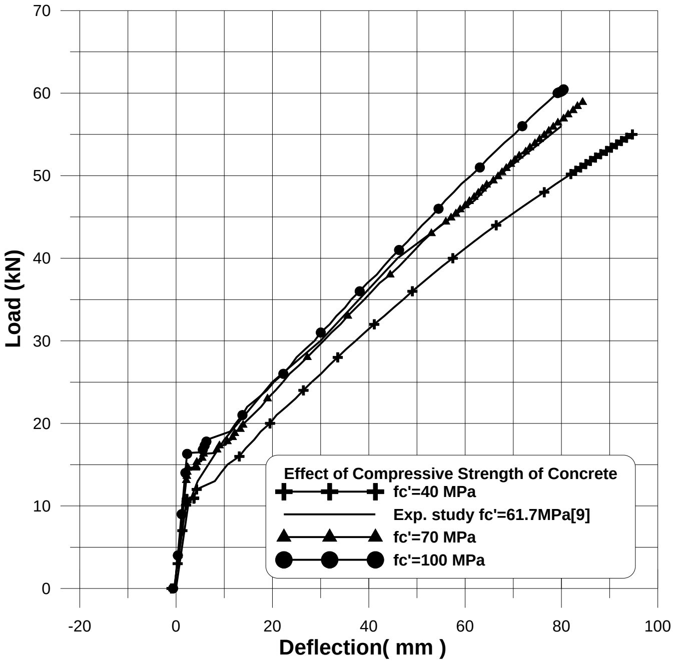
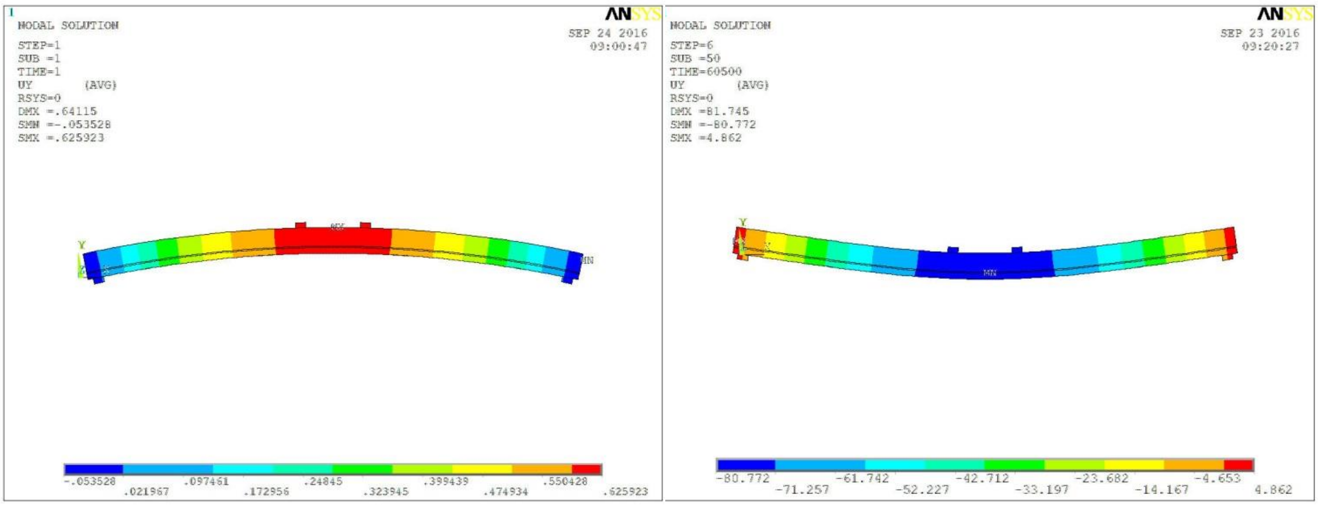
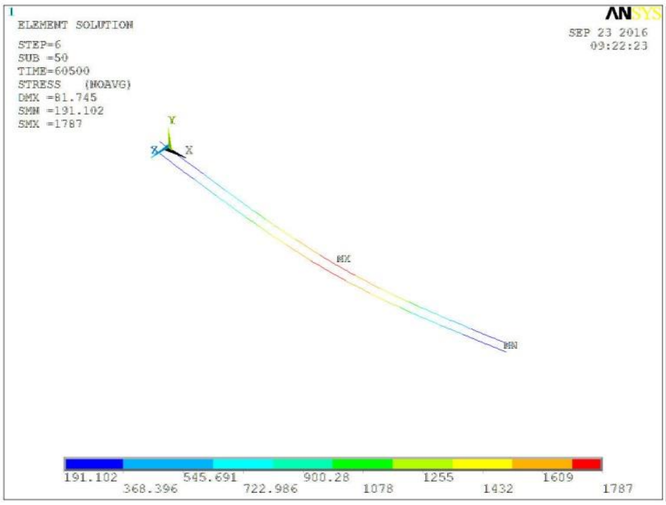


Fig.(10): Load – deflection curves of the prestressed concrete beams with various values of the compressive strength of concrete

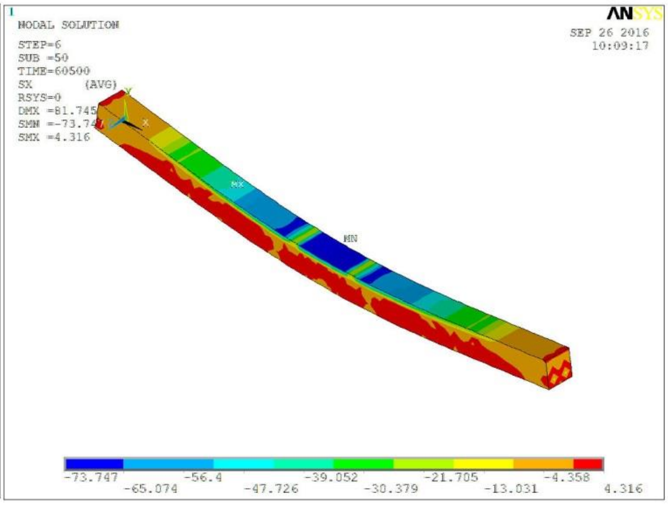


(a) At prestress force only (b) At failure load

Figure(11) Deflected shape of the prestressed concrete beam at different load step (fc'=100 MPa)



Figure(12) Stresses in the steel tendon of the prestressed beam (fc'=100 MPa)



Figure(13) Stresses in the concrete in x-direction of the prestressed beam (fc'=100 MPa)

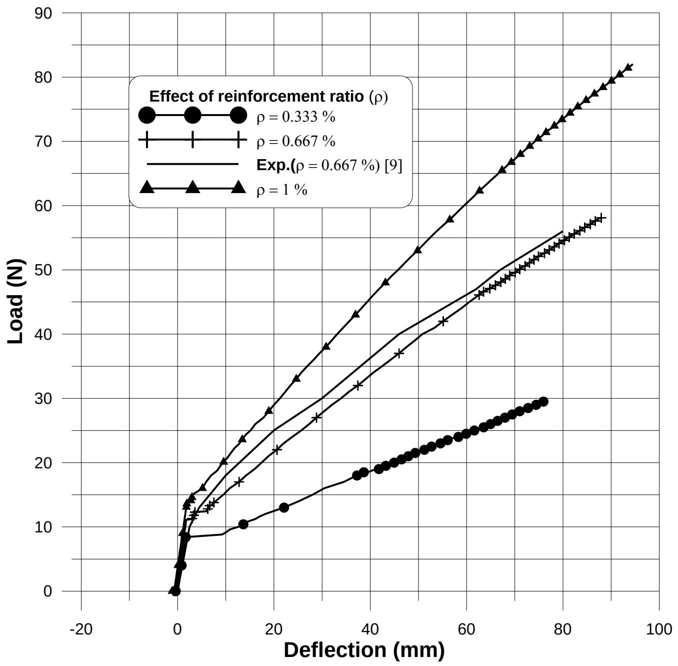
Figure(14) Crack pattern of the prestressed beam at failure load (fc'=100 MPa)

4.2 Effect of the Prestressing Reinforcement Ratio:

Three values of the prestressing reinforcement ratio (0.333 % , 0.667 %, and 1%) were used. The area of steel tendon used in the present study was equal to 100 mm^2.In the first reinforcement ratio , one steel tendon located in the central of the section with eccentricity equal to 50mm was used while two steel tendons were used in the second reinforcement ratio and three steel tendons were used in the final reinforcement ratio with the same eccentricity described in the first ratio.

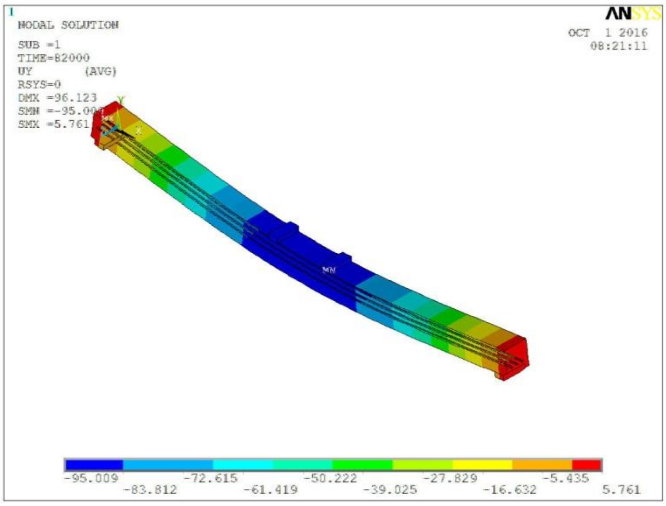
Curves in Figure(15) were drawn to explain the effect of the different ratios of prestressing reinforcement on the behavior of prestressed beam. From this figure can be noticed that the effect of increasing of the prestressing reinforcement ratio of the beam was very significantly on the ultimate load so that when the ratio of prestressing reinforcement increases to 0.667% , the ultimate load capacity increased by(96%) and when the ratio of prestressing reinforcement increases to 1% , the ultimate load capacity increased by(178 %) as comparison with the beam that has steel ratio equal to 0.333 % . Usually this behavior is interpreted as a result of the effectiveness of steel in increase flexural resistance and the stiffness of the beam.

Moreover, the deflection at ultimate load was increased by increasing of the prestressing reinforcement ratio.Increasing percentage of maximum midspan deflection was 15.35 % for the steel ratio 0.667% and was 24.47% for the steel ratio 1% as comparison with the beam that has steel ratio equal to 0.333 %. Also, influence of prestressing reinforcement ratio on maximum deflection reveals that increasing this ratio to 1% makes prestressed concrete beam more ductile and capable of undergoing large deflections before reaching ultimate load carrying capacity.In structural elements , this feature is very significant because it makes concrete give an alert before failure and prevent sudden crumpling.

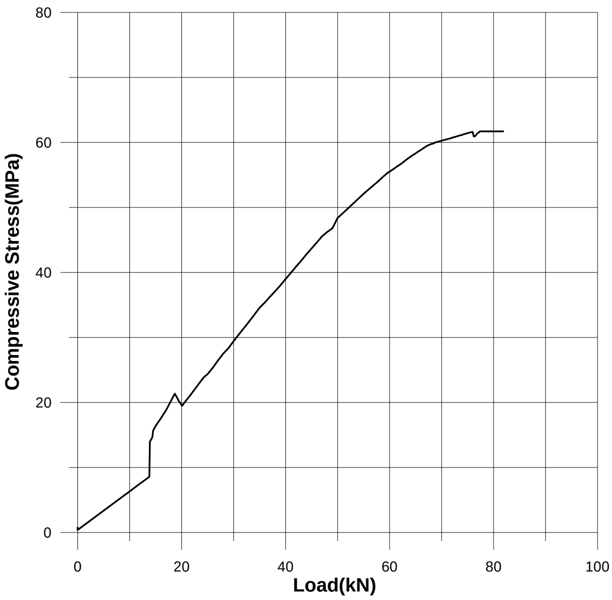


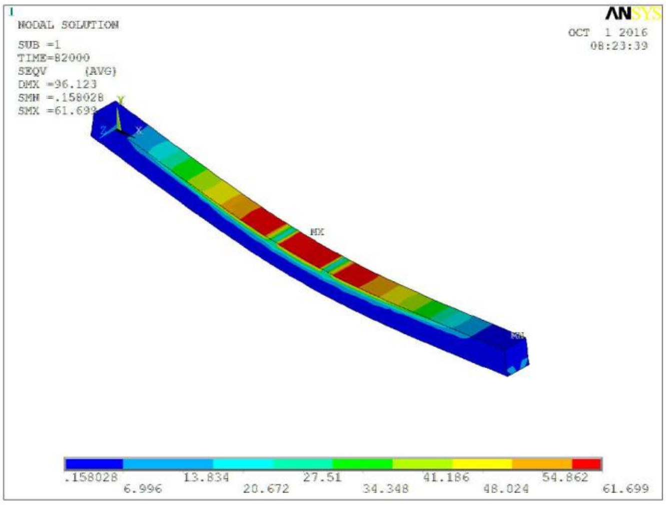
Figure(15) Load-deflection curves of various steel reinforcement ratios

Figure(16) shows deflected shape of the prestressed concrete beam at the failure load for the steel ratio 1% .The stresses in the concrete and steel tendons shown in the figures 17 and 18 respectively. The cracking behavior is quite similar to the behavior described in the above paragraph (4.1) and it was shown in Figure(19).



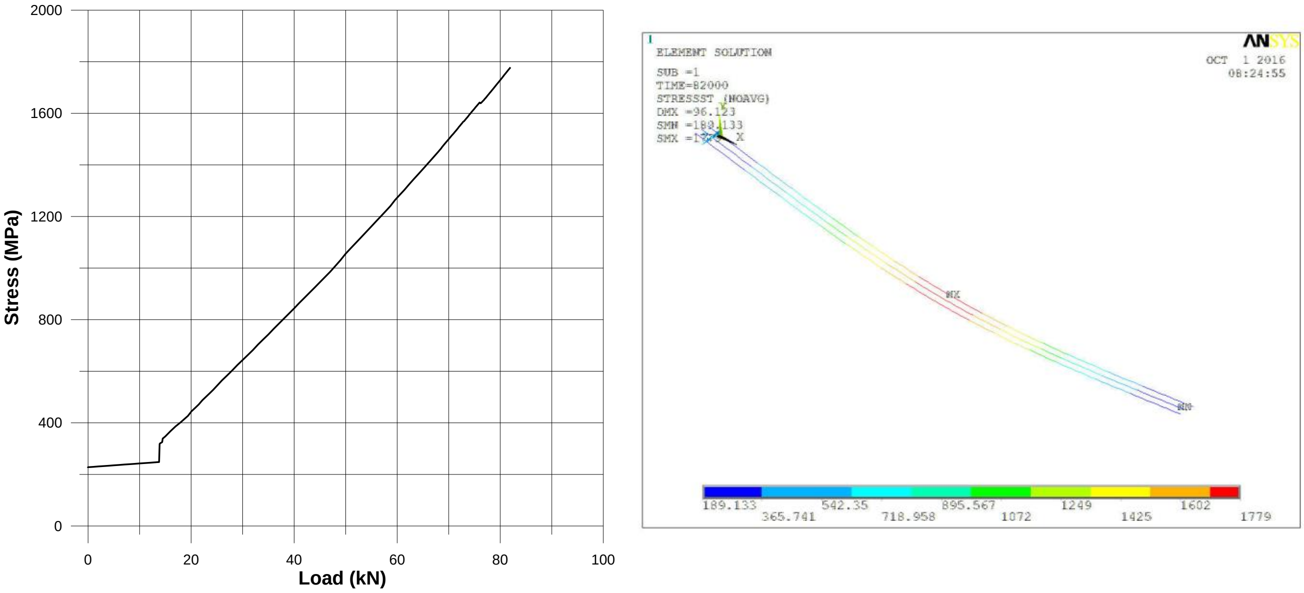
Figure(16): Deflected shape of prestressed beam of steel ratio 1%





(b): Stress-load curve at top face of mid-span (b): Von-Mises distribution stresses

Figure(17): Stresses in the concrete of prestressed beam of steel ratio 1%



(a) Stress-load curve at mid-length of tendon (b) Distribution stress along the beam

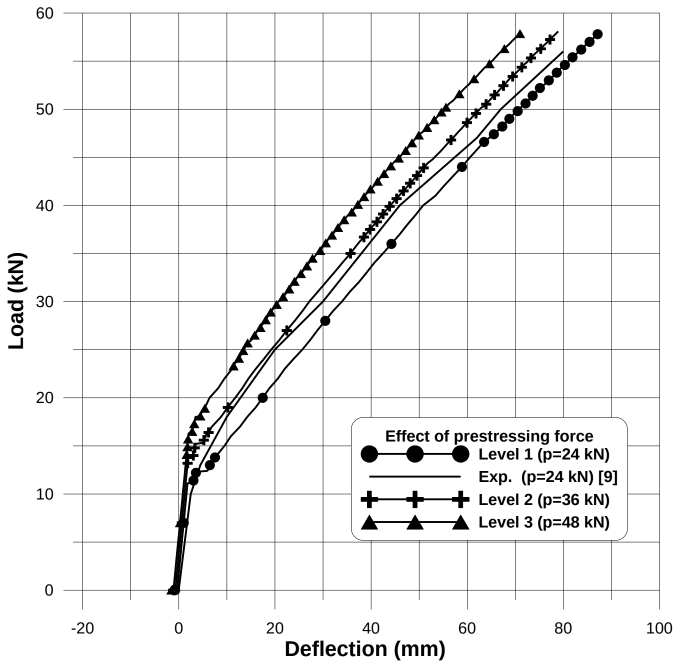
Figure(18) Stresses in the steel tendons of prestressed beam of steel ratio 1% at failure load

Figure(19) Crack pattern of the prestressed beam at failure load for steel ratio 1%

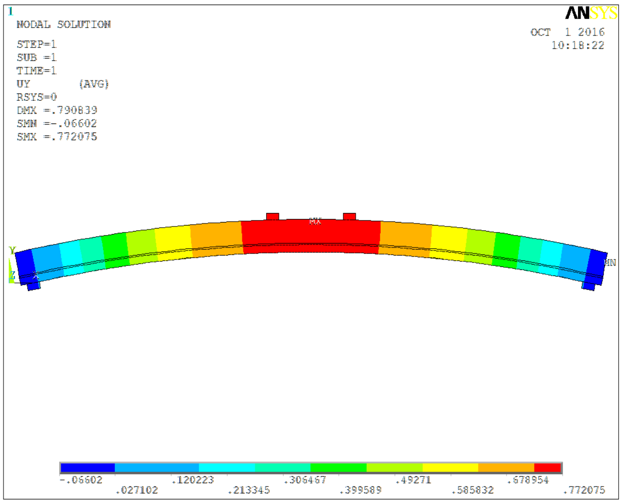
4.3 Effect of the Prestressing Force Level :

Different levels of prestressing force (24 kN, 36 kN ,and 48 kN) were used to determine the impact of this parameter on the overall behavior of the fully prestressed concrete beams. Curves in Figure (20) were drawn to the effect of prestressing force level. From this Figure it can be clearly seen that increase the force of prestressing to 36 kN, resulting in increased the cracking load by 21.6 % as comparison with the beam that has prestressing force equal to 24 kN, while increasing the prestressing force from 24 kN to 48 kN, resulted in increasing of the cracking load by 44.1 %. Meanwhile,the maximum deflection of prestressed concrete beam decreases with the increase of the prestressing force level , so that when the prestressing force increases to 36 kN, the maximum deflection decreases by 10.22 % and when the prestressing force level increases to 48 kN the maximum deflection decreases by 19.32%. Nevertheless, the stiffness of the beam after cracking stage as well as the ultimate load were not clearly sensitive to the effect of the change in the force of the prestressing so that the curves after cracking stage seemed parallel with each other with different values of the force of prestressing.This behavior occurs because of the effect of prestressing force is be clearly on increasing the initial deflection obtained from prestressing force only(without self weight for the beam and other service loads) as result to increase initial stress and initial strain in the steel tendons but the effect prestressing force is be not significantly on the stiffness of the prestressed beam after appearance of the cracks.

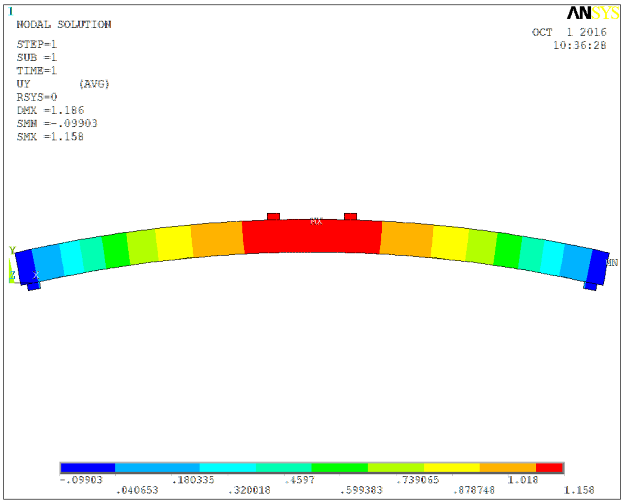
Figure(21) shows deflected shape of the prestressed concrete beam when the prestressing force is applied only (for different values of the prestressing force level) . The cracking behavior shown in figure(22) and can clearly seen that it is similar to the behavior described in the above previous items , but was noticed forming new cracks at the ends of the beams as result to increasing the prestressing force in the tendons.



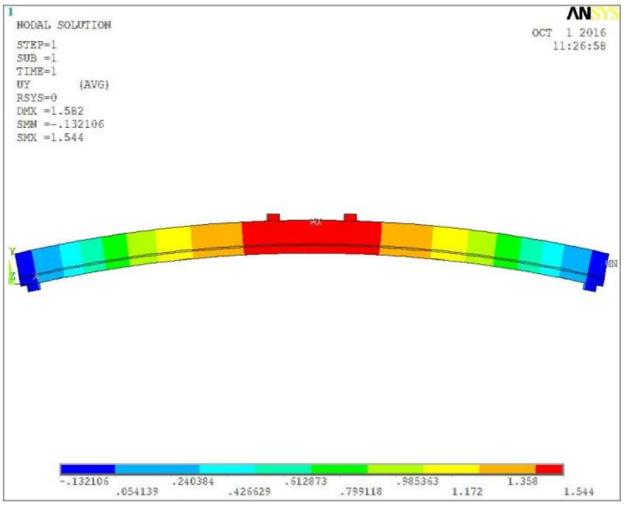
Figure(20): Load-deflection curves of the beam for various levels of prestressing focrce



(a) At prestressing force = 24 kN

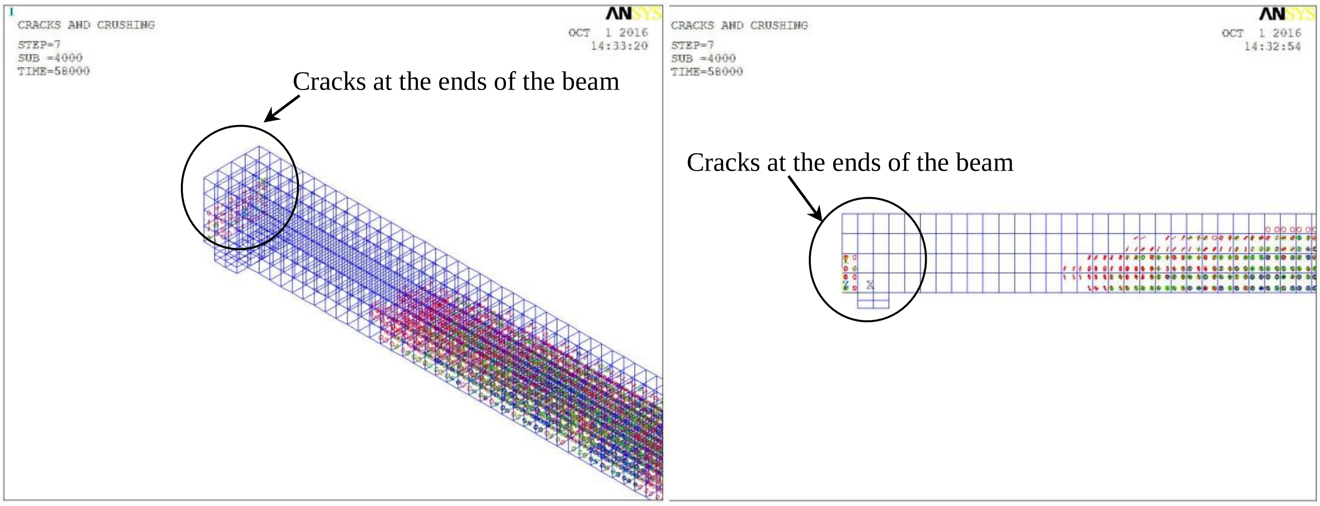


(b) At prestressing force = 36 kN



(c) At prestressing force = 48 kN

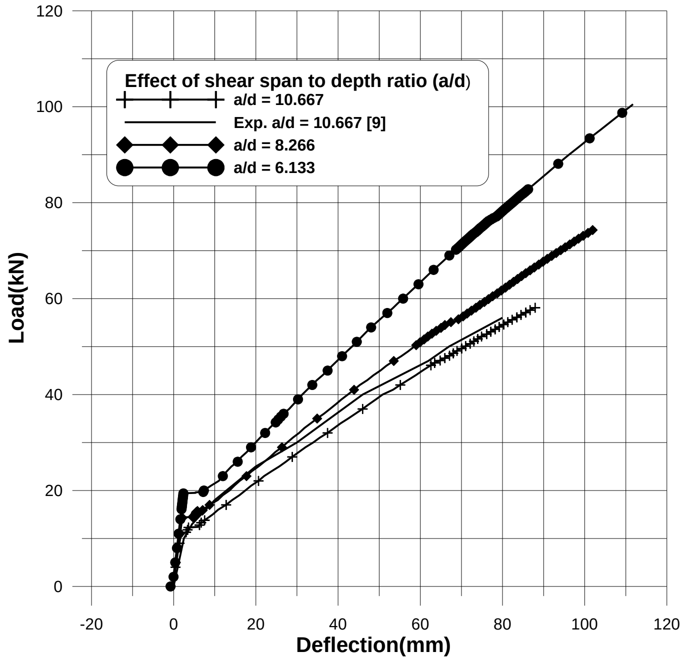
Figure(21): Deflected shape of the prestressed beam at prestressing force stage only



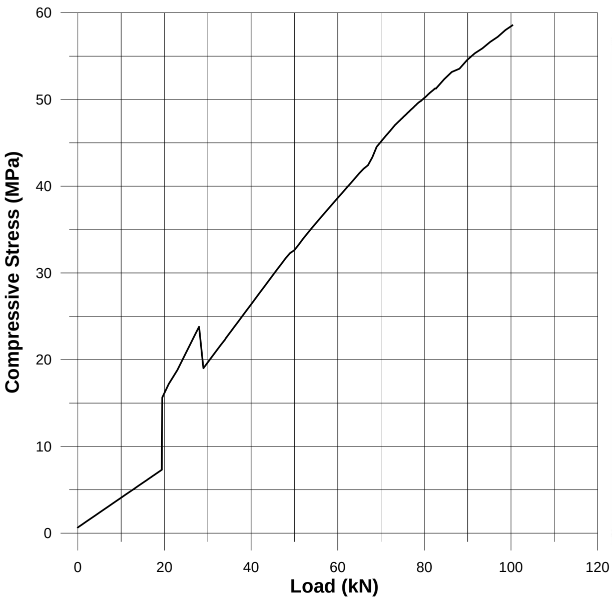
Figure(22) Crack pattern of the prestressed beam at failure load (at prestressed force=48 kN)

4.4 Effect of the Shear Span to Effective Depth Ratio(a/d) :

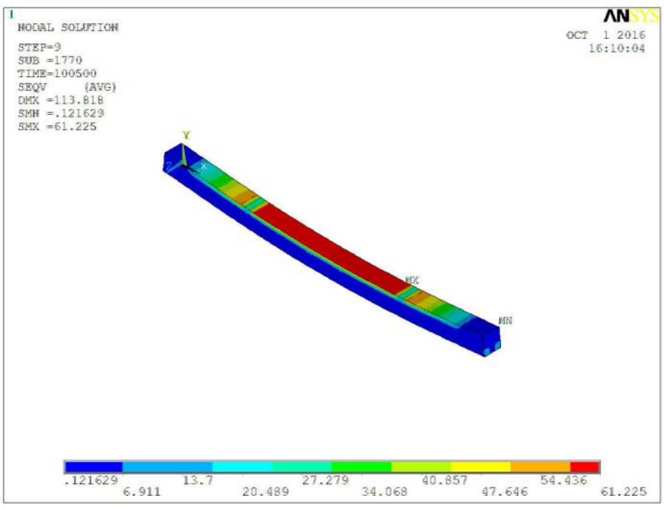
In the present parametreic study three values of the shear span to effective depth ratio were chosen. The ratios adopted are 10.667, 8.266, and 6.133. Load deflection curves for different ratios of shear span to effective depth were plotted as shown in figure(23). From these curves can be observed that as the shear span to depth ratio decreases, the behavior of the prestressed beam is affected. Load carrying capacity of the beam increases with decreasing shear span to depth ratio , and meanwhile the deflection increases.For the beam of 10.667 ratio , the load at the first crack is 11.1 kN ,the ultimate load is 58 kN , and the maximum deflection is about 88 mm, while for the beam of 8.226 ratio , the load at the first crack is 14.43 kN ,the ultimate load is 74.7 kN , and the maximum deflection is about 103 mm, also for the beam of 6.133 ratio , the load at the first crack is 19.5 kN ,the ultimate load is 100.5 kN , and the maximum deflection is about 111.7 mm. This behavior is because the shear resistance and flexural stiffness for the prestresssed beam increase with decreasing the shear span to effective depth ratio. The stresses in the concrete and steel tendons shown in the figures 24 and 25 respectively.Cracking pattren for the prestressed concrete beam at the ratio(a/d= 6.133) explained in Figure(26). For this beam,when the load reached up to 19.5 kN was observed the emergence of the first crack in the region where the bending moment constant , and by increasing the load was observed additional cracks are extended towards the shear span zone. Far ahead, when additional loads applied, great shear cracks were observed often at an angle of 45º.



Figure(23): Load-deflections curves for different values of shear span to effective depth ratios

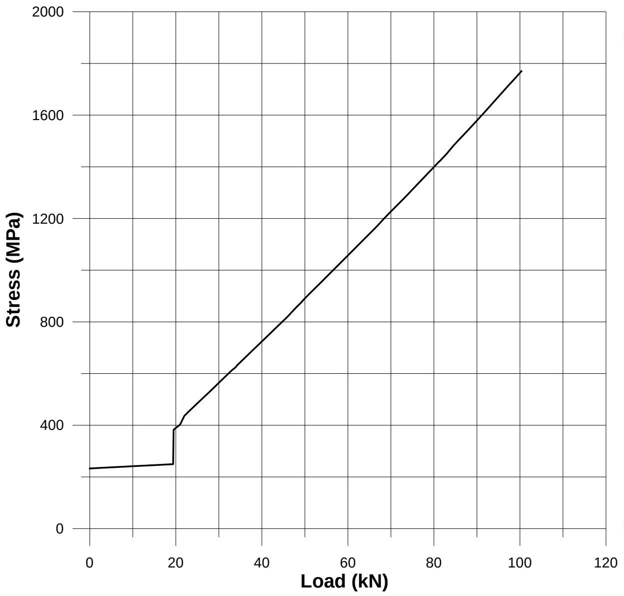


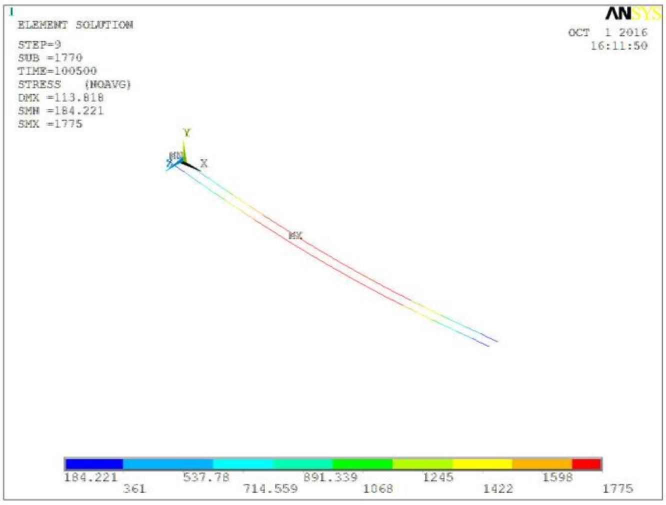
(a): Stress-load curve at top face of mid-span



(b): Von-Mises distribution stresses

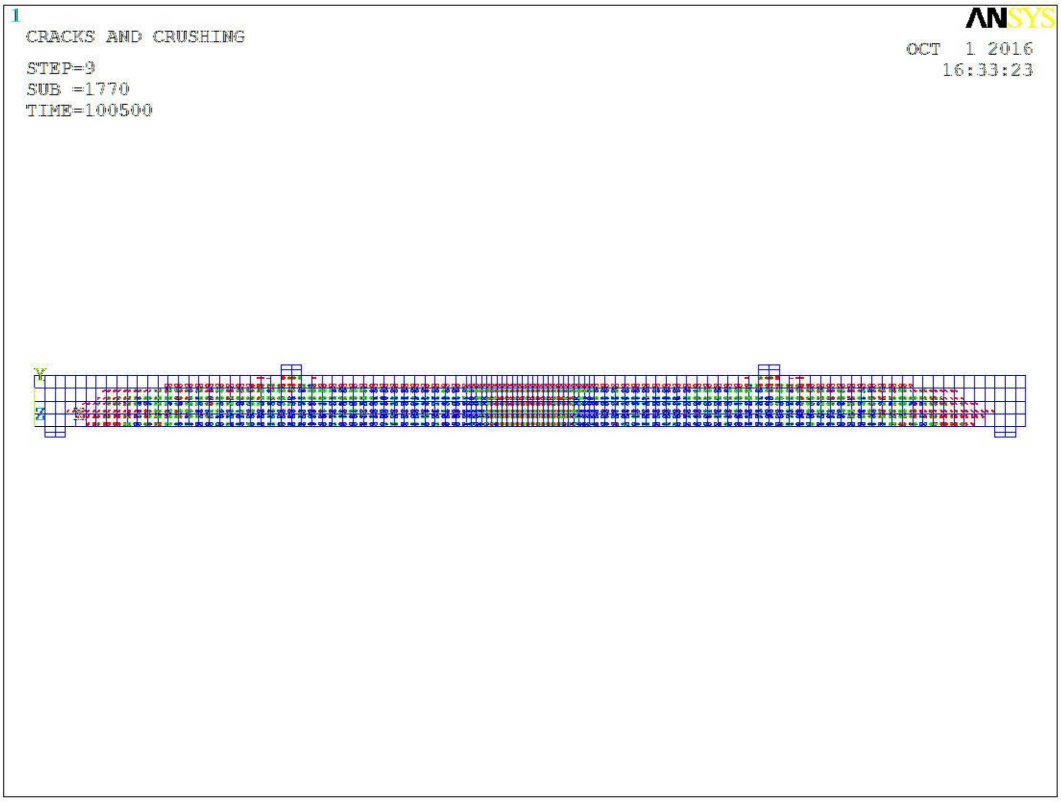
Figure(24): Stresses in the concrete of prestressed beam of a/d =6.133





(a) Stress-load curve at mid-length of tendon (b) Distribution stress along the beam

Figure(25) Stresses in the steel tendons of prestressed beam of a/d = 6.133



Figure(26) Crack pattern of the prestressed beam at failure load (at a/d =6.133)

5. CONCLUSIONS :

Many conclusions can be deduced from the current study and can be summarized as follows :

1. The ultimate load capacity of the prestressed concrete beams increases by (10%) when the compressive strength of concrete increases by (150 %) and meanwhile the maximum deflection at midspan of the beam decreases by 17.6%.
2. Increasing of the prestressing reinforcement ratio leads to improvement the overall performance of the prestressed concrete beam due to increase flexural resistance and the stiffness of the beam. Also, effect of prestressing reinforcement ratio on maximum deflection shows that increasing this ratio makes prestressed concrete beam more ductile and has the ability to pass high deflections before reaching failure load.
3. The stiffness and the load capacity of the prestressed concrete beams were not affected noticeably by prestressed force level especially after appearance of the cracks.
4. The ultimate load capacity increases by (28.79 %) when the shear span to effective depth ratio decreases by (22.5 %) , while the ultimate load capacity increases by (73.3%) when the shearspan to effective depth ratio decreases by (42.5 %). This behavior can be interpreted to the increasing in the shear resistance and flexural stiffness for the prestressed concrete beam as result to decreasing the shear span to effective depth ratio.

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