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The study of Laser -Lorenz System

W.H.Jbaer[(1)]D.J.Akoosh[(1)]D. S. Breesam[(1)]C.A.Emshary[(2)]

(1) The-Qar province Education Directorate, The-Qar

(2) Physics Department , Education College For Pure Sciences ,

Basrah University , Basrah , Iraq .

Abstract:

The generalized Lorenz system i.e Laser –Lorenz system is studied numerically where chaotic behaviors are possible to occur in class Clasers. Helical chaos appeared in the relation between imaginary part of laser field , real part of the same field and in the relation between imaginary part of polarization of the medium and real part of the laser field.

Keywords: Lorenz system, Laser-Lorenz system, Chaos, Helical chaos

دراسة نظام الليزر –لورنتز

وطن حمود جبر[1]، ضياء جبار عكوش[1]، ضياء صيوان بريسم[1]، جاسب عبد الحسين مشاري[2]

(1) مديرية تربية محافظة ذي قار/ وزارة التربية / ذي قار.

(2) قسم الفيزياء/ كلية التربية للعلوم الصرفة/ جامعة البصرة / وزارة التعليم العالي / البصرة / العراق .

الخلاصة :

تمت دراسة انموذج لورنتز معمم ، أي انموذج الليزر –لورنتز عددياً حيث لوحظ حصول تصرف فوضوي في ليزرات الصنف (C) .نتجت حركيات فوضوية حلزونية في العلاقة بين الجزئيين الحقيقي والخيالي للمجال الكهربائي وفي علاقةالجزئيين الخيالي لاستقطاب الوسط والحقيقي للمجال الكهربائي .

الكلمات المفتاحية : نظام لورنتز ،نظام الليزر- لورنتز، الفوضى ، الفوضى الحلزونية .

1- Introduction:

Since 1963 when E.N Lorenz derived the most astonishing mathematical model [1] used indirectly in the study of all types of lasers dynamics, a revolution started in the study of all types of lasers instabilities.This model, Lorenz, then rederived by Haken in 1975[2] where he suggested analogy between higher instabilities in fluids and lasers.Both Lorenz model [1] or "Hakemodel" with real quantitiesi.e temperature dependent quantities in the former and electric field and polarization of the medium in the later. With advances in the laser studies, it appears some time that the oscillating modefrequency in the laserdoes not coincide with that of the light frequency so that the field and polarization of the laser can not be treated as real quantities.This required a generalization of the Lorenz-Haken model, by the introduction of a quantity that make the L.H. model complex so that the laser field andpolarization can be written as real and imaginary parts.[2]

2- Mathematical model:

When laser works, first the population inversion ,Y(t), established and almost no light is available X(t) in the laser cavity, hence nopolarization Z(t) of the laser medium appeared .As time lapse , photon numbergrows and population is reduced.So there exists arelation between the three quantities i.eX(t),Y(t),and Z(t), so that laser is theoretically described by these threevariables, the electric field in the laser , the polarization of the laser medium ,and thepopulation inversion to induce the laser oscillation using the normalized variables of electric field X,polarization Y, andpopulation inversion, Z .The differential equations to describe the laseroscillation in the presence of condition of non resonance between the laser frequency ,ω_a, and the cavity one ,ω_c,they are given by [3],:

$$\frac{dx(t)}{dt} = -k[x(t) - y(t)] \quad (1)$$

$$\frac{dY(t)}{dt} = -(1 - i\Delta)y(t) + [r - z(t)]x(t) \quad (2)$$

$$\frac{dZ(t)}{dt} = -\gamma_{\parallel} z(t) + Re[x^{*}(t)y(t)] \quad (3)$$

$$\Delta = \omega_a - \omega_c \quad (4)$$

k is the decay rate of the field , Δ is the atomic detuning , r is the pump parameter ,and $\gamma_{\parallel}$ is the decay rate ofthe population inversion. * means complex conjugate and Re is the real part of X(t) or Y(t).

3-Simulation results:

To obtain results set of equations (1-3) was solved using the Rung-Kuttanumerical method and the Mat lab system together with certain initial conditions .The control parameters in the system (1-3) are the field decay rate, k, the population decay rate, $\gamma_{\parallel}$, the pump parameter ,r, and the atomic detuning, Δ. Three quantities are examined viz., temporal variation of population inversion, Z(t), relation of imaginary ImX(t) and real Re.X(t) parts of the laser field, to produce attractors and relation between imaginary part of the material polarization ImY(t) and the real part of the laser field Re. X(t) to create another attractors . Each one of the previous mentioned relations are studied under the effect of varying the control parameters, detuning, Δ , pump parameter ,r, laser field decay rate ,k, and decay rate of the population inversion,$\gamma_{\parallel}$,.

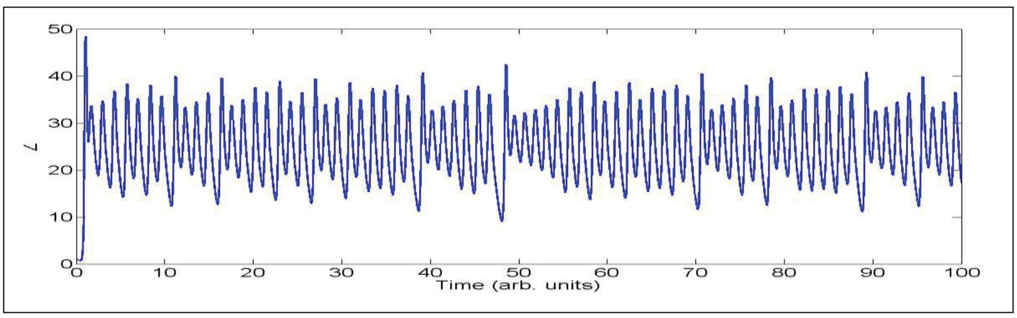
(a) Effect of atomic detuning Δ: when k=3, r=28, and$\gamma_{\parallel}$=1, Δ was varied through a wide range ranged from 0.00001 to 0.01. For Δ≥0.01 ,increases Z monotonically and reaches values that is not accepted physically.It shows negative values too. When Δ≤ 0.001 the usually butterfly attractors in the relations Im. X(t)v.s. Re. X(t) and Im. Y(t)v.s. Re X(t) appears and Z(t) show chaotic behavior especially the helical ones, as can be seen in figs(1and 2).

(b) Effect of pump parameter ,r: when k=3, $\gamma_{\parallel}$=1and Δ=0.001, r was varied from 2.8 to 2800 where Z values increased from 1.7 and reaches 2800 and its behavior started constant with time for low r then shows chaotic behavior for 28≤ r ≤ 280 then it shows periodic behaviors for r ≥ 280 as can be seen in fig (3). Special types if attractors appears for both relations i.e. ImX(t)v.s. Re X(t) and ImY(t)v.s. Re X(t) as can be seen in fig (4). The frequency of oscillation of Z(t) increases with the increases of pump parameters as can be deduced from fig (3) and fig (4) too through the density of lines in each attractor.

(c) Effect of decay rate of field, k: when r=28 ,$\gamma_{\parallel}$=1 and Δ=0.001, k was varied from 0.3 to 6 . For low values of k simple relations appears among the examined quantities. Temporal variations of Z is simple which evolves in to complex ones when k ≥ 3 and the famous butterfly attractor appears for the relations ImY(t)v.s. Re X(t) and ImX(t)v.s. Re X(t) as can be seen in fig(5) when k= 4.5.

(d) Effect of population inversion decay rate,$\gamma_{\parallel}$:when r =28, k=3 and Δ= 0.001, $\gamma_{\parallel}$was varied from 0.1 to 3 .Varieties of dynamics appeared started with deformed period one which switch when $\gamma_{\parallel}$=0.4 to chaos thenhelical chaos appear when b=0.8 up to $\gamma_{\parallel}$=2 then for $\gamma_{\parallel}$≥ 2 settle down where Z(t) shows steady state behavior .See figs (6-8) for more details

a



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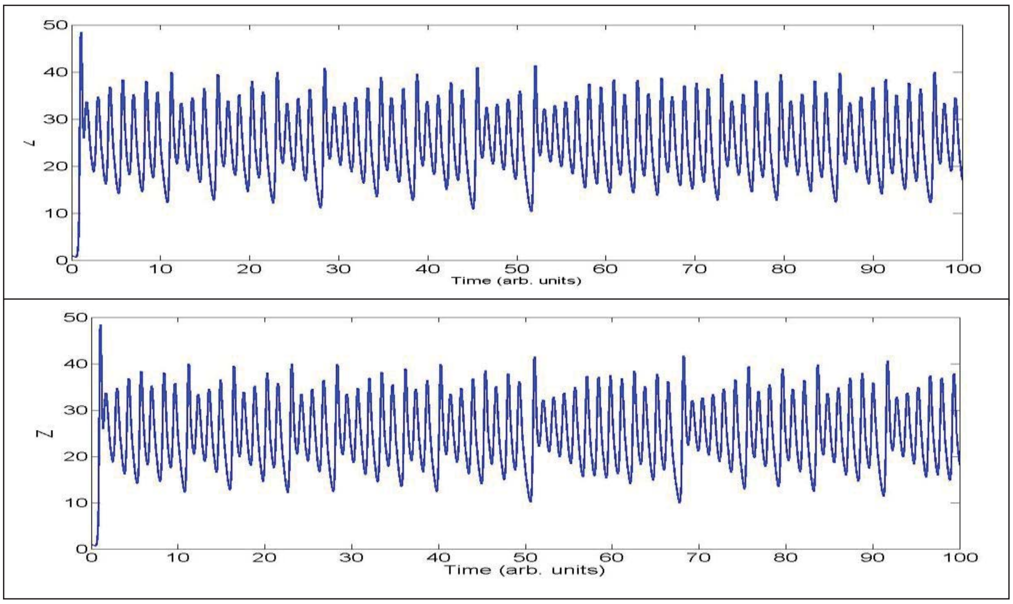


Fig (1): Temporal behavior of population difference,Z(t)

for (a) Δ=0.001 , (b) Δ=0.0001 , (c) Δ=0.00001

a

b

c

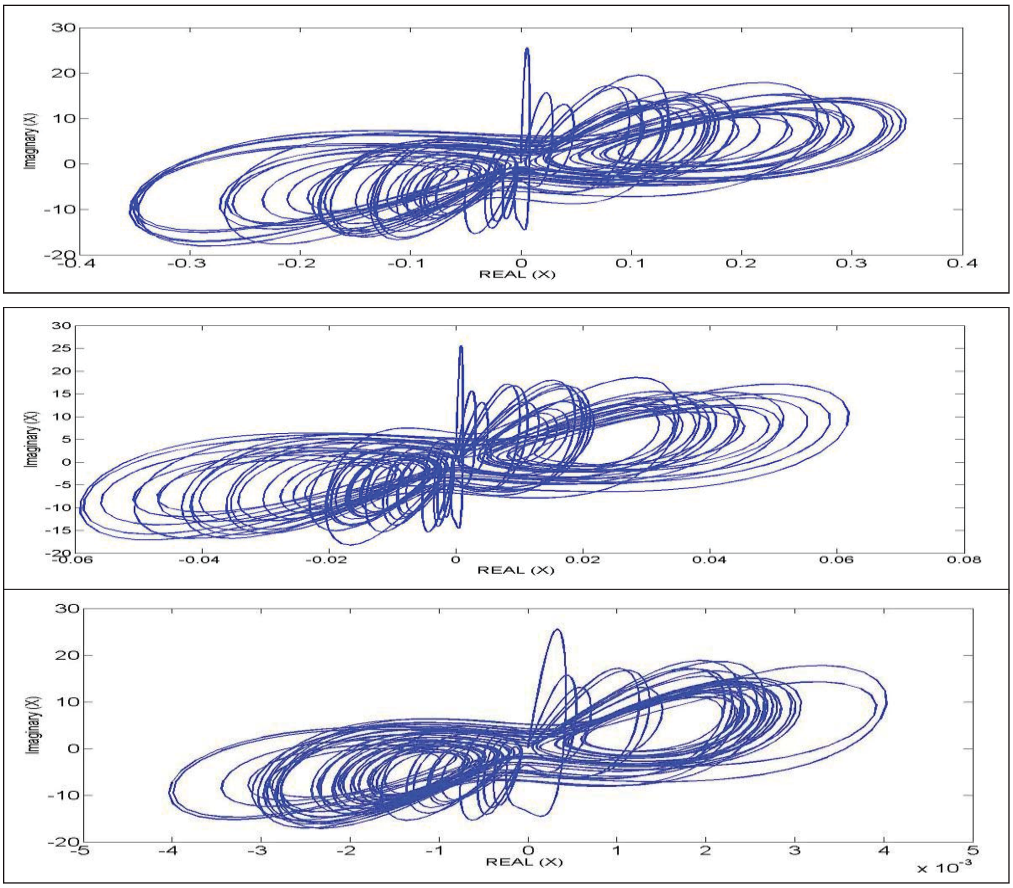
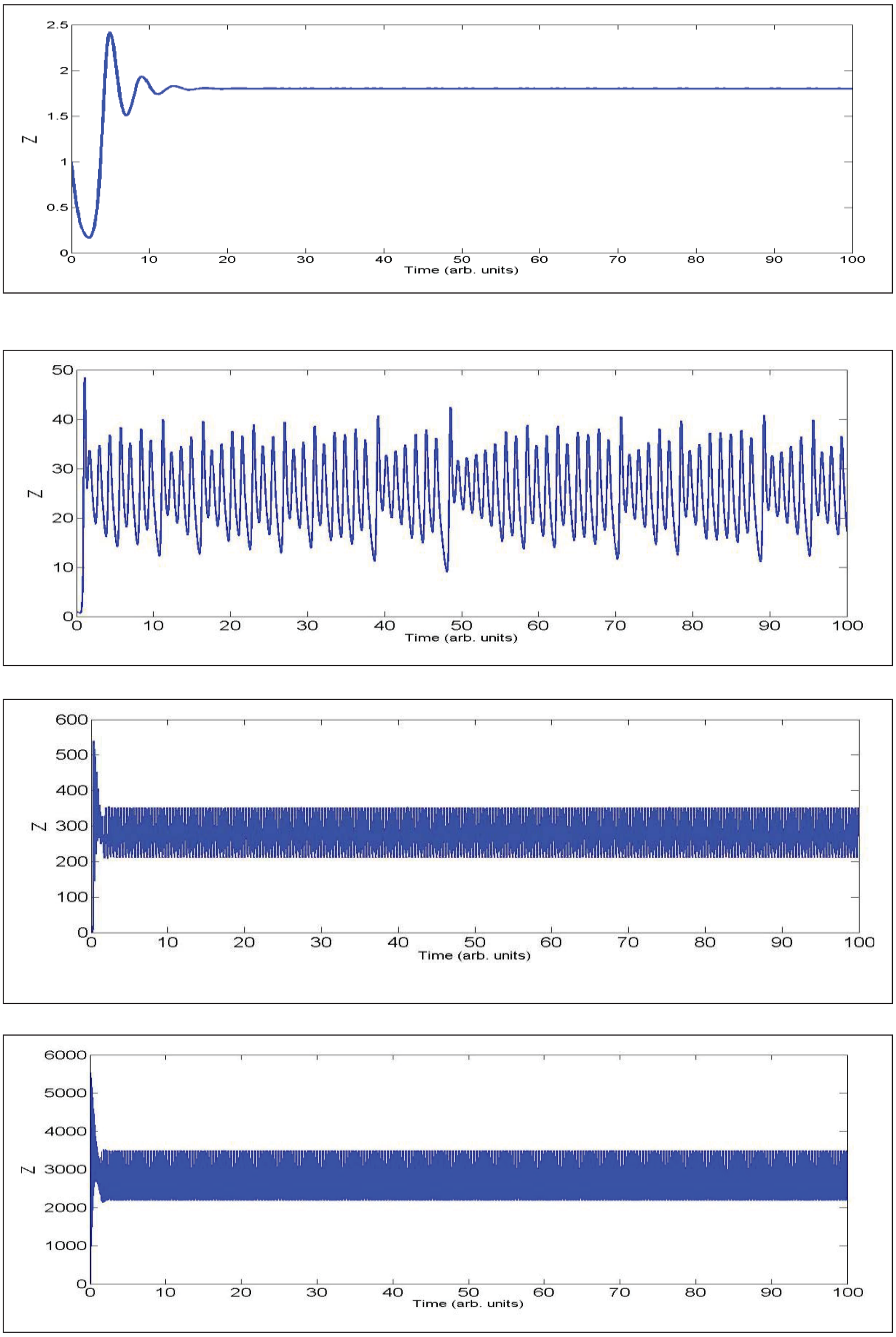


Fig (2):Attractors resulted in the relation ImX(t)v.s Re X(t)

for (a) Δ=0.001 , (b) Δ=0.0001 , (c) Δ=0.00001



a

b

c

d

Fig (3): Population variation with Tim,Z(t)

for Δ=0.001,$\gamma_{\|}$= 1,k=3and(a) r=2.8,

(b) r=28 ,(c) r=280, (d) r=2800

a

b

c

d

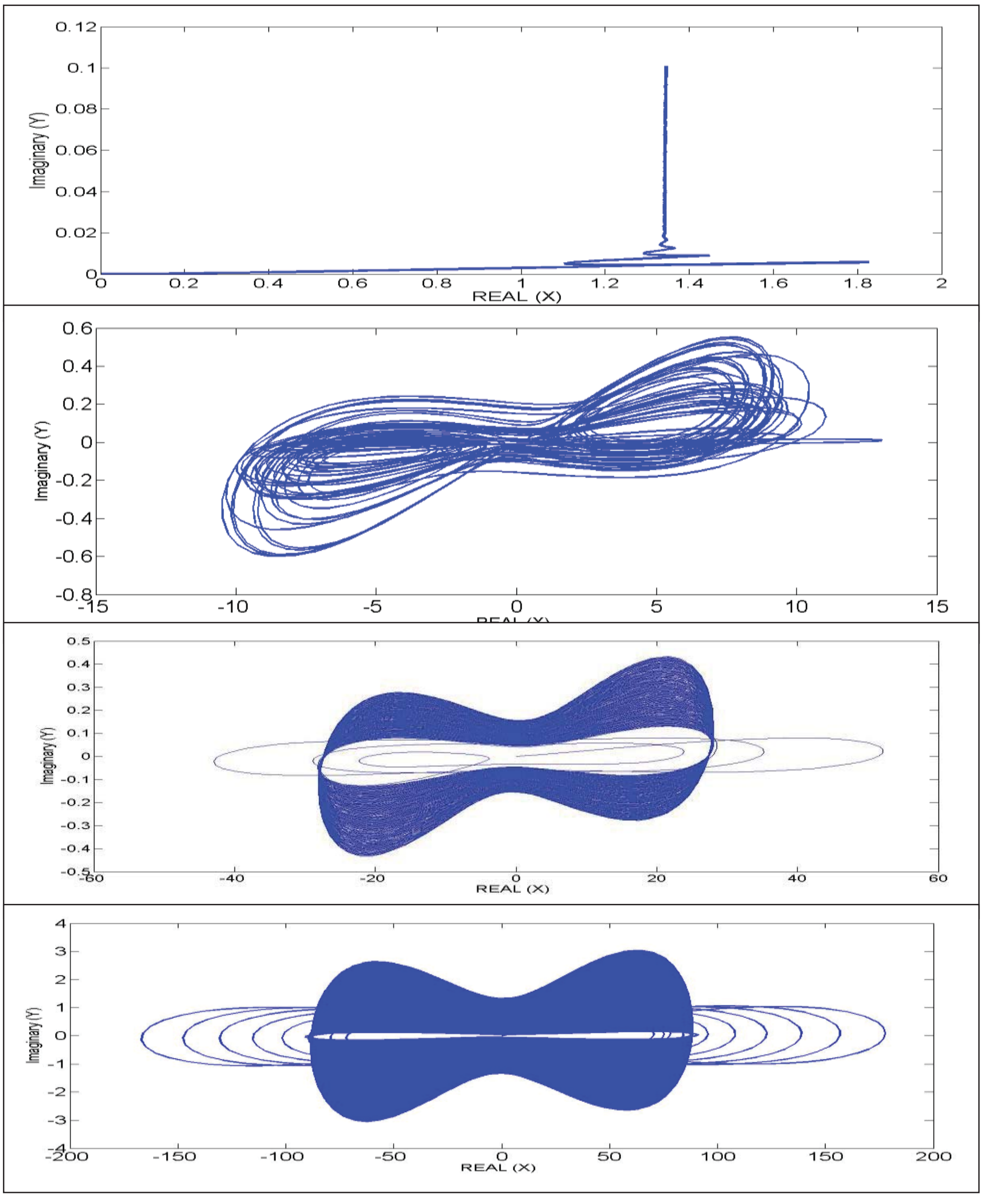
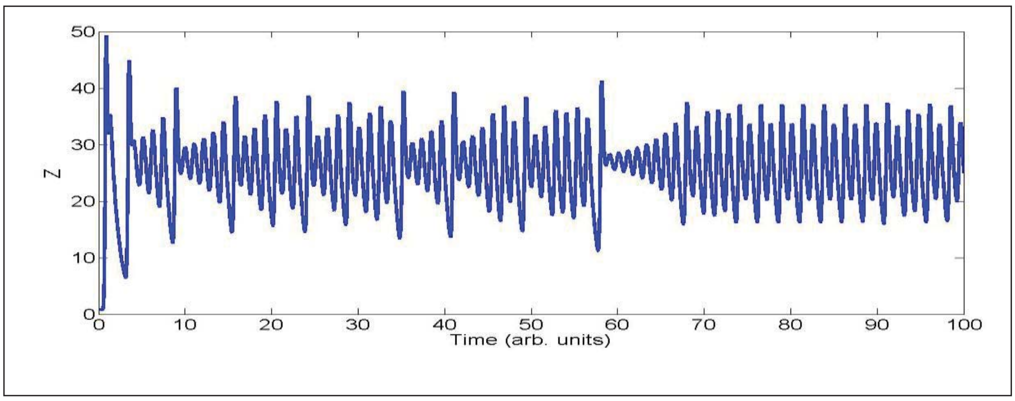


Fig (4): Attractors in the relation ImY(t)v.s Re X(t)

ForΔ= 0.001, $\gamma_{\|}$= 1, k=3and (a) r=2.8,

(b) r=28 ,(c) r=280, (d) r=2800,



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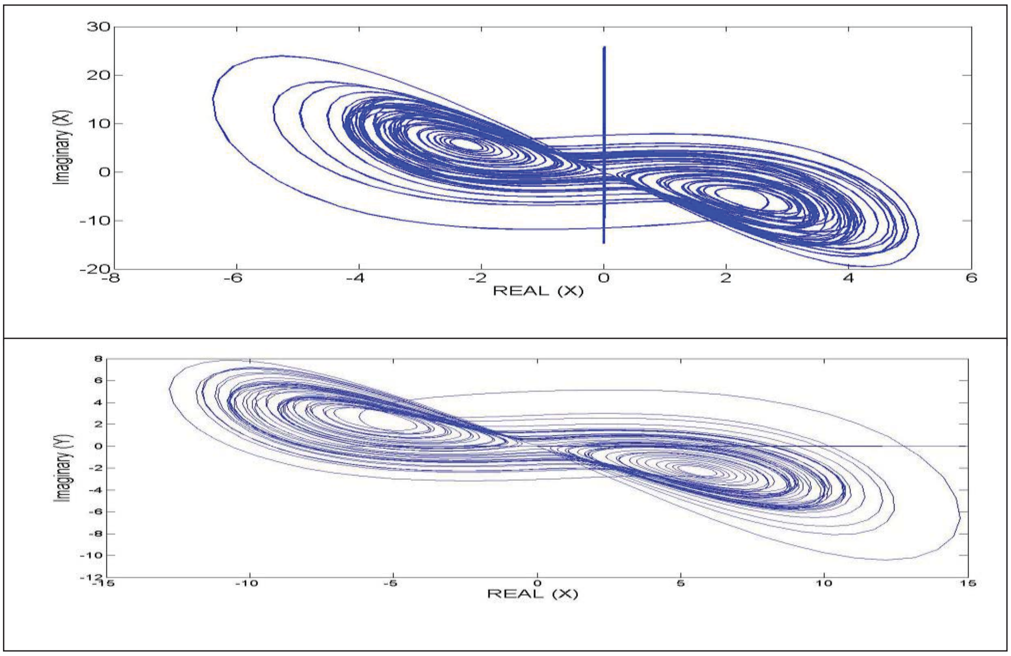

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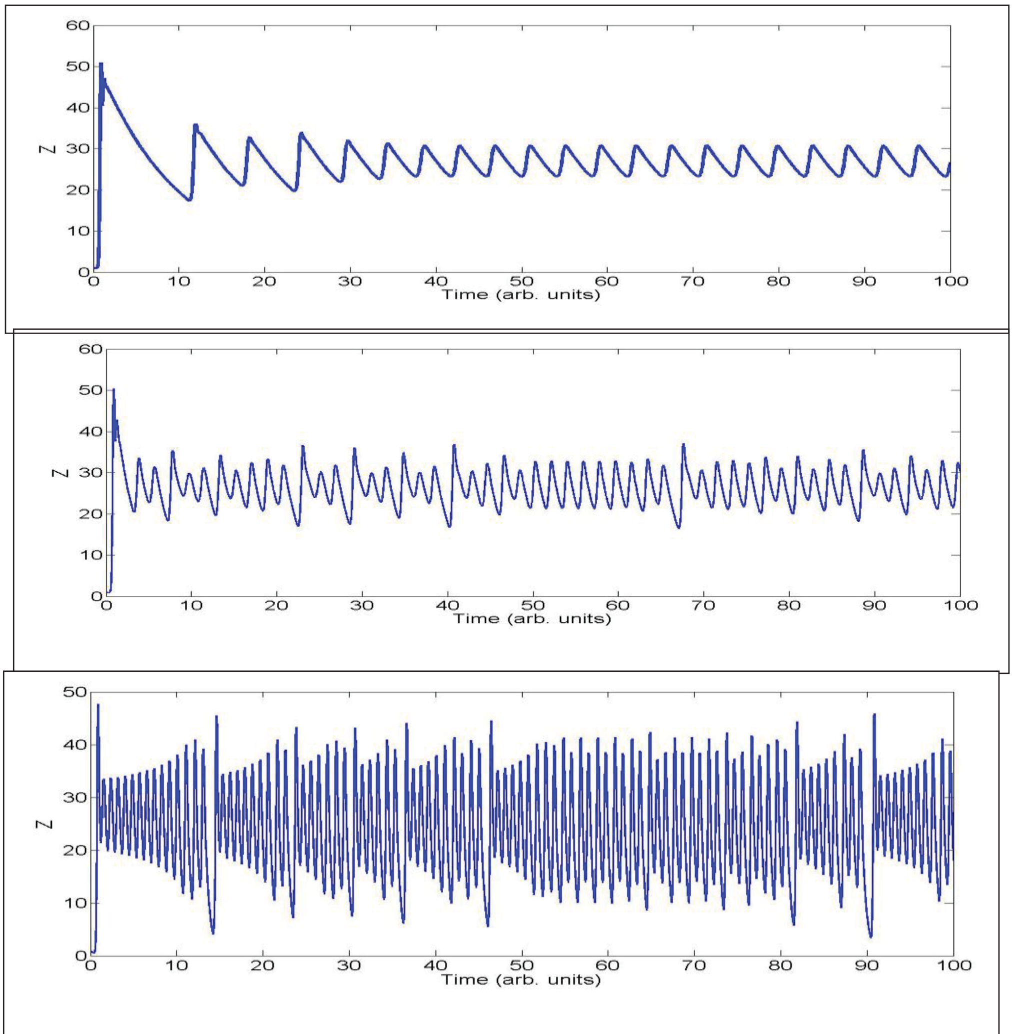
Fig(5):(a)Temporal variation of Z(b)Butterflyattractors resulted in the relation ImX(t)v.s Re X(t), (c)Another butterflyattractors resulted in the relation ImY(t) v.s Re X(t) for, r=28,$\gamma_{\parallel}$= 1,Δ=0.001and k = 4.5



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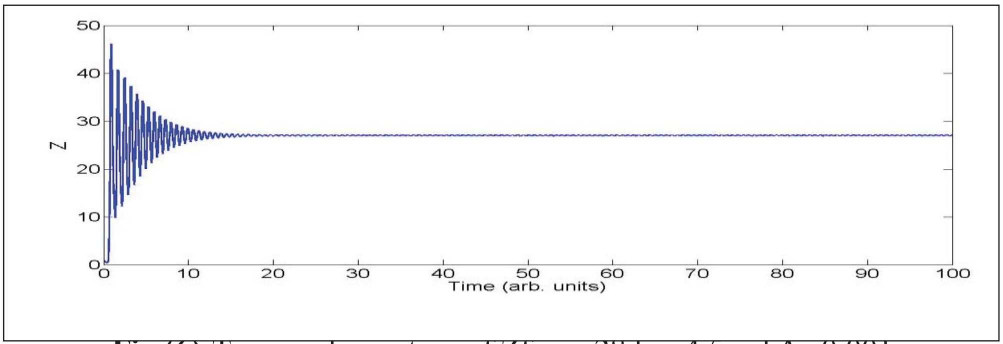
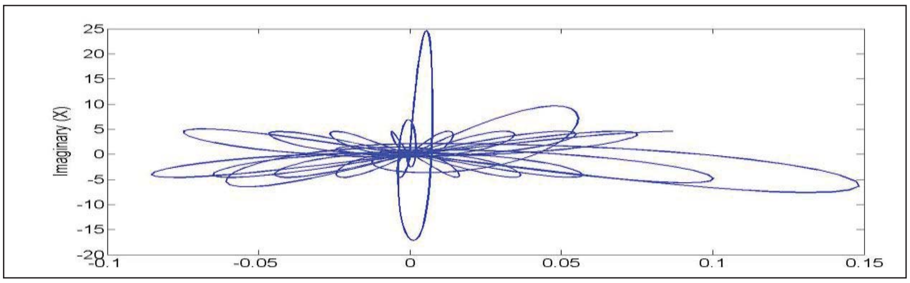


Fig (6): Temporal variation of Zfor,r=28,k = 4.5and Δ=0.001

(a)$\gamma_\parallel$= 0. 1 , (b) $\gamma_\parallel$= 0.4, (c)$\gamma_\parallel$= 2 , (d)$\gamma_\parallel$= 3

a



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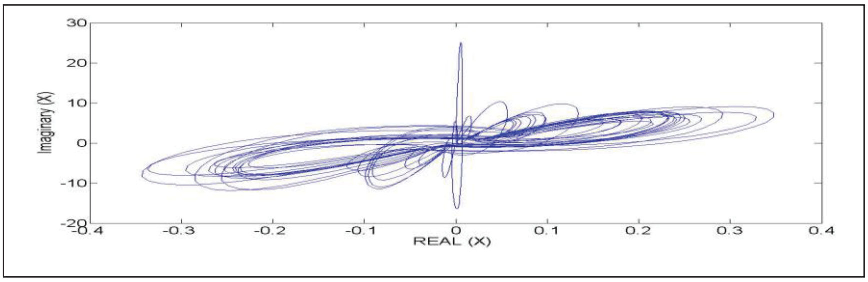

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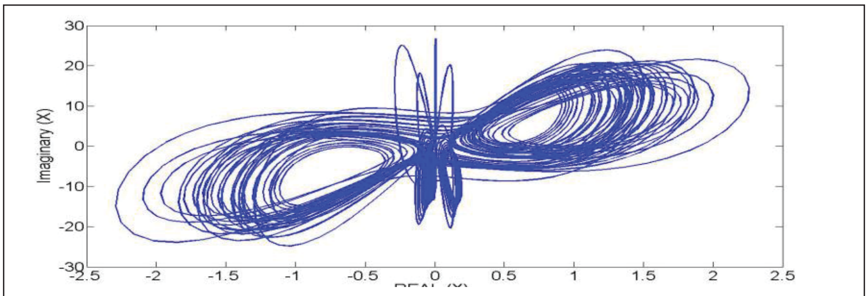

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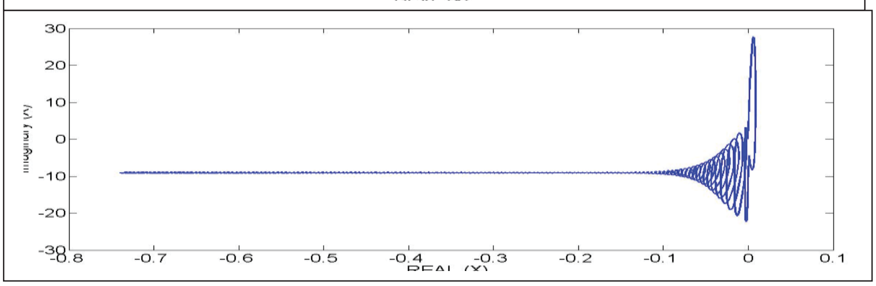
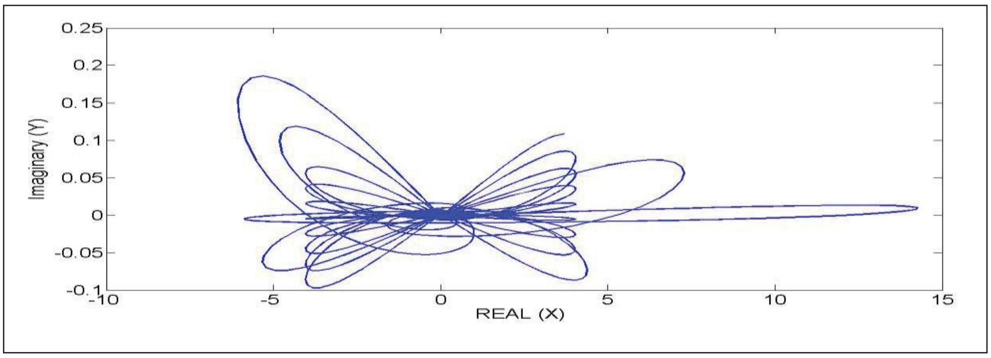
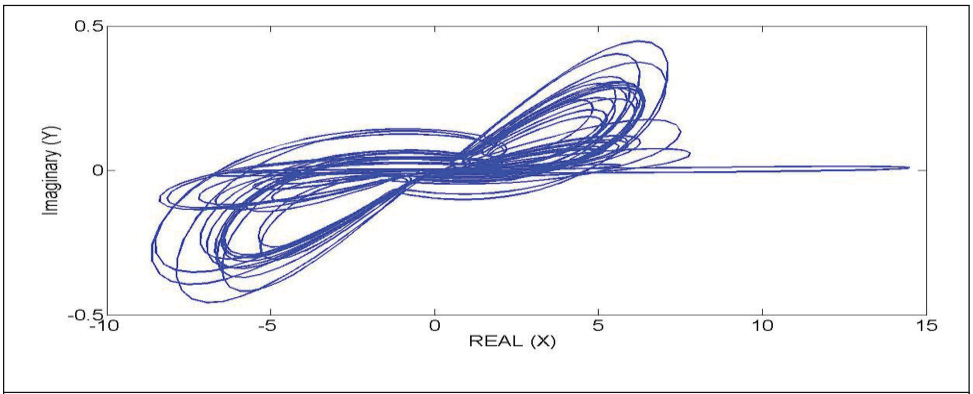


Fig (7): Attractors for,r =28,k = 4.5 and Δ=0.001, ofIm X(t)v.sRe X(t)

and (a) $\gamma_\parallel$= 0. 1 , (b) $\gamma_\parallel$= 0.4, (c) $\gamma_\parallel$= 2 , (d) $\gamma_\parallel$= 3



a



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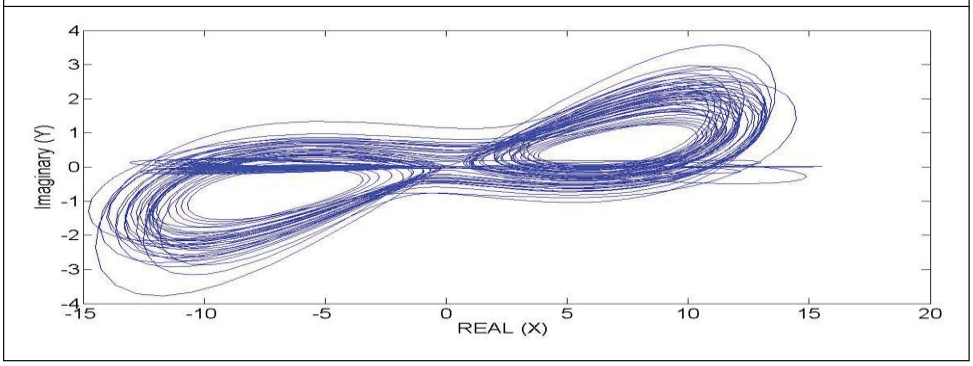

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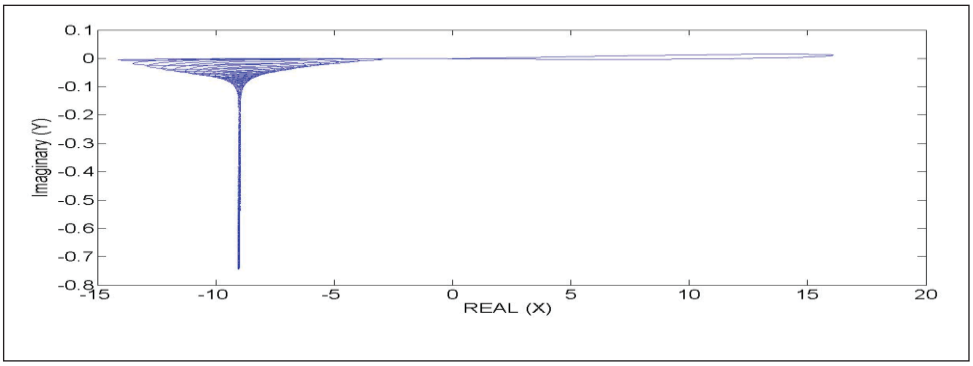

d

Fig (8) Attractorfor r =28, k = 4.5, Δ=0.001

ofIm Y(t) v.s. Re X(t) and (a) $\gamma_{\parallel}$= 0. 1 ,

(b) $\gamma_{\parallel}$= 0.4, (c) $\gamma_{\parallel}$= 2 , (d) $\gamma_{\parallel}$=3

4- Discussion:

Depending on each parameter appeared in the set of equations(1-3) or the combinations of them rich varieties of chaotic oscillations can be generated in lasers especially of the laser system belong to class C[4] where the relaxation rate of field ,k, equals that of polarization,$\gamma_\perp$, and population inversion,$\gamma_\parallel$, can be written as k $\approx\gamma_\parallel\approx\gamma_\perp$. Although there exist detuning in the system (1-3) the famous butterfly attractor appeared which ensures that the Lorenz system [1] is a special case of the generalized Laser-Lorenz system , since no meaning of detuning in theLorenz system .In other two classes of lasers viz , class A both population inversion ,Z, and polarization of the laser medium ,Y, and class B when the polarization of the laser medium ,y, vanishes quickly i.ek>>$\gamma_\perp$ and $\gamma_\parallel$for class A and$\gamma_\perp$>>k and$\gamma_\parallel$ for class B. To enhance instabilities in each class the laser system (or model) must be supplied with two degrees of freedom (class A) and one degree of freedom for class B.

5- Conclusion:

Chaos and butterfly attracters were noticed to occur in a generalizedLaser-Lorenz system for wide range of the four control parameters that appeared in L - L system .

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