مجلة كلية التربية للعلوم الصرفة - جامعة ذي قار
المجلد ٨، العدد ٢، حزيران ٢٠١٨
Website: jceps.utq.edu.iq Email: jceps@eps.utq.edu.iq

On Intuitionistic Fuzzy Soft α Separation Axioms

Mohammed JassimTuaimah

mohammedjassim255@gmail.com

Assistant Lecturer/General Directorate of Education in Thi-Qar Province

Abstract:

In this paper we define intuitionistic fuzzy soft (IFS, in short)α open (closed) sets, IFSα interior (closure) operators , IFSα continuous (open, closed and irresolute) mappings and IFSα separation axioms. We also give some the basic theorems and properties of these concepts.

Keywords:Soft set, fuzzy soft set, IFS set, IFS topology, IFSα open set and IFSαT_i spaces; $i = 1,2,3,4$.

Journal of College of Education for Pure Sciences
Volume 8, Number 2, June 2018
Website: jceps.utq.edu.iq Email: jceps@eps.utq.edu.iq

On Intuitionistic Fuzzy Soft α Separation Axioms

Mohammed JassimTuaimah
mohammedjassim255@gmail.com
Assistant Lecturer/General Directorate of Education in Thi-Qar Province

الخلاصة

في هذا البحث عرَفنا المجموعات الضبابية الحدسية المرنة α المفتوحة (المغلقة)، مؤثرات الداخل الضبابي الحدسي المرن α (الانغلاق)، التطبيقات المستمرة الضبابية الحدسية المرنة α (المفتوحة، المغلقة و المترددة)وبديهيات الفصل الضبابية الحدسية المرنةα. كذلك أعطينا بعض المبرهنات والخواص الأساسية لتلك المفاهيم.

1. *Introduction*

The "fuzzy sets" was firstly defined by Zadeh [13] in (1965). Atanassov [2] in (1986) initiated the study of "intuitionistic fuzzy sets". In (1999), Molodtsov [7] introduced the concept of "soft set". Maji, Biswas and Roy [5,6] in (2001), investigated the "fuzzy soft sets" and " intuitionistic fuzzy soft sets". In (2011), Shabir and Naz [10] presented "soft topology" and "soft separation axioms". The concepts of "fuzzy soft topology" and "fuzzy soft mappings" constructed by Pazar and Aygun [9] in (2012). Yin, Li and Jun [12] in (2012) gave the " intuitionistic fuzzy soft mappings". The notions of "intuitionistic fuzzy soft topology" and "a continuity of an intuitionistic fuzzy soft mappings" studied by Turanli and Es [11] in (2012). The "intuitionistic fuzzy soft interior (closure)" discussed by Bayramov and Gunduz [3] in (2014). Ismail and Deniz [8] in (2013) defined the "intuitionistic fuzzy soft separation axioms". In (2014) Kandil, Tantawy, El-Sheikh and Abd El-latif [4] investigated the "intuitionistic fuzzy softα separation axioms". Abd El-latif and Rodyna [1] in (2016) studied the properties of " fuzzy soft α T_i spaces", $i = 1,2,3,4$.In the present paper we study of the properties of "intuitionistic fuzzy soft α separation axioms" with some base theorems.

2. *Preliminaries*

In this section we recall the fundamental definitions and properties which it is needed in our paper.

Definition (2.1)[7]:

Let Xbe an initial universe set, E be a set of parameters, A be a non-empty subset of E and $P(X)$ denote the power of X. A pair (f, A) denoted by f_A is called soft set over X, where f is a mapping given by $f: A \to P(X)$.

Definition (2.2)[2]:

An intuitionistic fuzzy (IF, in short) set A over the universe X can be defined as follows: $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$, where the mappings $\mu_A: X \to I, \nu_A: X \to I$ denote the degree of membership and the degree of non-membership to the set A respectively, with the property $0 \le \mu_A(x) + \nu_A(x) \le 1$, for each $x \in X$. The set of all intuitionistic fuzzy sets over X denoted by $IF(X)$.

Definition(2.3)[2]:

Let X be a non-empty set. If $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$,$B = \{\langle x, \mu_B(x), \nu_B(x)\rangle : x \in X\}$

be two IF sets in X. Consider the following symbols: (1) $A \subseteq B \Leftrightarrow \mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x), \forall\, x \in X$.

(2) $A = B \Leftrightarrow A \subseteq B$ and $B \subseteq A$.

$$(3) A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}.$$

$$(4)\ A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle : x \in X\}.$$

$$(5)\ A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle : x \in X\}.$$

(6) An IF null set defined as: $\{\langle x, 0,1\rangle : x \in X\}$ and is denoted by $\tilde{0}$.

(7) An IF absolute set defined as: $\{\langle x, 1,0\rangle : x \in X\}$ and is denoted by $\tilde{1}$.

***Definition* (2.4)[6]:**

Let X be an initial universe set, E be a set of parameters, A be a non-empty subset of E and $IF(X)$ denote the collection of all IF subsets of X. A pair (F, A) is called intuitionistic fuzzy soft (IFS, in short) set over X, where F is a mapping given by $F: A \to IF(X)$. In general, for every $e \in A$, $F(e)$ is an IF set of X . Clearly $F(e)$ can be written as $\{\langle x, \mu_{F(e)}(x), \nu_{F(e)}(x)\rangle : x \in X , e \in A \subseteq E\}$. The set of all intuitionistic fuzzy soft sets over X with parameters from E denoted by $IFS(X_E)$.

***Definition* (2.5)[6]:**

Let (F, A) and (G, B) be two IFS sets over X . Then:

(1) Union: $(F, A)\ \tilde{\cup}\ (G, B) = (H, C)$, where $C = A \cup B$ and $\forall\, e \in C$,

$$H(e) = \begin{cases} F(e)\ , & if\ \ e \in A - B \\ G(e)\ , & if\ \ e \in B - A \\ F(e) \cup G(e)\ , & if\ \ e \in A \cap B \end{cases}$$

(2) Intersection: $(F, A)\ \tilde{\cap}\ (G, B) = (H, C); C = A \cap B$ and $\forall\, e \in C$, $H(e) = F(e) \cap G(e)$.

(3) Subset: $(F, A)\ \tilde{\subseteq}\ (G, B)$, where $(i)\ A \subseteq B\, (ii)\ \forall\, e \in A, F(e) \subseteq G(e)$

(4) Complement: $(F, A)^c = (F^c, A)$, where $F^c: A \to IF(X)$ is a mapping given by $F^c(e) = [F(e)]^c , \forall\, e \in A$.

i.e. If $F(e) = \{\langle x, \mu_{F(e)}(x), \nu_{F(e)}(x)\rangle : x \in X , e \in A \subseteq E\}$, then

$F^c(e) = [F(e)]^c = \{\langle x, \nu_{F(e)}(x) , \mu_{F(e)}(x)\rangle : x \in X , e \in A \subseteq E\}$.

(5) Absolute IFS set: (F, A) is said to be absolute IFS set denoted by $\tilde{1}_E$ if $\forall\, e \in A$, $F(e)$ is the absolute IF set $\tilde{1}$ of X.

(6) Null IFS set: (F, A) is said to be null IFS set denoted by $\tilde{0}_E$ if $\forall\, e \in A$, $F(e)$ is the null IF set $\tilde{0}$ of X.

Definition (2.6)[12]:

Let $IFS(X_E)$, $IFS(Y_D)$ be two IFS classes and $\varphi: X \to Y$, $\phi: E \to D$ be two mappings. Then a mapping $(\varphi, \phi): IFS(X_E) \to IFS(Y_D)$ is defined as: (i) For $(F, A) \in IFS(X_E)$, the image of (F, A) under (φ, ϕ) denoted by $(\varphi, \phi)(F, A) = (\varphi(F), \phi(A))$ is an IFS set in $IFS(Y_D)$ given by

$$\mu_{\varphi(F)(d)}(y) = \begin{cases} Sup_{x\in\varphi^{-1}(y), e\in A\cap\phi^{-1}(d)}\, \mu_{F(e)}(x) \;; & \varphi^{-1}(y) \neq \emptyset \\ 0 \;; & otherwise \end{cases}$$ and

$$\nu_{\varphi(F)(d)}(y) = \begin{cases} Inf_{x\in\varphi^{-1}(y), e\in A\cap\phi^{-1}(d)}\, \nu_{F(e)}(x) \;; & \varphi^{-1}(y) \neq \emptyset \\ 1 \;; & otherwise \end{cases}$$ for all $d \in \phi(A)$ and $y \in Y$.

(ii) For $(G, B) \in IFS(Y_D)$, the inverse image of (G, B) under (φ, ϕ) denoted by $(\varphi, \phi)^{-1}(G, B) = (\varphi^{-1}(G), \phi^{-1}(B))$ is an IFS set in $IFS(X_E)$ given by: $\mu_{\varphi^{-1}(G)(e)}(x) = \mu_{G(\phi(e))}(\varphi(x))$ and$\nu_{\varphi^{-1}(G)(e)}(x) = \nu_{G(\phi(e))}(\varphi(x))$, for all $e \in \phi^{-1}(B)$ and $x \in X$.

The IFS mapping (φ, ϕ) is called surjective (resp. injective) if φ and ϕ are surjective (resp. injective).

Theorem (2.7)[12]:

Let $(\varphi, \phi): IFS(X_E) \to IFS(Y_D)$ be an IFS mapping. Then the following statements hold:

(1)$(F, A) \,\tilde{\subseteq}\, (\varphi, \phi)^{-1}((\varphi, \phi)(F, A)), \forall (F, A) \in IFS(X_E)$. If (φ, ϕ) is injective, then the equality holds.

(2)$(\varphi, \phi)((\varphi, \phi)^{-1}(G, B)) \,\tilde{\subseteq}\, (G, B), \forall (G, B) \in IFS(Y_D)$. If (φ, ϕ) issurjective, then the equality holds.

(3)$((\varphi, \phi)(F, A))^c \,\tilde{\subseteq}\, (\varphi, \phi)((F, A)^c), \forall (F, A) \in IFS(X_E)$. If (φ, ϕ) isbijective, then the equality holds.

(4)$(\varphi, \phi)^{-1}((G, B)^c) = ((\varphi, \phi)^{-1}(G, B))^c , \forall (G, B) \in IFS(Y_D)$.

(5)$(\varphi, \phi)(\tilde{0}_E) = \tilde{0}_D$, $(\varphi, \phi)(\tilde{1}_E) \,\tilde{\subseteq}\, \tilde{1}_D$. If (φ, ϕ) issurjective, then the equality holds.

(6)$(\varphi, \phi)^{-1}(\tilde{0}_E) = \tilde{0}_D$, $(\varphi, \phi)^{-1}(\tilde{1}_E) = \tilde{1}_D$.

(7)If $(F, A) \,\tilde{\subseteq}\, (G, A)$, then $(\varphi, \phi)((F, A)) \,\tilde{\subseteq}\, (\varphi, \phi)((G, A)), \forall (F, A), (G, A) \in IFS(X_E)$.

(8)If $(F, B) \,\tilde{\subseteq}\, (G, B)$, then $(\varphi, \phi)^{-1}((F, B)) \,\tilde{\subseteq}\, (\varphi, \phi)^{-1}((G, B)), \forall (F, B), (G, B) \in IFS(Y_D)$.

(9)$(\varphi, \phi)^{-1}(\tilde{\bigcup}_{j\in J} (G, B)_j) = \tilde{\bigcup}_{j\in J} (\varphi, \phi)^{-1}((G, B)_j)$ and $(\varphi, \phi)^{-1}(\tilde{\bigcap}_{j\in J} (G, B)_j) = \tilde{\bigcap}_{j\in J} (\varphi, \phi)^{-1}((G, B)_j), \forall (G, B)_j \in IFS(Y_D)$.

$(10)(\varphi,\phi)(\tilde{\cup}_{j\in J}(F,A)_j)=\tilde{\cup}_{j\in J}(\varphi,\phi)((F,A)_j)$ and $(\varphi,\phi)(\tilde{\cap}_{j\in J}(F,A)_j)\tilde{\subseteq}\tilde{\cap}_{j\in J}(\varphi,\phi)((F,A)_j), \forall (F,A)_j \in IFS(X_E)$. If (φ,ϕ) is injective, then the equality holds.

Definition (2.8)[11]:

Let $\tau \subseteq IFS(X_E)$, then τ is said to be intuitionistic fuzzy soft topology (IFST, in short) on X if the following conditions hold:

$(i)\tilde{0}_E, \tilde{1}_E \in \tau$, (ii) If $(F,A),(G,B) \in \tau$, then $(F,A)\ \tilde{\cap}\ (G,B) \in \tau$,

(iii) If $(F,A)_j \in \tau$, then $\tilde{\cup}_{j\in J}(F,A)_j \in \tau$.

In this case the pair (X_E,τ) is called intuitionistic fuzzy soft topological space (IFSTS, in short) and each IFS set in τ is known as an intuitionistic fuzzy soft open set (IFSOS, for short). An IFS set is called intuitionistic fuzzy soft closed set (IFSCS, for short) if and only if its complement is IFSOS.

Definition (2.9)[11]:

Let $(X_E,\tau_1),(Y_D,\tau_2)$ be two IFSTSs. An IFS mapping $(\varphi,\phi):(X_E,\tau_1)\to(Y_D,\tau_2)$ is called:

(i) IFS continuous if $\forall (G,B)\in\tau_2, (\varphi,\phi)^{-1}((G,B))\in\tau_1$.

(ii) IFS open if $\forall (F,A)\in\tau_1, (\varphi,\phi)((F,A))\in\tau_2$.

Definition (2.10)[8]:

Let (X_E,τ) be an IFSTS and Y be a non-empty subset of X. Then:$\tau_Y=\{(H,C):(H,C)= Y_E\ \tilde{\cap}\ (F,A), \forall (F,A)\in\tau\}$is said to be IFST on Y and (Y_E,τ_Y) is called IFS subspace of (X_E,τ).

Definition (2.11)[8]:

An IFS set (F,A) is said to be IFS point denoted by e_F if for the element $e\in A, F(e)\neq\tilde{0}$ and $F(e`)=\tilde{0}, \forall e` \in A-\{e\}$.

Definition (2.12)[8]:

An IFS point e_F is said to be in IFS set (G,A) denoted by $e_F\in(G,A)$ if for the element $e\in A, F(e)\subseteq G(e)$. i.e. $\mu_{F(e)}(x)\leq\mu_{G(e)}(x)$and $\nu_{F(e)}(x)\geq\nu_{G(e)}(x), \forall\ x\in X, e\in A$.

Definition (2.13)[3]:

Let (X_E,τ) be an IFSTS and $(F,A)\in IFS(X_E)$. (i) The IFS interior of (F,A), defined as: $IFS\ int(F,A)=\tilde{\cup}\{(H,C):(H,C)\ is\ IFSOS,(H,C)\tilde{\subseteq}(F,A)\}$. (ii) The IFS closure of (F,A), defined as: $IFS\ cl(F,A)=\tilde{\cap}\{(H,C):(H,C)\ is\ IFSCS,(F,A)\tilde{\subseteq}(H,C)\}$.

Theorem (2.14)[3]:

Let (X_E,τ) be an IFSTS and $(F,A),(G,B)\in IFS(X_E)$. Then:

(1)$IFS\ int(F,A)\ \tilde{\cap}\ IFS\ int(G,B)=IFS\ int((F,A)\ \tilde{\cap}\ (G,B))$.

(2)$IFS\ int(F,A)\ \tilde{\cup}\ IFS\ int(G,B)\ \tilde{\subseteq}\ IFS\ int((F,A)\ \tilde{\cup}\ (G,B))$.

(3)$IFS\ cl(F,A)\ \tilde{\cup}\ IFS\ cl(G,B)=IFS\ cl((F,A)\ \tilde{\cup}\ (G,B))$.

(4)$IFS\ cl\big((F,A)\ \tilde{\cap}\ (G,B)\big)\ \tilde{\subseteq}\ IFS\ cl(F,A)\ \tilde{\cap}\ IFS\ cl(G,B)$.

(5)$(IFS\ int(F,A))^c=IFS\ cl((F,A)^c)$.

(6)$(IFS\ cl(F,A))^c=IFS\ int((F,A)^c)$.

3. Intuitionistic Fuzzy Soft α Open(Closed) Sets

In this section we define the intuitionistic fuzzy soft α open (closed) sets and we study the interior (closure) of them.

Definition (3.1):

Let (X_E,τ) be an IFSTS . An IFS set (F,A) over X is said to be $IFS\alpha$ open set if $(F,A)\ \tilde{\subseteq}\ IFS\ int(IFS\ cl(IFS\ int(F,A)))$. The complement of an $IFS\alpha$ open set is called $IFS\alpha$ closed set. We denote the set of all $IFS\alpha$ open sets and all $IFS\alpha$ closed sets by $IFS\alpha OS(X_E)$ and $IFS\alpha CS(X_E)$ respectively.

Theorem (3.2):

Let (X_E,τ) be an IFSTS and $(F,A)\in IFS\alpha OS(X_E)$. Then:

(i) The union of $IFS\alpha$ open sets is $IFS\alpha$ open set.

(ii) The intersection of $IFS\alpha$ closed sets is $IFS\alpha$ closed set.

Proof:

(i) Let $\{(F,A)_j, j\in J\}\ \tilde{\subseteq}\ IFS\alpha OS(X_E)\Rightarrow\forall j\in J,(F,A)_j\ \tilde{\subseteq}\ IFS\ int(IFS\ cl(IFS\ int(F,A)_j))$

$\Rightarrow\tilde{\cup}_{j\in J}\ (F,A)_j\ \tilde{\subseteq}\tilde{\cup}_{j\in J}\ (IFS\ int(IFS\ cl(IFS\ int(F,A)_j)))$

$\tilde{\subseteq}\ IFS\ int(\tilde{\cup}_{j\in J}\ IFS\ cl(IFS\ int(F,A)_j))$ (Theorem: 2.14(2))

$\tilde{\subseteq}\ IFS\ int(IFS\ cl(\tilde{\cup}_{j\in J}\ IFS\ int(F,A)_j))$ (Theorem: 2.14(3))

$\widetilde{\subseteq} IFS\, int(IFS\, cl(IFS\, int\, \widetilde{\cup}_{j\in J}\, (F,A)_j))$ (Theorem: 2.14(2))

$\Rightarrow \widetilde{\cup}_{j\in J}\, (F,A)_j \in IFS\alpha OS(X_E), \forall j \in J$.

(ii) Let $\{(G,B)_j\, , j \in J\} \widetilde{\subseteq} IFS\alpha CS(X_E) \Rightarrow (G,B)_j^{\ c}$ is $IFS\alpha$ open set, $\forall j \in J \Rightarrow$ from (i) we get: $\widetilde{\cup}_{j\in J}\, (G,B)_j^{\ c} \in IFS\alpha OS(X_E) \Rightarrow [\widetilde{\cup}_{j\in J}\, (G,B)_j^{\ c}]^c \in IFS\alpha CS(X_E)$

$\Rightarrow \widetilde{\cap}_{j\in J}\, (G,B)_j$ is $IFS\alpha$closed set, $\forall j \in J$. ■

***Definition* (3.3):**

Let (X_E, τ) be an IFSTS and $(F,A) \in IFS(X_E)$. Then:

(i) The IFSα interior of (F,A), denoted by $IFS\alpha\, int(F,A)$ and is defined as: $IFS\alpha\, int(F,A) = \widetilde{\cup}\, \{(H,C): (H,C) \in IFS\alpha OS(X_E), (H,C)\, \widetilde{\subseteq}\, (F,A)\}$

(ii) The IFSα closure of (F,A), denoted by $IFS\alpha\, cl(F,A)$ and is defined as: $IFS\alpha\, cl(F,A) = \widetilde{\cap}\, \{(H,C): (H,C) \in IFS\alpha CS, (F,A)\, \widetilde{\subseteq}\, (H,C)\}$.

***Theorem* (3.4):**

Let (X_E, τ) be an IFSTS and $(F,A), (G,B) \in IFS(X_E)$. Then:

(1)$IFS\alpha\, int(\tilde{1}_E) = \tilde{1}_E$ and $IFS\alpha\, int(\tilde{0}_E) = \tilde{0}_E$. (2)$IFS\alpha\, int(F,A)\, \widetilde{\subseteq}\, (F,A)$.

(3)$IFS\alpha\, int(F,A)$ is the largest $IFS\alpha$ open set contained in (F,A).

(4)(F,A) is $IFS\alpha$ open set if and only if $IFS\alpha\, int(F,A) = (F,A)$.

(5) If $(F,A)\, \widetilde{\subseteq}\, (G,B)$, then $IFS\alpha\, int(F,A)\, \widetilde{\subseteq}\, IFS\alpha\, int(G,B)$.

(6)$IFS\alpha\, int(IFS\alpha\, int(F,A)) = IFS\alpha\, int(F,A)$.

(7)$IFS\alpha\, int(F,A)\, \widetilde{\cup}\, IFS\alpha\, int(G,B)\, \widetilde{\subseteq}\, IFS\alpha\, int((F,A)\, \widetilde{\cup}\, (G,B))$.

(8)$IFS\alpha\, int((F,A)\, \widetilde{\cap}\, (G,B))\, \widetilde{\subseteq}\, IFS\alpha\, int(F,A)\, \widetilde{\cap}\, IFS\alpha\, int(G,B)$.

***Proof*:**It's clear.■

***Theorem* (3.5):**

Let (X_E, τ) be an IFSTS and $(F,A), (G,B) \in IFS(X_E)$. Then:

(1)$IFS\alpha\, cl(\tilde{1}_E) = \tilde{1}_E$ and $IFS\alpha\, cl(\tilde{0}_E) = \tilde{0}_E$. (2)$(F,A)\, \widetilde{\subseteq}\, IFS\alpha\, cl(F,A)$.

(3)$IFS\alpha\, cl(F,A)$ is the smallest $IFS\alpha$ closed set contains (F,A).

(4)(F,A) is $IFS\alpha$ closed set if and only if $IFS\alpha\, cl(F,A) = (F,A)$.

(5) If $(F,A)\, \widetilde{\subseteq}\, (G,B)$, then $IFS\alpha\, cl(F,A)\, \widetilde{\subseteq}\, IFS\alpha\, cl(G,B)$.

(6)$IFS\alpha\, cl(IFS\alpha\, cl(F,A)) = IFS\alpha\, cl(F,A)$.

(7)$IFS\alpha\, cl(F,A)\, \widetilde{\cup}\, IFS\alpha\, cl(G,B)\, \widetilde{\subseteq}\, IFS\alpha\, cl((F,A)\, \widetilde{\cup}\, (G,B))$.

(8)$IFS\alpha\, cl((F,A) \tilde{\cap} (G,B)) \tilde{\subseteq} IFS\alpha\, cl(F,A) \tilde{\cap} IFS\alpha\, cl(G,B)$.

Proof: Obvious.■

Lemma **(3.6)**:

Let (X_E,τ) be an IFSTS. Then:(i) Every IFS open set in (X_E,τ) is $IFS\alpha$ open set.

(ii) Every IFS closed set in (X_E,τ) is $IFS\alpha$ closed set.

Proof:

(i) Let (F,A) be an IFS open set in $(X_E,\tau) \Rightarrow IFS\ int(F,A) = (F,A)$

Since $(F,A) \tilde{\subseteq} IFS\ cl(F,A) \Rightarrow IFS\ int(F,A) \tilde{\subseteq} IFS\ int(IFS\ cl(F,A))$

$\Rightarrow (F,A) \tilde{\subseteq} IFS\ int(IFS\ cl(IFS\ int(F,A))) \Rightarrow (F,A)$ is $IFS\alpha$ open set

(ii) Let (G,B) be an IFS closed set in $(X_E,\tau) \Rightarrow (G,B)^c$ is IFS open set

$\Rightarrow$ By (i), we have: $(G,B)^c$ is $IFS\alpha$ open set $\Rightarrow (G,B)$ is $IFS\alpha$ closed set.■

Theorm **(3.7)**:

Let (X_E,τ) be an IFSTS and $(F,A) \in IFS(X_E)$. Then:

$(i)[IFS\alpha\, int(F,A)]^c = IFS\alpha\, cl(F,A)^c$. $(ii)[IFS\alpha\, cl(F,A)]^c = IFS\alpha\, int(F,A)^c$.

Proof:

(i) Since $IFS\alpha\, int(F,A) = \tilde{\cup}\{(H,C): (H,C) \in IFS\alpha OS, (H,C) \tilde{\subseteq} (F,A)\} \Rightarrow [IFS\alpha\, int(F,A)]^c =$ $\tilde{\cap}\{(H,C)^c: (H,C)^c \in IFS\alpha CS, (F,A)^c \tilde{\subseteq} (H,C)^c\} = IFS\alpha\, cl(F,A)^c$

(ii) Similar as (i)■

Theorm **(3.8)**:

Let (X_E,τ) be an IFSTS and $(F,A) \in IFS(X_E)$. Then:

$(i)(F,A)$is IFS open set if and only if $IFS\ cl(IFS\ int(F,A)) = IFS\ cl(F,A)$.

(ii) If $(G,B) \in \tau$, then $IFS\ cl(F,A) \tilde{\cap} (G,B) \tilde{\subseteq} IFS\ cl[(F,A) \tilde{\cap} (G,B)]$.

Proof: It's clear. ■

Theorm **(3.9)**:

Let (X_E,τ) be an IFSTS, $(F,A) \in IFSOS(X_E)$ and $(G,B) \in IFS\alpha OS(X_E)$ Then, $(F,A) \tilde{\cap} (G,B) \in IFS\alpha OS(X_E)$.

Proof:

Let $(F,A) \in IFSOS(X_E)$ and $(G,B) \in IFS\alpha OS(X_E)$

$\Rightarrow (F,A) \tilde{\cap} (G,B) \tilde{\subseteq} IFS\ int(F,A) \tilde{\cap} IFS\ int(IFS\ cl(IFS\ int(G,B)))$

$= IFS\ int[(F,A) \tilde{\cap} IFS\ cl(IFS\ int(G,B))]$ (Theorem: 2.14(1))

$\tilde{\subseteq} IFS\ int[IFS\ cl((F,A) \tilde{\cap} IFS\ int(G,B))]$ (Theorem: 3.8 (ii))

$\tilde{\subseteq} IFS\ int[IFS\ cl(F,A) \tilde{\cap} IFS\ cl(IFS\ int(G,B))]$ (Theorem: 2.14(4))

$= IFS\ int[IFS\ cl(F,A)] \tilde{\cap} IFS\ int[IFS\ cl(IFS\ int(G,B))]$ (Theorem: 2.14(1))

$= IFS\ int[IFS\ cl(IFS\ int(F,A))] \tilde{\cap} IFS\ int[IFS\ cl(IFS\ int(G,B))]$(Theorem: 3.8 (i))

$= IFS\ int(IFS\ cl(IFS\ int[(F,A) \tilde{\cap} (G,B)])) \Rightarrow (F,A) \tilde{\cap} (G,B) \in IFS\alpha OS(X_E)$. ■

***Theorm* (3.10)**:

Let (X_E,τ) be an IFSTS, $(F,A) \in IFS(X_E)$. Then, $(F,A) \in IFS\alpha CS(X_E)$ if and only if $IFS\ cl(IFS\ int(IFS\ cl(F,A))) \tilde{\subseteq} (F,A)$.

Proof: It's clear. ■

***Corollary* (3.11)**:

Let (X_E,τ) be an IFSTS, $(F,A) \in IFS(X_E)$. Then, $(F,A) \in IFS\alpha CS(X_E)$ if and only if $(F,A) = (F,A) \tilde{\cup} IFS\ cl(IFS\ int(IFS\ cl(F,A)))$.

Proof: It's obvious from Theorem (3.10) . ■

4. *Intuitionistic Fuzzy Soft* α *Continuous Mappings*

In this section we define the intuitionistic fuzzy soft α continuous (open, closed and irresolute) mappings and we prove some results of them. We denote the intuitionistic fuzzy soft mapping (φ,ϕ) by Φ .

***Definition* (4.1)**:

Let $(X_E,\tau_1),(Y_D,\tau_2)$ be two IFSTSs. An IFS mapping $\Phi = (\varphi,\phi):(X_E,\tau_1) \rightarrow (Y_D,\tau_2)$ is called:

(1)$IFS\alpha$ continuous if $\Phi^{-1}((G,B)) \in IFS\alpha OS(X_E), \forall (G,B) \in \tau_2$.

(2)$IFS\alpha$ open if $\Phi((F,A)) \in IFS\alpha OS(Y_D), \forall (F,A) \in \tau_1$.

(3)$IFS\alpha$ closed if $\Phi((F,A)) \in IFS\alpha CS(Y_D), \forall (F,A) \in IFSCS(X_E)$.

(4)$IFS\alpha$ irresolute if $\Phi^{-1}((G,B)) \in IFS\alpha OS(X_E), \forall (G,B) \in IFS\alpha OS(Y_D)$.

***Theorm* (4.2):**

(1) Every IFS continuous map is $IFS\alpha$ continuous.

(2) Every IFS open map is $IFS\alpha$ open. (3) Every IFS closed map is $IFS\alpha$ closed.

***Proof*:** It's clear from Lemma (3.6). ■

***Theorem* (4.3):**

Let $(X_E,\tau_1),(Y_D,\tau_2)$ be two IFSTSs and $\Phi:(X_E,\tau_1)\to(Y_D,\tau_2)$ be an $IFS\alpha$ mapping . Then the following statements are equivalent: (1)Φ is $IFS\alpha$ continuous.

(2)$\Phi^{-1}((G,B))\in IFS\alpha CS(X_E),\forall(G,B)\in IFSCS(Y_D)$.

(3)$\Phi[IFS\alpha\, cl(F,A)]\,\widetilde{\subseteq}\, IFS\, cl[\Phi((F,A))],\forall(F,A)\in IFS(X_E)$.

(4)$IFS\alpha\, cl[\Phi^{-1}((G,B))]\,\widetilde{\subseteq}\,\Phi^{-1}[IFS\, cl(G,B)],\forall(G,B)\in IFS(Y_D)$.

(5)$\Phi^{-1}[IFS\, int(G,B)]\,\widetilde{\subseteq}\, IFS\alpha\, int[\Phi^{-1}((G,B))],\forall(G,B)\in IFS(Y_D)$.

***Proof*:**

$(1\Rightarrow 2)$ Let $(G,B)\in IFSCS(Y_D)\Rightarrow(G,B)^c\in IFSOS(Y_D)$ and $\Phi^{-1}((G,B)^c)\in IFS\alpha OS(X_E)$ (Definition: 4.1). Since $\Phi^{-1}((G,B)^c)=[\Phi^{-1}((G,B))]^c$ (Theorem: 2.7)

$\Rightarrow\Phi^{-1}((G,B))\in IFS\alpha CS(X_E)$.

$(2\Rightarrow 3)$ Let $(F,A)\in IFS(X_E)\Rightarrow$ from (2) and (Theorem: 2.7), we get:

$$(F,A)\,\widetilde{\subseteq}\,\Phi^{-1}[\Phi((F,A))]\,\widetilde{\subseteq}\,\Phi^{-1}[IFS\, cl\,\Phi((F,A))]\in IFS\alpha CS(X_E)$$

$$\Rightarrow(F,A)\,\widetilde{\subseteq}\, IFS\alpha\, cl\,(F,A)\,\widetilde{\subseteq}\,\Phi^{-1}[IFS\, cl\,\Phi((F,A))]$$

$$\Rightarrow\Phi[IFS\alpha\, cl\,(F,A)]\,\widetilde{\subseteq}\,\Phi[\Phi^{-1}(IFS\, cl\,\Phi((F,A)))]$$

$\Rightarrow$ from (Theorem: 2.7), we get: $\Phi[IFS\alpha\, cl\,(F,A)]\,\widetilde{\subseteq}\, IFS\, cl\,\Phi((F,A))$.

$(3\Rightarrow 4)$ Let $(G,B)\in IFS(Y_D)$ and $(F,A)=\Phi^{-1}((G,B))\Rightarrow$ from (3), we get:

$\Phi[IFS\alpha\, cl\,\Phi^{-1}((G,B))]\,\widetilde{\subseteq}\, IFS\, cl\,\Phi[\Phi^{-1}((G,B))]\Rightarrow$ by (Theorem: 2.7), we have:

$$IFS\alpha\, cl\,\Phi^{-1}((G,B))\,\widetilde{\subseteq}\,\Phi^{-1}[\Phi(IFS\alpha\, cl\,\Phi^{-1}((G,B)))]\,\widetilde{\subseteq}\,\Phi^{-1}[IFS\, cl\,\Phi\,(\Phi^{-1}((G,B)))]$$

$\widetilde{\subseteq}\,\Phi^{-1}(IFS\, cl(G,B))\Rightarrow IFS\alpha\, cl\,\Phi^{-1}((G,B))\,\widetilde{\subseteq}\,\Phi^{-1}(IFS\, cl(G,B))$.

$(4\Rightarrow 5)$ Let $(G,B)\in IFS(Y_D)\Rightarrow(G,B)^c\in IFS(Y_D)\Rightarrow$ from (4), we get: $IFS\alpha\, cl\,\Phi^{-1}((G,B)^c)\,\widetilde{\subseteq}\,\Phi^{-1}(IFS\, cl(G,B)^c)\Rightarrow$ by (Theorem: 2.7), (Theorem: 2.14) and (Theorem: 3.7), we have:$\Phi^{-1}(IFS\, int(G,B))\,\widetilde{\subseteq}\, IFS\alpha\, int\,\Phi^{-1}((G,B))$.

$(5\Rightarrow 1)$ Let $(G,B)\in IFSOS(Y_D)\Rightarrow IFS\, int(G,B)=(G,B)$ and$\Phi^{-1}(IFS\, int(G,B))=$

$\Phi^{-1}((G,B))\,\widetilde{\subseteq}\, IFS\alpha\, int\,\Phi^{-1}((G,B))$ (by 5)

Since $IFS\alpha\, int\,\Phi^{-1}((G,B))\,\widetilde{\subseteq}\,\Phi^{-1}((G,B))$ (Theorem: 3.4)

$\Rightarrow IFS\alpha\ int\ \Phi^{-1}((G,B)) = \Phi^{-1}((G,B)) \in IFS\alpha OS(X_E) \Rightarrow \Phi$ is $IFS\alpha$ continuous.■

***Theorem* (4.4):**

Let $(X_E,\tau_1),(Y_D,\tau_2)$ be two IFSTSs and $\Phi:(X_E,\tau_1) \to (Y_D,\tau_2)$ be an $IFS\alpha$ mapping.Then, Φ is an $IFS\alpha$ open if and only if $\Phi[IFS\ int(F,A)] \,\tilde{\subseteq}\, IFS\alpha\ int[\Phi((F,A))], \forall (F,A) \in IFS(X_E)$.

***Proof*:**

$(\Rightarrow)$ Let Φ be an $IFS\alpha$ open map and $(F,A) \in IFS(X_E) \Rightarrow IFS\ int(F,A) \in \tau_1$ and $\Phi[IFS\ int(F,A)] \in IFS\alpha OS(Y_D)$ (Definition:4.1) $\Rightarrow \Phi[IFS\ int(F,A)] = IFS\alpha\ int[\Phi(IFS\ int(F,A))] \,\tilde{\subseteq}\, IFS\alpha\ int[\Phi((F,A))]$

$(\Leftarrow)$ Let $(F,A) \in \tau_1$. By the condition we get:

$\Phi[IFS\ int(F,A)] = \Phi((F,A)) \,\tilde{\subseteq}\, IFS\alpha\ int[\Phi((F,A))] \in IFS\alpha OS(Y_D)$

Since $IFS\alpha\ int[\Phi((F,A))] \,\tilde{\subseteq}\, \Phi((F,A))$ $\Rightarrow$ $IFS\alpha\ int[\Phi((F,A))] = \Phi((F,A)) \in IFS\alpha OS(Y_D), \forall\ (F,A) \in \tau_1 \Rightarrow \Phi$ is $IFS\alpha$ open map. ■

***Theorem* (4.5):**

Let $(X_E,\tau_1),(Y_D,\tau_2)$ be two IFSTSs and $\Phi:(X_E,\tau_1) \to (Y_D,\tau_2)$ be an $IFS\alpha$ mapping.Then, Φ is an $IFS\alpha$ closed if and only if $IFS\alpha\ cl[\Phi((F,A))] \,\tilde{\subseteq}\, \Phi[IFS\ cl(F,A)], \forall (F,A) \in IFS(X_E)$.

***Proof*:** Similar as Theorem (4.4). ■

5. *Intuitionistic Fuzzy Soft* α *Separation Axioms*

In this section we define the intuitionistic fuzzy soft αT_i spaces, $i = 1,2,3,4$and we introduce some of its basic properties.

***Definition* (5.1):**

An IFSTS (X_E,τ) is said to be $IFS\alpha\, T_0$ space if for every pair of distinct IFS points e_S, e_W there exists an $IFS\alpha$ open set (F,A) such that:$e_S \in (F,A), e_W \notin (F,A)$ or $e_W \in (F,A), e_S \notin (F,A)$.

***Example*(5.2):**

Let $X = \{x_1,x_2,x_3,x_4\}, E = \{e_1,e_2,e_3\}$ and τ be the discrete intuitionistic fuzzy soft topology on X. Then(X_E,τ) is $IFS\alpha\, T_0$ space.

***Theorem* (5.3):**

An IFS subspace (Y_E,τ_Y) of an $IFS\alpha\, T_0$ space(X_E,τ) is $IFS\alpha\, T_0$ space.

Proof:

Let e_S, e_W be two distinct IFS points in (Y_E, τ_Y). Since (Y_E, τ_Y) be a subspace of $(X_E, \tau) \Rightarrow e_S, e_W$ are two distinct IFS points in (X_E, τ)

Since (X_E, τ) be $IFS\alpha\, T_0$ space $\Rightarrow \exists IFS\alpha$ open set (F, A) in τ such that: $e_S \in (F, A), e_W \notin (F, A)$ or $e_W \in (F, A), e_S \notin (F, A) \Rightarrow (H, C) = Y_E \tilde{\cap} (F, A), \forall (F, A) \in \tau$ is $IFS\alpha$ open set in τ_Y such that: $e_S \in (H, C), e_W \notin (H, C)$ or $e_W \in (H, C), e_S \notin (H, C)$

Hence (Y_E, τ_Y) is $IFS\alpha\, T_0$ space. ■

***Definition* (5.4)**:

An IFSTS (X_E, τ) is said to be $IFS\alpha\, T_1$ space if for every pair of distinct IFS points e_S, e_W there exist an $IFS\alpha$ open sets $(F, A), (G, B)$such that:$e_S \in (F, A), e_W \notin (F, A)$ and $e_W \in (G, B), e_S \notin (G, B)$.

***Example* (5.5)**:

Let $X = \{x_1, x_2, x_3\}$, $E = \{e_1, e_2, e_3\}$ and τ be the discrete intuitionistic fuzzy soft topology on X. Then(X_E, τ) is $IFS\alpha\, T_1$ space.

***Theorem* (5.6)**:

An IFS subspace (Y_E, τ_Y) of an $IFS\alpha\, T_1$ space(X_E, τ) is $IFS\alpha\, T_1$ space.

Proof: Similar as Theorem (5.3). ■

***Theorem* (5.7)**:

If every IFS point of an IFSTS (X_E, τ) is $IFS\alpha$ closed set, then (X_E, τ) is $IFS\alpha\, T_1$ space.

Proof:

Let e_S, e_W be two distinct IFS points of $(X_E, \tau) \Rightarrow e_S, e_W$ are $IFS\alpha$ closed sets $\Rightarrow e_S^c, e_W^c$are distinct $IFS\alpha$ open sets such that: $e_S \in e_W^c, e_W \notin e_W^c$ and $e_W \in e_S^c, e_S \notin e_S^c \Rightarrow (X_E, \tau)$ is $IFS\alpha\, T_1$ space. ■

***Definition* (5.8)**:

An IFSTS (X_E, τ) is said to be $IFS\alpha\, T_2$ space if for every pair of distinct IFS points e_S, e_W there exist disjoint $IFS\alpha$ open sets $(F, A), (G, B)$ such that: $e_S \in (F, A)$ and $e_W \in (G, B)$.

Example **(5.9):**

Let $X = \{x_1, x_2, x_3, x_4\}$, $E = \{e_1, e_2, e_3\}$ and τ be the discrete intuitionistic fuzzy soft topology on X. Then(X_E, τ) is $IFS\alpha\, T_2$ space.

Theorem **(5.10):**

An IFS subspace (Y_E, τ_Y) of an $IFS\alpha\, T_2$ space(X_E, τ) is $IFS\alpha\, T_2$ space.

***Proof*:**Similar as Theorem (5.3). ■

Theorem **(5.11):**

If every IFS point of an IFSTS (X_E, τ) is $IFS\alpha$ closed set, then (X_E, τ) is $IFS\alpha\, T_2$ space.

***Proof*:** Similar as Theorem (5.7). ■

Proposition **(5.12):**

(1) Every $IFS\alpha T_2$ space is $IFS\alpha T_1$ space.

(2) Every $IFS\alpha T_1$ space is $IFS\alpha T_0$ space.

(3) Every $IFS\alpha T_2$ space is $IFS\alpha T_0$ space.

***Proof*:**

(1) Let (X_E, τ)be an $IFS\alpha T_2$ space and e_S, e_W be two distinct IFS points $\Rightarrow$ $\exists$ disjoint $IFS\alpha$ open sets $(F, A), (G, B)$ such that: $e_S \in (F, A)$ and $e_W \in (G, B)$. Since $(F, A) \,\tilde{\cap}\, (G, B) = \tilde{0}_E \Rightarrow e_S \notin (G, B)$ and $e_W \notin (F, A)$ $\Rightarrow$ $e_S \in (F, A), e_W \notin (F, A)$ and $e_W \in (G, B), e_S \notin (G, B)$. Thus, $(X_E, \tau)$is $IFS\alpha T_1$ space.

(2) Let (X_E, τ)be an $IFS\alpha T_1$ space and e_S, e_W be two distinct IFS points $\Rightarrow \exists IFS\alpha$ open sets $(F, A), (G, B)$ such that: $e_S \in (F, A), e_W \notin (F, A)$ and $e_W \in (G, B), e_S \notin (G, B)$. Then, there exists an $IFS\alpha$ open set containing one of the IFS point but not the other.
Thus, $(X_E, \tau)$is $IFS\alpha T_0$ space.

(3) Let (X_E, τ)be an $IFS\alpha T_2$ space$\Rightarrow$ By (1), we have: (X_E, τ) is $IFS\alpha T_1$ space.
$\Rightarrow$ From (2), we get: (X_E, τ) is$IFS\alpha T_0$ space. ■

Theorem **(5.13):**

For every pair of distinct IFS points e_S, e_W of an $IFS\alpha T_2$ space (X_E, τ), there exists an $IFS\alpha$ closed set (H, C) such that: $e_S \in (H, C), e_W \notin (H, C)$ and $e_W \notin IFS\alpha\, cl(H, C)$.

***Proof*:**

Let e_S, e_W be two distinct IFS points of an $IFS\alpha T_2$ space(X_E, τ)

$\Rightarrow \exists$ disjoint $IFS\alpha$ open sets $(F, A), (G, B)$ such that: $e_S \in (F, A)$ and $e_W \in (G, B) \Rightarrow e_S \in (G, B)^c$ and $e_W \notin (G, B)^c \Rightarrow (G, B)^c = (H, C)$ is $IFS\alpha$ closed set containing e_S but not e_W and $e_W \notin IFS\alpha\, cl(H, C) = (H, C)$. ■

***Definition* (5.14):**

Let (X_E, τ) be an IFSTS, (H, C) be an $IFS\alpha$ closed set and e_S be an IFS point such that $e_S \notin (H, C)$. If there exist disjoint $IFS\alpha$ open sets $(F, A), (G, B)$ such that: $e_S \in (F, A)$ and $(H, C) \widetilde{\subseteq} (G, B)$, then (X_E, τ) is called $IFS\alpha$ regular space. An IFSTS (X_E, τ) is said to be $IFS\alpha T_3$ space if it is $IFS\alpha$ regular and $IFS\alpha T_1$ space.

***Theorem* (5.15):**

Let (X_E, τ) be an $IFS\alpha$ regular space, (H, C) be an $IFS\alpha$ open set and e_S be an IFS point such that $e_S \in (H, C)$, then there exists an $IFS\alpha$ open set (F, A) such that: $e_S \in (F, A)$ and $IFS\alpha\, cl(F, A) \widetilde{\subseteq} (H, C)$.

***Proof*:**

Let (H, C) be an $IFS\alpha$ open set containing IFS point e_S in $IFS\alpha$ regular space$(X_E, \tau) \Rightarrow (H, C)^c$ is $IFS\alpha$ closed set such that $e_S \notin (H, C)^c$. By hypothesis, there exist disjoint $IFS\alpha$ open sets $(F, A), (G, B)$ such that: $e_S \in (F, A)$ and $(H, C)^c \widetilde{\subseteq} (G, B) \Rightarrow (G, B)^c \widetilde{\subseteq} (H, C)$ and $(F, A) \widetilde{\subseteq} (G, B)^c \Rightarrow IFS\alpha\, cl(F, A) \widetilde{\subseteq} (G, B)^c \widetilde{\subseteq} (H, C)$ $\Rightarrow$we get an $IFS\alpha$ open set (F, A) containing e_S and $IFS\alpha\, cl(F, A) \widetilde{\subseteq} (H, C)$■

***Theorem* (5.16):**

An IFS subspace (Y_E, τ_Y) of an $IFS\alpha\, T_3$ space(X_E, τ) is $IFS\alpha\, T_3$ space.

***Proof*:**

By Theorem (5.6), we get: (Y_E, τ_Y) is $IFS\alpha\, T_1$ space.

To prove that (Y_E, τ_Y) is $IFS\alpha$ regular space let (H, C) be an $IFS\alpha$ closed set and e_S be an IFS point in (Y_E, τ_Y) such that $e_S \notin (H, C)$. Since (X_E, τ) be an $IFS\alpha\, T_3$ space $\Rightarrow (X_E, \tau)$ is $IFS\alpha$ regular space $\Rightarrow$ there exist disjoint $IFS\alpha$ open sets $(F, A), (G, B)$ in τ such that: $e_S \in (F, A)$ and $(H, C) \widetilde{\subseteq} (G, B) \Rightarrow Y_E \widetilde{\cap} (F, A), Y_E \widetilde{\cap} (G, B)$ are disjoint $IFS\alpha$ open sets in τ_Y such that:

$e_S \in Y_E \widetilde{\cap} (F, A)$ and $(H, C) \widetilde{\subseteq} Y_E \widetilde{\cap} (G, B) \Rightarrow (Y_E, \tau_Y)$ is $IFS\alpha$ regular space. Therefore (Y_E, τ_Y) is $IFS\alpha\, T_3$ space. ■

***Definition* (5.17):**

Let (X_E,τ) be an IFSTS and $(H,C),(K,D)$ be disjoint $IFS\alpha$ closed sets. If there exist disjoint $IFS\alpha$ open sets $(F,A),(G,B)$ such that: $(H,C)\tilde{\subseteq}(F,A)$ and $(K,D)\tilde{\subseteq}(G,B)$, then (X_E,τ) is called $IFS\alpha$ normal space. An IFSTS (X_E,τ) is said to be $IFS\alpha T_4$ space if it is $IFS\alpha$ normal and $IFS\alpha T_1$ space.

***Theorem* (5.18):**

An IFSTS (X_E,τ) is $IFS\alpha$ normal space if and only if for every $IFS\alpha$ closed set (H,C) and $IFS\alpha$ open set (K,D) such that $(H,C)\tilde{\subseteq}(K,D)$, there exists $IFS\alpha$ open set (F,A) such that $(H,C)\tilde{\subseteq}(F,A)$ and $IFS\alpha\, cl(F,A)\tilde{\subseteq}(K,D)$.

***Proof*:**

$(\Rightarrow)$ Suppose that (X_E,τ) be an $IFS\alpha$ normal space, (H,C) be an $IFS\alpha$ closed set and (K,D) be an $IFS\alpha$ open set such that $(H,C)\tilde{\subseteq}(K,D)\Rightarrow(H,C),(K,D)^c$ are disjoint $IFS\alpha$ closed sets. By hypothesis, there exist disjoint $IFS\alpha$ open sets $(F,A),(G,B)$ such that: $(H,C)\tilde{\subseteq}(F,A)$ and $(K,D)^c\tilde{\subseteq}(G,B)$. Since $(F,A)\tilde{\subseteq}(G,B)^c\Rightarrow IFS\alpha\, cl(F,A)\tilde{\subseteq}IFS\alpha\, cl(G,B)^c=(G,B)^c$. Since$(G,B)^c\tilde{\subseteq}(K,D)\Rightarrow IFS\alpha\, cl(F,A)\tilde{\subseteq}(K,D)$.

$(\Leftarrow)$ Suppose that the condition holds and $(J,S),(L,W)$ be disjoint $IFS\alpha$ closed sets $\Rightarrow(J,S)\tilde{\subseteq}(L,W)^c$. By condition, there exists $IFS\alpha$ open set (F,A) such that $(J,S)\tilde{\subseteq}(F,A)$ and $IFS\alpha\, cl(F,A)\tilde{\subseteq}(L,W)^c\Rightarrow(L,W)\tilde{\subseteq}[IFS\alpha\, cl(F,A)]^c$ and $[IFS\alpha\, cl(F,A)]^c\tilde{\cap}(F,A)=\tilde{0}_E$, where (F,A) and $[IFS\alpha\, cl(F,A)]^c$ are $IFS\alpha$ open sets$\Rightarrow(X_E,\tau)$ is $IFS\alpha$ normal space. ■

***Theorem* (5.19):**

An $IFS\alpha$ closed subspace (Y_E,τ_Y) of an $IFS\alpha$ normal space(X_E,τ) is $IFS\alpha$ normal space.

***Proof*:**

Let $(H,C),(K,D)$ be disjoint $IFS\alpha$ closed sets in (Y_E,τ_Y)

$\Rightarrow Y_E\tilde{\cap}(H,C)$ and $Y_E\tilde{\cap}(K,D)$ are $IFS\alpha$ closed sets in (X_E,τ)

Since (X_E,τ) be an $IFS\alpha$ normal space $\Rightarrow$ there exist disjoint $IFS\alpha$ open sets$(F,A),(G,B)$ in τ such that: $Y_E\tilde{\cap}(H,C)\tilde{\subseteq}(F,A)$ and $Y_E\tilde{\cap}(K,D)\tilde{\subseteq}(G,B)\Rightarrow Y_E\tilde{\cap}(H,C)\tilde{\subseteq}Y_E\tilde{\cap}(F,A)$ and $Y_E\tilde{\cap}(K,D)\tilde{\subseteq}Y_E\tilde{\cap}(G,B)$, for some disjoint $IFS\alpha$ open sets$Y_E\tilde{\cap}(F,A),Y_E\tilde{\cap}(G,B)$ in τ_Y . Thus (Y_E,τ_Y) is $IFS\alpha$ normal space.■

***References*:**

[1] A. M. Abd El-Latif and Rodyna A. Hosny; *"On Soft separation Axioms via Fuzzy* α −*Open Sets"*, Inf. Sci. Lett. 5, N.1, 1-9 (2016).

[2] K. Atanassov; *"Intuitionistic Fuzzy Sets"*, Fuzzy Sets and Systems. V.20, 87-96 (1986).

[3] S. Bayramov and C. Gunduz(Aras); *"On Intuitionistic Fuzzy Soft Topological Spaces"*, J. Pure Appl. Math. V.5, N.1, 66-79 (2014).

[4] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh and A. M. Abd El-latif; *"Soft Semi Separation Axioms and Irresolute Soft Functions"*, Ann. Fuzzy Math. Inform. 8 (2) 305-318 (2014).

[5] P. K.Maji, R. Biswas and A. R. Roy; *"Fuzzy Soft Sets"*, Journal of Fuzzy Mathematics, 9 (3) 589-602 (2001).

[6] P. K.Maji, R. Biswas and A. R. Roy, *"Intuitionistic Fuzzy Soft Sets"*, Journal of Fuzzy Mathematics, 9 (3) 677-691 (2001).

[7] D. A.Molodtsov; *"Soft Set Theory-firs tresults"*, Comput.Math. Appl. 37, 19-31 (1999).

[8] I. Osmanoglu and D. Tokat; *"On Intuitionistic Fuzzy Topology"*, Gen. Math. Notes, V.19, N.2, 59-70 (2013).

[9] B. Pazarvarol and H. Aygun; *"Fuzzy Soft Topology"*, HacettepeJournal of Mathematics and Statistics, 41 (3), 407-419, (2012).

[10] M. Shabir and M. Naz; *"On Soft Topological Spaces"*, Comput. Math. Appl. 61, 1786-1799 (2011).

[11] N. Turanli and A.H. Es, *"A Note On Compactness in Intuitionistic Fuzzy Soft Topological Spaces"*, World Applied Sciences Journals, 19(9), 1355-1359 (2012).

[12] Y. Yin, H. Li and Y.B. Jun, *"On Algebraic Structure of Intuitionistic Fuzzy Soft Sets"*, Comput. Math. Appl. 64, 2896-2911, (2012).

[13] L.A. Zadeh ; *"Fuzzy Sets Information and Control"*. Vol. 8, 338- 353. (1965).

Assessment of Monthly Surface Air Temperature in Iraq Using General Circulation Model.

Ahmed S. Hassan, Jasim Hameed Kadhum, Ali H. Hashem.

Department of Atmospheric Sciences, College of Sciences,

Al- Mustansiriyah University.

Abstract