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Numerical Analysis of Free-Convection from an Inclined Isothermal Heated Plate

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Abstract

In this work, Boundary layer flow of laminar free-convection heat transfer along an inclined flat plate with constant face temperature is numerically analyzed and solved . The unsteady, two-dimensional , non-linear and coupled partial differential boundary layer equations of continuity, momentum, and energy governing the flow are transformed into a set of non-dimensional equations and solved by using Crank-Nicolson method (CNM) of finite difference. The results of dimensionless velocity and temperature, and local Nusselt number (Nu_x) are shown graphically and discussed. The influence of thermal stratification parameter (S), Prandtl number (Pr), and inclination angle of the plate (ϕ) on the dimensionless velocity and temperature profiles on the flow and heat transfer properties have been plotted and discussed . Results of the present work are compared with previously theoretical work . It can be seen good agreement .

Keywords: Crank-Nicolson Method (CNM) , Inclined , Free-Convection , Isothermal .

التحليل العددي للحمل الحر من صفيحة مائلة مسخنة و متساوية في درجة الحرارة

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الخلاصة :

في هذا البحث ، أجريت دراسة عددية لانتقال الحرارة بالحمل الحر الطباقي من صفيحة مستوية مائلة ذات سطح متساوي في درجة الحرارة . تم تحويل المعادلات التفاضلية الجزئية الحاكمة للجريان الثنائي الأبعاد (معادلات الاستمرارية والعزوم والطاقة) الى معادلات لا بعدية وحلها عدديا باستخدام طريقة كرانك- نيكلسون للفروق المحددة .

تم تمثيل وتوضيح النتائج المستحصلة للسرعة ودرجة الحرارة ورقم نيسلت الموضعي (Nu_x) بيانيا . تم دراسة تأثير كل من معامل التدرج الحراري (S) و رقم برانتل (Pr) و زاوية ميل الصفيحة المعدنية (ϕ) على منحنيات السرعة و درجة الحرارة وعلى خصائص الجريان و الأنتقال الحراري . تم مقارنة نتائج البحث الحالي مع بحوث نظرية سابقة وكان التطابق جيد .

Nomenclature

Symbol	Meaning
A	Area **(m^2)**
g	Gravitational acceleration **(m/s^2)**
Gr	Grashof number
h_x	Local convection heat-transfer coefficient **($W/m^2.K$)**
k	Thermal conductivity **(W/m.K)**
L	Length **(m)**
Nu_x	Local Nusselt number
Pr	Prandtl number
q	Heat transfer rate **(W)**
q''	Heat flux **(W/m^2)**
S	Thermal stratification parameter
T	Non-dimensional temperature
t	Non-dimensional time
T'	Temperature **(K)**
t'	Time **(sec)**
U , V	Non-dimensional velocity components in x- and y-directions
u ,v	Velocity components in x- and y-directions **(m/s)**
X	Non-dimensional coordinate along the plate
x	Coordinate along the plate **(m)**
Y	Non-dimensional horizontal space coordinate
y	Horizontal space coordinate **(m)**
Greek Letters :	
α	Thermal diffusivity **(m^2/s)**
β	Volumetric coefficient of thermal expansion **(1/K)**
φ	Angle of inclination of the plate with vertical **(deg)**
υ	Kinematic viscosity **(m^2/s)**
Subscript Symbols :	
∞	Free stream condition
∞ , x	Location away from the wall at any x
∞ , **0**	Location away from the wall at x= 0
w	Conditions on the wall

1. Introduction

Free-convection heat transfer frequently occurs in natural environment , many physical problems and engineering applications such as solar energy collectors , petroleum reservoir operations , drying processes , nuclear and chemical industries , packed beds , compact heat exchangers , geothermal energy conversion , etc.

In recent years , considerable attention has been devoted to the investigate of transient natural convection heat transfer from a different geometries (plane or coaxial bodies like cylinder and cone) , but very little attention has been given for the problem of free-convection heat transfer from an inclined plate .

Yang et al. (1972) [1] investigated natural convection heat transfer from a non-isothermal vertical flat plate immersed in a thermal stratified medium for a wide range of Prandtl number . **Cheng and Minkowycz (1977)** [2] investigated free convection about a vertical flat plate embedded in a porous medium with application to heat transfer from a dike . **Kulkarni et al. (1987)** [3] studied the problem of natural convection flow over an isothermal vertical wall immersed in thermally stratified medium using the Von-Karaman-Pohlhausen integral method . **Grosan and Pop (2001)** [4] studied a boundary layer analysis for the free convection flow over a vertical flat plate embedded in a porous medium by a power-law non-Newtonian fluid. **Takhar et al. (2002)** [5] used an implicit finite difference scheme as a numerical solution for the problem of natural convection on a vertical cylinder embedded in a thermally stratified high-porosity medium . **Ebrahim (2005)** [6] used alternating direction implicit method (ADI) to solution and investigated the problem of transient laminar natural convection heat transfer from a wall to a thermally linear stratified media (water and air) . **Palani (2008)** [7] numerically studied the convection effects on flow past an inclined plate in water at (4 ^{o}C). **Saha et al. (2011)** [8] discussed a theoretical investigation of natural convection from a vertical plate with uniform heat source and obtained a numerical solution using implicit finite difference method for a wide range of the stratification parameter . Other researches dealt the problem numerically using another methods of the numerical solution such as **Hussain and Ali (2012)** [9] presented a numerical simulation for the studied free convection from a vertical cone using Crank-Nicolson finite difference approach , and **Pullepu and Immanuel (2012)** [10] studied the transient free convection over a vertical cone in a thermally stratified medium using implicit finite-difference scheme of Crank-Nicolson .

The present work focuses on influence of some parameters like Prandtl number (Pr), thermal stratification parameter (S), and inclination angle of the heated plate (ϕ) on laminar free-convection from an inclined isothermal heated plate. The implicit finite difference of Crank-Nicolson method (CNM) used for a numerical solution .

2. Mathematical Analysis

2.1 The Governing Equations

The physical model and the coordinate system of the case study are given in **Figure. (1),** the x-axis measures the distance along the plate and the y-axis is perpendicular to the plate . The inclination angle of the plate with the vertical is assumed to be (ϕ) . Consider two dimensional transient laminar free-convection flow of a viscous and incompressible fluid along an inclined heated flat plate with constant temperature surface placed in a thermally stratified environment . It is assumed that the viscous dissipation effects are negligible. The pressure gradient along the boundary layer is neglected . The properties of fluid are assumed to be constant except in the buoyancy force term consideration of the momentum equation. The governing boundary layer equations of laminar free-convection heat transfer under the mentioned assumptions and the Boussinesq approximation can be written as follows [11,12,13] :-

Continuity Equation :

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad \text{………………….. (1)}$$

Momentum Equation in x-direction :

$$\frac{\partial u}{\partial t'}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=n\frac{\partial^2 u}{\partial y^2}+gb\cos j\,(T'-T'_{\infty,x}) \qquad \text{…………………..(2)}$$

Energy Equation :

$$\frac{\partial T'}{\partial t'}+u\frac{\partial T'}{\partial x}+v\frac{\partial T'}{\partial y}=a\frac{\partial^2 T'}{\partial y^2} \qquad \text{………………….. (3)}$$

The initial and boundary conditions are :-

$t' \le 0$: u= v = 0 $\quad T' = T'_\infty$ $\quad$ for all x and y ………………….. **(4a)**

$t' > 0$: u= v = 0 $\quad T' = T'_w$ $\quad$ at y= 0 , $x \ge 0$ ………………….. **(4b)**
u= 0 $\quad T' = T'_\infty$ $\quad$ at x= 0
$u \to 0$ $\quad T' \to T'_\infty$ $\quad$ as $y \to \infty$

The local Nusselt number (Nu_x) at the plate is computed by [11 , 12,13] :

$$Nu_x=\frac{h_x x}{k} \qquad \text{………………….. (5)}$$

and , the local heat flux q'' (rate of heat-transfer per unit area) is [11 , 12,13] :

$$q''=\frac{q}{A}=-k\left(\frac{\partial T'}{\partial y}\right)_{y=0}=h_x(T'_W-T'_{\infty,x}) \qquad \text{…………………..6)}$$

$$\therefore h_x = \frac{-k(\partial T' / \partial y)_{y=0}}{(T'_W - T'_{\infty,x})} \qquad \text{………………..} \ \mathbf{(7)}$$

By substituting equation (7) in equation (5) :

$$Nu_x = \frac{x}{(T'_W - T'_{\infty,x})}\left(-\frac{\partial T'}{\partial y}\right)_{y=0} \qquad \text{………………..} \ \mathbf{(8)}$$

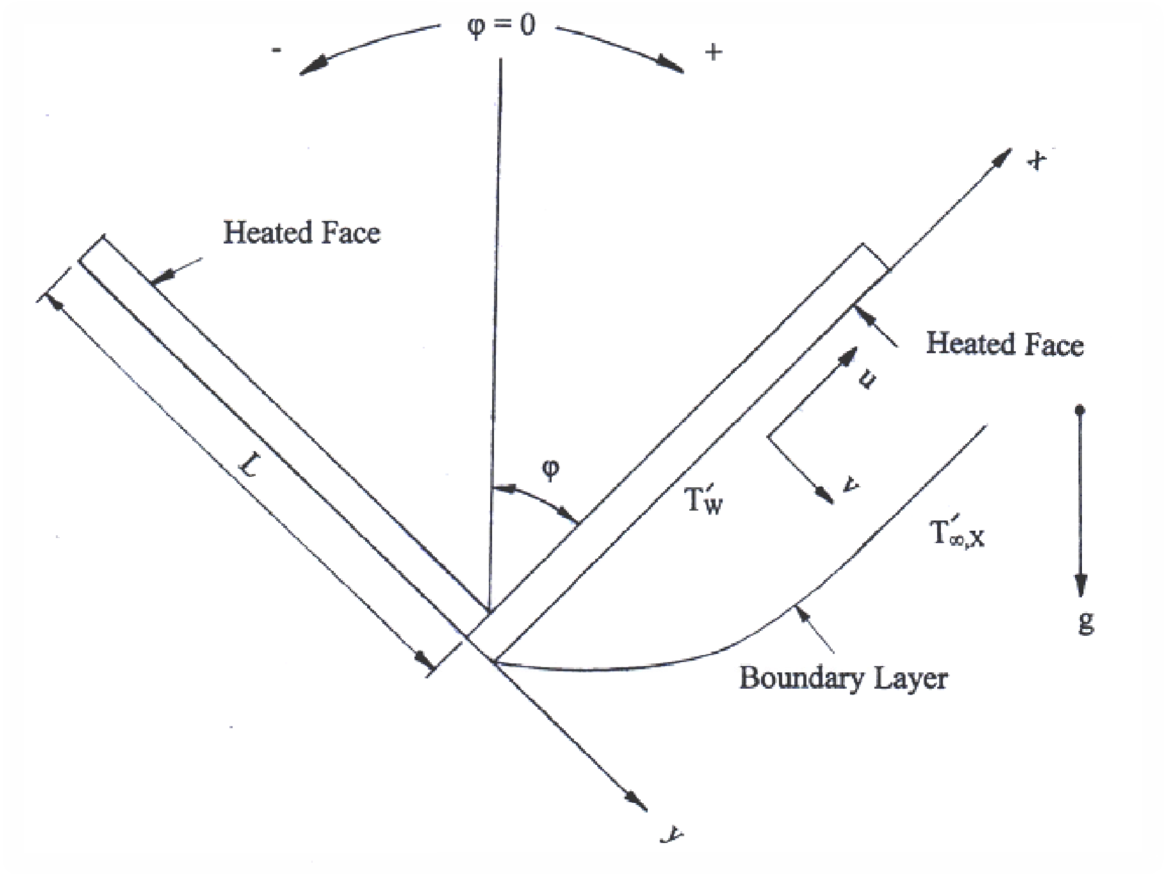


Fig .(1) Physical Model and Coordinate System

2.2 Non-Dimensional Expressions of the Governing Equations

Introducing the following non-dimensional quantities to substituting in the governing equations :-

$X = x/L$, $Y = (y/L)\,Gr^{1/4}$, $U = (uL/\nu)\,Gr^{-1/2}$, $V = (vL/\nu)\,Gr^{-1/4}$,

$t = \nu\, t'\, Gr^{1/2} / L^2$, $Pr = \nu/\alpha$, $Gr = g\,\beta\cos\phi\,\Delta T_O\, L^3/\nu^2$,

$\Delta T_O = T'_w - T'_{\infty,o}$, $\Delta T_X = T' - T'_{\infty,x}$, $T = \Delta T_X / \Delta T_O$,

$S = (dT'_{\infty,x} / dX) / \Delta T_O$

Rewrite the governing boundary layer equations (1) , (2) , and (3) in non-dimensional form :

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \qquad \text{………………..(9)}$$

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = \frac{\partial^2 U}{\partial Y^2} + T \qquad \text{……………….. (10)}$$

$$\frac{\partial T}{\partial t} + U\frac{\partial T}{\partial X} + V\frac{\partial T}{\partial Y} = \frac{1}{\Pr}\left(\frac{\partial^2 T}{\partial Y^2}\right) - SU \qquad \text{. ……………….. (11)}$$

The corresponding initial and boundary conditions in non-dimensional form are :

$t \le 0$: U=V=0 T=0 for all X and Y ……………….. **(12a)**

$t > 0$: U=V=0 $T=T_W$ at Y=0 , $X \ge 0$ ……………….. **(12b)**

U=0 T=0 at X=0
$U \rightarrow 0$ $T \rightarrow 0$ as $Y \rightarrow \infty$

The local Nusselt number (Nu_X) in non-dimensional form is :

$$Nu_X = -\left(\frac{X}{T_W}\right)\left(\frac{\partial T}{\partial Y}\right)_{Y=0} \qquad \text{……………….. (13)}$$

3. Numerical Analysis of the Governing Equations

An implicit finite difference scheme of Crank-Nicolson method (CNM) has been used to solve the governing dimensionless equations (9 , 10 , and 11) with the initial and boundary conditions (12a and 12b) for various values of the Prandtl number (Pr) , thermal stratification parameter (S) , and inclination angle of the plate (ϕ) . Replaced the partial derivatives in the governing dimensionless equations by using finite-difference approximations , the first order derivative in the time is approximated using a forward-difference , and the central-difference approximation being used in the space derivatives . The finite difference method of non-dimensional governing equations is reduced to tri-diagonal system of equations.

The entire domain is subdivided into a mesh system , size (m x n) , with uniform divisions ΔX and ΔY, in the X and Y directions respectively. The grid point (X , Y , t) are given by ($i\Delta X$) , ($j\Delta Y$) , and ($p\Delta t$) as shown in **Figure. (2).** The region is considered as a rectangle with (X_{max}= 1.0) ,and (Y_{max}=10) where Y_{max} corresponds to ($Y= \infty$) which lies far away outside the momentum and thermal boundary layers . The grid sizes are taken as ΔX= 0.02 , ΔY= 0.2 , and time step Δt= 0.01 . The system of discretized equations are solved numerically starting from initial time , and using small time step (Δt= 0.01) that allows for the stability of the solution . Computations are repeated until the steady state is reached. Steady state solution is assumed to have been reached when the absolute difference between the values of velocity (U) as well as temperature (T) at two respectively time steps is less than (10^{-5}) . The Crank-Nicolson method (CNM) is unconditionally stable and convergent **[6,9,14]** .

A computer program was built by using Fortran 90 language to solve the mathematical model of the governing boundary layer equations and to compute the local Nusselt number (Nu_x).

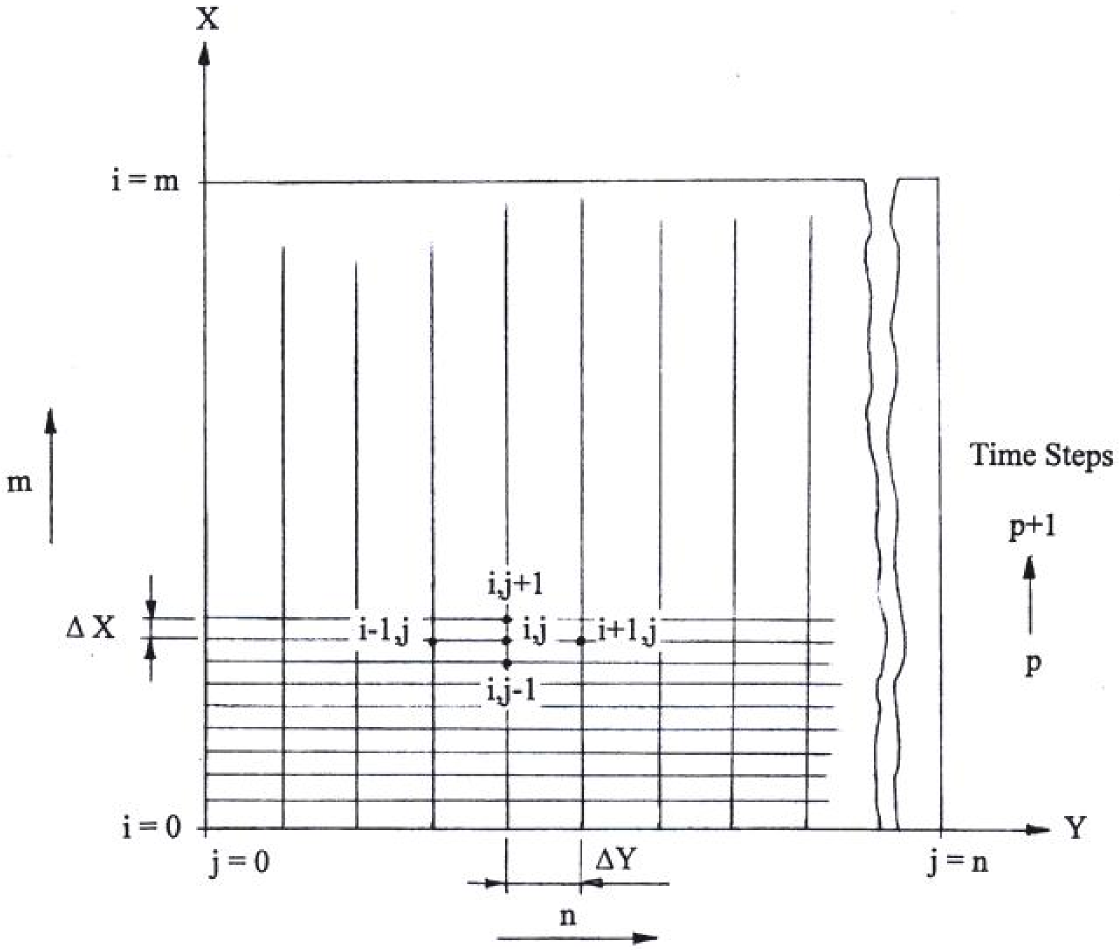
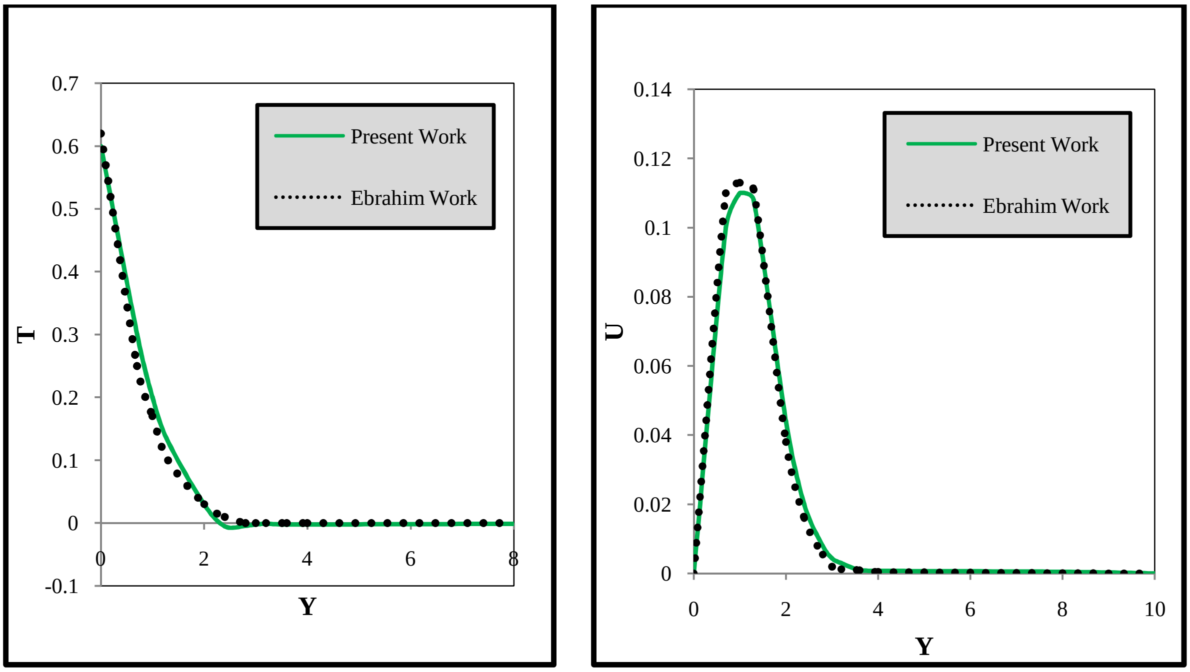


Fig .(2) Crank-Nicolson Computational Molecule and Mesh System

4. Results and Discussion

results for a previous investigations. The velocity and temperature profiles when Pr= 6.0 , S= 0.4 , and Gr= 10^6 at X= 1.0 are compared with **Ebrahim (2005)** [6] , as shown in **Figure. (3)** and are found to be in good agreement .



a. Steady State Temperature Profile **b. Steady State Velocity Profile**

Fig .(3) Comparison the Results of Present Work with Ebrahim Work [6] When Pr= 6.0 , S= 0.4 , and Gr= 10^6 at X= 1.0

Figures.(4) to (9) show the velocity and temperature profiles at (X=1.0) for different values of the Prandtl number **(Pr= 0.7 , 1.0 , 5.4 , 7.0)** , thermal stratification parameter **(S= 0.2 , 0.4 , 0.6)** , and inclination angle of the plate **(φ =30° , 15° , -15° , -30°)** . All solutions were carried out at Grashof number **(Gr= 10^6)** . In Figs. (4) and (5), velocity and temperature profiles are plotted for various values of Prandtl number (Pr) when stratification parameter (S=0.4), the increase of Prandtl number (Pr) causes a reduction in the velocity and temperature profiles because the thermal boundary layer and momentum boundary layer thickness reduces with increasing Prandtl number (Pr) . The negative values of dimensionless temperature (T) appears as wings profile with increasing Prandtl number (Pr) as shown in **Figure. (5).**

Figures. (6) and (7), show that an increase in the thermal stratification parameter (S) causes a reduction in the thermal boundary layer thickness . Hence , the velocity and temperature and their gradients reduce with increasing thermal stratification parameter (S) . Profiles of the velocity and temperature appear as wings in the outer region because temperature defect in the flow . **Figure. (8)** illustrates the influence of the inclination angle (φ) on velocity profile for stratification parameter (S=0.4) , and it is observed that in spite of the influence of thermal stratification the velocity profile will be less effected by the inclination angle of the plate (φ) . **Figure. (9)** illustrates the effect of the inclination angle (φ) on temperature profile for (S=0.4) , where it is observed that in spite of the influence of thermal stratification the temperature profile will be less effected by the inclination angle of the plate (φ).

Figures. (10) and (11), illustrate the effect of the stratification parameter (S) and angle of inclination (φ) on the local Nusselt number (Nu_x). Fig. (10) , shows the decreases in local Nusselt number (Nu_x) with the stratification parameter (S) because the local Nusselt number (Nu_x) dependence on the temperature profile which decreased with increase in thermal stratification parameter (S) as shown in **Figure (7).** Local Nusselt number (Nu_x) decreases with the inclination angle of plate (φ) as shown in **Figure. (11)** .

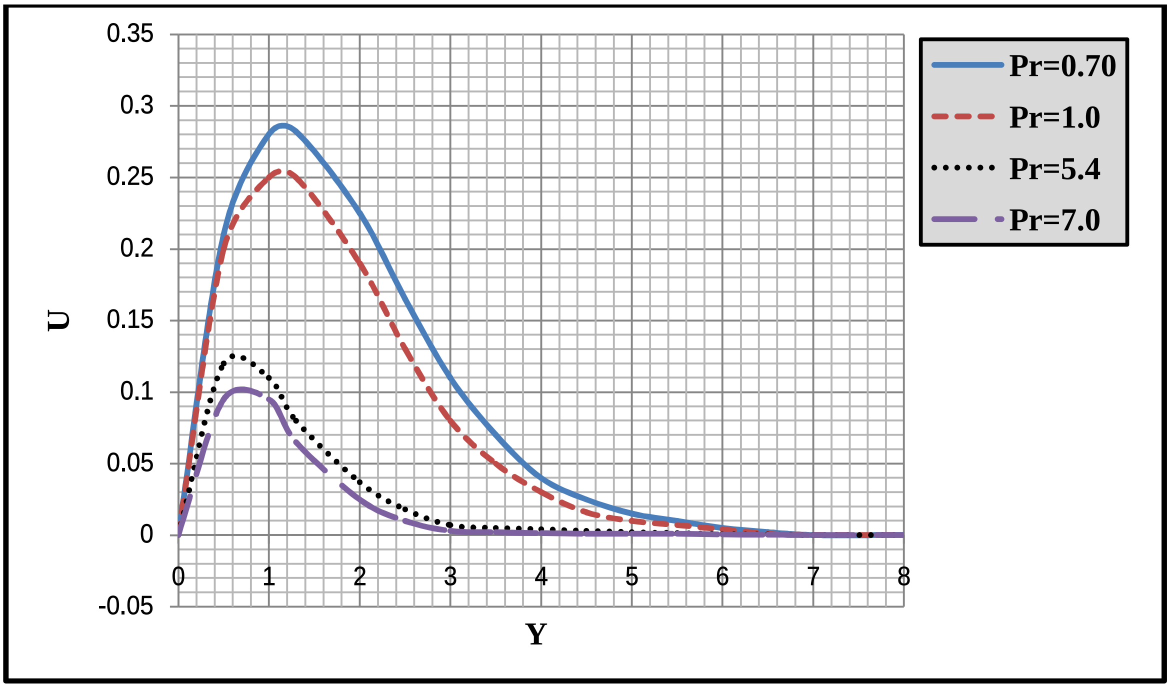


Fig .(4) Velocity Profiles at X= 1.0 for Different Values of Prandtl Number (Pr) and Stratification Parameter (S=0.4)

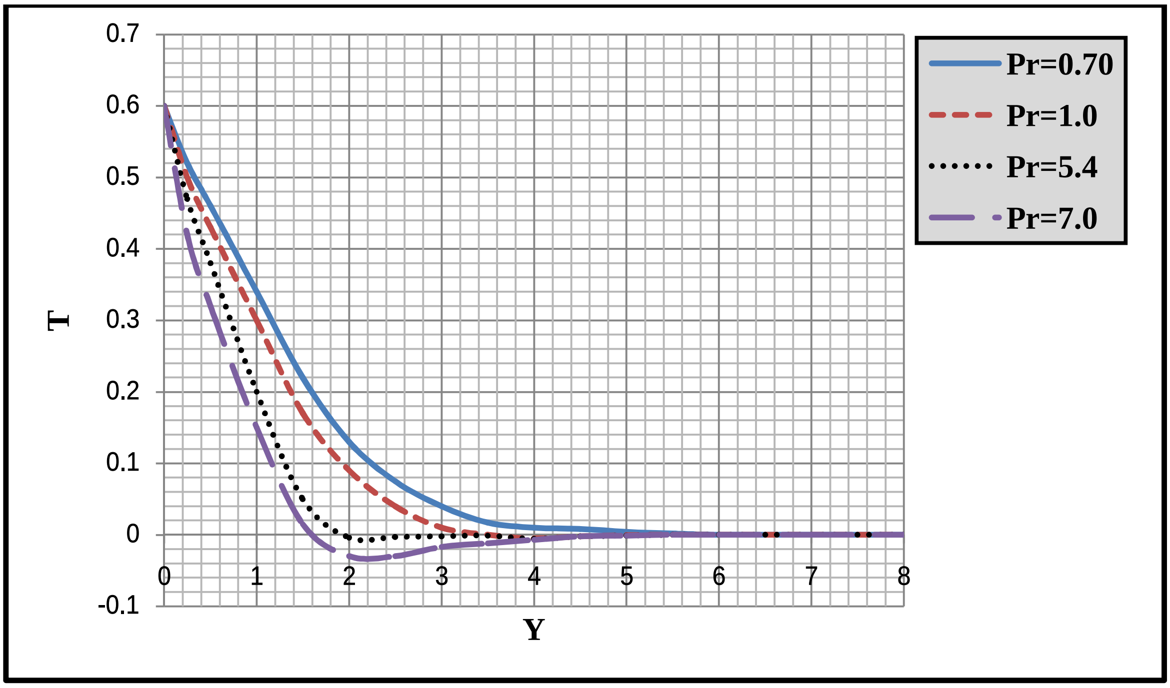


Fig .(5) Temperature Profiles at X= 1.0 for Different Values of Prandtl Number (Pr) and Stratification Parameter (S=0.4)

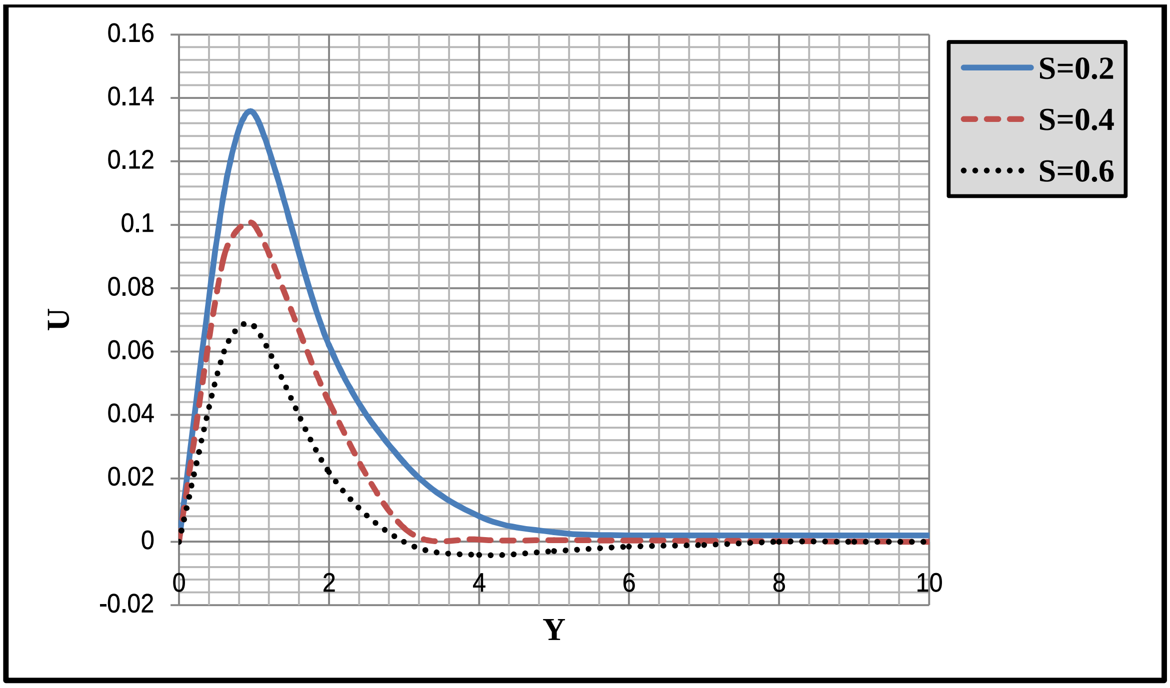


Fig .(6) Velocity Profiles at X= 1.0 for Different Values of Stratification Parameter (S) and Prandtl Number (Pr=7.0)

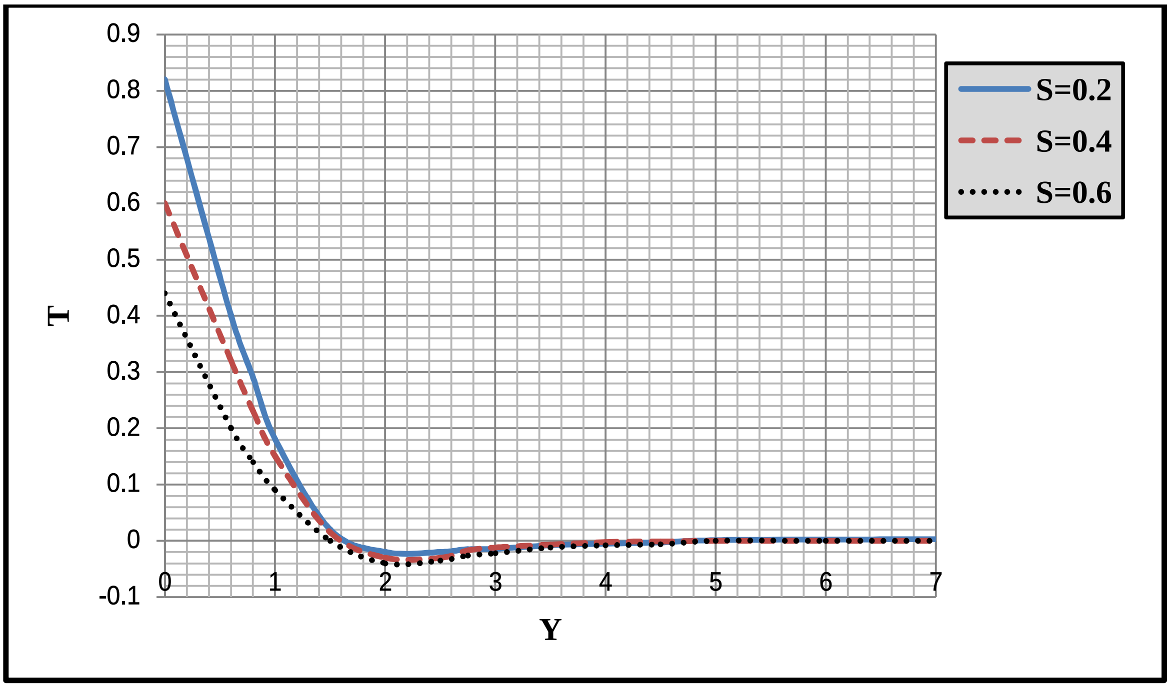


Fig .(7) Temperature Profiles at X= 1.0 for Different Values of Stratification Parameter (S) and Prandtl Number (Pr=7.0)

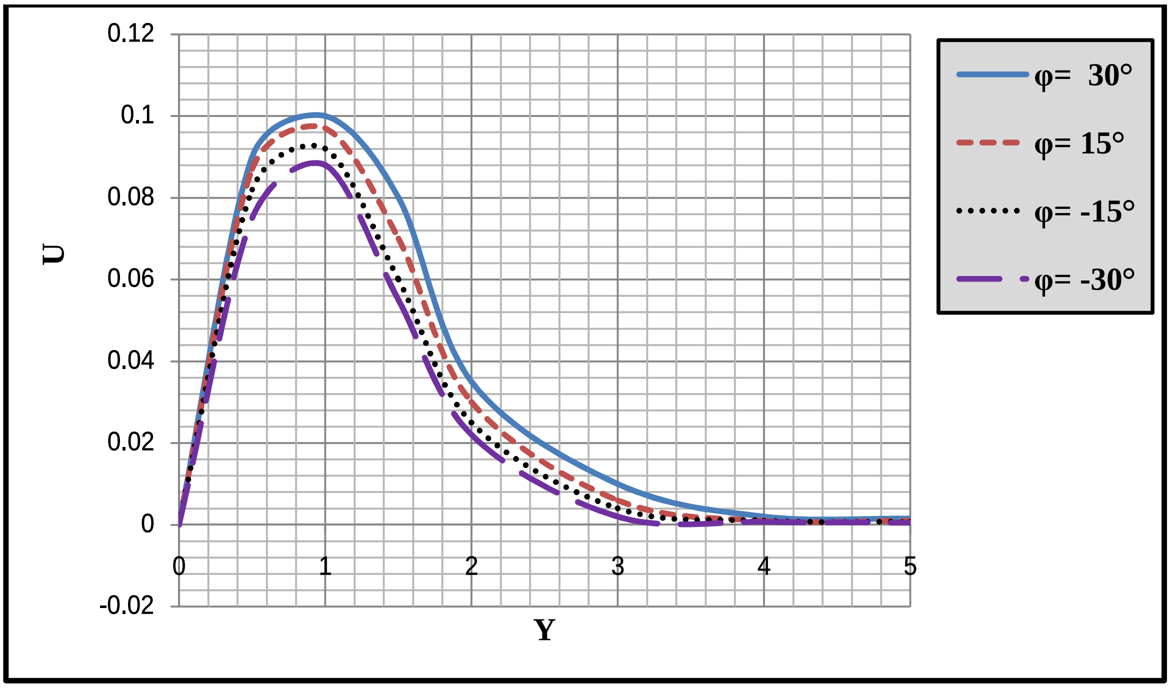


Fig .(8) Steady State Velocity Profiles at X= 1.0 for Different Values of an Inclined Angle (φ) , Prandtl Number (Pr=7.0) , and Stratification Parameter (S= 0.4)

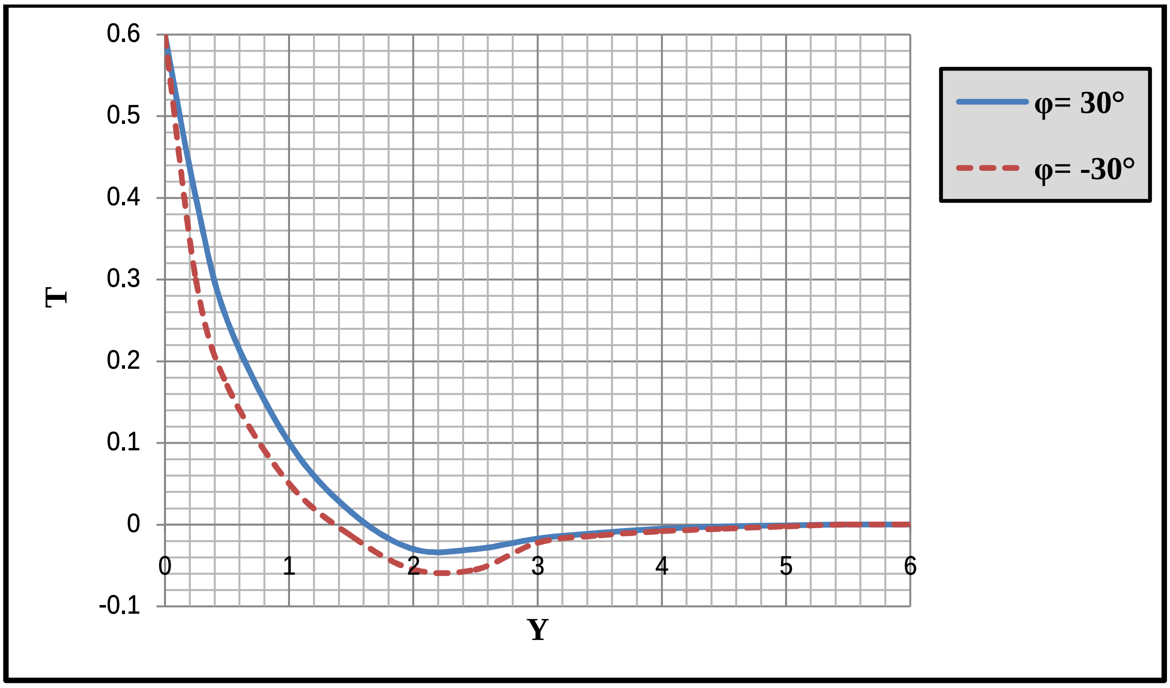


Fig .(9) Steady State Temperature Profiles at X= 1.0 for Different Values of an Inclined Angle (φ) , Prandtl Number (Pr=7.0) , and Stratification Parameter (S= 0.4)

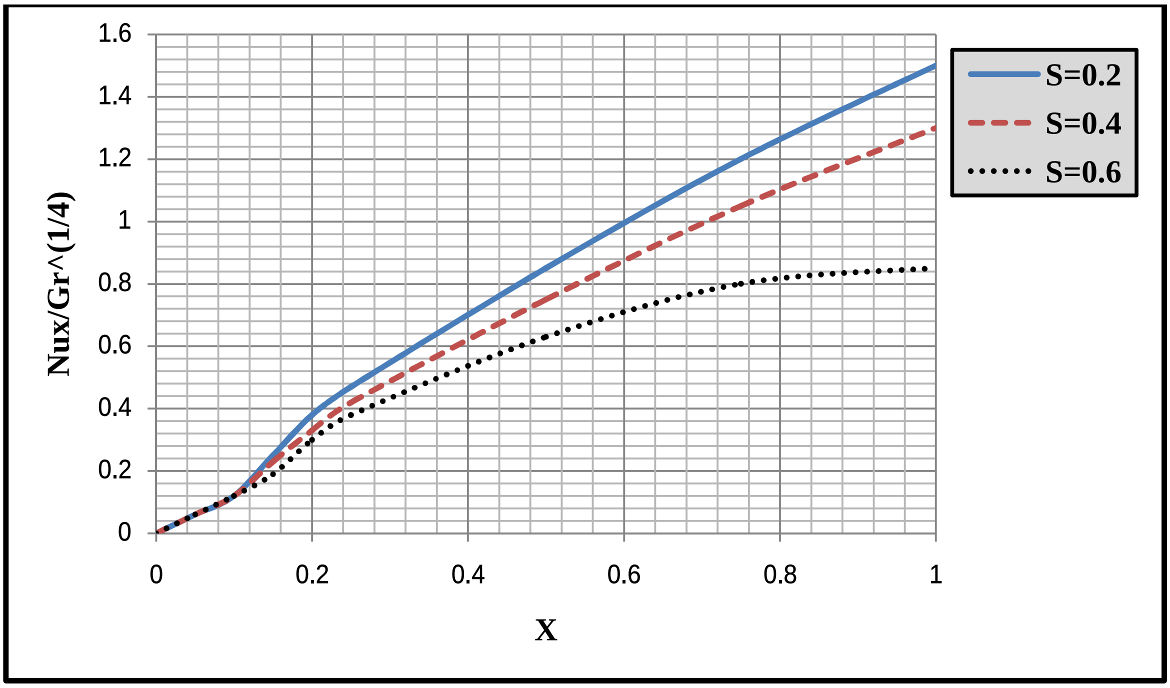


Fig .(10) Local Nusselt Number (Nu_X) for Different Values of a Stratification Parameter (S) Prandtl Number (Pr=7.0)

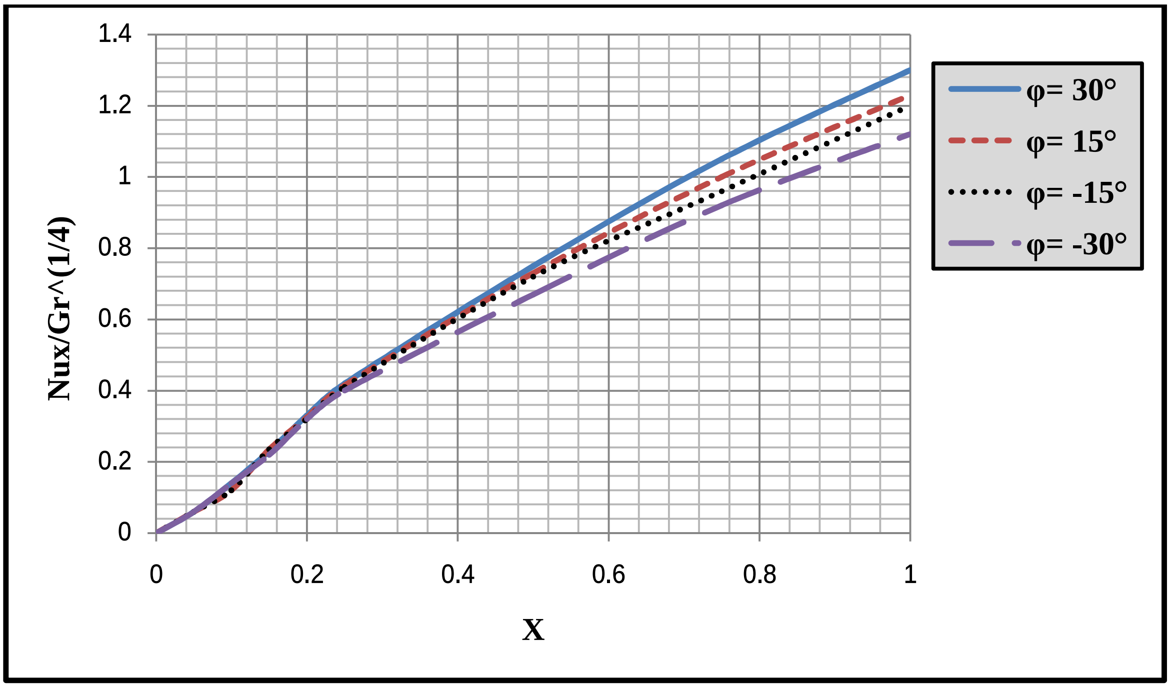


Fig .(11) Local Nusselt Number (Nu_X) for Different Values of an Inclined Angle (φ) , Prandtl Number (Pr=7.0) , and Stratification Parameter (S= 0.4)

5. Conclusions

The velocity and temperature profiles reduce when the thermal stratification parameter (S) and Prandtl number (Pr) are increased . The peak velocity decreases with increasing of the thermal stratification parameter (S) because of the decrease in the buoyancy force . The momentum boundary layers become thinner when thermal stratification parameter (S) and Prandtl number (Pr) are increased while the thermal boundary layer becomes thicker when values of the Prandtl number (Pr) are decreased .

The negative values of non-dimensional temperature (T) appears as wings profile with increasing Prandtl number (at Pr= 7.0) and thermal stratification parameter (S). While the negative values of non-dimensional velocity (U) appears as wings profile with increasing stratification parameter (at S= 0.6).

The local Nusselt number (Nu_X) decreases when the thermal stratification parameter (S) increases , and inclination angle of the plate (φ) decreases .

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