Al-Mustansiriyah Journal of Science
ISSN: 1814-635X (print), ISSN:2521-3520 (online) Volume 33, Issue 4, 2022 DOI: http://doi.org/10.23851/mjs.v33i4.1163

Research Article Open Access

Geometric Properties of Meromorphic Functions Involving Convolution Operator

Ismael I. Hameed*, Abdul Rahman S. Juma

University of Anbar, College of Education for Pure Sciences, Department of Mathematics, Anbar, IRAQ

*Correspondent contact: ism20u2004@uoanbar.edu.iq

Article Info

Received 01/06/2022

Accepted 26/06/2022

Published 30/12/2022

ABSTRACT

We introduce and study a subclass of meromorphic univalent functions with positive coefficients defined by a novel operator $I_{\rho,m}^{\varsigma}f(w)$ and obtain coefficient estimates, closure theorems, convolution properties, partial sums, and δ- neighborhood for the class $H_{\rho,m}^{\varsigma}(\varsigma,\rho,m,V,C,d)$.

KEYWORDS: Meromorphic functions; coefficient estimates; partial sums; starlike functions; convex functions.

الخلاصة

قدمنا وندرسنا فئة فرعية من الدوال أحادية الشكل ذات المعاملات الموجبة التي يحددها مؤثر جديد $I_{\rho,m}^{\varsigma}f(z)$ والحصول على تقديرات المعامل، ونظريات الإغلاق، وخصائص الالتفاف، والمجموعات الجزئية، والمجاورة للفصل $H_{\rho,m}^{\varsigma}(\varsigma,\rho,m,V,C,d)$.

INTRODUCTION

Let $\sum$ denote by class of meromorphic functions of analytic in unit disk $U^{*}=\{w: w\in C, 0<|w|<1\}=U\backslash\{0\}$ of the form

$$f(w)=w^{-1}+\sum_{\kappa=1}^{\infty}a_{\kappa}w^{\kappa}, a_{\kappa}\geq 0 \qquad (1)$$

We denote by $\sum_{S}(\sigma)$, $\sum_{K}(\sigma)$, $0\leq\sigma<1$, the subclass of $\sum$ that are meromorphic univalent meromorphically, meromorphically starlike functions of order σ and meromorphically convex functions of order σ, respectively.

A function $f\in\sum_{K}(\sigma)$ if and only if

$$\Re\left\{-\left(1+\frac{wf''(w)}{f'(w)}\right)\right\}>\sigma, w\in U^{*} \qquad (2)$$

Similarly, a function $f\in\sum_{S}(\sigma)$ if and only if

$$\Re\left\{\frac{-wf'(w)}{f(w)}\right\}>\varepsilon,\in U^{*},(0\leq\sigma<1) \qquad (3)$$

There are many other classes of meromorphically univalent functions that has been extensively studied (see [1, 2, 3, 4, 5, 6, 7]).

For functions $f(w)=w^{-1}+\sum_{\kappa=1}^{\infty}a_{\kappa}w^{\kappa}$ and $g(w)=w^{-1}+\sum_{\kappa=1}^{\infty}b_{\kappa}w^{\kappa}$, we define the Hadamard product of f and g is given by

$$(f*g)(w)=w^{-1}+\sum_{\kappa=1}^{\infty}a_{\kappa}b_{\kappa}w^{\kappa}, a_{\kappa}b_{\kappa}\geq 0 \qquad (4)$$

Let ς,ρ be positive real numbers. Motivated by the Salagean operator [8]. We consider the linear operator $H_{\rho}^{\varsigma}(w):\sum\rightarrow\sum$ defined by [9]

$$H_{\rho}^{\varsigma}(w)=w^{-1}+\sum_{\kappa=1}^{\infty}\left(\frac{\kappa+\rho+1}{\rho}\right)^{\varsigma}w^{\kappa} \qquad (5)$$

We think that a linear operator $Q_{m}f(w):\sum\rightarrow\sum$ which defined by the following Hadamard product (or convolution):

$$Q_{m}f(w)=\phi_{m}(w)*f(w)$$

where $\phi_{m}(w)=w^{-2}Li_{m}(w)$[10] and

$$Li_m(w) = \sum_{k=1}^{\infty} \frac{w^k}{k^m}, m \geq 2.$$

$$Q_m f(w) = w^{-1} + \sum_{\kappa=0}^{\infty} \frac{1}{(\kappa+2)^m} a_\kappa w^\kappa.$$

Next, we define the linear operator $T_m f(w): \sum \to \sum$ as follows [10]:

$$T_m f(w) = \left\{ Q_m f(w) - \frac{1}{2^m} a_0 \right\}$$

$$w^{-1} + \sum_{\kappa=1}^{\infty} \frac{1}{(\kappa+2)^m} a_\kappa w^\kappa, (m \in N). (6)$$

We consider operator $I_{\rho,m}^{\varsigma} f(w): \sum \to \sum$ which defined by the following Hadamard product of operator $H_\rho^\varsigma(w)$ and the operator $T_m f(w)$

$$I_{\rho,m}^{\varsigma} f(w) = H_\rho^\varsigma(w) * T_m f(w)$$

$$I_{\rho,m}^{\varsigma} f(w) = w^{-1} + \sum_{\kappa=1}^{\infty} \frac{(\kappa+\rho+1)^\varsigma}{\rho^\varsigma (\kappa+2)^m} a_k w^k, (7)$$

For ς, ρ be positive real numbers and $(m \in N)$.

Definition 1.1. [11]. Let $H_{\rho,m}^{\varsigma}(\varsigma, \rho, m, V, C, d)$ denote a Subclass of $\sum$ consisting functions of the form (1) that satisfy the requirement

$$1 - \frac{1}{d} \left\{ \frac{w \left(I_{\rho,m}^{\varsigma} f(w) \right)'}{I_{\rho,m}^{\varsigma} f(w)} + 1 \right\} \prec \frac{1 + Cw}{1 + Vw},$$

or, equivalently, to:

$$\left| \frac{\frac{w \left(I_{\rho,m}^{\varsigma} f(w) \right)'}{I_{\rho,m}^{\varsigma} f(w)} + 1}{V \frac{w \left(I_{\rho,m}^{\varsigma} f(w) \right)'}{I_{\rho,m}^{\varsigma} f(w)} + [(C-V)d + V]} \right| < 1, (8)$$

For $-1 \leq V < C \leq 1, d \in C/\{0\}, m \in N$ and ς, ρ be positive real numbers.

Coefficient Estimates

We obtain necessary and sufficient condition for a function $f(w)$ for the class $H_{\rho,m}^{\varsigma}(\varsigma, \rho, m, V, C, d)$.

Theorem 2.1. Let $f \in \sum$ by given by (1). Then $f \in H_{\rho,m}^{\varsigma}(\varsigma, \rho, m, V, C, d)$ if

$$\sum_{\kappa=1}^{\infty} \frac{[2\kappa + 1 + (C-V)d + V](\kappa+\rho+1)^\varsigma}{\rho^\varsigma (\kappa+2)^m} |a_\kappa| \leq (C-V)|d| (9)$$

Proof. If $f \in H_{\rho,m}^{\varsigma}(\varsigma, \rho, m, V, C, d)$, then by (8) we get

$$\left| \frac{\frac{w \left(I_{\rho,m}^{\varsigma} f(w) \right)'}{I_{\rho,m}^{\varsigma} f(w)} + 1}{V \frac{w \left(I_{\rho,m}^{\varsigma} f(w) \right)'}{I_{\rho,m}^{\varsigma} f(w)} + [(C-V)d + V]} \right| < 1$$

$$\left| \frac{w \left(I_{\rho,m}^{\varsigma} f(w) \right)' + I_{\rho,m}^{\varsigma} f(w)}{Vw \left(I_{\rho,m}^{\varsigma} f(w) \right)' + [(C-V)d + V] \left(I_{\rho,m}^{\varsigma} f(w) \right)} \right| < 1$$

$$\left| \frac{w \left(w^{-1} + \sum_{\kappa=1}^{\infty} \frac{(\kappa+\rho+1)^\varsigma}{\rho^\varsigma (\kappa+2)^m} a_k w^k \right)' + w^{-1} + \sum_{\kappa=1}^{\infty} \frac{(\kappa+\rho+1)^\varsigma}{\rho^\varsigma (\kappa+2)^m} a_k w^k}{Vw \left(w^{-1} + \sum_{\kappa=1}^{\infty} \frac{(\kappa+\rho+1)^\varsigma}{\rho^\varsigma (\kappa+2)^m} a_k w^k \right)' + [(C-V)d + V] \left(w^{-1} + \sum_{k=1}^{\infty} \frac{(\kappa+\rho+1)^\varsigma}{\rho^\varsigma (\kappa+2)^m} a_k w^k \right)} \right| < 1$$

$$\left| \frac{\sum_{\kappa=1}^{\infty} \frac{(k+\rho+1)^\varsigma}{\rho^\varsigma (\kappa+2)^m} (\kappa+1) a_\kappa w^\kappa}{(C-V) d w^{-1} + [(C-V)d + V + k] \sum_{\kappa=1}^{\infty} \frac{(\kappa+\rho+1)^\varsigma}{\rho^\varsigma (\kappa+2)^m} a_\kappa w^\kappa} \right| < 1$$

$$\sum_{k=1}^{\infty} \frac{[2\kappa + 1 + (C-V)d + V](\kappa+\rho+1)^\varsigma}{\rho^\varsigma (\kappa+2)^m} |a_\kappa| |w^\kappa| \leq (C-V) d |w|^{-1},$$

when w⟶ 1, we obtain

$$\sum_{\kappa=1}^{\infty} \frac{[2\kappa + 1 + (C-V)d + V](\kappa+\rho+1)^\varsigma}{\rho^\varsigma (\kappa+2)^m} |a_\kappa| \leq (C-V)|d|,$$

then

$$\sum_{\kappa=1}^{\infty} \frac{[2\kappa + 1 + (C-V)d + V](\kappa+\rho+1)^\varsigma}{\rho^\varsigma (\kappa+2)^m} |a_\kappa| \leq (C-V)|d|.$$

Thus, $f(w) \in H_{\rho,m}^{\varsigma}(\varsigma, \rho, m, V, C, d)$.

Corollary 2.2. If $f \in H_{\rho,m}^{\varsigma}(\varsigma, \rho, m, V, C, d)$, then

$$|a_k| \leq \frac{\rho^\varsigma (k+2)^m (C-V)|d|}{[2k + 1 + (C-V)d + V](k+\rho+1)^\varsigma}. (10)$$

The result is sharp for the function

$$f(w) = w^{-1} + \frac{\rho^\varsigma (k+2)^m (C-V)|d|}{[2k + 1 + (C-V)d + V](k+\rho+1)^\varsigma} w^k. (11)$$

Closure Theorems

ISSN: 1814-635X (print), ISSN:2521-3520 (online) Volume 33, Issue 4, 2022 DOI: http://doi.org/10.23851/mjs.v33i4.1163

The function f_j be defined, for $j = 1,2,3,\cdots,n$, by

$$f_j(w) = w^{-1} + \sum_{k=1}^{\infty}|a_{k,j}|w^k, a_{k,j} \geq 0. \quad (12)$$

Theorem 3.1. Let the function $f_j(w) = w^{-1} + \sum_{k=1}^{\infty}|a_{k,j}|w^k$, be in the class $H_{\rho,m}^{\varsigma}(\varsigma,\rho,m,V,C,d)$. Then the function h defined by

$$h(w) = w^{-1} + \sum_{k=1}^{\infty}\left(\frac{1}{n}\sum_{j=1}^{\infty}|a_{k,j}|\right)w^k, \quad (13)$$

also belongs to the class $H_{\rho,m}^{\varsigma}(\varsigma,\rho,m,V,C,d)$.

Proof. Since $f_j \in H_{\rho,m}^{\varsigma}(\varsigma,\rho,m,V,C,d)$, it follows from Theorem 2.1, that

$$\sum_{k=1}^{\infty}\frac{[2k+1+(C-V)d+V](k+\rho+1)^{\varsigma}}{\rho^{\varsigma}(k+2)^m}|a_k| \leq (C-V)|d|,$$

For every $j = 1,2,3,\cdots,n$. Hence

$$\sum_{k=1}^{\infty}\frac{[2k+1+(C-V)d+V](k+\rho+1)^{\varsigma}}{\rho^{\varsigma}(k+2)^m}\left(\frac{1}{n}\sum_{j=1}^{\infty}|a_{k,j}|\right)$$

$$\frac{1}{n}\sum_{j=1}^{\infty}\left(\sum_{k=1}^{\infty}\frac{[2k+1+(C-V)d+V](k+\rho+1)^{\varsigma}}{\rho^{\varsigma}(k+2)^m}|a_{k,j}|\right) \leq (C-V)|d|.$$

By Theorem 2.1, it follows that $h(w) \in H_{\rho,m}^{\varsigma}(\varsigma,\rho,m,V,C,d)$.

Theorem 3.2. The class $H_{\rho,m}^{\varsigma}(\varsigma,\rho,m,V,C,d)$ is closed under convex linear combinations.

Proof. Assume

$$f_j(w) = w^{-1} + \sum_{k=1}^{\infty}|a_{k,j}|w^k, j = 1,2,3,\cdots,n,$$

are in class $H_{\rho,m}^{\varsigma}(\varsigma,\rho,m,V,C,d)$. Then demonstrating that the function exists is sufficient.

$$h(w) = \zeta f_1(w) + (1-\zeta)f_2(w), 0 \leq \zeta < 1,$$

is in the class $H_{\rho,m}^{\varsigma}(\varsigma,\rho,m,V,C,d)$. Since for $0 \leq \zeta < 1$,

$$h(w) = w^{-1} + \sum_{k=1}^{\infty}\left[\zeta|a_{k,1}| + (1-\zeta)|a_{k,2}|\right]w^k.$$

In view of Theorem 2.1, we have:

$$\sum_{\kappa=1}^{\infty}\frac{[2\kappa+1+(C-V)d+V](\kappa+\rho+1)^{\varsigma}}{\rho^{\varsigma}(\kappa+2)^m}[\zeta|a_{\kappa,1}| + (1-\zeta)|a_{\kappa,2}|]$$

$$\zeta\sum_{\kappa=1}^{\infty}\frac{[2\kappa+1+(C-V)d+V](\kappa+\rho+1)^{\varsigma}}{\rho^{\varsigma}(\kappa+2)^m}|a_{\kappa,1}| + (1-\zeta)\sum_{\kappa=1}^{\infty}\frac{[2\kappa+1+(C-V)d+V](\kappa+\rho+1)^{\varsigma}}{\rho^{\varsigma}(\kappa+2)^m}|a_{\kappa,2}|$$

$$\leq \zeta(C-V)|d| + (1-\zeta)(C-V)|d| = (C-V)|d|,$$

which implies that $h(w) \in H_{\rho,m}^{\varsigma}(\varsigma,\rho,m,V,C,d)$.

Convolution Properties

Theorem 4.1. If the functions $f(w)$ and $g(w)$are in the class $H_{\rho,m}^{\varsigma}(\varsigma,\rho,m,V,C,d)$, then

$$(f*g)(w) = w^{-1} + \sum_{\kappa=1}^{\infty}a_\kappa b_\kappa w^\kappa,$$

is in the class $H_{\rho,m}^{\varsigma}(\varsigma,\rho,m,V,C,d)$.

Proof. Suppose $f(w)$ and $g(w)$ are in the class $H_{\rho,m}^{\varsigma}(\varsigma,\rho,m,V,C,d)$. By Theorem 2.1, we have

$$\sum_{\kappa=1}^{\infty}\frac{[2\kappa+1+(C-V)d+V](\kappa+\rho+1)^{\varsigma}}{\rho^{\varsigma}(\kappa+2)^m(C-V)|d|}|a_\kappa| \leq 1,$$

and

$$\sum_{\kappa=1}^{\infty}\frac{[2\kappa+1+(C-V)d+V](\kappa+\rho+1)^{\varsigma}}{\rho^{\varsigma}(\kappa+2)^m(C-V)|d|}|b_\kappa| \leq 1.$$

So is $(f*g)(w)$. Furthermore,

$$\sum_{\kappa=1}^{\infty}\frac{[2\kappa+1+(C-V)d+V](\kappa+\rho+1)^{\varsigma}}{\rho^{\varsigma}(\kappa+2)^m(C-V)|d|}|a_\kappa||b_\kappa|$$

$$\leq \sum_{\kappa=1}^{\infty}\left(\frac{[2\kappa+1+(C-V)d+V](\kappa+\rho+1)^{\varsigma}}{\rho^{\varsigma}(\kappa+2)^m(C-V)|d|}\right)^2|a_\kappa||b_\kappa|$$

$$\leq \left(\sum_{\kappa=1}^{\infty}\frac{[2\kappa+1+(C-V)d+V](\kappa+\rho+1)^{\varsigma}}{\rho^{\varsigma}(\kappa+2)^m(C-V)|d|}|a_\kappa|\right)$$

$$\times\left(\sum_{\kappa=1}^{\infty}\frac{[2\kappa+1+(C-V)d+V](\kappa+\rho+1)^{\varsigma}}{\rho^{\varsigma}(\kappa+2)^{m}(C-V)|d|}|b_{\kappa}|\right)$$
$\leq 1.$

Hence by Theorem 2.1,

$$(f*g)(w)\in H_{\rho,m}^{\varsigma}(\varsigma,\rho,m,V,C,d).$$

Partial Sums

In this section, we investigate the ratio of a function of the form (1) to its sequence of partial sums

$$f_n(w)=w^{-1}+\sum_{k=1}^{n}a_k w^k,$$

when the coefficients are small enough to satisfy condition (9). We'll establish clear lower bounds for

$$\Re\left(\frac{f(w)}{f_n(w)}\right),\Re\left(\frac{f_n(w)}{f(w)}\right).$$

Unless otherwise stated, we will assume that f is of the form 1 and that its sequence of partial sums is denoted by

$$f_n(w)=w^{-1}+\sum_{k=1}^{n}a_k\, w^k.$$

Theorem 5.1. If f of the form (1) satisfies condition (9), then

$$\Re\left(\frac{f(w)}{f_n(w)}\right)\geq 1-\frac{1}{d_{n+1}},(14)$$

and

$$\Re\left(\frac{f_n(w)}{f(w)}\right)\geq\frac{d_{n+1}}{1+d_{n+1}},(15)$$

where

$$d_n=\frac{[2\kappa+1+(C-V)d+V](\kappa+\rho+1)^{\varsigma}}{\rho^{\varsigma}(\kappa+2)^{m}(C-V)|d|}.$$

Proof. In order to demonstrate inequality 14, we must first establish

$$d_{n+1}\left[\frac{f(w)}{f_n(w)}-\left(1-\frac{1}{d_{n+1}}\right)\right]$$

$$d_{n+1}\left[\frac{w^{-1}+\sum_{\kappa=1}^{\infty}a_\kappa\, w^\kappa}{w^{-1}+\sum_{\kappa=1}^{n}a_\kappa\, w^\kappa}+\frac{1-d_{n+1}}{d_{n+1}}\right]$$

$$\frac{1+\sum_{\kappa=1}^{n}a_\kappa\, w^{\kappa+1}+d_{n+1}\sum_{\kappa=n+1}^{\infty}a_\kappa\, w^{\kappa+1}}{1+\sum_{\kappa=1}^{n}a_\kappa\, w^{\kappa+1}}$$

$$\frac{1+h_1(w)}{1+h_2(w)}.$$

If we set

$$\frac{1+h_1(w)}{1+h_2(w)}=\frac{1+z(w)}{1-z(w)},$$

then

$$2z(w)+z(w)h_1(w)+z(w)h_2(w)=h_1(w)-h_2(w),$$

$$z(w)=\frac{h_1(w)-h_2(w)}{2+h_1(w)+h_2(w)}.$$

Thus

$$z(w)=\frac{d_{n+1}\sum_{\kappa=n+1}^{\infty}a_\kappa\, w^{\kappa+1}}{2+2\sum_{\kappa=1}^{n}a_\kappa\, w^{\kappa+1}+d_{n+1}\sum_{\kappa=n+1}^{\infty}a_\kappa\, w^{\kappa+1}},$$

and

$$|z(w)|\leq\frac{d_{n+1}\sum_{\kappa=n+1}^{\infty}|a_\kappa|}{2-2\sum_{\kappa=1}^{n}|a_\kappa|-d_{n+1}\sum_{\kappa=n+1}^{\infty}|a_\kappa|}.$$

Now one can see that
$|z(w)|\leq 1$, if and only if

$$2d_{n+1}\sum_{\kappa=n+1}^{\infty}|a_\kappa|\leq 2-2\sum_{\kappa=1}^{n}|a_\kappa|,$$

which implies that

$$\sum_{\kappa=1}^{n}|a_\kappa|+d_{n+1}\sum_{\kappa=n+1}^{\infty}|a_\kappa|\leq 1.(16)$$

Finally, to demonstrate the inequality in 14, it suffices to show that the left-hand side of 16 is constrained above by $\sum_{\kappa=1}^{\infty}d_\kappa|a_\kappa|$, which is equivalent to

$$\sum_{\kappa=1}^{n}(1-d_\kappa)|a_\kappa|+\sum_{\kappa=n+1}^{\infty}(d_{n+1}-d_\kappa)|a_\kappa|\geq 0.(17)$$

The inequality proof in 14 is now complete by using equation 17.

To prove inequality 15, we set

$$(1+d_{n+1})\left(\frac{f_n(w)}{f(w)}-\frac{d_{n+1}}{1+d_{n+1}}\right)$$

$$\frac{1+\sum_{\kappa=1}^{n}a_\kappa\, w^{\kappa+1}-d_{n+1}\sum_{\kappa=n+1}^{\infty}a_\kappa\, w^{\kappa+1}}{1+\sum_{\kappa=1}^{\infty}a_\kappa\, w^{\kappa+1}}$$

$$=\frac{1+z(w)}{1-z(w)},$$

Where:

$$|z(w)| \leq \frac{(1+d_{n+1})\sum_{\kappa=n+1}^{\infty}|a_\kappa|}{2-2\sum_{\kappa=1}^{n}|a_\kappa|-(1+d_{n+1})\sum_{\kappa=n+1}^{\infty}|a_\kappa|} \leq 1. (18)$$

This last inequality in 18 is equivalent to

$$\sum_{\kappa=1}^{n}|a_\kappa| + (1+d_{n+1})\sum_{\kappa=n+1}^{\infty}|a_\kappa| \leq 1. (19)$$

Finally, we can observe that the left-hand side of the inequality in 19 is bounded above $\sum_{\kappa=1}^{\infty} d_\kappa|a_\kappa|$,we have completed the proof of (15), which concludes the proof of Theorem 5.1.

Neighborhoods for the Class $H_{\rho,m}^{\varsigma}(\varsigma,\rho,m,V,C,d)$

In this section, we define the $\delta-$ neighborhood of a function $f(w)$ and establish a relation between $\delta-$ neighborhood and $H_{\rho,m}^{\varsigma}(\varsigma,\rho,m,V,C,d,\gamma)$ class of a function.

Definition.6.1. A function $f \in \sum_\rho$ is said to be in the class $H_{\rho,m}^{\varsigma}(\varsigma,\rho,m,V,C,d,\gamma)$ if there exists a function $g \in H_{\rho,m}^{\varsigma}(\varsigma,\rho,m,V,C,d)$ such that

$$\left|\frac{f(w)}{g(w)}-1\right| < 1-\gamma, 0 \leq \gamma < 1. (20)$$

Following up on previous works on neighborhoods of analytic functions by Goodman [12], Shinde et al [13] and Ruschweyh [14]. We defined the $\delta-$ neighborhood of a function $f \in \sum_\rho$ by

$$N_\delta(f) = \left\{ \begin{array}{c} g \in \sum_\rho : g(w) = w^{-1} + \sum_{\kappa=1}^{\infty} b_\kappa w^\kappa \\ \sum_{\kappa=1}^{\infty} \kappa|a_\kappa - b_\kappa| \leq \delta \end{array} \right\} (21)$$

Theorem .6.1. If $g \in H_{\rho,m}^{\varsigma}(\varsigma,\rho,m,V,C,d)$ and

$$\gamma = 1 - \frac{\delta(3+[(C-V)|d|+V])(2+\rho)^\varsigma}{(3+[(C-V)|d|+V])(2+\rho)^\varsigma - \rho^\varsigma(3)^m(C-V)|d|}. (22)$$

Then $N_\delta(g) \subset H_{\rho,m}^{\varsigma}(\varsigma,\rho,m,V,C,d,\gamma)$.

Proof. Let $f \in N_\delta(g)$. Then we find from (21) that

$$\sum_{\kappa=1}^{\infty}\kappa|a_\kappa - b_\kappa| \leq \delta,$$

this implies that the coefficient of inequality $\sum_{\kappa=1}^{\infty}|a_\kappa - b_\kappa| \leq \delta. (\kappa \in N)$.

Since $g \in H_{\rho,m}^{\varsigma}(\varsigma,\rho,m,V,C,d)$, we have $\sum_{\kappa=1}^{\infty} b_\kappa < \frac{\rho^\varsigma(3)^m(C-V)|d|}{(3+[(C-V)|d|+V])(2+\rho)^\varsigma}$. So that

$$\left|\frac{f(w)}{g(w)}-1\right| \leq \frac{\sum_{\kappa=1}^{\infty}|a_\kappa - b_\kappa|}{1-\sum_{\kappa=1}^{\infty} b_\kappa} \leq \frac{\delta\big((3+[(C-V)|d|+V])\big)(2+\rho)^\varsigma}{(3+[(C-V)|d|+V])(2+\rho)^\varsigma - \rho^\varsigma(3)^m(C-V)|d|} = 1-\gamma.$$

Provided γ is given by (22).

Hence, by Inequality 20, $f \in H_{\rho,m}^{\varsigma}(\varsigma,\rho,m,V,C,d,\gamma)$ for γ given by (22), which completes the proof of Theorem.

CONCLUSIONS

This study looked at some basic features of geometric function theory and presented a new linear operator. As a result, various conclusions about coefficient estimates, closure theorems, convolution properties, partial sums, and neighborhoods were reached.

Disclosure and conflict of interest: The authors declare that they have no conflicts of interest.

REFERENCES

[1] R. S. Juma. H. H. Ebrahim and S. I. Ahmed. On subclass of meromorphic univalent function defined by derivative operator. Gen, (2015). 26(2), 79-96.
[2] M. K. Aouf. On a certain class of meromorphic univalent functions with positive coefficients. Rend. Mat.,(7), (1991). 11, 209-219.
[3] M. Darus Meromorphic functions with positive coefficients. International Journal of Mathematics and Mathematical Sciences, (2004), (6), 319-324. https://doi.org/10.1155/S0161171204306198
[4] O. Altintas, H. Irmak, and H. M. Srivastava. Family of meromorphically univalent functions with positive coefficients, (1994).
[5] P. T. Reddy. G. Swapna and B. Venkateswarlu. Certain subclass of meromorphic functions associated with Bessel function.
[6] S. Owa and N. N. Pascu. Coefficient inequalities for certain classes of meromorphically starlike and meromorphically convex functions. J. Inequal. Pure Appl. Math, (2003). 4(1), 1-6.
[7] S. Mahmood. Q. Z. Ahmad. H. M. Srivastava. N. Khan. B. Khan and M. Tahir. A certain subclass of meromorphically q-starlike functions associated with the Janowski functions. Journal of Inequalities and Applications, (2019). (1), 1-11. https://doi.org/10.1186/s13660-019-2020-z

[8] G. S. Salagean. Subclasses of univalent functions. In Complex Analysis—Fifth Romanian-Finnish Seminar (pp. (1983). 362-372). Springer, Berlin, Heidelberg. https://doi.org/10.1007/BFb0066543

[9] Dziok, J. Classes of meromorphic functions defined by the extended Salagean operator. Studia universitatis babes-bolyai mathematica. (2010),55(3), 123-130.

[10] G. Swapna. B. Venkateswarlu and P. Thirupathi Reddy. Notes on meromorphic functions with positive coefficients involving polylogarithm function. Известия Иркутского государственного университета. Серия «Математика»,(2020). 33(0), 51-63. https://doi.org/10.26516/1997-7670.2020.33.51

[11] S. Elhaddad and M. Darus. On meromorphic functions defined by a new operator containing the Mittag–Leffler function. Symmetry, (2019). 11(2), 210. https://doi.org/10.3390/sym11020210

[12] A. W. Goodman. Univalent functions and nonanalytic curves. Proceedings of the American Mathematical Society, (1957). 8(3), 598-601. https://doi.org/10.1090/S0002-9939-1957-0086879-9

[13] A. S. Shinde. R. N. Ingle and P. T. Reddy. On Certain Subclass Of Meromorphic Functions With Positive Coefficients. Palestine Journal of Mathematics, (2021). 10(2), 685–693.

[14] S. Ruscheweyh. Neighborhoods of univalent functions. Proceedings of the American Mathematical Society, (1981). 81(4), 521-527. https://doi.org/10.1090/S0002-9939-1981-0601721-6

How to Cite

I. I. Hameed and A. R. S. Juma, “Geometric Properties of Meromorphic Functions Involving Convolution Operator”, *Al-Mustansiriyah Journal of Science*, vol. 33, no. 4, pp. 80–85, Dec. 2022.