The Rational Valued Characters Table of the Group ($Q_{2m}\times D_5$) When m = p, p>2, is prime number

جدول الشواخص النسبية للزمرة ($Q_{2m}\times D_5$) عندما m عدد اولي اكبر من 2

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بحث مستل

1. Abstract

The main purpose of this paper is to find the rational valued characters table of the group $Q_{2m}\times D_5$ When m= p>2, p is prime number, which is denoted by $\equiv^*(Q_{2m}\times D_5)$, When Q_{2m} is denoted to quaternion group, and D_5 is the dihedral group of order 10.

الخلاصة

الهدف الرئيسي من هذا البحث هو ايجاد جدول الشواخص النسبية للزمرة ($Q_{2m}\times D_5$) عندما m عدد اولي اكبر من 2 والذي يرمز له بالرمز ($Q_{2m}\times D_5$) $\equiv^*$.

2. Introduction

Let G be a finite group, two elements of G are said to be Γ- conjugate if the cyclic subgroups they generate are conjugate in G, this relation is an equivalence relation on G. and this equivalence relation called Γ- classes.

The Z-valued class function on the group G, which is constant on the Γ- classes forms a finitely generated abelian group cf(G,Z) of a rank equal to the number of Γ- classes . the intersection of cf(G,Z) with the group of all generalized characters of G, R(G) is a normal subgroup of cf(G,Z) denoted by $\bar{R}(G)$ each element in $\bar{R}(G)$ can be written as $u_1\theta_1+u_2\theta_2+\ldots\ldots+u_i\theta_i$, where i is the number of Γ- classes, $u_1,u_2,\ldots,u_i\in Z$ and $\theta_i=\sum_{\sigma\in Gal(Q(\chi_i)/Q)}\sigma(\chi_i)$ where χ_i is an irreducible character of the group G and σ is any element in Galios group $Gal(Q(\chi_i)/Q)$. let $\equiv^*(G)$ denotes the i×i matrix which corresponds to the θ_i's and columns correspond to the Γ- classes of G the matrix expressing $\bar{R}(G)$ basis in terms of the cf(G,Z) basis is $\equiv^*(G)$ In 1995 N. R. Mahmood [1] studied the factor group $cf(Q_{2m},Z)/\bar{R}(Q_{2m})$. the aim of this Paper is to find $\equiv^*(Q_{2m}\times D_5)$ and determine general 15×15 matrix form of the rational valued characters table of the group $Q_{2m}\times D_5$ when m = p, p>2 is prime number.

3. preliminaries

The Generalized Quaternion Group Q_{2m}(3.1) [1]

For each positive integer m, The generalized quaternion group Q_{2m} of order 4m with two generators x and y satisfies $Q_{2m}=\{x^hy^k\ ,\ 0\le h\le 2m-1\ ,\ k=0,1\}$

which has the following properties$\{x^{2m}=y^4=I\ ,\ yx^my^{-1}=x^{-m}\}$

Definition (3.2): [2]

Let F be a field .The general linear group GL(n,F) is a multiplicative group of all non-singular n×n matrices over F .

Definition (3.3): [2]

Let F be a field .A matrix representation of G is homomorphism $T: G \rightarrow GL(n, F)$, is called the degree of representation T .

Definition (3.4): [23]

A matrix representation $T: G \rightarrow GL(n,F)$ is said to be reducible if there exists a non-singular matrix A over F such that:

$$A^{-1} T(g) A = \begin{bmatrix} T_1(g) & T(g) \\ 0 & T_2(g) \end{bmatrix}, \text{ for all } g \in G.$$

Where $T_1(g)$ and $T_2(g)$ are matrices representations over F of the dimensions r×r, s×s respectively and E(g) is a matrix of the dimensions r×s such that $0 < r < n$ and $r+s = n$.

If no such reducible matrix exists , then T(g) is called an irreducible matrix representation.

Definition (3.5): [4]

The trace of an $i \times i$ matrix A is the sum of main diagonal elements ,denoted by tr(A)

Definition (3.6): [5]

Let T be a matrix representation of G over the field F. The character χ of a matrix representation T is the mapping $\chi : G \rightarrow F$ defined by $\chi(g)=tr(T(g))$ for all $g \in G$ .The degree of T is called the degree of χ .

Definition (3.7): [6]

The character of an irreducible representation is called an irreducible characters **.**

Definition (3.8): [2]

Let χ and ψ are characters of a group G, then :

1. The sum of characters is defined by: $(\chi+\psi)(g) = \chi(g)+\psi(g)$, for all $g \in G$.
2. The product of characters is defined by : $(\chi.\psi)(g) = \chi(g).\psi(g)$, for all $g \in G$.

Theorem (3.9):[2]

Let $T_1 : G_1 \rightarrow GL(n,F)$ and $T_2 : G_2 \rightarrow GL(m,F)$ are two irreducible representations Of the groups G_1 and G_2 with characters $\chi_1$and χ_2 respectively , then $T_1 \otimes T_2$ is Irreducible representation of the group $G_1 \times G_2$ with the character $\chi_1 \cdot \chi_2$.

Definition (3.10):[7]

A class function f on G is a function $\boldsymbol{f} : G \rightarrow C$ which is constant on conjugacy classes ,that is: $\boldsymbol{f}(g^{-1} h g) = \boldsymbol{f}(h)$, for all $g,h \in G$, the set of all class functions on G is denoted by cf(G) .
If all values of f are in Z, then the class function is called Z-valued class function.

proposition(3.11):[8]

The character of group is a class function .

Definition (3.12):[9]

Let $\chi_1, \chi_2, \ldots, \chi_k$ be the irreducible characters of the finite group G and let $g_1, g_2, \ldots, g_k$ be representatives of the conjugacy classes of G .
the $k \times k$ matrix whose *ij*-entry is $\chi_i(g_j)$ (for all i , j with $1 \le i , j \le k$) is called character table of G which is denoted by ≡(G).

Definition (3.13):[10]

A rational valued character θ of G is a character whose values are in Z, which is $\theta(g) \in Z$, for all $g \in G$.

Definition (3.14): [11]

Let K a subfield of a field F . The Galios group of F over K ,denoted by Gal(F/K) , is the set of all those automorphisms of F that fix K .
If $f(x) \in K[x]$, and if $F=K(Z_1,Z_2,\ldots,Z_n)$ is a splitting field , then the Galois group of f(x) over K is defined to be Gal(F/K) .

Definition (3.15): [12]

The information about rational valued characters of a finite group G is displayed in a table called the rational valued characters table of G.

We denote it by $\equiv^*(G)$ which is $l \times l$ matrix whose columns are Γ-classes and rows are the values of all rational valued characters of G, where l is the number of Γ-classes.

The characters table of The Quaternion Group Q_{2m} when m is an odd Number(3.16)[1]

There are two types of irreducible characters one of them is the character of the Linear representation R_1, R_2, R_3 and R_4 which are denoted by Ψ_1, Ψ_2, Ψ_3 and Ψ_4 Respectively as In the following table :

	x^k	y^k
Ψ_1	1	1
Ψ_2	1	-1
Ψ_3	$(-1)^k$	$i(-1)^k$
Ψ_4	$(-1)^k$	$i(-1)^{k+1}$

Table (1)

where $0 \le k \le 2m-1$.
the other characters of irreducible representations T_h of degree 2 are denoted by χ_h such that:
$\chi_h(x^k) = \omega^{hk} + \omega^{-hk} = e^{\pi ihk/m} + e^{-\pi ihk/m} = 2\cos(\pi hk/m)$
we are denoted to $(\omega^{hk}+\omega^{-hk})$ by V_{hk} , thus $V_{hk}=V_{2m-hk}$, $V_m=-2$, $V_{2m}=2$, also we will write $V_{J(hk)}$ such that $J(hk) = \min\{hk \pmod{2m}, 2m-hk \pmod{2m}\}$ in the Character table of the quaternion group Q_{2m} when m is an odd number ,such that: $V_{J(hk)}=2\cos(\pi J(hk)/m)$, $\chi_h(x^k y) = 0$,
where, $1 \le h \le m-1$ and $\omega=e^{2\pi i/2m}$. So, there are m+3 irreducible characters of Q_{2m}. Then the general form of the characters table of Q_{2m} when m is an odd number is given in the following table:

$\equiv(Q_{2m})$

CL_α	[I]	$[x^2]$	$[x^4]$	….	$[x^{m-1}]$	$[x^m]$	[x]	$[x^3]$	….	$[x^{m-2}]$	[y]	[xy]
$\lvert CL_\alpha\rvert$	1	2	2	….	2	1	2	2	….	2	m	m
$\lvert C_{Q2m}(CL_\alpha)\rvert$	4m	2m	2m	….	2m	4m	2m	2m	….	2m	4	4
Ψ_1	1	1	1	….	1	1	1	1	….	1	1	1
χ_2	2	$V_{J(4)}$	$V_{J(8)}$	….	$V_{J(2m-2)}$	2	V_2	$V_{J(6)}$	….	$V_{J(2m-4)}$	0	0
⋮	⋮	⋮	⋮	⋱	⋮	⋮	⋮	⋮	⋱	⋮	⋮	⋮
$\chi_{(m-1)}$		$V_{J(2m-2)}$	$V_{J(4m-4)}$	….	$V_{J((m-1)(m-1))}$	2	$V_{J(m-1)}$	$V_{J(3m-3)}$	….	$V_{J((m-1)(m-2))}$	0	0
Ψ_2	1	1	1	….	1	1	1	1	….	1	-1	-1
χ_1	2	V_2	$V_{J(4)}$	….	$V_{J(m-1)}$	-2	V_1	$V_{J(3)}$	….	$V_{J(m-2)}$	0	0
⋮	⋮	⋮	⋮	⋱	⋮	⋮	⋮	⋮	⋱	⋮	⋮	⋮
$\chi_{(m-2)}$	2	$V_{J(2m-4)}$	$V_{J(4m-8)}$	….	$V_{J((m-2)(m-1))}$	-2	$V_{J((m-2)}$	$V_{J((3m-6)}$	….	$V_{J((m-2)(m-2))}$	0	0
Ψ_3	1	1	1	….	1	-1	-1	-1	….	-1	i	-i
Ψ_4	1	1	1	….	1	-1	-1	-1	….	-1	-i	i

Table(2)
The characters table of matrix from degree (m+3)·(m+3)

The Group $Q_{2m}\times D_5$(3.18):

The group $Q_{2m}\times D_5$ is the direct product group of the quaternion group Q_{2m} of order 4m and the group D_5 is the dihedral group of order 10 then The order of it is 4m×10 = 40m. the characters table of D_5 is given in the table (3):

$\equiv D_5$ =

CL_α	$[I^*]$	[r]	$[r^2]$	[s]
$\lvert CL_\alpha\rvert$	1	2	2	5
$\lvert C_{D5}(CL_\alpha)\rvert$	10	5	5	2
χ'_1	1	1	1	1
χ'_2	1	1	1	-1
χ'_3	2	ϵ_1	ϵ_2	0
χ'_4	2	ϵ_2	ϵ_1	0

Table(3)

according to theorem (3.3), each irreducible character χ_i of Q_{2m} defines four characters $\chi_{(i,1)}$, $\chi_{(i,2)}$, $\chi_{(i,3)}$ and $\chi_{(i,4)}$ such that $\chi_{(i,1)}=\chi_i\chi'_1$, $\chi_{(i,2)}=\chi_i\chi'_2$, $\chi_{(i,3)}=\chi_i\chi'_3$ and $\chi_{(i,4)}=\chi_i\chi'_4$ of $Q_{2m}\times D_5$. then

$\equiv(Q_{2m}\times D_5) = \equiv(Q_{2m})\otimes\equiv(D_5)$.

where $\epsilon_1 = \omega+\omega^4 = 2\cos(2\pi/5)$, $\epsilon_2 = \omega^2+\omega^3 = 2\cos(4\pi/5)$ and $\omega = e^{\frac{2\pi i}{5}}$.

Example (3.19)

To find characters table of $Q_{14}\times D_5$ from (3.3) we have the characters table of D_5 in the table (3) and the characters table of Q_{14} as table (4), as follows: $\equiv(Q_{14}\times D_5) = \equiv(Q_{14}) \otimes \equiv(D_5)$.
then $\equiv(Q_{14}\times D_5)$ is given in the table (5)

$\equiv(Q_{14}\times D_5)$

CL_α	[I]	$[x^2]$	$[x^4]$	$[x^6]$	$[x^7]$	[x]	$[x^3]$	$[x^5]$	[y]	[xy]
$\lvert CL_\alpha\rvert$	1	2	2	2	1	2	2	2	7	7
$\lvert C_{Q14}(CL_\alpha)\rvert$	28	14	14	14	28	14	14	14	4	4
Ψ_1	1	1	1	1	1	1	1	1	1	1
χ_2	2	V_4	V_6	V_2	2	V_2	V_6	V_4	0	0
χ_4	2	V_6	V_2	V_4	2	V_4	V_2	V_6	0	0
χ_6	2	V_2	V_4	V_6	2	V_6	V_4	V_2	0	0
Ψ_2	1	1	1	1	1	1	1	1	-1	-1
χ_1	2	V_2	V_4	V_6	-2	V_1	V_3	V_5	0	0
χ_3	2	V_6	V_2	V_4	-2	V_3	V_5	V_1	0	0
χ_5	2	V_4	V_6	V_2	-2	V_5	V_1	V_3	0	0
Ψ_3	1	1	1	1	-1	-1	-1	-1	i	-i
Ψ_4	1	1	1	1	-1	-1	-1	-1	-i	i

Table (4)

where $V_i = 2\cos(\pi i/7)$, $V_{14} = 2$, $V_7 = -2$.
by theorem (3.3), the characters table of $Q_{14}\times D_5$ can be written as follows:

CL_α	$[I,I^*]$	$[I,r]$	$[I,r^2]$	$[I,s]$	$[x^2,I^*]$	$[x^2,r]$	$[x^2,r^2]$	$[x^2,s]$	$[x^4,I^*]$	$[x^4,r]$	$[x^4,r^2]$	$[x^4,s]$
$\Psi_{(1,1)}$	1	1	1	1	1	1	1	1	1	1	1	1
$\Psi_{(1,2)}$	1	1	1	-1	1	1	1	-1	1	1	1	-1
$\Psi_{(1,3)}$	2	ϵ_1	ϵ_2	0	2	ϵ_1	ϵ_2	0	2	ϵ_1	ϵ_2	0
$\Psi_{(1,4)}$	2	ϵ_2	ϵ_1	0	2	ϵ_2	ϵ_1	0	2	ϵ_2	ϵ_1	0
$\chi_{(2,1)}$	2	2	2	2	V_4	V_4	V_4	V_4	V_6	V_6	V_6	V_6
$\chi_{(2,2)}$	2	2	2	-2	V_4	V_4	V_4	$-V_4$	V_6	V_6	V_6	$-V_6$
$\chi_{(2,3)}$	4	$2\epsilon_1$	$2\epsilon_2$	0	$2V_4$	$V_4\epsilon_1$	$V_4\epsilon_2$	0	$2V_6$	ϵ_1V_6	ϵ_2V_6	0
$\chi_{(2,4)}$	4	$2\epsilon_2$	$2\epsilon_1$	0	$2V_4$	$V_4\epsilon_2$	$V_4\epsilon_1$	0	$2V_6$	ϵ_2V_6	ϵ_1V_6	0
$\chi_{(4,1)}$	2	2	2	2	V_6	V_6	V_6	V_6	V_2	V_2	V_2	V_2
$\chi_{(4,2)}$	2	2	2	-2	V_6	V_6	V_6	$-V_6$	V_2	V_2	V_2	$-V_2$
$\chi_{(4,3)}$	4	$2\epsilon_1$	$2\epsilon_2$	0	$2V_6$	ϵ_1V_6	ϵ_2V_6	0	$2V_2$	ϵ_1V_2	ϵ_2V_2	0
$\chi_{(4,4)}$	4	$2\epsilon_2$	$2\epsilon_1$	0	$2V_6$	ϵ_2V_6	ϵ_1V_6	0	$2V_2$	ϵ_2V_2	ϵ_1V_2	0
$\chi_{(6,1)}$	2	2	2	2	V_2	V_2	V_2	V_2	V_4	V_4	V_4	V_4
$\chi_{(6,2)}$	2	2	2	-2	V_2	V_2	V_2	$-V_2$	V_4	V_4	V_4	$-V_4$
$\chi_{(6,3)}$	4	$2\epsilon_1$	$2\epsilon_2$	0	$2V_2$	ϵ_1V_2	ϵ_2V_2	0	$2V_4$	$V_4\epsilon_1$	$V_4\epsilon_2$	0
$\chi_{(6,4)}$	4	$2\epsilon_2$	$2\epsilon_1$	0	$2V_2$	ϵ_2V_2	ϵ_1V_2	0	$2V_4$	$V_4\epsilon_2$	$V_4\epsilon_1$	0
$\Psi_{(2,1)}$	1	1	1	1	1	1	1	1	1	1	1	1
$\Psi_{(2,2)}$	1	1	1	-1	1	1	1	-1	1	1	1	-1
$\Psi_{(2,3)}$	2	ϵ_1	ϵ_2	0	2	ϵ_1	ϵ_2	0	2	ϵ_1	ϵ_2	0
$\Psi_{(2,4)}$	2	ϵ_2	ϵ_1	0	2	ϵ_2	ϵ_1	0	2	ϵ_2	ϵ_1	0
$\chi_{(1,1)}$	2	2	2	2	V_2	V_2	V_2	V_2	V_4	V_4	V_4	V_4
$\chi_{(1,2)}$	2	2	2	-2	V_2	V_2	V_2	$-V_2$	V_4	V_4	V_4	$-V_4$
$\chi_{(1,3)}$	4	$2\epsilon_1$	$2\epsilon_2$	0	$2V_2$	ϵ_1V_2	ϵ_2V_2	0	$2V_4$	$V_4\epsilon_1$	$V_4\epsilon_2$	0
$\chi_{(1,4)}$	4	$2\epsilon_2$	$2\epsilon_1$	0	$2V_2$	ϵ_2V_2	ϵ_1V_2	0	$2V_4$	$V_4\epsilon_2$	$V_4\epsilon_1$	0
$\chi_{(3,1)}$	2	2	2	2	V_6	V_6	V_6	V_6	V_2	V_2	V_2	V_2
$\chi_{(3,2)}$	2	2	2	-2	V_6	V_6	V_6	$-V_6$	V_2	V_2	V_2	$-V_2$
$\chi_{(3,3)}$	4	$2\epsilon_1$	$2\epsilon_2$	0	$2V_6$	ϵ_1V_6	ϵ_2V_6	0	$2V_2$	ϵ_1V_2	ϵ_2V_2	0
$\chi_{(3,4)}$	4	$2\epsilon_2$	$2\epsilon_1$	0	$2V_6$	ϵ_2V_6	ϵ_1V_6	0	$2V_2$	ϵ_2V_2	ϵ_1V_2	0
$\chi_{(5,1)}$	2	2	2	2	V_4	V_4	V_4	V_4	V_6	V_6	V_6	V_6
$\chi_{(5,2)}$	2	2	2	-2	V_4	V_4	V_4	$-V_4$	V_6	V_6	V_6	$-V_6$
$\chi_{(5,3)}$	4	$2\epsilon_1$	$2\epsilon_2$	0	$2V_4$	$V_4\epsilon_1$	$V_4\epsilon_2$	0	$2V_6$	ϵ_1V_6	ϵ_2V_6	0
$\chi_{(5,4)}$	4	$2\epsilon_2$	$2\epsilon_1$	0	$2V_4$	$V_4\epsilon_2$	$V_4\epsilon_1$	0	$2V_6$	ϵ_2V_6	ϵ_1V_6	0
$\Psi_{(3,1)}$	1	1	1	1	1	1	1	1	1	1	1	1
$\Psi_{(3,2)}$	1	1	1	-1	1	1	1	-1	1	1	1	-1
$\Psi_{(3,3)}$	2	ϵ_1	ϵ_2	0	2	ϵ_1	ϵ_2	0	2	ϵ_1	ϵ_2	0
$\Psi_{(3,4)}$	2	ϵ_2	ϵ_1	0	2	ϵ_2	ϵ_1	0	2	ϵ_2	ϵ_1	0
$\Psi_{(4,1)}$	1	1	1	1	1	1	1	1	1	1	1	1
$\Psi_{(4,2)}$	1	1	1	-1	1	1	1	-1	1	1	1	-1
$\Psi_{(4,3)}$	2	ϵ_1	ϵ_2	0	2	ϵ_1	ϵ_2	0	2	ϵ_1	ϵ_2	0
$\Psi_{(4,4)}$	2	ϵ_2	ϵ_1	0	2	ϵ_2	ϵ_1	0	2	ϵ_2	ϵ_1	0

$[x^6,I^*]$	$[x^6,r]$	$[x^6,r^2]$	$[x^6,s]$	$[x^7,I^*]$	$[x^7,r]$	$[x^7,r^2]$	$[x^7,s]$	$[x,I^*]$	$[x,r]$	$[x,r^2]$	$[x,s]$	$[x^3,I^*]$	$[x^3,r]$
1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	-1	1	1	1	-1	1	1	1	-1	1	1
2	ϵ_1	ϵ_2	0	2	ϵ_1	ϵ_2	0	2	ϵ_1	ϵ_2	0	2	ϵ_1
2	ϵ_2	ϵ_1	0	2	ϵ_2	ϵ_1	0	2	ϵ_2	ϵ_1	0	2	ϵ_2
V_2	V_2	V_2	V_2	2	2	2	2	V_2	V_2	V_2	V_2	V_6	V_6
V_2	V_2	V_2	$-V_2$	2	2	2	-2	V_2	V_2	V_2	$-V_2$	V_6	V_6
$2V_2$	ϵ_1V_2	ϵ_2V_2	0	4	$2\epsilon_1$	$2\epsilon_2$	0	$2V_2$	ϵ_1V_2	ϵ_2V_2	0	$2V_6$	ϵ_1V_6
$2V_2$	ϵ_2V_2	ϵ_1V_2	0	4	$2\epsilon_2$	$2\epsilon_1$	0	$2V_2$	ϵ_2V_2	ϵ_1V_2	0	$2V_6$	ϵ_2V_6
V_4	V_4	V_4	V_4	2	2	2	2	V_4	V_4	V_4	V_4	V_2	V_2
V_4	V_4	V_4	$-V_4$	2	2	2	-2	V_4	V_4	V_4	$-V_4$	V_2	V_2
$2V_4$	$V_4\epsilon_1$	$V_4\epsilon_2$	0	4	$2\epsilon_1$	$2\epsilon_2$	0	$2V_4$	$V_4\epsilon_1$	$V_4\epsilon_2$	0	$2V_2$	ϵ_1V_2
$2V_4$	$V_4\epsilon_2$	$V_4\epsilon_1$	0	4	$2\epsilon_2$	$2\epsilon_1$	0	$2V_4$	$V_4\epsilon_2$	$V_4\epsilon_1$	0	$2V_2$	ϵ_2V_2
V_6	V_6	V_6	V_6	2	2	2	2	V_6	V_6	V_6	V_6	V_4	V_4
V_6	V_6	V_6	$-V_6$	2	2	2	-2	V_6	V_6	V_6	$-V_6$	V_4	V_4
$2V_6$	ϵ_1V_6	ϵ_2V_6	0	4	$2\epsilon_1$	$2\epsilon_2$	0	$2V_6$	ϵ_1V_6	ϵ_2V_6	0	$2V_4$	$V_4\epsilon_1$
$2V_6$	ϵ_2V_6	ϵ_1V_6	0	4	$2\epsilon_2$	$2\epsilon_1$	0	$2V_6$	ϵ_2V_6	ϵ_1V_6	0	$2V_4$	$V_4\epsilon_2$
1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	-1	1	1	1	-1	1	1	1	-1	1	1
2	ϵ_1	ϵ_2	0	2	ϵ_1	ϵ_2	0	2	ϵ_1	ϵ_2	0	2	ϵ_1
2	ϵ_2	ϵ_1	0	2	ϵ_2	ϵ_1	0	2	ϵ_2	ϵ_1	0	2	ϵ_2
V_6	V_6	V_6	V_6	-2	-2	-2	-2	V_1	V_1	V_1	V_1	V_3	V_3
V_6	V_6	V_6	$-V_6$	-2	-2	-2	2	V_1	V_1	V_1	$-V_1$	V_3	V_3
$2V_6$	ϵ_1V_6	ϵ_2V_6	0	-4	$-2\epsilon_1$	$-2\epsilon_2$	0	$2V_1$	ϵ_1V_1	ϵ_2V_1	0	2V3	V3ϵ1
$2V_6$	ϵ_2V_6	ϵ_1V_6	0	-4	$-2\epsilon_2$	$-2\epsilon_1$	0	$2V_1$	ϵ_2V_1	ϵ_1V_1	0	2V3	V3ϵ2
V_4	V_4	V_4	V_4	-2	-2	-2	-2	V_3	V_3	V_3	V_3	V_5	V_5
V_4	V_4	V_4	$-V_4$	-2	-2	-2	2	V_3	V_3	V_3	$-V_3$	V_5	V_5
$2V_4$	$V_4\epsilon_1$	$V_4\epsilon_2$	0	-4	$-2\epsilon_1$	$-2\epsilon_2$	0	$2\ V_3$	ϵ_1V_3	ϵ_2V_3	0	$2V_5$	$\epsilon_1\ V_5$
$2V_4$	$V_4\epsilon_2$	$V_4\epsilon_1$	0	-4	$-2\epsilon_2$	$-2\epsilon_1$	0	$2\ V_3$	ϵ_2V_3	ϵ_1V_3	0	$2V_5$	$\epsilon_2\ V_5$
V_2	V_2	V_2	V_2	-2	-2	-2	-2	V_5	V_5	V_5	V_5	V_1	V_1
V_2	V_2	V_2	$-V_2$	-2	-2	-2	2	V_5	V_5	V_5	$-V_5$	V_1	V_1
$2V_2$	ϵ_1V_2	ϵ_2V_2	0	-4	$-2\epsilon_1$	$-2\epsilon_2$	0	$2V_5$	$\epsilon_1\ V_5$	$\epsilon_2\ V_5$	0	$2V_1$	ϵ_1V_1
$2V_2$	ϵ_2V_2	ϵ_1V_2	0	-4	$-2\epsilon_2$	$-2\epsilon_1$	0	$2V_5$	$\epsilon_2\ V_5$	$\epsilon_1\ V_5$	0	$2V_1$	ϵ_2V_1
1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
1	1	1	-1	-1	-1	-1	1	-1	-1	-1	1	-1	-1
2	ϵ_1	ϵ_2	0	-2	$-\epsilon_1$	$-\epsilon_2$	0	-2	$-\epsilon_1$	$-\epsilon_2$	0	-2	$-\epsilon_1$
2	ϵ_2	ϵ_1	0	-2	$-\epsilon_2$	$-\epsilon_1$	0	-2	$-\epsilon_2$	$-\epsilon_1$	0	-2	$-\epsilon_2$
1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
1	1	1	-1	-1	-1	-1	1	-1	-1	-1	1	-1	-1
2	ϵ_1	ϵ_2	0	-2	$-\epsilon_1$	$-\epsilon_2$	0	-2	$-\epsilon_1$	$-\epsilon_2$	0	-2	$-\epsilon_1$
2	ϵ_2	ϵ_1	0	-2	$-\epsilon_2$	$-\epsilon_1$	0	-2	$-\epsilon_2$	$-\epsilon_1$	0	-2	$-\epsilon_2$

$[x^3,r^2]$	$[x^3,s]$	$[x^5,I^*]$	$[x^5,r]$	$[x^5,r^2]$	$[x^5,s]$	$[y,I^*]$	$[y,r]$	$[y,r^2]$	$[y,s]$	$[xy,I^*]$	$[xy,r]$	$[xy,r^2]$	$[xy,s]$
1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	-1	1	1	1	-1	1	1	1	-1	1	1	1	-1
ϵ_2	0	2	ϵ_1	ϵ_2	0	2	$\epsilon 1$	$\epsilon 2$	0	2	$\epsilon 1$	$\epsilon 2$	0
ϵ_1	0	2	ϵ_2	ϵ_1	0	2	$\epsilon 2$	$\epsilon 1$	0	2	$\epsilon 2$	$\epsilon 1$	0
V_6	V_6	V_4	V_4	V_4	V_4	0	0	0	0	0	0	0	0
V_6	$-V_6$	V_4	V_4	V_4	$-V_4$	0	0	0	0	0	0	0	0
$\epsilon_2 V_6$	0	$2V_4$	$V_4\epsilon_1$	$V_4\epsilon_2$	0	0	0	0	0	0	0	0	0
$\epsilon_1 V_6$	0	$2V_4$	$V_4\epsilon_2$	$V_4\epsilon_1$	0	0	0	0	0	0	0	0	0
V_2	V_2	V_6	V_6	V_6	V_6	0	0	0	0	0	0	0	0
V_2	$-V_2$	V_6	V_6	V_6	$-V_6$	0	0	0	0	0	0	0	0
$\epsilon_2 V_2$	0	$2V_6$	$\epsilon_1 V_6$	$\epsilon_2 V_6$	0	0	0	0	0	0	0	0	0
$\epsilon_1 V_2$	0	$2V_6$	$\epsilon_2 V_6$	$\epsilon_1 V_6$	0	0	0	0	0	0	0	0	0
V_4	V_4	V_2	V_2	V_2	V_2	0	0	0	0	0	0	0	0
V_4	$-V_4$	V_2	V_2	V_2	$-V_2$	0	0	0	0	0	0	0	0
$V_4\epsilon_2$	0	$2V_2$	$\epsilon_1 V_2$	$\epsilon_2 V_2$	0	0	0	0	0	0	0	0	0
$V_4\epsilon_1$	0	$2V_2$	$\epsilon_2 V_2$	$\epsilon_1 V_2$	0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1
1	-1	1	1	1	-1	-1	-1	-1	1	-1	-1	-1	1
ϵ_2	0	2	ϵ_1	ϵ_2	0	-2	$-\epsilon 1$	$-\epsilon 2$	0	-2	$-\epsilon 1$	$-\epsilon 2$	0
ϵ_1	0	2	ϵ_2	ϵ_1	0	-2	$-\epsilon 2$	$-\epsilon 1$	0	-2	$-\epsilon 2$	$-\epsilon 1$	0
V_3	V_3	V_5	V_5	V_5	V_5	0	0	0	0	0	0	0	0
V_3	$-V_3$	V_5	V_5	V_5	$-V_5$	0	0	0	0	0	0	0	0
$\epsilon_2 V_3$	0	$2V_5$	$\epsilon_1\ V_5$	$\epsilon_2\ V_5$	0	0	0	0	0	0	0	0	0
$\epsilon_1 V_3$	0	$2V_5$	$\epsilon_2\ V_5$	$\epsilon_1\ V_5$	0	0	0	0	0	0	0	0	0
V_5	V_5	V_1	V_1	V_1	V_1	0	0	0	0	0	0	0	0
V_5	$-V_5$	V_1	V_1	V_1	$-V_1$	0	0	0	0	0	0	0	0
$\epsilon_2\ V_5$	0	$2V_1$	$\epsilon_1 V_1$	$\epsilon_2 V_1$	0	0	0	0	0	0	0	0	0
$\epsilon_1\ V_5$	0	$2V_1$	$\epsilon_2 V_1$	$\epsilon_1 V_1$	0	0	0	0	0	0	0	0	0
V_1	V_1	V_3	V_3	V_3	V_3	0	0	0	0	0	0	0	0
V_1	$-V_1$	V_3	V_3	V_3	$-V_3$	0	0	0	0	0	0	0	0
$\epsilon_2 V_1$	0	$2\ V_3$	$\epsilon_1 V_3$	$\epsilon_2 V_3$	0	0	0	0	0	0	0	0	0
$\epsilon_1 V_1$	0	$2\ V_3$	$\epsilon_2 V_3$	$\epsilon_1 V_3$	0	0	0	0	0	0	0	0	0
-1	-1	-1	-1	-1	-1	i	i	i	i	-i	-i	-i	-i
-1	1	-1	-1	-1	1	i	i	i	-i	-i	-i	-i	i
$-\epsilon_2$	0	-2	$-\epsilon_1$	$-\epsilon_2$	0	2i	$i\epsilon 1$	$i\epsilon 2$	0	-2i	$-i\epsilon 1$	$-i\epsilon 2$	0
$-\epsilon_1$	0	-2	$-\epsilon_2$	$-\epsilon_1$	0	2i	$i\epsilon 2$	$i\epsilon 1$	0	-2i	$-i\epsilon 2$	$-i\epsilon 1$	0
-1	-1	-1	-1	-1	-1	-i	-i	-i	-i	i	i	i	i
-1	1	-1	-1	-1	1	-i	-i	-i	i	i	i	i	-i
$-\epsilon_2$	0	-2	$-\epsilon_1$	$-\epsilon_2$	0	-2i	$-i\epsilon 1$	$-i\epsilon 2$	0	2i	$i\epsilon 1$	$i\epsilon 2$	0
$-\epsilon_1$	0	-2	$-\epsilon_2$	$-\epsilon_1$	0	-2i	$-i\epsilon 2$	$-i\epsilon 1$	0	2i	$i\epsilon 2$	$i\epsilon 1$	0

Table(5)

4.The main results

Proposition (4.1): [12]

The rational valued characters $\theta_i = \sum_{\sigma \epsilon Gal(Q(\chi_i)/Q)} \sigma(\chi_i)$ form basis for $\overline{R}(G)$,where χ_i are the irreducible characters of G and their numbers are equal to the number of all distinct Γ- classes of G.

Proposition (4.2): [1]

The rational valued characters table of Q_{2m} when m= p>2, p is a prime number is given as follows :

$\equiv^*(Q_{2m})$ =

Γ- classes	[I]	$[x^2]$	$[x^m]$	[x]	[y]
θ_1	1	1	1	1	1
θ_2	p-1	-1	p-1	-1	0
θ_3	1	1	1	1	-1
θ_4	p-1	-1	1-p	1	0
θ_5	2	2	-2	-2	0

Table (6)

Example(4.3):

To calculate the rational valued characters table of $Q_{14}\times D_5$ from example (3.5) and proposition(4.1) we have to do the following:

$\theta_{(1,1)}= \psi_{(1,1)}$, $\theta_{(1,2)}= \psi_{(1,2)}$, $\theta_{(1,3)}= \psi_{(1,3)} + \psi_{(1,4)}$,

$\theta_{(3,1)}= \psi_{(2,1)}$, $\theta_{(3,2)}= \psi_{(2,2)}$, $\theta_{(3,3)}= \psi_{(2,3)} + \psi_{(2,4)}$,

$\theta_{(5,1)}= \psi_{(3,1)} + \psi_{(4,1)}$, $\theta_{(5,2)}= \psi_{(3,2)} + \psi_{(4,2)}$, $\theta_{(5,3)}= \psi_{(3,3)}+ \psi_{(3,4)}+ \psi_{(4,3)} + \psi_{(4,4)}$.

now the elements of Gal $(\chi_{(1,i)})$ /Q, are: $\{\sigma_{(1,i)}, \sigma_{(3,i)}, \sigma_{(5,i)}\}$,

where $\sigma_{(1,i)}(\chi_{(1,i)}) = \chi_{(1,i)}$, $\sigma_{(3,i)}(\chi_{(1,i)}) = \chi_{(3,i)}$ and $\sigma_{(5,i)}(\chi_{(1,i)}) = \chi_{(5,i)}$, where i=1,2,3 .

to calculate $\theta_{(2,i)}$

<u>(I) If i=1</u>

$\theta_{(2,1)}([I,I^*])=2+2+2=6$

$\theta_{(2,1)}([I,r])=2+2+2=6$

$\theta_{(2,1)}([I,s])=2+2+2=6$

$\theta_{(2,1)}([x^2,I^*])=V_4+V_6+V_2=-1$

$\theta_{(2,1)}([x^2,r])=V_4+V_6+V_2=-1$

$\theta_{(2,1)}([x^2,s])=V_4+V_6+V_2=-1$

$\theta_{(2,1)}([x^7,I^*])=2+2+2=6$

$\theta_{(2,1)}([x^7,r])=2+2+2=6$

$\theta_{(2,1)}([x^7,s])=2+2+2=6$

$\theta_{(2,1)}([x,I^*])=V_2+V_4+V_6=-1$

$\theta_{(2,1)}([x,r])=V_2+V_4+V_6=-1$

$\theta_{(2,1)}([x,s])=V_2+V_4+V_6=-1$

$\theta_{(2,1)}([y,I^*]) = 0$

$\theta_{(2,1)}([y,r]) = 0$

$\theta_{(2,1)}([y,s]) = 0$.

(II) If i=2

$\theta_{(2,2)}([I,I^*])=2+2+2=6$

$\theta_{(2,2)}([I,r])=2+2+2=6$

$\theta_{(2,2)}([I,s])=-2+(-2)+(-2)=-$

$\theta_{(2,2)}([x^2,I^*])=V_4+V_6+V_2=-1$

$\theta_{(2,2)}([x^2,r])=V_6+V_2+V_4=-1$

$\theta_{(2,2)}([x^2,s])=-V_2+(-V_4)+(-V_6)=1$

$\theta_{(2,2)}([x^7,I^*])=2+2+2=6$

$\theta_{(2,2)}([x^7,r])=2+2+2=6$

$\theta_{(2,2)}([x^7,s])=-2+(-2)+(-2)=-6$

$\theta_{(2,2)}([x,I^*])=V_2+V_4+V_6=-1$

$\theta_{(2,2)}([x,r])=V_6+V_2+V_4=-1$

$\theta_{(2,2)}([x,s])=-V_4+(-V_6)+(-V_2)=1$

$\theta_{(2,1)}([y,I^*])=0$

$\theta_{(2,1)}([y,r])=0$

$\theta_{(2,1)}([y,s])=0$.

(III) If i=3

$\theta_{(2,3)}([I,I^*])=4+4+4+4+4+4=24$

$\theta_{(2,3)}([I,r])=2\epsilon_1+2\epsilon_2+2\epsilon_1+2\epsilon_2+2\epsilon_1+2\epsilon_2=-6$

$\theta_{(2,3)}([I,s])=0$

$\theta_{(2,3)}([x^2,I^*])=2V_4+2V_4+2V_6+2V_6+2V_2+2V_2=-4$

$\theta_{(2,3)}([x^2,r])=\epsilon_1V_4+\epsilon_2V_4+\epsilon_1V_6+\epsilon_2V_6+\epsilon_1V_2+\epsilon_2V_2=1$

$\theta_{(2,3)}([x^2,s])=0$

$\theta_{(2,3)}([x^7,I^*])=4+4+4+4+4+4=24$

$\theta_{(2,3)}([x^7,r])=2\epsilon_1+2\epsilon_2+2\epsilon_1+2\epsilon_2+2\epsilon_1+2\epsilon_2=-6$

$\theta_{(2,3)}([x^7,s])=0$

$\theta_{(2,3)}([x,I^*])=2V_2+2V_2+2V_4+2V_4+2V_6+2V_6=-4$

$\theta_{(2,3)}([x,r])=\epsilon_1V_2+\epsilon_2V_2+\epsilon_1V_4+\epsilon_2V_4+\epsilon_1V_6+\epsilon_2V_6=1$

$\theta_{(2,3)}([x,s])=0$

$\theta_{(2,3)}([y,I^*])=0$

$\theta_{(2,3)}([y,r])=0$

$\theta_{(2,3)}([y,s])=0$.

To calculate $\theta_{(4,i)}$

(I) If i=1

$\theta_{(4,1)}([I,I^*])=2+2+2=6$

$\theta_{(4,1)}([I,r])=2+2+2=6$

$\theta_{(4,1)}([I,s])=2+2+2=6$

$\theta_{(4,1)}([x^2,I^*])=V_2+V_6+V_4=-1$

$\theta_{(4,1)}([x^2,r])=V_2+V_6+V_4=-1$

$\theta_{(4,1)}([x^2,s])=V_2+V_6+V_4=-1$

$\theta_{(4,1)}([x^7,I^*])=-2+(-2)+(-2)=-6$

$\theta_{(4,1)}$ ([x^7,r])=-2+(-2)+(-2)=-6
$\theta_{(4,1)}$([x^7,s])=-2+(-2)+(-2)=-6
$\theta_{(4,1)}$([x,I^*])=$V_1+V_3+V_5$=1
$\theta_{(4,1)}$([x,r])=$V_3+V_5+ V_1$=1
$\theta_{(4,1)}$([x,s])=$V_5+V_1+V_3$=1
$\theta_{(4,1)}$([y,I^*])=0
$\theta_{(4,1)}$([y,r])=0
$\theta_{(4,1)}$([y,s])=0.

(II) If i=2
$\theta_{(4,2)}$([I,I^*])=2+2+2=6
$\theta_{(4,2)}$([I,r])=2+2+2=6
$\theta_{(4,2)}$([I, s])=-2+(-2)+(-2)=-6
$\theta_{(4,2)}$([x^2,I^*])=$V_2+V_6+V_4$=-1
$\theta_{(4,2)}$([x^2,r])=$V_2+V_6+V_4$=-1
$\theta_{(4,2)}$([x^2,s])=$-V_2+(-V_6)+(-V_4)$=-1
$\theta_{(4,2)}$([x^7,I^*])=-2+(-2)+(-2)=-6
$\theta_{(4,2)}$ ([x^7,r]) =-2+(-2)+(-2)=-6
$\theta_{(4,2)}$ ([x^7,s]) =2+2+2=6
$\theta_{(4,2)}$([x,I^*])=$V_1+V_3+V_5$=1
$\theta_{(4,2)}$ ([x,r]) = $V_3+V_5+ V_1$=1
$\theta_{(4,2)}$([x,s])=$-V_5+(-V_1)+(-V_3)$=1
$\theta_{(4,2)}$([y,I^*])=0
$\theta_{(4,2)}$([y,r])=0
$\theta_{(4,2)}$([y,s])=0 .

(III) If i=3
$\theta_{(4,3)}$ ([I,I^*]) = 4+4+4+4+4+4=24
$\theta_{(4,3)}$ ([I,r])=$2\epsilon_1+2\epsilon_2+2\epsilon_1+2\epsilon_2+2\epsilon_1+2\epsilon_2$=-6
$\theta_{(4,3)}$ ([I, s]) =0
$\theta_{(4,3)}$([x^2,I^*])=$2V_2+2V_2+2V_6 +2V_6 +2V_4 +2V_4$=-4
$\theta_{(4,3)}$([x^2,r])=$\epsilon_1V_2+\epsilon_2V_2+\epsilon_1V_6 +\epsilon_2V_6 +\epsilon_1V_4+\epsilon_2V_4$ =-1
$\theta_{(4,3)}$ ([x^2,s])=0
$\theta_{(4,3)}$ ([x^7,I^*])=-4+(-4)+(-4)+(-4)+(-4)+(-4)=-24
$\theta_{(4,3)}$([x^7,r])=$-2\epsilon_1+(-2\epsilon_2)+(-2\epsilon_1)+(-2\epsilon_2)+(-2\epsilon_1)+(-2\epsilon_2)$=6
$\theta_{(4,3)}$ ([x^7,s])=0
$\theta_{(4,3)}$ ([x,I^*])=$2V_1+2V_1+2V_3+ 2V_3+2V_5+2V_5$=-4
$\theta_{(4,3)}$ ([x,r]) =$\epsilon_1V_1+\epsilon_2V_1+\epsilon_1V_3+\epsilon_2V_3+\epsilon_1V_5+\epsilon_2V_5$=1
$\theta_{(4,3)}$ ([x,s]) = 0
$\theta_{(4,3)}$ ([y,I^*]) = 0
$\theta_{(4,3)}$ ([y,r]) = 0
$\theta_{(4,3)}$ ([y,s])=0 .

Theorem(4.4):

The rational valued characters table of the group $Q_{2m}\times D_5$ when m is a prime number is given as follows: $\equiv^*(Q_{2m}\times D_5) = \equiv^*(Q_{2m}) \otimes \equiv^*(D_5)$

Proof :-

The characters table of D_5 is :

$\equiv D_5=$

CL_α	$[t'_1]$	$[t'_2]$	$[t'_3]$	$[t'_4]$
χ'_1	1	1	1	1
χ'_2	1	1	1	-1
χ'_3	2	ϵ_1	ϵ_2	0
χ'_4	2	ϵ_2	ϵ_1	0

Table(7)

where $t'_1=\{(I^*)\}$, $t'_2=\{ r , r^4\}$,$t'_3 = \{ r^2, r^3 \}$, $t'_4 =\{ s , sr , sr^2, sr^3, sr^4\}$
the rational valued character table of D_5 is equal to:

$\equiv^* D_5 =$

CL_α	$[t'_1]$	$[t'_2]$	$[t'_3]$
θ'_1	1	1	1
θ'_2	1	1	-1
θ'_3	4	-1	0

Table(8)

then,

$\chi'_1(t'_1)=\chi'_1(t'_2) = \chi'_1(t'_3) = \chi'_1(t'_4) = \theta'_1(t'_1) =\theta'_1(t'_2) = \theta'_1(t'_3) = 1$

$\chi'_2(t'_1) =\chi'_2(t'_2)) =\chi'_2(t'_3) = \theta'_2(t'_1) =\theta'_2(t'_2) = 1, \chi'_2(t'_4) = \theta'_2(t'_3)$

$\chi'_3(t'_1) + \chi'_4(t'_1) = \theta'_3(t'_1) = 4, \chi'_3(t'_2) + \chi'_4(t'_2) = \chi'_3(t'_3) + \chi'_4(t'_3) = \theta'_3(t'_2) = -1$

$\chi'_3(t'_4) + \chi'_4(t'_4) = \theta'_3(t'_3) = 0.$

from the definition of $Q_{2m}\times D_5$ and theorem(3.3) $(\equiv Q_{2m}\times D_5) = (\equiv Q_{2m}) \otimes (\equiv D_5)$
each element in $Q_{2m}\times D_5$ $t_{nk}= t_n \cdot t_k$ $\forall t_n\in Q_{2m}$, $t_k\in D_5$, n = 1,2,3,…,4m,
$k\in \{I^*,r,r^2,r^3,r^4,s,sr,sr^2,sr^3,sr^4\}$.
and each irreducible character of $Q_{2m}\times D_5$ is $\chi_{(i,j)} =\chi_i \cdot \chi'_j$
where χ_i is an irreducible character of Q_{2m} and χ'_j is the irreducible character of D_5 ,then

$$\chi_{(i.j)}(t_{nk}) = \begin{cases} \chi_i(t_n) & \text{if } j = 1 \text{ and } k \in D_5 \\ \chi_i(t_n) & \text{if } j = 2 \text{ and } k \in \{I. r. r^2. r^3. r^4\} \\ -\chi_i(t_n) & \text{if } j = 2 \text{ and } k \in \{S. Sr. Sr^2. Sr^3. Sr^4\} \\ 4\chi_i(t_n) & \text{if } j = 3 \text{ and } k \in \{I^*\} \\ -\chi_i(t_n) & \text{if } j = 3 \text{ and } k \in \{r. r^2. r^3. r^4\} \\ 0 & \text{if } j = 3 \text{ and } k \in \{S. Sr. Sr^2. Sr^3. Sr^4\} \end{cases}$$

from proposition (4.1) $\theta_{(i.j)} = \sum_{\sigma\epsilon Gal(Q(\chi_{(i.j)})/Q)}\sigma(\chi_{(i.j)})$

where $\theta_{(i,j)}$is the rational valued character of $Q_{2m}\times D_5$Then,

$$\theta_{(i.j)}(t_{nk}) = \sum_{\sigma\epsilon Gal(Q(\chi_{(i.j)}(t_{nk}))/Q)}\sigma(\chi_{(i.j)}(t_{nk}))$$

(I) If j=1 and $k\in D_5$

$$\theta_{(i.j)}(t_{nk}) = \sum_{\sigma\epsilon Gal(Q(\chi_i(t_n))/Q)}\sigma(\chi_i(t_n)) = \theta_i(t_n)\cdot 1 = \theta_i(t_n)\cdot \theta'_j(t'_k)$$

where θi is the rational valued character of Q_{2m} .

(II) (a) If j=2 and $k\in\{I^*,r,r^2,r^3,r^4\}$

$$\theta_{(i.j)}(t_{nk}) = \sum_{\sigma\epsilon Gal(Q(\chi_i(t_n))/Q)}\sigma(\chi_i(t_n)) = \theta_i(t_n)\cdot 1 = \theta_i(t_n)\cdot \theta'_j(t'_k)$$

(b) If j=2 and $k\in\{s,sr,sr^2,sr^3,sr^4\}$

$$\theta_{(i.j)}(t_{nk}) = \sum_{\sigma\epsilon Gal(Q(\chi_i(t_n))/Q)}\sigma(-\chi_i(t_n)) = -\sum_{\sigma\epsilon Gal(Q(\chi_i(t_n))/Q)}\sigma(\chi_i(t_n))$$
$$= \sum_{\sigma\epsilon Gal(Q(\chi_i(t_n))/Q)}\sigma(\chi_i(t_n))\cdot -1 = \theta_i(t_n)\cdot -1 = \theta_i(t_n)\cdot \theta'_j(t'_k)$$

(III) (a) If j=3 and $k\in\{I^*\}$

$$\theta_{(i.j)}(t_{nk}) = \sum_{\sigma\epsilon Gal(Q(\chi_i(t_n))/Q)}\sigma(4\chi_i(t_n)) = 4\sum_{\sigma\epsilon Gal(Q(\chi_i(t_n))/Q)}\sigma(\chi_i(t_n))$$
$$= \sum_{\sigma\epsilon Gal(Q(\chi_i(t_n))/Q)}\sigma(\chi_i(t_n))\cdot 2 = \theta_i(t_n)\cdot 2 = \theta_i(t_n)\cdot \theta'_j(t'_k)$$

(b) If j=3 and $k\in\{r,r^2,r^3,r^4\}$

$$\theta_{(i.j)}(t_{nk}) = \sum_{\sigma\epsilon Gal(Q(\chi_i(t_n))/Q)}\sigma(-\chi_i(t_n)) = \sum_{\sigma\epsilon Gal(Q(\chi_i(t_n))/Q)}\sigma(\chi_i(t_n))\cdot -1 = \theta_i(t_n)\cdot \theta'_j(t'_k)$$

(c) If j=3 and $k\in\{s,sr,sr^2,sr^3,sr^4\}$

$$\theta_{(i.j)}(t_{nk}) = \sum_{\sigma\epsilon Gal(Q(\chi_i(t_n))/Q)}\sigma(0\cdot\chi_i(t_n)) = 0\cdot\sum_{\sigma\epsilon Gal(Q(\chi_i(t_n))/Q)}\sigma(\chi_i(t_n))$$
$$= \sum_{\sigma\epsilon Gal(Q(\chi_i(t_n))/Q)}\sigma(\chi_i(t_n))\cdot 0 = 0 = \theta_i(t_n)\cdot \theta'_j(t'_k)$$

From [I], [II] and [III] we have $\theta_{(i,j)}= \theta_i\cdot\theta'_j$. Then $\equiv^*(Q_{2m}\times D_5) = \equiv^*(Q_{2m})\otimes\equiv^*(D_5)$

the rational character table of the quaternion group $Q_{2m}\times D_5$ when m = p>2, p is prime number(4.5) from theorem (4,4) and from of $\equiv^*(Q_{2m})$ in the table (6) then the rational character table of the quaternion Group $Q_{2m}\times D_5$ when m = p>2, p is prime number is given in the general (15×15) matrix form $\equiv^*(Q_{2m}\times D_5)$ as table (9)

$\equiv^*(Q_{2m}\times D_5)=$

	$[I,I^*]$	$[I,r]$	$[I,s]$	$[x^2,I^*]$	$[x^2,r]$	$[x^2,s]$	$[x^p,I^*]$	$[x^p,r]$	$[x^p,s]$	$[x,I^*]$	$[x,r]$	$[x,s]$	$[y,I^*]$	$[y,r]$	$[y,s]$
$\theta_{(1,1)}$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$\theta_{(1,2)}$	1	1	-1	1	1	-1	1	1	-1	1	1	-1	1	1	-1
$\theta_{(1,3)}$	4	-1	0	4	-1	0	4	-1	0	4	-1	0	4	-1	0
$\theta_{(2,1)}$	(p-1)	(p-1)	(p-1)	-1	-1	-1	(p-1)	(p-1)	(p-1)	-1	-1	-1	0	0	0
$\theta_{(2,2)}$	(p-1)	(p-1)	-(p-1)	-1	-1	1	(p-1)	(p-1)	-(p-1)	-1	-1	1	0	0	0
$\theta_{(2,3)}$	4(p-1)	-(p-1)	0	-4	1	0	4(p-1)	-(p-1)	0	-4	1	0	0	0	0
$\theta_{(3,1)}$	1	1	1	1	1	1	1	1	1	1	1	1	-1	-1	-1
$\theta_{(3,2)}$	1	1	-1	1	1	-1	1	1	-1	1	1	-1	-1	-1	1
$\theta_{(3,3)}$	4	-1	0	4	-1	0	4	-1	0	4	-1	0	-4	1	0
$\theta_{(4,1)}$	(p-1)	(p-1)	(p-1)	-1	-1	-1	-(p-1)	-(p-1)	-(p-1)	1	1	1	0	0	0
$\theta_{(4,2)}$	(p-1)	(p-1)	-(p-1)	-1	-1	1	-(p-1)	-(p-1)	(p-1)	1	1	-1	0	0	0
$\theta_{(4,3)}$	4(p-1)	-(p-1)	0	-4	1	0	-4(p-1)	(p-1)	0	4	-1	0	0	0	0
$\theta_{(5,1)}$	2	2	2	2	2	2	-2	-2	-2	-2	-2	-2	0	0	0
$\theta_{(5,2)}$	2	2	-2	2	2	-2	-2	-2	2	-2	-2	2	0	0	0
$\theta_{(5,3)}$	8	-2	0	8	-2	0	-8	2	0	-8	2	0	0	0	0

Table(9)

Example (4.3):

To find the rational valued characters table of $Q_{14}\times D_5$, we can use theorem (4.4) since p=7 then we have as table (10)

$\equiv^*(Q_{14}\times D_5)=$

CL_α	$[I,I^*]$	$[I,r]$	$[I,s]$	$[x^2,I^*]$	$[x^2,r]$	$[x^2,s]$	$[x^7,I^*]$	$[x^7,r]$	$[x^7,s]$	$[x,I^*]$	$[x,r]$	$[x,s]$	$[y,I^*]$	$[y,r]$	$[y,s]$
$\lvert CL_\alpha\rvert$	1	4	5	2	8	10	1	4	5	2	8	10	7	28	35
$\lvert C_{Q_{14}\times D_5}(CL_\alpha)\rvert$	280	70	56	140	35	28	280	70	56	140	35	28	40	10	8
$\theta_{(1,1)}$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$\theta_{(1,2)}$	1	1	-1	1	1	-1	1	1	-1	1	1	-1	1	1	-1
$\theta_{(1,3)}$	4	-1	0	4	-1	0	4	-1	0	4	-1	0	4	-1	0
$\theta_{(2,1)}$	6	6	6	-1	-1	-1	6	6	6	-1	-1	-1	0	0	0
$\theta_{(2,2)}$	6	6	-6	-1	-1	1	6	6	-6	-1	-1	1	0	0	0
$\theta_{(2,3)}$	24	-6	0	-4	1	0	24	-6	0	-4	1	0	0	0	0
$\theta_{(3,1)}$	1	1	1	1	1	1	1	1	1	1	1	1	-1	-1	-1
$\theta_{(3,2)}$	1	1	-1	1	1	-1	1	1	-1	1	1	-1	-1	-1	1
$\theta_{(3,3)}$	4	-1	0	4	-1	0	4	-1	0	4	-1	0	-4	1	0
$\theta_{(4,1)}$	6	6	6	-1	-1	-1	-6	-6	-6	1	1	1	0	0	0
$\theta_{(4,2)}$	6	6	-6	-1	-1	1	-6	-6	6	1	1	-1	0	0	0
$\theta_{(4,3)}$	24	-6	0	-4	1	0	-24	6	0	4	-1	0	0	0	0
$\theta_{(5,1)}$	2	2	2	2	2	2	-2	-2	-2	-2	-2	-2	0	0	0
$\theta_{(5,2)}$	2	2	-2	2	2	-2	-2	-2	2	-2	-2	2	0	0	0
$\theta_{(5,3)}$	8	-2	0	8	-2	0	-8	2	0	-8	2	0	0	0	0

Table (10)

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