On IFI$_{S^*}$ g_Continuous Functions in Intuitionistic Fuzzy Ideal Topological Spaces

حول الدوال الحدسية الضبابية المستمرة من النمط I_{S^*} في الفضاءات التبولوجية المثالية الحدسية الضبابية

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Abstract

By using IFI_{S^*} g_closed $sets$ we introduce the notion of IFI_{S^*} $g\_continuous$ functions in intuitionistic fuzzy ideal Topological spaces . we obtain several properties of IFI_{S^*} g – continuity and the relationship between this function and other related functions.

set,**key words and phrases** : Intuitionistic fuzzy local – function , IFI_{S^*} $g-closed$ IFI_{S^*} $g-continuous$, $IFstrong\ I_{S^*}$ $g-continuous$, IFI_{S^*} g-continuous, $IF\ weakly\ I_{S^*}$ $g-continuous$, $IFT_{1/2}$ $space$.

الملخص:

باستخدام المجاميع الحدسية الضبابية IFI_{S^*} g_closed $sets$ في الفضاءات التبولوجية الحدسية الضبابية ذات المثالي الحدسي الضبابي عرفنا مفهوم الدوال المستمرة الحدسية الضبابية IFI_{S^*} $g\_continuous$ وحصلنا على خواص هذه الدوال وارتباطها بالدوال الاخرى ذات العلاقة .

1. Introduction

After the introduction of fuzzy sets by Zadeh in 1965[1] and fuzzy topology by Chang in 1967 [2],there have been a number of generalizations of this fundamental concept.The notion of intuitionistic fuzzy sets introduced by Atanssov in 1983 [3] is one among them .

Using the notion of intuitionistic fuzzy sets Coker [4] introduced the notion of intuitionistic fuzzy topological spaces .

Coker and Demirci [5] introduced the basic definitions and properties of intuitionistic fuzzy topological spaces in Sostak's sense , which is generalized form of " fuzzy topological space " developed by Sostak [6 , 7] .

In 2006 the concepts of fuzzy g – closed sets and fuzzy g – continuous mappings due to Thakur and Malviya [8] was been extended in intuitionistic fuzzy topology space by Thakur and Rekha chaturvedi [9] .

In 2011 Khan and Hamza [10] introduced and investigated the $notion\ of$ I_{S^*} $g-closed$ set in ideal topological spaces as a generalization of I g – closed sets .

In this paper , we introduce the intuitionistic fuzzy I_{S^*} $g-closed$ sets and use it to introduce the intuitionistic fuzzy I_{S^*} $g-$ continuous functions , intuitionistic fuzzy strongly I_{S^*} g-continuous functions and intuitionistic fuzzy weakly I_{S^*} g – continuous functions ,weaker than intuitionistic It turns out that intuitionistic fuzzy weak I_{S^*} $g-continuity$ is

continuity . we obtain several properties of intuitionistic fuzzy fuzzy weak I –and other related functions . I_{S^*} *g – continuity and the relationship between this function*

2 – preliminaries :

Definition 2.1. [1] :

Let X be a non – empty set and $I = [0 , 1]$ be the closed interval of the real numbers . A fuzzy subset μ of X is defined to be membership function $\mu : X \longrightarrow I$,such that $\mu(X) \in I$ for every $x \in X$. The set of all fuzzy subsets of X denoted by I^X .

Definition 2.2. [3] :-

An intuitionistic fuzzy set (IFs , for short) A is an object have the form :

$A = \{< x , \mu_{A(x)} , \nu_{A(x)} > ; x \in X\}$, where the functions $\mu_A : X \to I , \nu_A : X \to I$ denote the degree of membership and the degree of non – membership of each element $x \in X$ to the set A respectively , and $0 \le \mu_A(x) + \nu_A(x) \le 1$, for each $x \in X$. The set of all intuitionistic fuzzy sets in X denoted by IFs (X) .

Definition 2.3. [4] :-

$0_\sim =< x , 0 , 1 >$, $1_\sim =< x , 1 , 0 >$ are the intuitionistic sets corresponding to empty set and the entire universe respectively .

Definition 2.4. [11] :-

Let X be a non – empty set . An intuitionistic fuzzy point (IFP , for short) denoted by $x (\alpha , \beta)$ is an intuitionistic fuzzy set have the form

$x (\alpha , \beta)(y) = \begin{cases} < x , \alpha , \beta > & ; \quad x = y \\ < x , 0 , 1 > & ; \quad x \neq y \end{cases}$, where $x \in X$ is a fixed point , and $\alpha , \beta \in [0 , 1]$ satisfy $\alpha + \beta \le 1$. The set of all IFPs denoted by IFP (x) . If $A \in$ IFs (x) . We say the $x(\alpha , \beta) \in A$ if and only if $\alpha \le \mu_A(x)$ and $\beta \ge \nu_A(x)$, for each $x \in X$.

Definition 2.5. [11] :-

Let $A = \{< x , \mu_A (x) , \nu_A(x) >: x \in X\}$ and $B = \{< x , \mu_B (x) , \nu_A(x) >: x \in X\}$ be two intuitionistic fuzzy sets in X . A is said to be quasi – coincident with B (written AqB) if and only if , there exists an element $x \in X$ such that $\mu_A (x) > \nu_B(x)$ or $\nu_A(x) < \mu_B (x)$, otherwise A is not quasi – coincident with B and denoted by $A\tilde{q}B$.

Definition 2.6. [11] :-

Let $x (\alpha , \beta) \in$ IFP(x) and $A \in$ IFs (x) . We say that $x (\alpha , \beta)$ quasi–coincident with A denoted $x (\alpha , \beta)q$ A if and only if , $\alpha > \nu_A(x)$ or $\beta < \mu_A (x)$, other wise $x (\alpha , \beta)$ is not quasi – coincident with A and denoted by $x (\alpha , \beta)\tilde{q}$ A .

Definition 2.7. [4] :-

An intuitionistic fuzzy topology (IFT , for short) on a nonempty set X is a family τ of an intuitionistic fuzzy set in X such that

(i) $0_\sim , 1_\sim \in \tau$,

(ii) $G_1 \cap G_2 \in \tau$, for any $G_1 , G_2 \in \tau$,

(iii) $\cup G_i \in \tau$, for any arbitrary family $\{ G_i : i \in J \} \subseteq \tau$.

Definition 2.8. [4] :-

Let (X, τ) be an intuitionistic fuzzy topological space and

$A = \{< x, \mu_A(x), \nu_A(x) >, x \in X\}$ be an intuitionistic fuzzy set in X then , an intuitionistic fuzzy interior and intuitionistic fuzzy closure of A are respectively defined by

$int(A) = \cup \{G : G$ is an IFos in X and $G \subseteq A\}$

$cl(A) = \cap \{K : K$ is an IFcs in X and $A \subseteq K\}$.

Proposition 2.9. [4] :-

Let A be an intuitionistic fuzzy set in X , then we have :

(1) A is an intuitionistic fuzzy open set in X if and only if , $A = int(A)$.

(2) A is an intuitionistic fuzzy closed set in X if and only if , $A = cl(A)$.

Proposition 2.10. [4] :-

let(X, τ)be an intuitionistic fuzzy topological space and A ,B be an intuitionistic fuzzy sets in X , then the following properties hold :

(a) $\text{int}(A) \subseteq A$.

(b) $A \subseteq cl(A)$.

(c) $If \ \ A \subseteq B \Rightarrow \text{int}(A) \subseteq \text{int}(B)$.

(d) $If \ \ A \subseteq B \Rightarrow cl(A) \subseteq cl(B)$.

(e) $\text{int}(\text{int}(A)) = \text{int}(A)$.

(f) $cl(cl(A)) = cl(A)$.

(g) $\text{int}(A \cap B) = \text{int}(A) \cap \text{int}(B)$.

(h) $cl(A \cup B) = cl(A) \cup cl(B)$.

(i) $\text{int}(1_\sim) = 1_\sim$.

(j) $cl(0_\sim) = 0_\sim$.

Definition 2.11. [12] :-

A non – empty collection of intuitionistic fuzzy sets L of a set X is called intuitionistic fuzzy ideal on X (IFI , for short) such that :

(i) If $A \in L$ and $B \leq A \Longrightarrow B \in L$ (heredity)

(ii) If $A \in L$ and $B \in L \Longrightarrow A \vee B \in L$ (finite additivity) . If (X, τ) be an IFTS , then the triple (X, τ, L) is called an intuitionistic fuzzy ideal topological (IFITS , for short) .

Definition 2.12. [12] :-

Let (X, τ, L) be an IFITS . If $A \in IFs(X)$. Then the intuitionistic fuzzy local function $A^*(L, \tau)$ (A^* , for short) of A in (X, τ, L) is the union of all intuitionistic fuzzy points $x(\alpha, \beta)$ such that :

$A^*(L, \tau) = \vee \{x(\alpha, \beta) : A \wedge U \notin L$, for every $U \in N(x(\alpha, \beta), \tau)\}$, where

$N(x(\alpha, \beta), \tau)$ is the set of all quasi – neighborhoods of an IFP $x(\alpha, \beta)$ in τ . The intuitionistic fuzzy closure operator of an IFs A is defined by

$cl^*(A) = A \vee A^*$, and $\tau^*(L)$ is an IFT finer than τ generated by $cl^*(\cdot)$ and defined as

$\tau^*(L) = \{A : cl^*(A^C) = A^C\}$

Theorem 2.13. [12] :-

Let (X, τ) be an IFTS and L_1 , L_2 be two intuitionistic fuzzy ideals on X . Then for any intuitionistic fuzzy sets A , B of X . Then the following statements are verified

(i) $A \subseteq B \Longrightarrow A^*(L, \tau) \subseteq B^*(L, \tau)$,

(ii) $L_1 \subseteq L_2 \Longrightarrow A^*(L_2, \tau) \subseteq A^*(L_1, \tau)$.

(iii) $A^* = cl(A^*) \subseteq cl(A)$.

(iv) $A^{**} = A^*$.

(v) $(A \vee B)^* = A^* \vee B^*$.
(vi) $(A \wedge B)^*(L) \leq A^*(L) \wedge B^*(L)$.
(vii) $\ell \in L \Longrightarrow (A \vee \ell)^* = A^*$.
(viii) $A^*(L, \tau)$ is intuitionistic fuzzy closed set .

Theorem 2.14. [12] :-

Let τ_1, τ_2 be two intuitionistic fuzzy topologies on X . Then for any intuitionistic fuzzy ideal L on X , $\tau_1 \leq \tau_2$ implies
(i) $A^*(L, \tau_2) \subseteq A^*(L, \tau_1)$, for every $A \in L$.
(ii) $\tau_1^* \subseteq \tau_2^*$.

Definition 2.15. [12] :-

For an IFTS (X, τ) , $A \in$ IFss . Then A is called
i) Intuitionistic fuzzy dense if $cl(A) = 1_\sim$.
ii) Intuitionistic fuzzy nowhere dense subset if $Int(cl(A)) = 0_\sim$.
iii) Intuitionistic fuzzy codense subset if $Int(A) = 0_\sim$.
v) Intuitionistic fuzzy countable subset if it is a finite or has the some cardinal number .
iv) Intuitionistic fuzzy meager set if it is an Intuitionistic fuzzy countable union of Intuitionistic fuzzy nowhere dense sets .

Definition 2.16 [13] :-

An IFs $A = < x, \mu_A, \nu_A >$ in an IFTS (X, τ) is said to be an
(i) Intuitionistic fuzzy semi – cloed set (IFScs in short) if $int(cl(A)) \subseteq A$. The complement of Intuitionistic fuzzy semi – closed set is said to be Intuitionistic fuzzy semi – open set (IFSos for short) if $A \subseteq cl(int(A))$.
(ii) Intuitionistic fuzzy pre – closed set (IFPcs for short) if $cl(int(A)) \subseteq A$. The complement of Intuitionistic fuzzy pre – closed set is said to be Intuitionistic fuzzy pre – open set (IFPos for short) if $A \subseteq int(cl(A))$.
(iii) Intuitionistic fuzzy $\alpha -$ closed set (IFαcs in short) if $cl\left(int(cl(A))\right) \subseteq A$. The complement of intuitionistic fuzzy $\alpha -$ closed is said to be intuitionistic fuzzy $\alpha -$ open (IFαos for short) if $A \subseteq int\left(cl(int(A))\right)$.

Definition 2.17 [13] :-

An IFs A is an
(i) intuitionistic fuzzy regular closed set (IFRcs for short) if $A = cl(int(A))$. The complement of intuitionistic fuzzy regular closed set is intuitionistic fuzzy regular open set (IFRos for short) if $int(cl(A)) = A$.
(ii) intuitionistic fuzzy generalized closed set (IFg $-$closed for short) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFos . The complement of an IF g $-$closed set is said to be intuitionistic fuzzy generalized open set (IF g $-$open for short) if (X – A) is IF g $-$closed .

Remark 2.18. [9] :-

Every intuitionistic fuzzy closed set (intuitionistic fuzzy open set) is intuitionistic fuzzy g $-$closed set (intuitionistic fuzzy g $-$open set) .

Definition 2.19. [13] :-

An IFTS (X, τ) is said to be $IFT_{1/2}$ space if every IF g $-$closed set in (X, τ) is an IFcs in (X, τ) .

Since (X,τ,L) is an intuitionistic fuzzy ideal topological space and IFs A is subset of X, *then* (A,τ_A,L_A) *is an* intuitionistic fuzzy ideal topological space ,where τ_A *is the relative IF* topology A and $L_A=\{A\cap\ell : \ell\in L\}$.

3 – Main Results

3 . 1 Intuitionistic fuzzy I_{S^*} g – closed sets (IFI_{S^*} g – closed)

In this section we will give the definition of intuitionistic fuzzy I_{S^*} $g-$ closed sets *properties of* IFI_{S^*} $g-closed$ *sets in* (IFI_{S^*} $g-closed$ *for short*) *and we will view the* intuitionistic fuzzy ideal topological spaces (X , τ , L) .

Definition (3 . 1 . 1) : -

An intuitionistic fuzzy A (IFs A) which is subset of an IFTS (X, τ ,L) is said to be IFSos in X. $A\subset U$ *and* U *is* IFI_{S^*} $g-closed$ *if* $A^*\subset U$, *whenever*The complement of IF I_{S^*} $g-closed$ *set is said to be* intuitionistic fuzzy

I_{S^*} $g-open$ (IFI_{S^*} $g-open$ *for short*) *if* $F\subset \text{int}^*(A)$, *whenever*, $F\subset A$

For every intuitionistic fuzzy semi – closed set (IFScs) F in X .

Lemma (3 . 1 . 2) :-

Every intuitionistic fuzzy closed (intuitionistic fuzzy open set) is intuitionistic fuzzy I_{s^*} g − closed set (intuitionistic fuzzy I_{s^*} g − open set) .

Proof :- Let A be intuitionistic fuzzy closed set (IFcs) by remark (2.18)
$\Rightarrow$ A is IF g − closed set
i.e $cl(A)\subseteq U$ wherever $A\subseteq U$ and U is IFos
Since $A^*\subseteq cl(A)$ and $cl(A)\subseteq U$
Therefor $A^*\subseteq cl(A)\subseteq U \Rightarrow A^*\subseteq U$
Now U is IFos . By proposition (2 . 10)
$int(U)\subseteq U \Rightarrow cl(int(U))\subseteq cl(U)$
$\Rightarrow U\subseteq cl(int(U))\subseteq cl(U)$
$\Rightarrow U\subseteq cl(int(U))$
$\Rightarrow$ U is IFSos
$\Rightarrow A^*\subseteq U$, whenever $A\subset U$ and U is IFSos
$\Rightarrow$ A is IFI_{s^*} g −closed set .

Lemma (3 . 1 . 3) :-

Let (X , τ , L) be an intuitionistic fuzzy ideal topological space and $B\subset A\subset X$. *Then* ,

$B^*(I_A,\tau_A)=B^*(I,\tau)\cap A$.

Proof:-the proof is directly conclusion by the properties of the local function .

Lemma (3 . 1 . 4) : -

Intuitionistic fuzzy U is IFos and A is IFI_{S^*} $g-open$, *then* $U\cap A$ *is* IFI_{S^*} $g-open$.

proof:- we prove that $X-(U\cap A)$ *is* IFI_{S^*} $g-closed$

Let $X-(U\cap A)\subset G$, *where* G *is IFSo in* X . *This implies* $(X-U)\cup(X-A)\subset G$

since $(X-A)\subset G$ *and* $(X-A)$ *is* IFI_{S^*} g – closed in X , therefore $(X-A)^*\subset G$

Moreover X – U is IFcs and contained in G , therefore , $(X-U)^*\subseteq cl(X-U)\subset G$, *Hence*

$$(X-(U\cap A)^{*})=((X-U)\cup(X-A))^{*}$$
$$=(X-U)^{*}\cup(X-A)^{*}\subset G .$$

This prove that $U\cap A$ *is* IFI_{S^*} $g-open$.

Theorem (3 . 1 . 5) :-

Let (X , τ , L) be an intuitionistic fuzzy ideal topological space and A , B are two intuitionistic fuzzy subsets of X such that $B\subset A\subset X$. If B is an IFI_{S^*} $g-closed$ *set relative* .to A , where A is IF open set and IFI_{S^*} $g-closed$ *set in* X , then B is IFI_{S^*} g – closed set in X

proof : - *Let* $B\subset G$, *where* G *is an* intuitionistic fuzzy semi – open in X (IFSo) .

Then , $B\subset A\cap G$ *and* $A\cap G$ *is IFSo in* X *and hence* in A . Therefore $B^{*}_{A}\subset A\cap G$.

It follows from lemma (3.1.3) *that* $A\cap B^{*}_{X}\subset A\cap G$. *or* $A\subset G\cup(X-B^{*}_{X})$.

B^{*}_{X} *is IF closed set in* X *and* $G\cup(X-B^{*}_{X})$ *is IFSo in* X since A is IFI_{S^*} $g-closed$ *set in* X $A^{*}_{X}\subset G\cup(X-B^{*}_{X})$ *and hence*

$B^{*}=B^{*}\cap A^{*}\subset B^{*}\cap[G\cup(X-B^{*}_{X})]\subset G$.

Therefore , we obtain $B^{*}_{X}\subset G$.*This proves that* B *is* IFI_{S^*} $g-closed$ *in* X.

Theorem (3 . 1 . 6) :-

Let A be intuitionistic fuzzy semi – open set (IFSos) in an intuitionistic fuzzy ideal topological space (X , τ , L) and $B\subset A\subset X$. if B is IFI_{S^*} $g-closed$ *in* X , then B is IFI_{S^*} $g-closed$ relative to A .

proof: *-Let* $B\subset U$ *where* U *is IFSo in* A . *Then , there exists an IFSo set* V in X. such that $U=A\cap V$. *Thus* $B\subset A\cap V$. *Now* $B\subset V$ *implies* that $B^{*}_{X}\subset V$.

It follows that $A\cap B^{*}_{X}\subset A\cap V$. *By Lemma* (3.1.3) $B^{*}_{A}\subset A\cap V=U$

This proves that B is an IFI_{S^*} g-closed relative to A.

Corollary (3 . 1 . 7) :-

g – closed in IFTS intuitionistic fuzzy open set (IFos) and IFI_{S^*} $B\subset A\subset X$ *and* A *be* (X , τ ,L) .Then B is IFI_{S^*} $g-closed$ *relative to* A if and only if B is IFI_{S^*} $g-closed$ *in* X .

Theorem (3 . 1 . 8) :-

If B is an IF subset of IFTS (X , τ , L) such that $A\subset B\subset A^{*}$ *and* A *is* IFI_{S^*} $g-closed$ *in* X , *then* B *is also* IFI_{S^*} $g-closed$ *in* X .

proof :- Let G be IFSo set in X containing B , then $A\subset G$. since A is IFI_{S^*} g – closed ,therefore $A^{*}\subset G$ *and hence* $B^{*}\subset(A^{*})^{*}\subset A^{*}\subset G$.*This implies that* B *is* IFI_{S^*} $g-closed$ *in* X .

Theorem (3 . 1 . 9) :-

Let $B \subset A \subset X$ *and* suppose that B is $IFI_{S^*}\ g-open$ in X and A is an intuitionistic fuzzy semi – regular set in X (IFSR) , then B *is* $IFI_{S^*}\ g-open$ *relative to* A .

proof : - we prove that A–B is $IFI_{S^*}\ g-closed$ *relative to* A. *Let* $U \in IFSo(A)$ *such that* $(A-B) \subset U$. *Now* $(A-B) \subset (X-B) \subset U \bigcup (X-A)$, *where* $U \bigcup (X-A) \in IFSo(X)$ *because* $A \in IFSR(X)$. Since X-B *is* $IFI_{S^*}\ g-closed$ in X, therefor $(X-B)^*_X \subset U \bigcup (X-A)$ *or* $(X-B)^*_X \bigcap A \subset (U \bigcup (X-A)) \bigcap A \subset U$. By lemma(3.1.3) $(A-B)^*_A = (A-B)^*_X \bigcap A \subset (X-B)^*_X \bigcap A \subset U$ *and hence* $(A-B)^*_A \subset U$.*This proves that* B *is* $IFI_{S^*}\ g-open$ *relative to* A .

Theorem(3.1.10):- *Let* $B \subset A \subset X$, B *is an* $IFI_{S^*}\ g-open$ *in* A *and* A *is IFos in* X . *Then* B *is* $IFI_{S^*}\ g-open$ *in* X .

proof : - Let F be an intuitionistic fuzzy semi – closed (IFSc) subset of B in X . since A is IFo set, therefore $F \in IFSc(A)$. since B is $IFI_{S^*}\ g-open$ in A ,therefore, $F \subset \mathrm{int}^*_A(B) = A \bigcap \mathrm{int}^*_X(B) \subset \mathrm{int}^*_X(B)$ .This proves that B is $IFI_{S^*}\ g-open$ *in* X .

3 . 2 Intuitionistic fuzzy I_{S^*}g – continuous functions (IFI_{S^*} g – continuous functions)

In this section we will introduce the definition of intuitionistic fuzzy $I_{S^*}\ g-continuous$ Function ($IFI_{S^*}\ g-continuous$ *for short*) in intuitionistic fuzzy ideal topological space and its properties and the relationship between this function and other related functions .

Definition (3 . 2 . 1) :-

Afunction $f:(X,\tau,L) \longrightarrow (y,\Omega,J)$, *where* (X,τ,L) *is IFTS with IF ideal* L *on* X *and IFTS* (y,Ω,J) *is IFTS with IF ideal* J *on* Y , is said to be intuitionistic fuzzy weakly I – continuous (IF weakly I – continuous for short) if for each $x \in X$ *and each* IFos V in Y containing f (x) , there exists an IFos U containing x such that $f(U) \subset cl^*(V)$.

Definition (3 . 2 . 2) :-

A function $f:(X,\tau,L) \longrightarrow (y,\Omega)$ is said to be intuitionistic fuzzy $I_{S^*}\ g-continuous$ ($IFI_{S^*}\ g-continuous$ *for short*) if for every $U \in \Omega$, $f^{-1}(U)$ is $IFI_{S^*}\ g-open$ *in* (x,τ_X,L) .

Definition (3 . 2 . 3) :-

A function $f:(X,\tau) \longrightarrow (y,\Omega,J)$ *is said to be* intuitionistic fuzzy *strongly* I_{S^*} g – continuous (IF strongly $I_{S^*}\ g-continuous$ *for short*) *if for* every $IFI_{S^*}\ g-open$ set U in Y , $f^{-1}(U)$ is IF open in X.

Definition (3 . 2 . 4) :-

intuitionistic fuzzy weakly *A function* $f:(X,\tau,L)\longrightarrow(y,\Omega,J)$ *is said to be* I_{S^*} *g−continuous* (*IF weakly* I_{S^*} *g−continuous for* short) if for each $x\in X$ and each intuitionistic fuzzy open set V in Y containing f(x) , there exists an IFI_{S^*} *g−open* set U containing x such that *such that* $f(U)\subset cl^*(V)$.

Lemma (3 . 2 . 5) :-

By the above definitions , for a function $f:(X,\tau,L)\longrightarrow(y,\Omega,L)$
We obtain following implications :

$$\begin{array}{ccccc} IF\ strong\ I_{S^*}\ g-continuity & \Rightarrow & IF\ continuity & \Rightarrow & IFI_{S^*}\ g-continuity \\ & & \Downarrow & & \Downarrow \\ & & IF\ weak\ I-continuiy & \Rightarrow & IF\ weak\ I_{S^*}\ g-continuity \end{array}$$

Remark (3 . 2 . 6) :-

IFI_{S^*} *g−continuity and IF weak I−continuity are independent* of each other .

Theorem (3 . 2 . 7) :-

An intuitionistic fuzzy ideal topological space (X , τ , L) is said to be intuitionistic fuzzy T – dense (IFT – dense for short) if every IF subset A of X is *– dense in itself (i.e $A\subset A^*$) Proof: it is clear

Definition (3 . 2 . 8) :-

called an *N be an IF subset of an IFTS* (X,τ,L) *and* $x\in X$ *. Then , N is* Let IFI_{S^*} *g− open neighborhood of X if there exists an* IFI_{S^*} *g−open set U containing x such that* $U\subset N$.

Theorem (3 . 2 . 9) :-

Let (X,τ,L) *be IFT−dense . Then, for a function* $f:(X,\tau,L)\longrightarrow(y,\Omega)$ *the following are equivalent* :statement

1− *f is* IFI_{S^*} *g−continuous* .

2− *For each* $x\in X$ *and each IF open set V in Y with* $f(x)\in V$ *,there exists an* IFI_{S^*} *g−open set U containing x such that* $f(U)\subset V$.

3− *for each* $x\in X$ *and each IF open set V in Y with* $f(x)\in V$, $f^{-1}(v)$ *is an* IFI_{S^*} *g−open neighborhood of X* .

proof :- $(1)\Rightarrow(2)$
intuitionistic fuzzy open set (IFos) in Y such that $f(x)\in V$. *Let* $x\in X$ *and Let V be an*
Since f is an IFI_{S^*} *g−continuous* , $f^{-1}(V)$ *is* IFI_{S^*} *g−open in X . By putting*
$f(U)\subset V$. $U=f^{-1}(V)$, *we have* $x\in U$ *and*
$(2)\Rightarrow(3)$

$an\ IFI_{S^*}\,g-open$ exists $Let\ V\ be\ an\ IFo\ set\ in\ Y\ and\ let\ f(x)\in V$. $Then\ by$ (2) , $there$

$set\ U\ containing\ x\ such\ that\ f(U)\subset V$, $so\ x\in U\subset f^{-1}(v)$. $Hence\ f^{-1}(v)\ is\ an$

of X . $IFI_{S^*}\,g-open\ neighborhood$

$(3)\Rightarrow(1)$

Let V be an intuitionistic fuzzy open set in Y and let $f(x)\in V$. $Then\ by$ (3) , $f^{-1}(V)\ is\ an$

$IFI_{S^*}\,g-neighborhood\ of\ x\ thus\ for\ each\ x\in f^{-1}(V)$, $there\ exists\ an\ IFI_{S^*}\,g-open$

$set\ U_X\ containing\ x\ such\ that\ x\in U_X\subset f^{-1}(V)\ Hence\ f^{-1}(V)=U_{x\in f^{-1}(v)}U_x$

$IFI_{S^*}g-open\ in\ X\ .\Rightarrow f^{-1}(V)\ is\ an$

Theorem (3 . 2 . 10) :-

A function $f:(X,\tau)\longrightarrow(y,\Omega,J)$ $is\ IF\ strongly\ I_{S^*}\,g-containuous$ if and only if the inverse image of every $IFI_{S^*}\,g-closed\ set\ in\ Y\ is$ IF closed in X .

proof :by using the definition of $IFI_{S^*}\,g-closed\ set$,we can proof it directly.

Theorem (3 . 2 . 11) :-

$Let\ f:(X,\tau,L)\longrightarrow(y,\Omega)\ be\ IFI_{S^*}\,g-continuous\ and\ U\in IFRo(x)$. Then the restriction

$f/U:(U,T_U,L_U)\longrightarrow(Y,\Omega)\ is\ IFI_{S^*}$ g – continuous .

proof :- let V be any intuitionistic fuzzy open set of (Y, τ_Y) since f is $IFI_{S^*}\ g-continuous$,

$f^{-1}(V)\ is\ IFI_{S^*}g-open\ in\ X$, $f^{-1}(V)\bigcap U\ is\ IFI_{S^*}\ g-open\ in\ X$ .Thus by theorem(3.1.9)

$(f/U)^{-1}(V)=f^{-1}(V)\bigcap U\ is\ IFI_{S^*}\,g-open\ in\ U\ because\ U\ is\ IF\ regular-open\ in\ X$.

. This proves that $f/U:(U,\tau/U,\mathrm{I}/U)\longrightarrow(Y,\tau_Y)\ is\ IFI_{S^*}\,g-$ continuous

Theorem (3 . 2 . 12) :-

Let $f:(X,\tau,L)\longrightarrow(y,\Omega,J)$ $be\ function\ and$ $\{U_\alpha:\alpha\in\nabla\}$ be an intuitionistic fuzzy open cover of IFT – dense space X . If the restriction $f/U_\alpha\ is\ IFI_{S^*}\,g-continuous\ for\ each$ $\alpha\in\nabla$, $then\ f\ is\ IFI_{S^*}$ g – continuous .

proof:- suppose F is an arbitrary IFos in (Y,Ω,J). $Then\ for\ each\ \alpha\in\nabla$, $we\ have$

$(f/U_\alpha)^{-1}(V)=f^{-1}(V)\bigcap U_\alpha$. Because $f/U_\alpha\ is\ IFI_{S^*}\,g-continuous$, therefore $f^{-1}(V)\bigcap U_\alpha$ is $IFI_{S^*}\,g-open\ in\ X\ for\ each\ \alpha\in\nabla$. Since for each $\alpha\in\nabla$, U_α is IFo in X , by theorem (3 . 1 .10), $f^{-1}(V)\cap U_\alpha\ is\ IFI_{S^*}\,g-open$ in X .Now since X is IFT-dense ,

$U_{\alpha\in\nabla}f^{-1}(V)\bigcap U_\alpha=f^{-1}(V)\ is\ IFI_{S^*}$ g-open in X. This implies f is $IFI_{S^*}\,g-continuous$.

Theorem (3 . 2 . 13) :-

If (X,τ,L) *is an IFT − dense and* $f:(X,\tau,L)\longrightarrow(y,\Omega)$ *is* IFI_{S^*} g − continuous, then graph function $g:X\longrightarrow X\times Y$ *defined by* $g(x)=(x,f(x))$ *for each* $x\in X$, *is* IFI_{S^*} *g − continuous* .

proof :- let $x\in X$ *and W any IFos in* $X\times Y$ *containing* $g(x)=(x,f(x))$.
Then there exists a basic IF open set U×V such that $g(x)\subset U\times V\subset W$. *Since f is* IFI_{S^*} *g − continuous , there exists an* IFI_{S^*} *g − open set* U_1 *in X containing x.* *Such that* $f(U_1)\subset V$. *By Lemma* (3.1.4) $U_1\cap U$ *is* IFI_{S^*} *g − open in X and we have* $x\in U_1\cap U\subset U$ *and* , $g(U_1\cap U)\subset U\times V\subset W$. *Since X is IFT − dense,* therefor by theorem(3 . 2 . 9) , g is IFI_{S^*} *g − continuous* .

Theorem (3 . 2 . 14) :-

A function $f:(X,\tau,L)\longrightarrow(y,\Omega)$ *is* IFI_{S^*} *g − continuous if the* graph function $g:X\longrightarrow X\times Y$ *is* IFI_{S^*} *g − continuous* .

proof :- let V be an IFos in Y containing f (x) . then $X\times V$ *is an IFos in* $X\times Y$ *and by the* IFI_{S^*} *g − continuity of g , there exists an* IFI_{S^*} g-open set U in X containing x such that $g(U)\subset X\times V$. *Therefore* , we obtain $f(U)\subset V$. *This shows that f is* IFI_{S^*} *g − continuous* .

Theorem (3 . 2 . 15) :-

Let $\{X_\alpha:\alpha\in\nabla\}$ *be any family of* intuitionistic fuzzy topological spaces . If $f:(X,\tau,L)\longrightarrow\Pi_{\alpha\in\nabla}X_\alpha$ *is an* IFI_{S^*} *g − continuous function , then* $p_\alpha\circ f:X\longrightarrow X_\alpha$ *is* IFI_{S^*} *g − continuous for each* $\alpha\in\nabla$, *where* p_α is the projection of ΠX_α *on to* X_α .

proof : - we will consider a fixed $\alpha_0\in\nabla$. *let* G_{α_0} *be an IFos of* X_{α_0} .
then $(P_{\alpha_0})^{-1}(G_{\alpha_0})$ *is IFo in* ΠX_α . *Since f is* IFI_{S^*} *g − continuous* ,
$f^{-1}((P_{\alpha_0})^{-1}(G_{\alpha_0}))=(P_{\alpha_0}\circ f)^{-1}(G_{\alpha_0})$ *is* IFI_{S^*} *g − open in X* . *Thus* $P_\alpha\circ f$ *is* IFI_{S^*} *g − continuous*.

Corollary (3 . 2 . 16) :-

for any bijective function $f:(X,t)\longrightarrow(y,\Omega,J)$, *the following* are equivalent .

1 − $f^{-1}:(y,\Omega,J)\longrightarrow(X,\tau)$ *is* IFI_{S^*} *g − continuous* .

2 − $f(U)$ *is* IFI_{S^*} *g − open in Y for every IFo set U in X* .

3 − $f(U)$ *is* IFI_{S^*} *g − closed in Y for every IFc set U in X* .

proof :- It is clear.

Definition (3 . 2 . 17) :-

An intuitionistic fuzzy ideal topological space (X , τ , L) is an IFRI space , if for each x ϵ X and each intuitionistic fuzzy open neighborhood V of x, there exists an intuitionistic fuzzy open neighborhood U of x *such that* $x \in U \subset cl^*(U) \subset V$.

Theorem (3 . 2 . 18) :-

let (Y, Ω, J) *be an IFRI − space and* (X, l, τ) *be an IFT − dense*. Then

$f:(X, \tau, L) \longrightarrow (y, \Omega, J)$ *is IF weak* $I_{S^*}g$ *− continuous*, *if* and *only if* f *is*

$IFI_{S^*}g$ *− continuous* .

proof :- The sufficiency is clear .

Necessity . Let $x \in X$ *and* V *be an IFo set of* Y containing f(x).Since Y is an IFRI-space ,there exist an IFo set W of Y such that $f(x) \in W \subset cl^*(w) \subset V$, *Since* f *is IF weakly* $I_{S^*}g-$ Continuous, *there exists an* $IFI_{S^*}g-$open set U such that $x \in U$ and $f(U) \subset cl^*(W)$. Hence we obtain that $f(U) \subset cl^*(w) \subset V$. *By theorem* $(3,2,10)$ f *is* $IFI_{S^*}g-$*continuous*.

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