Q(QS)-algebras Defined by Fuzzy Points
الجبريات Q (QS) المعرفة على النقاط الضبابية

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بحث مستـــل

Abstract:

In this paper, recall the definitions of F-ideal and fuzzy F-ideal of QS-algebras, Q-ideal and fuzzy Q-ideal of Q-algebras. We discussed some of the relationships and characteristics associated and it is hiring some examples to illustrate unlike some theorems,

Also, we introduce a new ideas of fuzzy F-ideal of fuzzy point on QS-algebras and fuzzy Q-ideal of fuzzy point on Q-algebras and we give some properties and theorem of its.

The introduction of the concept of the normal fuzzy F-ideal of fuzzy point on QS-algebra and normal fuzzy Q-ideal of fuzzy point on Q-algebra and we study some of the properties related thereto,

ملخص:

في هذا البحث، قدمنا تعاريف -F المثالي والضبابي-F المثالي الأعلى لـ-QS الجبر، -Q-المثالي والضبابي-Q المثالي الأعلى لـ-Qالجبر. وناقشنا بعض العلاقات والخصائص المرتبطة بها، وذلك تم توظيف بعض الأمثلة لتوضيح عكس بعض النظريات،

أيضا، قمنا بتقديم الأفكار الجديدة من ضبابي-F مثالية من نقطة ضبابية على-QS الجبر وضبابي-Q مثالية من نقطة ضبابي على-Q الجبر ونعطي بعض الخصائص ونظرية من فيها.

قمنا بإدخال المفهوم العادي ضبابي-F مثالية من نقطة ضبابية على-QS الجبر وطبيعية ضبابي-Q مثالية من نقطة ضبابية على-Q الجبر ودرسنا بعض الخصائص المتعلقة بها،

Keywords:

QS-algebras, F-ideal, fuzzy F-ideal, fuzzy F-ideal of fuzzy point, Q-algebras, Q-ideal, fuzzy Q-ideal, fuzzy Q-ideal of fuzzy point, homomorphism of Q(QS)-algebra on fuzzy Q(F)-ideal, Q(QS)-sub-algebra of fuzzy point on Q(QS)-algebra, fuzzy Q(F)-ideal of fuzzy point on Q(QS)-algebra , normal fuzzy Q(F)-ideal of fuzzy point on Q(QS)-algebra, homomorphism of fuzzy Q(QS)-algebra on normal fuzzy Q(F)-ideal.

2000 Mathematics Subject Classification: 06F35, 03G25, 08A72.

Introduction

In 1999, S.S. Ahn and Kim H.S. were introduced the class of QS-algebras and give some properties of QS-algebras [2] and described connections between such QS-sub-algebra and congruences, see [4]. In 2006, A.B. Saeid considered the fuzzification of QS-sub-algebra to QS-algebras [10]. In 2012, M.A. Al-Kadhi, was introduce the F-ideals and fuzzy F-ideals [1].

In 2001, J. Neggers, S.S. Ahn and H.S. Kim [8] introduced the class of Q-algebras which is a generalization of BCK(BCI)-algebras. In 2009, S.M. Mostafa, M. A.Abd-Elnaby and O.R. Elgendy [7] applied the concept of fuzzy subset to Q-algebras.

In this paper, we introduced a definition of the Q(F)-ideal , fuzzy Q(F)-ideal and fuzzy Q(F)-ideal of fuzzy point. We study some of the related properties, homomorphism fuzzy Q(F)-ideal on fuzzy point, normal fuzzy Q(F)-ideal on fuzzy point, homomorphism normal fuzzy Q(F)-ideal of fuzzy point on Q(QS)-algebras .

1. Preliminaries

In this section ,we study the definition of QS-algebra ,QS-sub-algebra , F-ideals, fuzzy QS-sub-algebra , fuzzy F-ideals of QS-algebras and we give some properties of it. We study the definition of Q-algebra , Q-sub-algebra , Q-ideals, fuzzy Q-sub-algebra , fuzzy Q-ideals of Q-algebras and we give some properties of it.

Definition 1.1([2],[4]) :

Let (X∗;, 0) be a set with a binary operation (∗) and a constant (0).Then (X; ∗, 0) is called **a QS-algebra** if it satisfies the following axioms: for all x, y ,z∈X,

1 . $x * x = 0$,
2. $x * 0 = x$,
3. $(x * y) * z = (x * z) * y$,
4. $(x * z) * (x * y) = (y * z)$.

For brevity we also call X a QS-algebra, we can define a binary relation (≤) by putting $x \le y$ if and only if $x * y = 0$, as [6].

Proposition 1.2 ([3],[4]):

Let (X;∗,0) be a QS-algebra, then the following hold: for any x, y, z ∈ X,

(a) if $x * y = z$, then $x * z = y$,
(b) $x * y = 0$ implies $x = y$,
(c) $0 * (x * y) = y * x$,
(d) $x * (0 * y) = y * (0 * x)$.

Example 1.3 ([2]):

Let X ={0, a, b , c} in which (∗) be defined by the following table:

∗	0	a	b	c
0	0	c	b	a
a	a	0	c	b
b	b	a	0	c
c	c	b	a	0

Then (X;∗,0) is a QS-algebra .

Definition 1.4 ([9],[10]):

Let (X; ∗, 0) be a QS-algebra X and S be a nonempty subset of X. Then S is called a **QS-sub-algebra** of X if , $x * y \in$S, for any x, y ∈ S.

Definition 1.5 ([9],[10]):

Let X be a QS-algebra. A fuzzy subset μ of X is said to be **a fuzzy QS-sub-algebra** of X if it satisfies: $\mu(x * y) \ge \min\{\mu(x), \mu(y)\}$, for all x , y ∈ X.

Definition 1.6 ([1]):

Let (X; ∗, 0) be a QS-algebra and I be a non empty subset of X. I is called **a F-ideal of X** if it satisfies:

i. 0∈I
ii. $(x * y) * z \in I$ and $y \in I$ imply $x * z \in I$, for all x, y , z ∈ X.

Definition 1.7 ([1]):

Let X be a QS-algebra. A fuzzy subset μ of X is said to be **a fuzzy F-ideal of X** if it satisfies:

(1) $\mu(0) \geq \mu(x)$, for all x ∈X,

(2) $\mu(x * z) \geq \min\{\mu((x * y) * z), \mu(y)\}$, for all x, y, z ∈ X .

Proposition 1.8 ([1]):

Let (X; ∗, 0) be a QS-algebra, then every fuzzy F-ideal of QS-algebra is a fuzzy QS-sub-algebra.

Definition 1.9([5],[11]):

If ζ is the family of all fuzzy subsets on a BCK-algebra X , $x_\alpha \in \zeta$ is called **a fuzzy point** if and only if, there exists α ∈ (0,1] such that for all y ∈ X ,

$$x_\alpha(y) = \begin{cases} \alpha & \text{if } x = y \\ 0 & \text{otherwise} \end{cases}$$

Remark 1.10([3],[5]) :

If (X; ∗,0) is a BCK-algebra and $FP_q(X)$ denote the set of all fuzzy points of X,then $(FP_q(X), \odot, 0_q)$ is a BCK-algebra, which is called a fuzzy point BCK-algebra, where the value q , (0 < q ≤ 1).

Definition 1.11([3],[5]) :

For a fuzzy subset μ of a BCK-algebra X, we define the set FP(μ) of all fuzzy points of X covered by μ to be the set $FP(\mu) = \{x_\alpha \in FP(X) \mid \mu(x) \geq \alpha, 0 < \alpha \leq 1\}$, and

$FP_q(\mu) = \{x_q \in FP_q(X) \mid \mu(x) \geq q\}$, for q ∈ (0, 1] , x∈X .

Definition 1.12:

Let (X;∗,0) be a QS-algebra , the set of all **fuzzy points on X** denote by

$FP(X) = \{x_\alpha \mid x \in X, \alpha \in (0, 1]\}$. Define a binary operation (⊙) on FP(X) by :

$x_\alpha \odot y_\beta = (x * y)_{\min\{\alpha,\beta\}}$, for all $x_\alpha, y_\beta \in FP(X)$, then

$(QS_{1'})$: $x_\alpha \odot x_\alpha = 0_\alpha$,

$(QS_{2'})$: $x_\alpha \odot 0_\alpha = x_\alpha$,

$(QS_{3'})$: $(x_\alpha \odot y_\beta) \odot z_\gamma = (x_\alpha \odot z_\gamma) \odot y_\beta$,

$(QS_{4'})$: $(x_\alpha \odot z_\gamma) * (x_\alpha \odot y_\beta) = (y_\beta \odot z_\gamma)$.

In X we can define a binary relation (≤) by : $x_\alpha \leq y_\beta$ if and only if, $x_\alpha \odot y_\beta = 0_{\min\{\alpha,\beta\}}$.

Now, we give some properties and theorems of QS-algebras.

Theorem 1.13:

If (X;∗,0) is a QS-algebra, then the following hold: for all $x_\alpha, y_\beta, z_\gamma \in FP(X)$,

a) $x_\alpha \odot 0_\beta = x_{\min\{\alpha,\beta\}}$.

b) $(x_\alpha \odot y_\beta) \odot x_\alpha = 0_\alpha \odot y_\beta$,

c) $0_\alpha \odot (x_\alpha \odot y_\beta) = y_\beta \odot x_\alpha$,

d) $x_\alpha \odot 0_\alpha = 0_\alpha$ implies that $x_\alpha = 0_\alpha$,

e) $x_\alpha = (x_\alpha \odot 0_\alpha) \odot 0_\alpha$,

f) $(x_\alpha \odot y_\beta) \odot 0_{\min\{\alpha,\beta\}} = (x_\alpha \odot 0_\alpha) \odot (y_\beta \odot 0_\alpha)$,

g) $z_\gamma \odot x_\alpha = z_\gamma \odot y_\beta$ implies that $0_\gamma \odot x_\alpha = 0_\gamma \odot y_\beta$.

Proof:

a) It is clear by $(QS_{2'})$.

b) $(x_\alpha \odot y_\beta) \odot x_\alpha = (x_\alpha \odot x_\alpha) \odot y_\beta = 0_\alpha \odot y_\beta$.

c) $0_\alpha \odot (x_\alpha \odot y_\beta) = (x_\alpha \odot x_\alpha) \odot (x_\alpha \odot y_\beta) = y_\beta \odot x_\alpha$, by $(QS_{1'})$ and $(QS_{4'})$..

(d), (e) and (f) are clears by $(QS_{2'})$.

g) $0_\gamma \odot x_\alpha = (z_\gamma \odot z_\gamma) \odot x_\alpha = (z_\gamma \odot x_\alpha) \odot z_\gamma = (z_\gamma \odot y_\beta) \odot z_\gamma = (z_\gamma \odot z_\gamma) \odot y_\beta = 0_\gamma \odot y_\beta$. ⌂

Proposition 1.14:

Let (X; $*$,0) be a QS-algebra . Then the following holds: for any $x_\alpha, y_\beta, z_\gamma \in FP(X)$,

1. $x_\alpha \odot y_\beta \leq z_\gamma$ imply $x_\alpha \odot z_\gamma \leq y_\beta$,
2. $x_\alpha \leq y_\beta$ implies that $z_\gamma \odot x_\alpha \leq z_\gamma \odot y_\beta$,
3. $x_\alpha \leq y_\beta$ implies that $x_\alpha \odot z_\gamma \leq y_\beta \odot z_\gamma$,
4. $(x_\alpha \odot z_\gamma) \odot (y_\beta \odot z_\gamma) \leq (y_\beta \odot x_\alpha)$.

Proof:

1. It follows from ($QS_{4'}$) .
2. By ($QS_{1'}$), we obtain $[(z_\gamma \odot y_\beta) \odot (z_\gamma \odot x_\alpha)] = (x_\alpha \odot y_\beta)$, but $x_\alpha \leq y_\beta$ implies $x_\alpha \odot y_\beta = 0_{\min\{\alpha,\beta\}}$, then we get $(z_\gamma \odot y_\beta) \odot (z_\gamma \odot x_\alpha) = 0_{\min\{\alpha,\beta\}}$.
   i.e., $z_\gamma \odot x_\alpha \leq z_\gamma \odot y_\beta$.
3. It is clear.
4. By ($QS_{3'}$) , ($QS_{4'}$) and ($QS_{1'}$) , we have $[(y_\beta \odot z_\gamma) \odot (x_\alpha \odot z_\gamma)] \odot (y_\beta \odot x_\alpha)$
   $= [(y_\beta \odot z_\gamma) \odot (y_\beta \odot x_\alpha)] \odot (x_\alpha \odot z_\gamma) = (x_\alpha \odot z_\gamma) \odot (x_\alpha \odot z_\gamma) = 0_{\min\{\alpha,\gamma\}}$.

Thus $(x_\alpha \odot z_\gamma) \odot (y_\beta \odot z_\gamma) \leq (y_\beta \odot x_\alpha)$. ⌂

Definition 1.15 [8] :

Let (X;$*$,0) be an algebra with a single binary operation ($*$). Then (X;$*$,0) is called **a Q-algebra** if it satisfies the following axioms : for any x , y , z $\in$X ,

(Q_1) : $x * x = 0$,

(Q_2) : $x * 0 = x$,

(Q_3) : $(x * y) * z = (x * z) * y$.

For brevity we also call X a Q-algebra, we can define a binary relation ($\leq$) by putting $x \leq y$ if and only if $x * y = 0$.

Proposition 1.16 [8]:

Let (X;$*$,0) be a Q-algebra, then the following hold: for any x, y, z $\in$ X,

(Q_1) $\left(x * (x * y)\right) * y = 0$.

(Q_2) $\left((x * z) * \left((x * z) * y\right)\right) * y = 0$.

Example 1. 17 [11] :

Let X = {0, 1, 2 } be a set with a binary operation($*$) defined by the following table:

$*$	0	1	2
0	0	2	1
1	1	0	2
2	2	1	0

Then (X;$*$,0) is a Q-algebra.

Definition 1.18 ([7]):

Let (X; $*$, 0) be a Q-algebra X and S be a nonempty subset of X. Then S is called **a Q- sub-algebra** of X if , $x * y \in$ S, for any x, y $\in$ S.

Definition 1.19 ([7]):

Let X be a Q-algebra. A fuzzy subset μ of X is said to be a **fuzzy Q-sub-algebra of X** if it satisfies: $\mu(x * y) \geq \min\{\mu(x), \mu(y)\}$, for all x, y$\in$ X.

Definition 1.19 ([8]):

Let (X; $*$, 0) be a Q-algebra and I be a nonempty subset of X. I is called **a Q-ideal of X** if it satisfies:

i. $0 \in I$

ii. $(x * y) * z \in I$ and y $\in$ I imply $(x * z) \in$ I, for all x, y, z $\in$ X .

Definition 1.20 ([8]):

Let X be a Q-algebra. A fuzzy subset μ of X is said to be a **fuzzy Q-ideal of X** if it satisfies:

i. $\mu(0) \geq \mu(x)$, for all $x \in X$,

ii. $\mu(x * z) \geq \min\{\mu((x * y) * z), \mu(y)\}$, for all x, y, z ∈ X .

Proposition 1.21 ([8]):

Let (X; ∗, 0) be a Q-algebra, then every fuzzy Q-ideal of Q-algebra is a fuzzy Q-sub-algebra.

Definition 1. 22:

Let (X;∗,0) be a Q-algebra , the set of all fuzzy points on X denote by

FP(X) = { x_α | x ∈X, α ∈ (0, 1]}. Define a binary operation (⊙) on FP(X) by :

$x_\alpha \odot y_\beta = (x * y)_{\min\{\alpha,\beta\}}$, for all x_α, y_β, z_γ ∈ FP(X) , then

($Q_{1'}$) : $x_\alpha \odot x_\alpha = 0_\alpha$,

($Q_{2'}$) : $x_\alpha \odot 0_\alpha = x_\alpha$,

($Q_{3'}$) : $(x_\alpha \odot y_\beta) \odot z_\gamma = (x_\alpha \odot z_\gamma) \odot y_\beta$

In X we can define a binary relation (≤) by : $x_\alpha \leq y_\beta$ if and only if , $x * y = 0$

Remark 1.23:

If (X; ∗,0) is a Q-algebra and $FP_q(X)$ denote the set of all fuzzy points of X, then ($FP_q(X)$,⊙, 0_q) is a Q-algebra, which is called a fuzzy point Q-algebra, where the value q, (0 < q ≤ 1).

Definition 1.24 :

For a fuzzy subset μ of a Q-algebra X, we define the set FP(μ) of all fuzzy points of X covered by μ to be the set FP(μ) = {x_α∈ FP(X) | μ(x) ≥ α, 0 < α ≤ 1}, and

$FP_q(\mu)$ = {x_q∈ $FP_q(X)$ | μ(x) ≥ q} , for q ∈ (0, 1] , x∈X .

Now, we give some properties and theorems of Q-algebras.

Theorem 1.25:

If (X;∗,0) is a Q-algebra, then the following hold: for all x_α, y_β, z_γ∈FP(X),

a) $x_\beta \odot 0_\alpha = x_{\min\{\alpha,\beta\}}$.

b) $(x_\alpha \odot y_\beta) \odot x_\alpha = 0_\alpha \odot y_\beta$,

c) $0_\alpha \odot x_\alpha = 0_\alpha$ implies that $x_\alpha = 0_\alpha$,

d) $z_\gamma \odot x_\alpha = z_\gamma \odot y_\beta$ implies that $0_\gamma \odot x_\alpha = 0_\gamma \odot y_\beta$.

Proof:

a) It is clear by ($Q_{2'}$).

b) $(x_\alpha \odot y_\beta) \odot x_\alpha = (x_\alpha \odot x_\alpha) \odot y_\beta = 0_\alpha \odot y_\beta$,

c) It is clear.

d) $0_\gamma \odot x_\alpha = (z_\gamma \odot z_\gamma) \odot x_\alpha = (z_\gamma \odot x_\alpha) \odot z_\gamma = (z_\gamma \odot y_\beta) \odot z_\gamma = (z_\gamma \odot z_\gamma) \odot x_\alpha = 0_\gamma \odot y_\beta$. ⌂

Proposition 1.26 :

If (X;* ,0) is a Q-algebra, then $(x_\alpha \odot (x_\alpha \odot y_\beta)) \odot y_\beta = 0_{\min\{\alpha,\beta\}}$, for all x_α, y_β∈FP(X), and $(\alpha, \beta) \in (0,1]$

Proof:

$(x_\alpha \odot (x_\alpha \odot y_\beta)) \odot y_\beta = (x_\alpha \odot y_\beta) \odot (x_\alpha \odot y_\beta) = 0_{\min\{\alpha,\beta\}}$, by ($Q_{3'}$) and ($Q_{1'}$).

The following example shows that a fuzzy point Q-algebra may not satisfy the associative law.

Example 1.27 :

Let X : = {0,1,2} with the table as follows:

$*$	0	1	2
0	0	2	1
1	1	0	2
2	2	1	0

Then X is a fuzzy point Q-algebra, but associativity does not hold , since
$(0_p \odot 1_q) \odot 2_p = 0_{\min\{p,q\}} \neq 1_{\min\{p,q\}} = (0_p \odot (1_p \odot 2_q))$, for all {p, q}∈ (0,1]

Theorem 1.28:

Every fuzzy point Q-algebra X satisfying the associative law is a group under the operation " $\odot$ ".

Proof:

Putting $x_\alpha, y_\beta, z_\gamma \in FP(X)$ and $x_\alpha = y_\beta = z_\gamma$ in the associative law
$(x_\alpha \odot y_\beta) \odot z_\gamma = x_\alpha \odot (z_\gamma \odot y_\beta)$ and using ($Q_{1'}$) and ($Q_{2'}$), we obtain
$0_\alpha \odot x_\alpha = x_\alpha \odot 0_\alpha = x_\alpha$. This means that 0_α is the zero element of X. by ($Q_{1'}$), every element x of X has as its inverse the element x itself. Therefore (X;* ,0) is a group. ⌂

2. Fuzzy point Q(QS)-sub-algebras of Q(QS)-algebra

In this section, we introduce the concept of fuzzy point Q(QS)-sub-algebra of FP(μ) and give some examples and properties of its .

Definition 2.1 :

A subset S of X. FP(X) is called **a fuzzy point QS-sub-algebra** if $x_\alpha \odot y_\beta \in S$ whenever x_α , y_β ∈ S .

Example 2.2:

Let X = {0, 1, 2 } be a QS-algebra. In example (1.3) , it is routine to check that ($FP_{0.3}(X)$,$\odot$, $0_{0.3}$) is a fuzzy point QS-algebra, and that S = {$0_{0.3}$, $b_{0.3}$} is a fuzzy point QS-sub-algebra of $FP_{0.3}(X)$.

Definition 2.3 :

A subset S of X or $FP_q(X)$. FP(X) is called a **fuzzy point Q-sub-algebra** if
$x_\alpha \odot y_\beta \in S$ whenever x_α , y_β ∈ S .

Example 2.4:

Let X = {0, 1, 2 } be a QS-algebra. In example (1.17) , it is routine to check that ($FP_{0.2}(X)$,$\odot$, $0_{0.2}$) is a fuzzy point Q-algebra, and that S = {$0_{0.2}$, $b_{0.2}$} is a fuzzy point Q-sub-algebra of $FP_{0.2}(X)$.

Remark 2.5:

$FP_q(X)$ is a fuzzy point Q(QS)-sub-algebra of FP(X), for every q ∈ (0, 1].

Theorem 2.6:

Let μ be a fuzzy subset of a Q-algebra X. Then the following are equivalent:
(i) μ is a fuzzy Q-sub-algebra of X.
(ii) $FP_q(\mu)$ is a fuzzy point Q-sub-algebra of $FP_q(X)$, for every q ∈ (0, 1].
(iii) U(μ; t) is a Q-sub-algebra of X when it is nonempty, for every t ∈ (0, 1].
(iv) FP(μ) is a fuzzy point Q-sub-algebra of FP(X).

Proof.

(i) ⇒ (ii) Assume that μ is a fuzzy Q-sub-algebra of X and let x_q , y_q ∈$FP_q(\mu)$ where q ∈ (0, 1]. Then μ(x) ≥ q and μ(y) ≥ q. It follows that μ(x ∗ y) ≥ min{μ(x), μ(y)} ≥ q so that $(x_q \odot y_q) = (x * y)_q \in FP_q(\mu)$.Hence $FP_q(\mu)$ is a fuzzy point Q-sub-algebra of $FP_q(X)$.

(ii) ⇒(iii) Suppose that $FP_q(\mu)$ is a fuzzy point Q-sub-algebra of $FP_q(X)$, for every q ∈ (0, 1]. Let x, y ∈ U(μ; t), where t ∈ (0, 1]. Then μ(x) ≥ t and μ(y) ≥ t, and so x_t, $y_t \in FP_t(\mu)$. It follows that $(x * y)_t = (x_t \odot y_t) \in FP_t(\mu)$ so that μ(x ∗ y) ≥ t, i.e. (x ∗ y) ∈ U(μ; t). Therefore U(μ; t) is a Q-sub-algebra of X.

(iii) ⇒(iv) Suppose U(μ; t) (≠∅) is a Q-sub-algebra of X, for every t ∈ (0, 1]. Let x_p , y_q ∈ FP(μ) and let t = min{p, q}. Then μ(x) ≥ p ≥ t and μ(y) ≥ q ≥ t and thus x, y ∈ U(μ; t). It follows that (x ∗ y) ∈ U(μ; t) because U(μ; t) is a Q-sub-algebra of X. Thus μ(x∗y) ≥ t, which implies that

$(x_p \odot y_q) = (x * y)_{\min\{p,\ q\}} = (x_t \odot y_t) = (x * y)_t \in FP(\mu)$. Hence FP(μ) is a fuzzy point Q-sub-algebra of FP(X).

(iv)⇒(i) Assume that FP(μ) is a fuzzy point Q-sub-algebra of FP(X). For any x, y ∈ X, we have x_t, y_t ∈ FP(μ) which imply that $(x * y)_t = (x_t \odot y_t) \in FP(\mu)$, that is, μ(x ∗ y) ≥ min{μ(x), μ(y)}. Consequently, μ is a fuzzy Q-sub-algebra of X. ⌂

Corollary 2.7:

Let μ be a fuzzy subset of a QS-algebra X. Then the following are equivalent:

(i) μ is a fuzzy QS-sub-algebra of X.

(ii) $FP_q(\mu)$ is a fuzzy point QS-sub-algebra of $FP_q(X)$, for every q ∈ (0, 1].

(iii) U(μ; t) is a QS-sub-algebra of X when it is nonempty, for every t ∈ (0, 1].

(iv) FP(μ) is a fuzzy point QS-sub-algebra of FP(X).

Proof.

Similarly as theorem (2.6) . ⌂

Proposition 2.8:

Let μ be a fuzzy subset of a Q-algebra X. If FP(μ) is a fuzzy point Q-sub-algebra of FP(X), then $0_p \in$ FP(μ), for all p ∈Im(μ).

Proof. Let p ∈Im(μ), then there exists x ∈X such that μ(x) = p. Hence $x_p \in$ FP(μ), and so $0_p = (x * x)_p = x_p \odot x_p \in FP(\mu)$. ⌂

Corollary 2.9:

1) Let μ be a fuzzy subset of a QS-algebra X. If FP(μ) is a fuzzy point QS-sub-algebra of FP(X), then $0_p \in$ FP(μ), for all p ∈Im(μ).

Corollary 2.10:

1) If μ is a fuzzy Q-sub-algebra of a Q-algebra X, then $0_p \in$ FP(μ), for all p ∈Im(μ).

2) If μ is a fuzzy QS-sub-algebra of a QS-algebra X, then $0_p \in$ FP(μ), for all p ∈Im(μ).

3) If μ is a fuzzy Q(QS)-sub-algebra of a Q(QS)-algebra X, then $0_q \in FP_q(\mu)$, for all q ∈ (0, 1].

4) Let μ be a fuzzy subset of a Q(QS)-algebra X and let p, q ∈ (0, 1] with p ≥ q. If $x_p \in$ FP(μ), then $x_q \in$ FP(μ).

Proposition 2.11 :

Let μ be a fuzzy subset of Q-algebra X . If FP(μ) is a fuzzy point Q-sub-algebra of FP(X) if and only if , for every t ∈[0,1] , μ is either empty fuzzy Q-sub-algebra of X .

Proof:

Assume that FP(μ) is a fuzzy point Q-sub-algebra of FP(X). For any x, y ∈ X, we have x_p , y_q ∈ FP(μ), hence you get $(x * y)_{\min\{p, q\}= t} = x_p \odot y_q$ ∈ FP(μ), that is μ(x ∗ y) ≥ min{μ(x), μ(y)}. Consequently, μ is a fuzzy Q-sub-algebra of X .

Conversely, Assume that $\mu_t \neq \emptyset$ is a fuzzy Q-sub-algebra of X, for every t ∈ (0, 1]. Let x_p , y_q∈ FP(μ) and let t = min{p, q}. Then μ(x) ≥ p ≥ t and μ(y) ≥ q ≥ t, and thus x, y ∈ μ. It follows that (x ∗ y) ∈ μ because μ is a fuzzy Q-sub-algebra of X, which implies that μ(x ∗ y) ≥ min{μ(x), μ(y)}, thus μ(x∗y) ≥ t which implies that $x_p \odot y_q = (x*y)_{\min\{p, q\}} = (x * y)_t$∈ FP(μ).

Hence FP(μ) is a fuzzy point Q-sub-algebra of FP(X). ⌂

Corollary 2.12 :

Let μ be a fuzzy subset of QS-algebra X . If FP(μ) is a fuzzy point QS-sub-algebra of FP(X) if and only if , for every t ∈[0,1] , μ is either empty fuzzy QS-sub-algebra of X .

3. Fuzzy point Q(F)-ideal of Q(QS)-algebras

In this section, we introduce the concept of fuzzy point Q(F)-ideal of Q(QS)-algebra X and give some examples and properties of its .

Definition 3.1:

A subset FP(μ) of FP(X) is called **a fuzzy point Q-ideal of FP(X)** where X is Q-algebra, if

QI1) 0_λ∈FP(μ), for all λ ∈Im(μ) and

QI2) $\left((x_\alpha \odot y_\beta)_{\min\{\alpha,\beta\}} \odot z_\gamma\right)$, y_β∈ FP(μ) implies that $(x * z)_{\min\{\alpha,\beta,\gamma\}}$∈FP(μ), for all x_α, y_β , z_γ∈ X and α, β, γ∈ (0, 1] .

Definition 3.2:

A subset FP(μ) of FP(X) is called **a fuzzy point F-ideal of FP(X)** where X is QS-algebra, if

FI1) 0_λ∈FP(μ), for all λ ∈Im(μ) and

FI2) $\left((x_\alpha \odot y_\beta)_{\min\{\alpha,\beta\}} \odot z_\gamma\right)$, y_β∈ FP(μ) implies that $(x * z)_{\min\{\alpha,\beta,\gamma\}}$∈FP(μ), for all x_α, y_β , z_γ∈ X and α, β, γ∈ (0, 1] .

Theorem 3.3:

If μ is a fuzzy Q-ideal of a Q-algebra X, then FP(μ) is a fuzzy point Q-ideal of FP(X).

Proof.

Since μ(0) ≥ μ(x), for all x ∈X, we have μ(0) ≥ λ for all λ ∈Im(μ), hence 0_λ∈ FP(μ). Let x_α, y_β , z_γ∈ X and α, β, γ∈ (0, 1] such that $((x * y)_{\min\{\alpha,\beta\}} \odot z_\gamma)$, ∈ FP(μ) and y_β∈ FP(μ), then $((x * y)_{\min\{\alpha,\beta\}} \odot z_\gamma) \geq \min\{\alpha, \beta, \gamma\}$ and $\mu(y) \geq \beta$. Since μ is a fuzzy Q-ideal of X, it follows that $\mu(x * z) \geq \min\{\mu((x * y) * z), \mu(y)\} \geq \min\{\alpha, \beta, \gamma\}$, so that $(x * z)_{\min\{\alpha,\beta,\gamma\}}$ ∈ FP(μ). ⌂

Corollary 3.4:

If μ is a fuzzy F-ideal of a QS-algebra X, then FP(μ) is a fuzzy point F-ideal of FP(X).

Proposition 3.5:

$FP_q(X)$ is a fuzzy point Q-ideal of FP(X) where X is Q-algebra, for every q ∈ (0, 1].

Corollary 3.6:

$FP_q(X)$ is a fuzzy point F-ideal of FP(X) where X is QS-algebra, for every q ∈ (0, 1].

Theorem 3. 7:

Let μ be a fuzzy subset of a Q-algebra X. Then the following are equivalent:

(i) μ is a fuzzy Q-ideal of X.

(ii) $FP_q(\mu)$ is a fuzzy point Q-ideal of $FP_q(X)$, for every q ∈ (0, 1].

(iii) U(μ; t) is a Q-ideal of X when it is nonempty, for every t ∈ (0, 1].

(iv) FP(μ) is a fuzzy point Q-ideal of FP(X).

Proof.

(i) ⇒ (ii) Assume that μ is a fuzzy Q-ideal of X and let x, y , z∈ X and q∈(0, 1] , then $\mu((x*y) * z) \geq q$ and $\mu(y) \geq q$. It follows that $\mu(x*z) \geq \min\{\mu((x*y) * z)), \mu(y)\} \geq q$, so that $(x*z)_q \in FP_q(\mu)$. Hence $FP_q(\mu)$ is a fuzzy point Q-ideal of $FP_q(X)$.

(ii) ⇒(iii) Suppose that $FP_q(\mu)$ is a fuzzy point Q-ideal of $FP_q(X)$, for every q ∈ (0, 1]. Let x_α, y_β , $z_\gamma \in$ U(μ; t), where $\min\{\alpha,\beta,\gamma\} \in (0, 1]$. Let $t = \min\{\alpha,\beta,\gamma\}$. Then $\mu((x*y)_{\min\{\alpha,\beta\}} \odot z_\gamma) \geq t$ and $\mu(y) \geq t$. so $((x*y) * z)_t$ and $y_t \in FP_t(\mu)$. It follows that $(x*z)_t \in FP_t(\mu)$ so that $\mu(x*z) \geq t$, i.e.(x ∗ z) ∈ U(μ; t). Therefore U(μ; t) is a Q-ideal of X.

(iii) ⇒(iv) Suppose U(μ; t) (≠∅) is a Q-ideal of X for every t ∈ (0, 1]. Let $x_\alpha, y_\beta, z_\gamma \in$ X and α, β, γ ∈(0, 1] and let t = min { α, β, γ }. Then $\mu((x*y)_{\min\{\alpha,\beta\}} \odot z_\gamma) \geq t$ and $\mu(y) \geq \beta \geq t$, and thus $((x*y)*z), y \in$ U(μ; t). It follows that $(x*z) \in$ U(μ; t) because U(μ; t) is a Q-ideal of X. Thus

$\mu(x*z) \geq t$, which implies that $(x*z)_{\min\{\alpha,\gamma\}} = (x*z)_t \in$ FP(μ). Hence FP(μ) is a fuzzy point Q-ideal of FP(X).

(iv)⇒(i) Assume that FP(μ) is a fuzzy point Q-ideal of FP(X). For any x, y, z ∈ X, we have $x_\alpha, y_\beta, z_\gamma \in$ X and $\alpha, \beta, \gamma \in$(0, 1], which imply that$((x*y)_{\min\{\alpha,\beta\}} \odot z_\gamma) \geq t \in$ FP(μ) and $y_\alpha \in$ FP(μ), It follows that $(x*z)_{\min\{\alpha,\gamma\}} \in$ FP (μ) so that $\mu(x*z) \geq \min\{\alpha, \gamma\}$, that is $\mu(x*z) \geq \min\{\mu((x*y)*z)), \mu(y)\}$. Hence, μ is a fuzzy Q-ideal of X. ⌂

Corollary 3.8:

Let μ be a fuzzy subset of a QS-algebra X. Then the following are equivalent:

(i) μ is a fuzzy F-ideal of X.

(ii) $FP_q(\mu)$ is a fuzzy point F-ideal of $FP_q(X)$, for every q ∈ (0, 1].

(iii) U(μ; t) is a F-ideal of X when it is nonempty, for every t ∈ (0, 1].

(iv) FP(μ) is a fuzzy point F-ideal of FP(X).

Proposition 3.9 :

Let { $FP(\mu_i)$ | i∈Λ} be a family of fuzzy point Q-ideal of Q-algebra X , then $\cap_{i\in\Lambda} FP(\mu_i)$ is a fuzzy point Q-ideal of X.

Proof:

Since { $FP(\mu_i)$ | i∈Λ} is a family of fuzzy point Q-ideal of Q-algebra X ,we have

(1) $0_\lambda \in \cap_{i\in\Lambda} FP(\mu_i)$, for all i ∈Λ and λ ∈$Im(\mu_i)$, then $0_\lambda \in \cap_{i\in\Lambda} FP(\mu_i)$

(2) For any $x_\alpha, y_\beta, z_\gamma \in$X suppose $((x*y)_{\min\{\alpha,\beta\}} \odot z_\gamma)$, $\in \cap_{i\in\Lambda} FP(\mu_i)$ and $y_\beta \in \cap_{i\in\Lambda} FP(\mu_i)$, then $((x*y)_{\min\{\alpha,\beta\}} \odot z_\gamma) \in FP(\mu_i)$ and $y_\beta \in FP(\mu_i)$, for all i ∈Λ, but $FP(\mu_i)$ is a fuzzy point Q-ideal of Q-algebra X, for all i ∈Λ. Then$(x*z)_{\min\{\alpha,\gamma\}} \in FP(\mu_i)$, for all i ∈Λ, therefore, $(x*z)_{\min\{\alpha,\gamma\}} \in \cap_{i\in\Lambda} FP(\mu_i)$. Hence $\cap_{i\in\Lambda} FP(\mu_i)$ is a fuzzy point Q-ideal of Q-algebra X.⌂

Corollary 3.10 :

Let { $FP(\mu_i)$ | i∈Λ} be a family of fuzzy point F-ideal of QS-algebra X , then $\cap_{i\in\Lambda} FP(\mu_i)$ is a fuzzy point F-ideal of X.

Proposition3.11:

Let { FP(μ_i)| i∈Λ} be a family of fuzzy point Q-ideal of Q-algebra X, then $\cup_{i\in\Lambda}$FP(μ_i) is a fuzzy point Q-ideal of Q-algebra X, where FP(μ_i)⊆FP(μ_{i+1}), for all i ∈Λ .

Proof:

Since { FP(μ_i)| i∈Λ} is a family of fuzzy point Q-ideal of Q-algebra X, we have

(1) 0_λ∈FP(μ_i), for some i ∈Λ and λ ∈Im(μ_i), then $0_\lambda \in \cup_{i\in\Lambda}$FP($\mu_i$) .

(2) For any x_α, y_β , z_γ∈X suppose $((x*y)_{\min\{\alpha,\beta\}} \odot z_\gamma) \in \cup_{i\in\Lambda}$FP($\mu_i$), and $y_\beta \in \cup_{i\in\Lambda}$ FP(μ_i) $\Longrightarrow \exists i, j\in\Lambda$ such that $((x*y)_{\min\{\alpha,\beta\}} \odot z_\gamma)$∈FP($\mu_i$) and $y_\beta \in$ FP(μ_j) .By assumption FP(μ_i)⊆FP(μ_k), and FP(μ_j)⊆FP(μ_k) , k∈Λ, hence $((x*y)_{\min\{\alpha,\beta\}} \odot z_\gamma)$∈FP($\mu_k$) , y_β ∈ FP(μ_k) , but FP(μ_k) is a fuzzy point Q-ideal of Q-algebra X, then $(x*z)_{\min\{\alpha,\gamma\}}$∈FP($\mu_k$).Therefore $(x*z)_{\min\{\alpha,\ \gamma\}} \in \cup_{i\in\Lambda}$FP($\mu_i$) . Hence $\cup_{i\in\Lambda}$FP(μ_i) is a fuzzy point Q-ideal of Q-algebra X. ⌂

Corollary 3.12:

Let { FP(μ_i)| i∈Λ} be a family of fuzzy point F-ideal of QS-algebra X, then $\cup_{i\in\Lambda}$FP(μ_i) is a fuzzy point F-ideal of QS-algebra X, where FP(μ_i) ⊆ FP(μ_{i+1}), for all i ∈Λ .

Note that :

The converse of proposition (3.11) is not true as seen it the following example

Example 3.13:

Let X = {0,1,2,3} be a set with a binary operation (∗) defined by the following table:

∗	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Then (X; ∗, 0) is a Q(QS)-algebra.I_1={0,1} and I_2={0,2} are fuzzy Q(F)-ideal of Q(QS)-algebra X. But $I_1 \cup I_2$={0,1,2} since $(1*0)_\alpha$=$(1)_\alpha$∈$I_1\cup I_2$ and $(2*0)_\beta$= $(2)_\beta$∈$I_1\cup I_2$ for all $\alpha,\beta \in$ (0,1], but $(1*2)_{\min\{\alpha,\beta\}}$= $(3)_{\min\{\alpha,\beta\}} \notin I_1\cup I_2$.

Theorem 3.14:

Let μ be a fuzzy subset of Q-algebra X . If FP(μ) is a fuzzy point Q-ideal of FP(X) if and only if , for every t ∈[0,1] , μ is either empty or a fuzzy Q-ideal of Q-algebra X

Proof:

Assume that FP (μ) is a fuzzy point Q-ideal of FP (X) $\Longrightarrow 0_\lambda$∈FP(μ), for all λ ∈Im(μ) and x∈X, therefore $\mu(0) \geq \mu(x) \geq t$, for x ∈ μ and so 0 ∈ μ .

Let x, y, z ∈ X, where $\alpha,\beta,\gamma \in$ (0, 1], $((x*y)*z) \in \mu$, y ∈ μ and Let t = min{ α,β,γ }. Then $\mu((x*y)_{\min\{\alpha,\beta\}} \odot z_\gamma) \geq t$ and $\mu(y) \geq t$, so $((x*y)\ *z)_t$ and $y_t\in$ FP_t(μ). It follows that $(x*z)_t$∈FP_t(μ) such that $\mu(x\ *z) \geq \min\{\mu((x*y)*z), \mu(y)\} \Rightarrow \mu(x\ *z) \geq t$, then $(x\ *z) \in \mu$. Hence μ is fuzzy Q-ideal of X.

Conversely , Suppose μ ≠∅ is a fuzzy Q-ideal, let x, y , z∈ X and q∈(0, 1] .Then $\mu((x*y)\ *z) \geq q$ and $\mu(y) \geq q$. It follows that $\mu(x*z) \geq \min\{\mu((x*y)\ *z)), \mu(y)\} \geq q$, so that $(x*z)_q$∈FP_q(μ). Hence FP_q(μ) is a fuzzy point Q-ideal of FP_q(X). ⌂

Corollary 3.15:

Let μ be a fuzzy subset of QS-algebra X . If FP(μ) is a fuzzy point F-ideal of FP(X) if and only if, for every t ∈[0,1] , μ is either empty or a fuzzy F-ideal of QS-algebra X

Proposition 3.16:

Every fuzzy point Q-ideal of Q-algebra X is a fuzzy point Q-sub-algebra of X **.**

Proof:

Since FP(μ) is fuzzy point Q-ideal of a Q-algebra X, then by theorem (3.14), for every t ∈[0,1], μ is either empty or a fuzzy Q-ideal of X. By proposition (1.21), then Every fuzzy Q-ideal of Q-algebra X is a fuzzy Q-sub-algebra of X, for every t ∈[0,1], μ is either empty or a fuzzy Q-sub-algebra of X by Proposition (2.13**)**. Hence FP(μ) is a fuzzy point Q-sub-algebra of Q-algebra X. ⌂

Corollary 3.17:

Every fuzzy point F-ideal of QS-algebra X is a fuzzy point QS-sub-algebra of X **.**

Note that :

The converse of proposition (3.16) is not true as seen it the following example.

Example 3.18:

Let X = {0,1,2,3}be a set with a binary operation (∗)define a by the following table:

∗	**0**	**1**	**2**	**3**
0	**0**	**0**	**0**	**0**
1	**1**	**0**	**0**	**0**
2	**2**	**1**	**0**	**1**
3	**3**	**1**	**1**	**0**

Then (X;∗,0) is a Q(QS)-algebra .Define a fuzzy subset $\mu: X \to [0,1]$ by:

X	**0**	**1**	**2**	**3**
μ	**0.8**	**0.7**	**0.6**	**0.5**

Then μ is fuzzy point Q(QS)-sub-algebra of X, but not fuzzy point Q(F)-ideal of X. since I={0,1,2}∈ FP(μ). Let($(3_{0.5} \odot 2_{0,6}) \odot 0_{0.8}) = (1_{0.7})$∈FP(μ) and $(2_{0.6})$∈ FP(μ) but $(3_{0.5} \odot 0_{0.8}) = (3_{0.5})$∉FP(μ).

4- Homomorphism fuzzy point Q(F)-ideal of Q(QS)-algebras:

In this section ,we introduce the definition of homomorphism of Q(QS)-algebra and we study some properties of it.

Definition 4.1([2]):

Let (X; ∗, 0) and (Y; $\acute{*}$, $0'$)be Q(QS)-algebras. A mapping f : (X; ∗, 0) → (Y; $\acute{*}$, $0'$), is said to be a **homomorphism** if $f(x * y) = f(x) \acute{*} f(y)$, for all x, y∈ X.

Definition 4.2([5]) :

For any homomorphism f : (X; ∗, 0) → (Y; $\acute{*}$, $0'$), the set $\{x \in X \mid f(x) = 0'\}$ is called the **kernel of** f, denoted by $Ker(f)$.

Proposition 4.3:

Let (X;∗,0) and (Y; $\acute{*}$, $0'$) be Q-algebras and f : (X; ∗, 0) → (Y; $\acute{*}$, $0'$) be a homomorphism, then *Ker(f)* is fuzzy point Q-ideal of Q-algebra X .

Proof:

(1) Since $f(0_\lambda)$, then $0_\lambda \in Ker(f)$ and λ∈lm(μ)

(2) For any x, y, z ∈X ,let $\left((x * y)_{\min\{\alpha,\beta\}} \odot z_\gamma\right) \in Ker(f)$ and $y_\beta \in Ker(f)$

$f((x_\alpha \odot y_\beta) \odot z_\gamma) \in Ker(f)$ and $f(y_\beta) \in Ker(f) \Rightarrow f\left((x_\alpha \odot y_\beta) \odot z_\gamma\right) = 0'$ and

$f(y_\beta) = 0' \Rightarrow f(x_\alpha) \odot f(z_\gamma) = f\left((x_\alpha \odot z_\gamma)\right) \geq \min\{ f\left((x_\alpha \odot y_\beta) \odot z_\gamma\right), f(y_\beta)\} = 0'$

$\Rightarrow f\left((x_\alpha \odot z_\gamma)\right) = 0'$

That is $(x * z)_{\min\{\alpha,\gamma\}} \in Ker(f)$,then *Ker(f)* is a fuzzy point Q-ideal of X.⌂

Corollary 4.4:

Let (X; ∗,0) and (Y; $\acute{*}$, 0′) be QS-algebras and *f :* (X; ∗, 0) → (Y; $\acute{*}$, 0′)be a homomorphism, then *Ker(f)* is fuzzy point F-ideal of QS-algebra X .

Proposition 4.5:

Let (X; ∗,0) and (Y; $\acute{*}$, 0′) be Q-algebras , *f :* (X; ∗, 0) → (Y; $\acute{*}$, 0′)be a homomorphism ,onto and

(i) I be a fuzzy point Q-ideal of X, then *f* (I)is fuzzy point Q-ideal of Q-algebra Y.

(ii) S be a fuzzy point Q-sub-algebra of X, then *f* (S)is fuzzy point Q-sub-algebra of Y.

Proof:

(i) Since I is a fuzzy point Q-ideal of X $\Longrightarrow 0_\lambda \in$ I $\Longrightarrow f(0_\lambda) \in f(I)$ for all $\lambda \in$lm(μ). Let x ,y ,z ∈X and $(\alpha,\beta,\gamma) \in (0,1], f\left((x * y)_{\min\{\alpha,\beta\}} \odot z_\gamma\right) \in f(I), f(y_\beta) \in f(I)$

$\Longrightarrow \left((x * y)_{\min\{\alpha,\beta\}} \odot z_\gamma\right) \in$ I, $(y_\beta) \in$ I, Since I is a fuzzy point Q-ideal of X

$\Longrightarrow (x * z)_{\min\{\alpha,\gamma\}} \in$ I $\Longrightarrow f(x * z)_{\min\{\alpha,\gamma\}} \in f(I)$. Hence $f(I)$ is fuzzy point Q-ideal of Q-algebra Y.

(ii) Since S is a fuzzy point Q-sub-algebra of X, such that $x_p \odot y_q \in$ S whenever x_p , $y_q \in$ S, let (a∗b) $\in f(S)$ and $x_p = f$ (a), $y_q = f$(b) ,

f(a∗b) = f(a) $\acute{*} f$(b) = $x_p \odot' y_q = f(x_p \odot y_q) = f(x * y)_{\min\{p, q\}} \in f(S)$.

Hence $f(S)$ is fuzzy point Q-sub-algebra of Y.

Proposition 4.6:

Let (X; ∗,0) and (Y; $\acute{*}$, 0′) be QS-algebras , *f :* (X; ∗, 0) → (Y; $\acute{*}$, 0′)be a homomorphism ,onto and

(i) I be a fuzzy point F-ideal of X, then *f* (I)is fuzzy point F-ideal of QS-algebra Y.

(ii) S be a fuzzy point QS-sub-algebra of X, then *f* (S)is fuzzy point QS-sub-algebra of Y.

Proposition 4.7:

Let (X; ∗,0) and (Y; $\acute{*}$, 0′) be Q-algebras , *f :* (X; ∗, 0) → (Y; $\acute{*}$, 0′)be a homomorphism and B be a fuzzy point Q-ideal of Y, then f^{-1}(B)is fuzzy point Q-ideal of Q-algebra X.

Proof:

(1) Since B is a fuzzy point Q-ideal of Y $\Longrightarrow (0'_\lambda) \in$ B $\Longrightarrow f^{-1}(0'_\lambda) \in f^{-1}(B)$ since $0_\lambda = f^{-1}(0'_\lambda)$, then $0_\lambda \in f^{-1}(B)$ for all $\lambda \in$lm(μ).

(2) Let x ,y ,z ∈X and $(\alpha,\beta,\gamma) \in (0,1], \left((x * y)_{\min\{\alpha,\beta\}} \odot z_\gamma\right) \in f^{-1}(B), (y_\beta) \in f^{-1}(B) \Longrightarrow$ $f\left((x * y)_{\min\{\alpha,\beta\}} \odot z_\gamma\right) \in B, f(y_\beta) \in B$, Since B is a fuzzy point Q-ideal of Y

$\Longrightarrow f(x * z)_{\min\{\alpha,\gamma\}} \in B \Longrightarrow (x * z)_{\min\{\alpha,\gamma\}} \in f^{-1}(B)$. Hence f^{-1}(B)is fuzzy point Q-ideal of Q-algebra X.⌂

Proposition 4.8:

Let (X; ∗,0) and (Y; $\acute{*}$, 0′) be QS-algebras , *f :* (X; ∗, 0) → (Y; $\acute{*}$, 0′)be a homomorphism and B be a fuzzy point F-ideal of Y, then f^{-1}(B) is fuzzy point F-ideal of QS-algebra X.

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