N_α-Perfect Mappings In Topological Spaces

التطبيقات التامة من النمط -N_α في الفضاءات التبولوجية

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Abstract:

In this paper, we introduce new types of N_α-continuity mappings by using N_α-open sets in topological spaces, which is called N_α-perfect mappings; also we study some properties of these types. Some definitions are given.

Keywords: perfect mapping, N_α-open set, N_α- continuity mappings.

الخلاصة:

في هذا البحث قدمنا أنواع جديدة من التطبيقات المستمرة من النمط N_α بواسطة استخدام المجموعات المفتوحة من النمط N_α في الفضاءات التبولوجية التي تسمى التطبيقات التامة من نمط N_α وكذلك درسنا بعض خواص هذه الأنواع. بعض التعاريف أعطيت.

1.0 Introduction

One of the very important concepts in Mathematic, spatially in topology is the concept of continuous mapping ,there are several types of it one of them is called ″Perfect Mapping ″.A mapping f : X⟶ Y is called perfect mapping if it is continuous, closed, and has compact fibers $f^{-1}\{y\}$for each y $\in$Y. For more details see [1] , [2] and its′ references. In 1965, O. Njasted introduced the concept of α-open set in topological space X, see [3]. A subset A of a topological space X is called α- open set if A $\subseteq$int(cl(intA).The family of all α- open sets of a space X is denoted by τ_α ,is a topology on X finer than τ and its complement is called α-closed and denoted by $_\alpha C(X)$. .For more details see [4]. The concept of N_α-open set was first studied in 2015 by N. A. Dawood, N. M. Ali ,see [5] by using these sets we study some class of N_α- continuity mappings which is called N_α-perfect mappings and investigated some of their properties . In this paper mean that all spaces X and Y are topological spaces, also the closure (interior resp.) of a subset A of X is denoted by cl(A) (int(A) resp.).

2.0 Some Basic Concepts

Here, we shall give some basic concepts which we need in our work.

Definition (2.1): [5]

Let (X,τ) be a topological space, a subset A of X is called "N_α-open" set if there exists a non-empty α-open set B such that cl B$\subseteq$ A. The family of all N_α-open sets is denoted by $N_\alpha O(X)$, and its complement is called N_α-closed and denoted by $N_\alpha C(X)$.

Remark (2.2): [5]

A set A is called "N_α-closed" set if there exists a non-empty α-closed set B≠X such that A$\subseteq$int B.

Remark (2.3): [5]

In every topological space the set X and ϕ are N_α- clopen sets.

Remarks (2.4): [5]

(i) The concepts of open and N_α-open sets are independent.

(ii) Every clopen set is N_α-open set.

(iii) Any finite set in the usual topological space(R ,τ_u) on the real numbers R is N_α- closed set.

Theorem (2.5): [5]

Let (X_1,τ_1), (X_2,τ_2) be topological spaces. Then A_1 and A_2 are N_α-open(N_α-closed) sets in X_1 and X_2 resp. if and only if $A_1 \times A_2$ is N_α-open(N_α-closed) set in $X_1 \times X_2$.

Proposition (2.6): [5]

Let (X,τ) be a topological space. Then

(1) The finite union of N_α-open sets is N_α-open set.
(2) The finite intersection of N_α-open sets is N_α-open set.
(3) The finite union of N_α-closed sets is N_α-closed set.
(4) The finite intersection of N_α-closed sets is N_α-closed set.

Proposition (2.7): [5]

Let (Y,τ_Y) be a subspace of a topological space (X,τ) such that $A \subseteq Y \subseteq X$. Then:

(i) If $A \in N_\alpha O(X)(N_\alpha C(X))$, then $A \in N_\alpha O(Y)(N_\alpha C(X))$.

(ii) If $A \in N_\alpha O (Y)(N_\alpha C(Y))$ then $A \in N_\alpha O(X)(N_\alpha C(X))$,where Y is clopen set inX .

Definition (2.8): [5]

Let (X,τ) be a topological space. Then X is called N_α^{**}-regular space if for every $x \in X$,and every $N\alpha$- closed set F such $x \notin F$ **t**here exist two open sets A and B such that $x \in A$, $F \subset B$ and $A \cap B = \varnothing$

Definition (2.9): [4]

Let (X,τ) be a topological space. Then X is called α^{**} -regular space if for every $x \in X$,and every α- closed set F such $x \notin F$ **t**here exist two open sets A and B such that $x \in A$, $F \subset B$ and $A \cap B = \varnothing$

Proposition (2.10): [4]

Let (X,τ) be a topological space. Then X is α^{**} -regular space if and only if every α -open set A contains x, there exists open set B contains x such that $x \in B \subseteq cl\, B \subseteq A$.

Proposition (2.11): [5]

Let (X,τ) be a topological space . Then X is N_α^{**} -regular space if and only if every N_α-open set A contains x, there exists open set B contains x such that $x \in B \subseteq cl\, B \subseteq A$.

Proposition (2.12): [5]

Le*t* (X,τ) be α^{**}-regular space then every open (closed) set is N_α-open (N_α-closed) set. .

Proposition (2.13): [5]

Le*t* (X,τ) be N_α^{**} -regular space then any N_α-open (N_α-closed) set is open*(*closed*)*set.

Definition (2.14): [6]

Let (X_1 , τ_1) , (X_2,τ_2) be topological spaces, and $f : X_1 \longrightarrow X_2$ be a mapping, then f is called N_α, N_α^{*}-continuous if $f^{-1}(A)$ is N_α-open set in X_1 for every open (N_α-open) set A in X_2.

Proposition (2.15): [6]

Let (X_1 , τ_1) ,(X_2,τ_2) be topological spaces ,and F be N_α-open subset of X_1,if $f:X_1 \longrightarrow X_2$ is N_α, N_α^{*}-continuous then $f_{/F} :F \longrightarrow X_2$ is also, N_α, N_α^{*}-continuous.

Proposition (2.16): [6]

Let (X_1,τ_1) , (X_2,τ_2) be topological spaces, let $f : X_1 \longrightarrow X_2$,and $f_A: f^{-1}(A) \longrightarrow A$ which defined by , $f_A (x)=f(x)$ be mappings if f is N_α-continuous ,then f_A is also, N_α-continuous ,where A is an open set in X_2

Proposition (2.17): [6]
Let (X_1,τ_1),(X_2,τ_2) be two topological spaces, and $f:(X_1\ \tau_1) \longrightarrow (X_2,\tau_2)$ be a mapping, where A_1 and A_2 be subsets in X_1, such that $X_1= A_1 \cup A_2$,then f is N_α (N^*_α-continuous),such that

$f|_{A1}$, $f|_{A2}$ are N_α (N^*_α-continuous*)* mappings ,where A_1 and A_2 are disjoint clopen subsets in X_1.

Lemma (2.18): [7]
Let $A \subseteq Y \subseteq X$. Then A is compact set in X if and only if A is compact set in Y.

In follows, we shall introduce a new definitions that we shall use it in this work.

Definition (2.19)
Let (X_1 , τ_1) , (X_2,τ_2) be topological spaces, and $f : X_1 \longrightarrow X_2$ be a mapping, then f is called N_α , N^*_α- open mapping if f(A) is N_α-open set in X_2 for every open (N_α-open) set A in X_1.

Definition (2.20)
Let (X_1 , τ_1) , (X_2,τ_2) be topological spaces, and $f : X_1 \longrightarrow X_2$ be a mapping, then f is called N_α , N^*_α- closed mapping if f(A) is N_α-closed set in X_2 for every closed ,N_α-closed set A in X_1.

3.0 N_α-Perfect Mappings

In this section, the concept of N_α-open set will be used to define some new types of N_α-continuity which is called N_α-perfect mapping.

Definition (3.1):
Let (X ,τ_1), (Y,τ_2) be topological spaces. A surjective mapping $f : X \longrightarrow Y$ is called N_α-perfect mapping if f is N_α-continuous, N_α-closed, and all fibers $f^{-1}\{y\}$ is compact set in X for all y in Y.

To illustrate this concept see the following Examples:
Example (3.2)
Let X,Y be topological spaces where X={a,b,c}=Y, $\tau_{x=}${X,{a},{b,c},ϕ},$\tau_{y=}${Y,{a,b},{c},ϕ},
$_\alpha$C(Y)=C(Y) $_=${Y,{a, b},{c},ϕ}, let f:X $\longrightarrow$ s Y such that f(a)=c, f(b)=a, f(c)=b we observe f is surjective, all fibers $f^{-1}\{y\}$ is compact set in X for all y in Y, so f is N_α-continuous ,since $f^{-1}\{Y\}=X$, $f^{-1}\{\phi\}=\phi$ are N_α- open sets in X
see(Remark(2.3)) ,$f^{-1}\{a, b\}=\{b, c\}$ $f^{-1}\{c\}=\{a\}$ are clopen sets so are N_α- open sets in X see Remark(2.4), thus f is N_α-continuous, also f is N_α-closed mapping since ,
f{b, c}={a ,b},f{a}={c},f(X) =Y ,f {ϕ}=ϕ are closed sets so they are N_α-closed sets(since Y is α^{**}-regular space) see Remark(2.12).
Example (3.3)
Let X,Y be topological spaces where, X={1,2,3,4},Y={1,2,3,4} $\tau_{x=}${X,{1},{2},{1,2},ϕ},$N_\alpha O(X)$={{2,3,4},{1,3,4},$\tau_{y=}${Y,{1,2,3},ϕ}.$N_\alpha O(Y)=N_\alpha C(Y)$={Y, ϕ} , let f: X$\longrightarrow$Y , such that f(1)=4, f(2)=1, f(3)=2,f(4)=3 ,we observe that f is N_α-continuous, surjective, all fibers $f^{-1}\{y\}$ is compact set , but it is not N_α-closed mapping, since {3,4} is closed set in X but f{3,4}={2,3} which is not N_α-closed set in Y, thus f is not N_α-perfect mapping.
Proposition (3.4):
Let $f : X \longrightarrow Y$ be N_α-perfect mapping, then the restriction of f on clopen subset A in X is also, N_α-perfect mapping.
Proof: To prove $f|_A$:A $\longrightarrow$ Y is N_α-perfect mapping, since A is clopen ,then by (Remark$_{(2\text{-}4)}$) A is N_α-open set, thus, by (Proposition$_{(}$2.15$_{)}$) we get $f|_A$:A $\longrightarrow$ Y is

N_α-continuous mapping ….. (1) . Now, let B be closed subset in A, since A is clopen set ,thus A is closed set in X, thus B is closed set in X ,hence f(B) is N_α-closed set in Y, but $f|_{A(B)}= f_{(B)}$, thus $f|_A$ is N_α-closed mapping…..(2) , since f is surjective mapping , thus, $f|_A$ is surjective mapping also………(3). Now, to prove$(f|_A)^{-1}\{y\}$ is compact set in A for all $y\in Y$, we have, $(f|_A)^{-1}\{y\}= A \cap f^{-1}\{y\}$ where $f^{-1}\{y\}$is compact set in X then $f^{-1}\{y\}$is compact set in A, see Lemma (2.18)………(4).Hence, by (1),(2),(3) and(4), we obtain, $f|_A$ is N_α-perfect mapping.

Proposition(3.5)

Let $f:X \longrightarrow Y$ be N_α-perfect mapping, then $f_A : f^{-1}(A) \longrightarrow A$ is also, N_α-perfect mapping, where A is clopen set in Y and X is N_α^{**} -regular space .

Proof: Since f is N_α-continuous thus, by(Proposition(2.16))) f_A is N_α-continuous mapping……(1),since f is onto mapping, thus, $f_A : f^{-1}(A) \longrightarrow A$ is onto mapping also…..(2) . Now, let B be closed set in $f^{-1}(A)$, since A is clopen, and f is N_α-continuous then $f^{-1}(A)$is N_α-clopen set in X ,since X is N_α^{**} -regular space then by (Proposition(2.13))) we get

$f^{-1}(A)$is clopen in X ,thus it is closed set in X so B is closed set in X, thus f(B)is N_α-closed set in Y since $f^{-1}(A)$is closed set in X ,thus $f(f^{-1}(A))$ is N_α-closed in Y ,but $ff^{-1}(A)=A$

(since f is onto) ,thus we get A is N_α-closed in Y ,thus we get A ,f(B)are N_α-closed sets in Y so by (Proposition(2.6))) $A \cap f(B)$ is N_α-closed set in Y, thus by(Proposition(2.7))) $A\cap f(B)$ is N_α-closed set in A ,but $f_A(B)=A \cap f(B)$ .This shows f_A is N_α-closed mapping…….(3).Now, to prove $(f_A)^{-1}\{a\}$ is compact set in $f^{-1}(A)$for every $a\in A$. We have:

$(f_A)^{-1}\{a\}= f^{-1}(A) \cap f^{-1}\{a\}$,where $f^{-1}(A)$, $f^{-1}\{a\}$, are closed and compact sets in X respectively, so their intersection is compact set in X, since $(f_A)^{-1}\{a\} \subseteq f^{-1}(A)\subseteq X$,thus by Lemma(2.18)we obtain $(f_A)^{-1}\{a\}$ is compact set in $f^{-1}(A)$ for every $a\in A$. …..(4) .Thus by (1),(2),(3) and (4) we get f_A is N_α-perfect mapping.

Proposition (3.6)

Let X be topological space, where $X = A_1\cup A_2$ where A_1, A_2 are disjoint clopen sets, and $f : X \longrightarrow Y$ be a mapping. Then $f|_{A1}$, $f|_{A2}$ are N_α-perfect mappings if and only if f is N_α-perfect mapping.

Proof: For (if) it is immediate by using proposition (3.4).Now, for (only if),

Suppose $f|_{A1}$,$f|_{A2}$ are N_α-perfect mappings to prove f is N_α-perfect mapping. We have by (Proposition (2.17)) f is N_α-continuous mapping ……… (1).Now, to prove f is N_α-closed mapping. Let B be closed set in X .We have:

$f(B)=f(B\cap X)=f(B \cap(A_1\cup A_2))=f((B \cap A_1) \cap A_1)\cup(B \cap A_2) \cap A_2))=f|_{A1}(B \cap A1)\cup f|_{A2}(B\cap A2)$,where$(B \cap A_1)$,$(B\cap A_2)$are closed sets in A_1 , A_2 resp., since $f|_{A1}$ and $f|_{A2}$ are N_α- closed mappings, thus $f|_{A1}(B \cap A_1)$, $f|_{A2}(B\cap A_2)$ are N_α-closed sets in Y, so their union is also N_α-closed set in Y.Thus f(B)is N_α-closed set in Y………. (2).Now, since $f|_{A1}$, $f|_{A2}$ are N_α-perfect mappings then ,$(f|_{A1})^{-1}\{y\}$, $(f|_{A2})^{-1}\{y\}$ are compact sets in A_1, A_2 resp.thus by Lemma(2.18)we get$(f|_{A1})^{-1}\{y\}$,$(f|_{A2})^{-1}\{y\}$ are compact sets in X so their union is also compact set in X.(since the union of finite compact sets is compact set…………(3)..Now, it is obvious that f is onto ………(4). Thus, by(1), (2), (3) and (4),we get f is N_α-perfect mapping.

Proposition (3.7)

Let $f_1 : X_1\longrightarrow Y_1$, $f_2: X_2\longrightarrow Y_2$ be mappings, if $f_1\times f_2: X_1\times X_2 \longrightarrow Y_1\times Y_2$ is N_α-perfect mapping, then f_i is N_α-perfect for each i = 1, 2

Proof: We shall prove only $f_1 : X_1\longrightarrow Y_1$,is N_α-perfect mapping, to prove $f_1:X_1\longrightarrow Y_1$ is N_α-continuous mapping .Let A be an open set in Y_1, thus $A \times Y_2$ is an open set in $Y_1\times Y_2$, thus $(f_1\times f_2)^{-1}(A \times Y_2)$ is N_α- open set in $X_1\times X_2$,where $(f_1\times f_2)^{-1}(A\times Y_2) =(f_1)^1(A) \times (f_2)^{-1}(Y_2)=(f_1)^{-1}(A) \times X_2$, thus by (Th.(2.5)) we obtain $f_1^{-1}(A)$ is N_α- open set in X_1,thus$f_1:X_1\longrightarrow Y_1isN_\alpha$-continuous mapping…(1) Now, let B be closed set in X_1 ,thus $B \times X_2$ isclosed set in $X_1\times X_2$ so $f_1\times f_{2(B\times X2)}$

isN_α-closeset in $Y_1 \times Y_2$, where $f_1 \times f_{2(B} \times x f_{1(B)} \times f_{2(X2)}$, thus by((Th.(2.5)) $f_{1(B)}$ is N_α-closedset in Y_1….(2)
On the other hand, since $f_1 \times f_2$ is surjective mapping , thus f_1, f_2 are surjective also mappings…(3).
Now, the fourth condition. Let $y_1 \in Y_1$, to prove $(f_1)^{-1}\{y_1\}$ is compact set X_1 ,we have $(f_1 \times f_2)^{-1}\{(y_1, y_2)\} = (f_1)^{-1}\{y_1\}, (f_2)^{-1}\{y_2\}$ is compact set in $X_1 \times X_2$.
for every $(y_1, y_2) \in Y_1 \times Y_2$,thus $(f_1)^{-1}\{y_1\}, (f_2)^{-1}\{y_2\}$ are compact sets in X_1, X_2 resp. …(4).
Thus $f_1 : X_1 \longrightarrow Y_1$ is N_α-perfect mapping. In similar way, we can prove $f_2: X_2 \longrightarrow Y_2$ is N_α-perfect mapping.

Definition (3.8)
Let $f : X_1 \longrightarrow X_2$ be a mapping , then f is called N_α-proper mapping if f is:
(i) N_α-continuous .
(ii) $f \times I_{'X} : X_1 \times 'X \longrightarrow X_2 \times 'X$ is N_α-closed mapping for each α^{**}-regular topological space $'X$..

Example(3.9)
Let (R, τ_u) be usual topological space on the real numbers R, let f: $(R, \tau_u) \longrightarrow (R, \tau_u)$ such that f(x)=a for each $x \in R$, then f is N_α-continuous mapping, since for each open set G in (R, τ_u) then $f^{-1}(G)=\{R$ if $a \in R$, or ϕ if $a \notin R\}$ and by Remark(),f is N_α-continuous mapping .Now ,to prove $f \times I_{X'}: R \times 'X \longrightarrow R \times 'X$ is N_α-closed mapping for each α^{**}-regular topological space $'X$. Let F be closed set in $R \times 'X$ then $F = F_1 \times F_2$ is closed set where F_1 is closed in R, and F_2 is closed set in $'X$. ,then $f \times I_{X'}(F) = f \times I_{X'}(F_1 \times F_2) = f(F_1) \times F_2 = \{a\} \times F_2$, where $\{a\}$ is N_α-closed set in(R, τ_u) see Remark(2.4) also F_2 is N_α-closed set in $'X$,see Propo.(2.12),thus by Th.(2.5),we get $f(F_1) \times F_2$ is N_α-closed set in $R \times 'X$ .Thus f is N_α-proper mapping .

Example (3.10)
Let (R, τ_u) be usual topological space on the real numbers R, let f: $(R, \tau_u) \longrightarrow (R, \tau_u)$ such that f(x)=0 for each $x \in R$,let $I: R \longrightarrow R$,we observe f is N_α-continuous mapping (easy check). Now let $f \times I_R : R \times R \longrightarrow R \times R$, where $f \times I_R(x, y) = (0, y)$ for all $(x, y) \in R \times R$, let A={(x, y) such that x. y=1}is closed set in R×R, thus $f \times I_R(A) = \{0\} \times R \diagup \{0\} \approx R \diagup \{0\}$,but $R \diagup \{0\}$ is not N_α-closed set since the only α-closed set contains it is R and this contradiction with Remark(2.2).Thus f is not N_α-proper mapping .

Theorem (3.11)
Let f: $X \longrightarrow Y$ be surjective with all fibers $f^{-1}\{y\}$ is compact set in X for all y in Y. Then if f is N_α-proper mapping then f is N_α-perfect mapping.

Proof: We need to prove only the condition of N_α-closed mapping, since the other conditions are satisfying. Let f: $X_1 \longrightarrow X_2$ be a mapping since f is N_α-proper mapping ,thus $f \times I_{'X} : X_1 \times 'X \longrightarrow X_2 \times 'X$ is N_α-closed mapping for each α^{**}-regular topological space $'X$. Take $'X = \{t\}$,then by hypothesis the mapping $f \times I_{\{t\}}: X_1 \times \{t\} \longrightarrow X_2 \times \{t\}$ is N_α-closed mapping topological ,but $X_1 \times \{t\}$, $X_2 \times \{t\}$ are homeomorphism to X_1, X_2 thus $f: X_1 \longrightarrow X_2$ is N_α-closed mapping.

Now, we shall discuss the converse of above Theorem.

Proposition (3.12)
Every N_α-perfect mapping is N_α-proper mapping.

Proof: Let f: $X \longrightarrow Y$ be N_α-perfect mapping, thus f is N_α-continuous mapping, now to prove $f \times I_Z : X \times Z \longrightarrow Y \times Z$ is N_α-closed mapping for each α^{**}-regular topological

Space Z. Let $G=G_1 \times G_2$ be closed set in $X\times Z$,we have $f\times I_z(G_1 \times G_2)=f(G_1)\times G_2$,we have $f(G_1)$ is N_α-closed set in Y(since f is N_α-perfect mapping),also since Z is α^{**}-regular topological space ,then by Propo.(2.12) G_2 is N_α-closed set in Z ,thus $f\times I_z(G_1 \times G_2)=$
$f(G_1)\times G_2$ is N_α-closed set $Y\times Z$ see Th.(2.5).

.Now, we have by proposition(3.11) and proposition(3.12) we have the following result:

Corollary (3.13)
Let $f: X \longrightarrow Y$ be surjective with all fibers $f^{-1}\{y\}$ is compact set in X for all y in Y. Then f is N_α-proper mapping if and only if N_α-perfect mapping.

Proposition (3.14)
If X is compact set ,then $f : X \longrightarrow \{t\}$ is N_α-perfect mapping, $t\notin X$.
Proof: It is clearly that f is surjective . ……… (1).On the other hand f is N_α-continuous mapping, since $f^{-1}\{t\}= X$,and by Remark(2.3) X is N_α-open set, thus f is N_α-continuous mapping……… (2).Also, $f^{-1}\{t\}=X$, where X is compact set………… (3).Now ,let F be closed set ,then either $f(F)=\{t\}$ or $f(F)=\phi$,and by (Remark (2.3))we obtain f is N_α-closed mapping………… (4).Thus $f: X \longrightarrow \{t\}$ is N_α-perfect mapping.

Corollary (3. 15)
If $f: X \longrightarrow Y$ is N_α-perfect mapping, then $f_{\{y\}}: f^{-1}\{y\}\longrightarrow \{y\}$ is also N_α-perfect mapping for every $y \in Y$.

Proof: Since $f : X \longrightarrow Y$ is N_α-perfect mapping, thus $f^{-1}\{y\}$ is compact set for every $y\in Y$. Thus, by proposition (3. 14) $f_{\{y\}}: f^{-1}\{y\}\longrightarrow \{y\}$ is also N_α-perfect mapping.

Proposition (3.16)
Let $f : X \longrightarrow Y$ and $g : Y \longrightarrow Z$ be mappings such that $g\circ f$ is N_α-perfect mapping, where g is bijective, open, and N^*_α-continuous mapping, then f is N_α-perfect mapping.
Proof: Let B be open set in Y, since g is open mapping ,thus g(B)is open set in Z, since $g\circ f$ is N_α-continuous mapping ,then $(g\circ f)^{-1}(g(B)$ is N_α- open set in X, but :
$(g\circ f)^{-1}(g(B)= f^{-1}(g^{-1}g_{(B)})) = f^{-1}(B)$ since (g is$_{(1-1)}$) ,hence f is N_α-continuous mapping…… (1) let F be closed set in X, thus $g\circ f(F)$ is N_α-closed set in Z, since g is N^*_α-continuous mapping ,thus $g^{-1}(g\circ f(F))$ is N_α-closed set in Y, where $g^{-1}(g\circ f(F)) = f(F)$,thus f is N_α-closed mapping…… (2). Now, f is surjective (easy check)…… (3).Now ,to prove $f^{-1}{}_{\{y\}}$ is compact set in X for every $y \in Y$, let $y \in Y$, and g(y)=z , we have $(g\circ f)^{-1}{}_{\{z\}}$ is compact set in X, where $(g\circ f)^{-1}{}_{\{z\}})) = f^{-1}(y)$, (since g is (1-1)) ,thus $f^{-1}{}_{\{y\}}$ is compact set in X… . (4) .Thus by (1), (2) , (3) and (4) we get f is N_α-perfect mapping .

Proposition (3.17)
Let $f : X \longrightarrow Y$ $g : Y \longrightarrow Z$ be mappings such that $g\circ f$ is N_α-perfect mapping, where f is continuous surjective, N^*_α- open mapping ,then g is N_α-perfect mapping
Proof: Let B be open set in Z, since $g\circ f$ is N_α-perfect mapping, thus it is N_α-continuous mapping, thus $(g\circ f)^{-1}{}_{(B)}$ is N_α-open set in X ,where $(g\circ f)^{-1}{}_{(B)} = f^{-1}(g^{-1}{}_{(B)})$, since f is N^*_α- open mapping ,then $f f^{-1}(g^{-1}{}_{(B)})$ is N_α- open set in Y, since f is surjective mapping then:
$f f^{-1}(g^{-1}{}_{(B)}) = g^{-1}{}_{(B)}$,thus g is N_α- continuous mapping……....(1) Let F be closed set in Y since f is continuous mapping, thus $f^{-1}(F)$is closed set in X, since $g\circ f$ is N_α-perfect mapping,thus $(g\circ f)(f^{-1}{}_{(F)})$ is N_α-closed set in Z, but $(g\circ f)(f^{-1}{}_{(F)})$ is N_α-closed set in Z, but $=(g\circ f)(f^{-1}{}_{(F)}) = g_{(F)}$ (since f is surjective mapping),thus g is N_α-closed mapping……… (2) .Now, since $g\circ f$ is N_α-perfect mapping, then , $(g\circ f)^{-1}{}_{\{z\}}$ is compact set in X for every $z\in Z$,

where $(g \circ f)^{-1}\{z\} = f^{-1}(g^{-1}(Z))$,since f is continuous, then $f(g \circ f)^{-1}\{z\} = ff^{-1}(g^{-1}(Z))$ is compact set in Y ,since f is surjective mapping ,then $ff^{-1}(g^{-1}(Z)) = g^{-1}(Z)$…………..(3),clearly g is surjective mapping……(4).Thus by(1), (2) , (3) and (4) we get g is N_α-perfect mapping .

4.0 Future Work

We can use the concept of N_α-open sets to study a new kinds of N_α-perfect mapping such as:

(1) f is continuous mapping , N_α-closed mapping, $f^{-1}(y)$ is compact
(2) f is N_α- continuous mapping, closed mapping, $f^{-1}(y)$ is compact
(3) f is continuous mapping, closed, $f^{-1}(y)$ is N_α- compact
(4) f is N_α- continuous mapping,closed, $f^{-1}(y)$ is N_α- compact
(5) f is N_α- continuous mapping, N_α-closed, $f^{-1}(y)$ is N_α- compact
(6) f is continuous mapping, N_α-closed, $f^{-1}(y)$ is N_α- compact

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