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A New Notion For Intuitionistic Fuzzy Soft Normed Linear Space

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Abstract

In this paper the concept of intuitionistic fuzzy soft normed linear space of set of all soft points over a scalar field K is studied in a different way. Also the concepts: Cauchy, convergence are defined. Some theorems related to these concepts are proved.

Keywords: soft set, soft normed space, fuzzy set, fuzzy soft normed space.

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الخلاصة :
في هذا البحث عرفنا الفضاء المعياري المرن الضبابي الحدسي وكذلك قدمنا تعريف المتتابعة المتقاربة ومتتابعة كوشي وبعض المبرهنات المرتبطة بهذا الفضاء.

1. Introduction

The concept of fuzzy was introduced by Zadeh [18]. The definition and notion of intuitionistic fuzzy set introduced by Atanassov [1] in 1986.Katsaras [8]in 1984, introduced the concept of fuzzy normed space.R. Saadati and M.Vaezapour [14] in 2005,studied fuzzy Banach spaces. In 2006, Saadati and Park [13] introduced the concept as the intuitionistic fuzzy normed linear space. The definition of intuitionistic fuzzy n-normed linear spaces was introduced by S. Vijayabalaji, N. Thillaigovindan and Y. Jun [16] in 2007. T.Samanta and Iqbal H. Jebril [15] in 2009, introduced the definition of finite dimensional intuitionistic fuzzy normed space.In 2010, B. Dinda and T.K. Samanta [7] introduced the concept of intuitionistic fuzzy convergence and uniformly intuitionistic fuzzy convergence of a sequence of functions. T. Beaula and D. Esthar[2] in 2013, introduced the definition of strong fuzzy continuity and weak fuzzy continuity and boundedness in intuitionistic 2-fuzzy 2-normed space.In 1999, D. Molodostov [12], introduced soft set theory. In 2002, Maji[10] studied a new concept called fuzzy soft set. The same authors [11] also extended crisp soft sets to fuzzy soft sets. In2012, S. Das, P. Majumdar and S.K. Samanta[4],are introduced soft linear spaces and soft normed linear spaces.In 2013 Zahedi,A. Kilicman and A. RazakSalleh [9] coined fuzzy soft norm over a set and established the relationship between fuzzy soft norm and fuzzy norm over a set. T. Beaula and M. Merlin [3] in 2015,introduced the concept of fuzzy soft convergence and fuzzy soft Cauchy in fuzzy soft normed linear space.

2. Preliminaries

Definition (2.1)[18]:

Let X be a non-empty set and I be the closed interval $I = [0,1]$ of real numbers. A fuzzy set μin X (or a fuzzy subset from X) is a function from X into $I = [0\,,1]$."If μ is a fuzzy set in X then μ is described as characteristic function which connects every $x \in X$ to real number $\mu(x)$ in the interval I. $\mu(x)$is the grade of membership function to x inμ. μcan be described completely as":

$\mu = \{(x\,,\mu(x)): x \in X,\ 0 \le \mu(x) \le 1\}$ or $\mu = \left\{\frac{\mu(x)}{x} : x \in X\right\}$

where$\mu(x)$ is called the membership function for the fuzzy set μ.

The family of all fuzzy sets in X is denoted by I^X.

Definition (2.2)[1]:

"Let X be a non-empty set. An intuitionistic fuzzy set (In short, IFS) A is an object having the form: A={$\langle x, \mu_A(x), \upsilon_A(x)\rangle, x \in X$ }, where the functions $\mu_A: X \longrightarrow I$ and $\upsilon_A: X \longrightarrow I$ denote the degree of membership and the degree of non-membership of each element $x \in X$ to the set A (respectively) and $0 \leq \mu_A(x) + \upsilon_A(x) \leq 1$ for all $x \in X$. The family of all intuitionistic fuzzy sets denoted by IF(X).

Furthermore, we call:

$\pi_A(x) = 1 - \mu_A(x) - \upsilon_A(x)$, $x \in X$, the intuitionistic index or hesitancy degree of x in A. It is obvious that $0 \leq \pi_A(x) \leq 1$ for all$x \in X$. "

Definition (2.3)[13]:

The 5-tuple $(X, \mu, \upsilon, *, \lozenge)$ is said to be an intuitionistic fuzzy normed linear space (In short, IFNLS) if X is a linear space over field F,$*$ is a continuous t-norm , $\lozenge$ is a continuous t-conorm and μ , υ are fuzzy sets in $X \times (0, \infty)$($i.e.$ $\mu, \upsilon: X \times (0, \infty) \rightarrow [0,1]$) satisfying the following conditions: for all $x, y \in X$ and $t, s > 0$, "

$(\boldsymbol{IFN.1})$ $\mu(x,t) + \upsilon(x,t) \leq 1$,

$(\boldsymbol{IFN.2})$ $\mu(x,t) > 0$,

$(\boldsymbol{IFN.3})$ $\mu(x,t) = 1$ if and only if $x = 0$,

$(\boldsymbol{IFN.4})$ $\mu(\alpha x,t) = \mu\left(x, \frac{t}{|\alpha|}\right)$for each $F|\{0\}$,

$(\boldsymbol{IFN.5})$ $\mu(x+y, t+s) \geq \mu(x,t) * \mu(y,s)$,

$(\boldsymbol{IFN.6})$ $\mu(x ,\bullet) : (0 ,\infty) \rightarrow [0 ,1]$is continuous,

$(\boldsymbol{IFN.7})$ $\lim_{t\to\infty} \mu(x,t) = 1$ and$\lim_{t\to 0} \mu(x ,t) = 0$,

$(\boldsymbol{IFN.8})$ $\upsilon(x,t) < 1$,

$(\boldsymbol{IFN.9})$ $\upsilon(x,t) = 0$ if and only if $x = 0$,

$(\boldsymbol{IFN.10})$ $\upsilon(\alpha x,t) = \upsilon\left(x, \frac{t}{|\alpha|}\right)$for each $F|\{0\}$,

$(\boldsymbol{IFN.11})$ $\upsilon(x+y, t+s) \leq \upsilon(x,t) \ \lozenge \ \upsilon(y,s)$

$(\boldsymbol{IFN.12})$ $\upsilon(x ,\bullet) : (0,\infty) \rightarrow [0 ,1]$is continuous .

$(\boldsymbol{IFN.13})$ $\lim_{t\to\infty} \upsilon(x,t) = 0$ and$\lim_{t\to 0} \upsilon(x,t) = 1$.

Definition (2.4) [12]:

Let X be a universe and E be a set of parameters. Let $P(X)$ denote the power set of X.

A pair (F, E) is called a soft set over X, where F is a mapping given by $F: E \rightarrow P(X)$.

In other words, a soft set over X is a parameterized family of subsets of the universe X.
"

Definition (2.5) [11]:

(1)A soft set (F,E)over X is said to be null soft set denoted by $\emptyset$, if for all $e \in E, F(e) = \emptyset$.

(**2**)"A soft set (F,E)over X is said to be an absolute soft set denoted by $\tilde{X}$, if for all $e \in E, F(e) = X$."

Definition (2.6) [6]:

Let R be the set of real and $B(R)$ be the collection of all non-empty bounded subsets of R and E taken a set of parameters. Then a mapping $F: E \to B(R)$ is called a soft real set. If a soft real set is a singleton soft set, it will be called a soft real number and denoted by $\tilde{r}, \tilde{s}, \tilde{t}$ etc.
"

Remarks (2.7) [6]:

(1)$\tilde{0}, \tilde{1}$are the soft real numbers where $\tilde{0}(e) = 0\,, \tilde{1}(e) = 1, \forall e \in E\ (respectively)$.

(2) The set of all soft real numbers denoted by $R(E)$ and the set of all non-negative soft real numbers by $R(E)^*$.

Definition(2.8)[6]:

Let $\tilde{r}, \tilde{s}$ be two soft real numbers, then the following statements:

1) $\tilde{r} \,\tilde{\leq}\, \tilde{s}\ \ if\ \tilde{r}(e) \leq \tilde{s}(e)\ for\ all\ \ e \in E;$

2) $\tilde{r} \,\tilde{\geq}\, \tilde{s}\ \ if\ \tilde{r}(e) \geq \tilde{s}(e)\ for\ all\ \ e \in E;$

3) $\tilde{r} \,\tilde{<}\, \tilde{s}\ \ if\ \tilde{r}(e) < \tilde{s}(e)\ for\ all\ \ e \in E;$

4) $\tilde{r} \,\tilde{>}\, \tilde{s}\ \ if\ \tilde{r}(e) > \tilde{s}(e)\ for\ all\ \ e \in E;$

hold.

Definition(2.9)[17]:

Let X be a vector space over a field K and let E be a parameter set. Let (F,E) be a soft set over X. The soft set (F,E) is said to be a soft vector and denoted by $\tilde{x}_e$ if there is exactly one $e \in E$, such that $F(e) = \{x\}$ for some $x \in X$ and $F(e') = \emptyset, \forall e' \in E/\{e\}$.

The set of all soft vectors over $\tilde{X}$ will be denoted by $SV(\tilde{X})$.

Definition(2.10)[17]:

Two soft vectors $\tilde{x}_e , \tilde{y}_{e'}$ are said to be equal if $e = e'$ and $x = y$.Thus $\tilde{x}_e \neq \tilde{y}_{e'} \Leftrightarrow x \neq y$ or $e \neq e'$.

Proposition(2.11)[17]:

The set $SV(\tilde{X})$ is a vector space according to the following operations:

1) $\tilde{x}_e + \tilde{y}_{e'} = (\widetilde{x+y})_{(e+e')}$ $for\ every$ $\tilde{x}_e , \tilde{y}_{e'} \tilde{\in} SV(\tilde{X})$.

2) $\tilde{r}.\tilde{x}_e = (\widetilde{rx})_{(re)}$for every$\tilde{x}_e \tilde{\in} SV(\tilde{X})$and for every soft real number$\tilde{r}$.

Definition(2.12)[17.5]:

Let $SV(\tilde{X})$ be a soft vector space. Then a mapping $\|\cdot\|: SV(\tilde{X}) \to R^+(E)$ is said to be a soft norm on $SV(\tilde{X})$, if $\|\cdot\|$ satisfies the following conditions:

1) $\|\tilde{x}_e\| \tilde{\geq} \tilde{0}$ and $\|\tilde{x}_e\| = \tilde{0} \Leftrightarrow \tilde{x}_e = \tilde{\theta}_0$.

2) $\|\tilde{r}.\tilde{x}_e\| = |\tilde{r}|\|\tilde{x}_e\|$for all $\tilde{x}_e \tilde{\in} SV(\tilde{X})$ and for every soft scalar $\tilde{r}$.

3) $\|\tilde{x}_e + \tilde{y}_{e'}\| \tilde{\leq} \|\tilde{x}_e\| + \|\tilde{y}_{e'}\|$for all $\tilde{x}_e , \tilde{y}_{e'} \tilde{\in} SV(\tilde{X})$.

The soft vector space $SV(\tilde{X})$ with a soft norm $\|.\|$ on $\tilde{X}$ is said to be a soft normed linear space and is denoted by $(\tilde{X}, \|.\|)$.

Definition (2.13)[3]:

The 3-tuple $(\tilde{X}, \Gamma, *)$ is said to be a fuzzy soft normed linear space (In short, FSNLS) if $\tilde{X}$ be an absolute soft linear space over the field K, $*$ is a continuous t-norm, $R(E)^*$ is the set of all positive soft real numbers, $SSP(\tilde{X})$ denote the set of all soft points on $\tilde{X}$ and Γ is a fuzzy set in $\tilde{X} \times R(E)^*$$(i.e. \Gamma : \tilde{X} \times R(E)^* \to [0,1])$ satisfying the following conditions: for all $\tilde{x}_e , \tilde{y}_{e'} \tilde{\in} SSP(\tilde{X}), \tilde{t}, \tilde{s} \tilde{>} \tilde{0}$ and $\tilde{k} \tilde{\in} K$,

$(\boldsymbol{FSN.1})$ $\Gamma(\tilde{x}_e , \tilde{t}) > 0$,

$(\boldsymbol{FSN.2})\Gamma(\tilde{x}_e , \tilde{t}) = 1$ if and only if $\tilde{x}_e = \tilde{\theta}_0$,

$(\boldsymbol{FSN.3})\Gamma(\tilde{k}.\tilde{x}_e , \tilde{t}) = \Gamma\left(\tilde{x}_e , \frac{\tilde{t}}{|\tilde{k}|}\right)$and $\tilde{k} \neq \tilde{0}$, ,

$(\boldsymbol{FSN.4})\Gamma(\tilde{x}_e + \tilde{y}_{e'} , \tilde{t} + \tilde{s}) \tilde{\geq} \Gamma(\tilde{x}_e , \tilde{t}) * \Gamma(\tilde{y}_{e'} , \tilde{s})$,

$(\boldsymbol{FSN.5})\Gamma(\tilde{x}_e , \bullet)$isa continuous non-decreasing function of $R(E)^*$ and $\lim_{\tilde{t}\to\infty} \Gamma(\tilde{x}_e , \tilde{t}) = 1$

3. Main Results

Definition (3.1):

The 5-tuple $(\tilde{X}, \Delta, \nabla, *, \diamond)$ is said to be an intuitionistic fuzzy soft metric space (In short, IFSMS) if $\tilde{X}$is an arbitrary absolute soft set, $*,\diamond$ is a continuous t-norm, $R(E)^*$ is the set of all positive soft real numbers, $SSP(\tilde{X})$ denote the set of all soft points on $\tilde{X}$ and Δ,∇ is a fuzzy set in $SSP(\tilde{X}) \times SSP(\tilde{X}) \times R(E)^* (i.e.\ \Delta,\nabla: SSP(\tilde{X}) \times SSP(\tilde{X}) \times R(E)^* \to [0,1])$ satisfying the following conditions: for all$\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{z}_{e_3} \tilde{\in} SSP(\tilde{X}), \tilde{t}, \tilde{s} \tilde{>} \tilde{0}$,

$(\boldsymbol{IFSM.1})\ \Delta(\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{t}) + \nabla(\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{t}) \leq 1$

$(\boldsymbol{IFSM.2})\ \Delta(\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{t}) > 0$,

$(\boldsymbol{IFSM.3})\Delta(\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{t}) = 1$ if and only if $\tilde{x}_{e_1} = \tilde{y}_{e_2}$,

$(\boldsymbol{IFSM.4})\ \Delta(\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{t}) = \Delta(\tilde{y}_{e_2}, \tilde{x}_{e_1}, \tilde{t})$,

$(\boldsymbol{IFSM.5})\Delta(\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{t} + \tilde{s}) \tilde{\geq} \Delta(\tilde{x}_{e_1}, \tilde{z}_{e_3}, \tilde{t}) * \Delta(\tilde{z}_{e_3}, \tilde{y}_{e_2}, \tilde{s})$,

$(\boldsymbol{IFSM.6})\Delta(\tilde{x}_{e_1}, \tilde{y}_{e_2}, \bullet): R(E)^* \to [0,1]$is continuous;

$(\boldsymbol{IFSM.7})\ \nabla(\tilde{x}_e, \tilde{t}) < 1$,

$(\boldsymbol{IFSM.8})\nabla(\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{t}) = 0$ if and only if $\tilde{x}_{e_1} = \tilde{y}_{e_2}$,

$(\boldsymbol{IFSM.9})\ \nabla(\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{t}) = \nabla(\tilde{y}_{e_2}, \tilde{x}_{e_1}, \tilde{t})$, ,

$(\boldsymbol{IFSM.10})\nabla(\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{t} + \tilde{s}) \tilde{\leq} \nabla(\tilde{x}_e, \tilde{z}_{e_3}, \tilde{t}) \diamond \nabla(\tilde{z}_{e_3}, \tilde{y}_{e'}, \tilde{s})$,

$(\boldsymbol{IFSM.11})\nabla(\tilde{x}_{e_1}, \tilde{y}_{e_2}, \bullet): R(E)^* \to [0,1]$is continuous.

Definition (3.2):

The 5-tuple $(\tilde{X}, \zeta, \eta, *, \diamond)$ is said to be an intuitionistic fuzzy soft normed linear space (In short, IFSNLS) if $\tilde{X}$be an absolute soft linear space over the field K, $*,\diamond$ is a continuous
t-norm, $R(E)^*$ is the set of all positive soft real numbers, $SSP(\tilde{X})$ denote the set of all soft points on $\tilde{X}$ and ζ,η is a fuzzy set in $\tilde{X} \times R(E)^* (i.e. \zeta, \eta : SSP(\tilde{X}) \times R(E)^* \to [0,1])$ satisfying the following conditions: for all $\tilde{x}_e, \tilde{y}_{e'} \tilde{\in} SSP(\tilde{X}), \tilde{t}, \tilde{s} \tilde{>} \tilde{0}\ and\ \tilde{k} \tilde{\in}\ K$,

$(\boldsymbol{IFSN.1})\ \zeta(\tilde{x}_e, \tilde{t}) + \eta(\tilde{x}_e, \tilde{t}) \leq 1$

$(\boldsymbol{IFSN.2})\ \zeta(\tilde{x}_e, \tilde{t}) > 0$,

$(\boldsymbol{IFSN.3})\zeta(\tilde{x}_e, \tilde{t}) = 1$ if and only if $\tilde{x}_e = \tilde{\theta}_0$,

$(\boldsymbol{IFSN.4})\zeta(\tilde{k}.\tilde{x}_e, \tilde{t}) = \zeta\left(\tilde{x}_e, \frac{\tilde{t}}{|\tilde{k}|}\right)and\tilde{k} \neq \tilde{0}$,

$(\boldsymbol{IFSN.5})\zeta(\tilde{x}_e + \tilde{y}_{e'}, \tilde{t} + \tilde{s}) \tilde{\geq} \zeta(\tilde{x}_e, \tilde{t}) * \zeta(\tilde{y}_{e'}, \tilde{s})$,

$(\boldsymbol{IFSN.6})\zeta(\tilde{x}_e, \bullet)$is a continuous non-decreasing function of $R(E)^*$ and $\lim_{\tilde{t}\to\infty} \zeta(\tilde{x}_e, \tilde{t}) = 1$

$(\boldsymbol{IFSN.7})\eta(\tilde{x}_e, \tilde{t}) < 1$,

$(\boldsymbol{IFSN.8})\eta(\tilde{x}_e, \tilde{t}) = 0$ if and only if $\tilde{x}_e = \tilde{\theta}_0$,

$(\boldsymbol{IFSN.9})\eta(\tilde{k}.\tilde{x}_e, \tilde{t}) = \eta\left(\tilde{x}_e, \frac{\tilde{t}}{|\tilde{k}|}\right)$and $\tilde{k} \neq \tilde{0}$, ,

$(\boldsymbol{IFSN.10})\eta(\tilde{x}_e + \tilde{y}_{e'}, \tilde{t} + \tilde{s}) \lesseqq \eta(\tilde{x}_e, \tilde{t}) \diamond \eta(\tilde{y}_{e'}, \tilde{s})$,

$(\boldsymbol{IFSN.11})\eta(\tilde{x}_e, \bullet)$is a continuous non-increasing function of $R(E)^*$ and $\lim_{\tilde{t}\to\infty} \eta(\tilde{x}_e, \tilde{t}) = 0$

Furthermore, assume that $(\tilde{X}, \zeta, \eta, *, \diamond)$ satisfying the following conditions:

$(\boldsymbol{IFSN.12})$ $\alpha * \alpha = \alpha$ and $\alpha \lozenge \alpha = \alpha$, $\forall \alpha \in [0,1]$,

$(\boldsymbol{IFSN.13})\zeta(\tilde{x}_e, \tilde{t}) > 0$ and$\eta(\tilde{x}_e, \tilde{t}) < 1$, $\forall\, \tilde{t} \gtrsim \tilde{0} \implies \tilde{x}_e = \tilde{\theta}_0$. "

Example(3.3):

Let$(\tilde{X}, \|.\|)$be a soft normed linear space, and let $a * b = a.b$ and $a \diamond b = min\{1, a + b\}\ for\ all\ a, b \in [0,1]$, $\zeta, \eta$is a fuzzy set in $SSP(\tilde{X}) \times R(E)^*$ defined as:

$\zeta(\tilde{x}_e, \tilde{t}) = \frac{h(\tilde{t})^n}{h(\tilde{t})^n + m\|\tilde{x}_e\|}$ and $\eta(\tilde{x}_e, \tilde{t}) = \frac{m\|\tilde{x}_e\|}{k(\tilde{t})^n + m\|\tilde{x}_e\|}$for all $h, k, m, n \in R^+$.

Then $(\tilde{X}, \zeta, \eta, *, \diamond)$ is an IFSNLS.

In particular, if $h = k = m = n = 1$, we have $\zeta(\tilde{x}_e, \tilde{t}) = \frac{\tilde{t}}{\tilde{t} + \|\tilde{x}_e\|}$ and $\eta(\tilde{x}_e, \tilde{t}) = \frac{\|\tilde{x}_e\|}{\tilde{t} + \|\tilde{x}_e\|}$

for all $\tilde{t} \gtrsim \tilde{0}$,

In this case, we said $(\tilde{X}, \zeta, \eta, *, \diamond)$ standard intuitionistic fuzzy soft normed linear space.

Definition (3.4):

Let $(\tilde{X}, \zeta, \eta, *, \diamond)$ be an intuitionistic fuzzy soft normed linear space and let $\{\tilde{x}^n_{e_j}\}$ be a sequence of soft vectors in $\tilde{X}$, then:

1)A sequence $\{\tilde{x}^n_{e_j}\}$ is said to be converges to $\tilde{x}^0_{e_j}$ w.r.t. (ζ, η), if for each$\alpha \in (0,1)\ and\ \tilde{t} \gtrsim \tilde{0}$, there exists$n_0 \in Z^+$ such that $\zeta\left(\tilde{x}^n_{e_j} - \tilde{x}^0_{e_j}, \tilde{t}\right) \gtrsim 1 - \alpha$ and $\eta\left(\tilde{x}^n_{e_j} - \tilde{x}^0_{e_j}, \tilde{t}\right) \lesssim \alpha$ for every $n \geq n_0$.

(or equivalently $\lim_{\tilde{t}\to\infty} \zeta\left(\tilde{x}^n_{e_j} - \tilde{x}^0_{e_j}, \tilde{t}\right) = 1$ and$\lim_{\tilde{t}\to\infty} \eta\left(\tilde{x}^n_{e_j} - \tilde{x}^0_{e_j}, \tilde{t}\right) = 0, as\ \tilde{t} \to \infty$).

2) A sequence $\{\tilde{x}^n_{e_j}\}$ is said to be Cauchy sequence w.r.t. (ζ, η), if for each $\alpha \in (0,1)\ and\ \tilde{t} \gtrsim \tilde{0}$, there exists$n_0 \in Z^+$ such that $\zeta\left(\tilde{x}^n_{e_j} - \tilde{x}^m_{e_j}, \tilde{t}\right) \gtrsim 1 - \alpha$ and $\eta\left(\tilde{x}^n_{e_j} - \tilde{x}^m_{e_j}, \tilde{t}\right) \lesssim \alpha$ for every $n, m \geq n_0$.

(or equivalently $\lim_{\tilde{t}\to\infty} \zeta\left(\tilde{x}^n_{e_j} - \tilde{x}^m_{e_j}, \tilde{t}\right) = 1$ and $\lim_{\tilde{t}\to\infty} \eta\left(\tilde{x}^n_{e_j} - \tilde{x}^m_{e_j}, \tilde{t}\right) = 0, as\ \tilde{t} \to \infty$).

Definition (3.5):

Let $(\tilde{X}, \zeta, \eta, *, \diamond)$ be an intuitionistic fuzzy soft normed linear space. Then $(\tilde{X}, \zeta, \eta, *, \diamond)$ is said to be complete if every Cauchy sequence in $SSP(\tilde{X})$ converges to a soft vector of $SSP(\tilde{X})$.

Definition (3.6):

Let $(\tilde{X}, \zeta, \eta, *, \diamond)$ be an intuitionistic fuzzy soft normed linear space. Then open ball $B(\tilde{x}_{e_1}, r, \tilde{t})$, the closed ball $B[\tilde{x}_{e_1}, r, \tilde{t}]$ and a sphere $S(\tilde{x}_{e_1}, r, \tilde{t})$ with center at $\tilde{x}_{e_1} \tilde{\in} SSP(\tilde{X})$ and radius $0 < r < 1, \tilde{t} \tilde{>} \tilde{0}$ are defined as follows:

$B(\tilde{x}_{e_1}, r, \tilde{t}) = \{\tilde{y}_{e_2} \tilde{\in} SSP(\tilde{X}) : \zeta(\tilde{x}_{e_1} - \tilde{y}_{e_2}, \tilde{t}) \tilde{>} 1 - r\ and\ \eta(\tilde{x}_{e_1} - \tilde{y}_{e_2}, \tilde{t}) \tilde{<} r\}$

$B[\tilde{x}_{e_1}, r, \tilde{t}] = \{\tilde{y}_{e_2} \tilde{\in} SSP(\tilde{X}) : \zeta(\tilde{x}_{e_1} - \tilde{y}_{e_2}, \tilde{t}) \tilde{\geq} 1 - r\ and\ \eta(\tilde{x}_{e_1} - \tilde{y}_{e_2}, \tilde{t}) \tilde{\leq} r\}$

$S(\tilde{x}_{e_1}, r, \tilde{t}) = \{\tilde{y}_{e_2} \tilde{\in} SSP(\tilde{X}) : \zeta(\tilde{x}_{e_1} - \tilde{y}_{e_2}, \tilde{t}) = 1 - r\ and\ \eta(\tilde{x}_{e_1} - \tilde{y}_{e_2}, \tilde{t}) = r\}$.

Definition (3.7):

Let $(\tilde{X}, \zeta, \eta, *, \diamond)$ be an intuitionistic fuzzy soft normed linear space and A be a subset of $SSP(\tilde{X})$. Then:

1) A is said to be open set if for each $\tilde{x}_{e_1} \tilde{\in} A$, there exists $\tilde{t} \tilde{>} \tilde{0}\ and\ 0 < r < 1$ such that $B(\tilde{x}_{e_1}, r, \tilde{t}) \tilde{\subseteq} A$.

2) A is said to be closed set if for any sequence $\{\tilde{x}^n_{e_j}\}$ in A converges to $\tilde{x}^0_{e_j} \tilde{\in} A$.

3) A is said to be bounded set if there exists $\tilde{t} \tilde{>} \tilde{0}\ and\ 0 < r < 1$ such that $\zeta(\tilde{x}_e, \tilde{t}) \tilde{>} 1 - r\ and\ \eta(\tilde{x}_e, \tilde{t}) \tilde{<} r, \forall\ \tilde{x}_e \tilde{\in} A$.

4) A is said to be compact set if for any sequence $\{\tilde{x}^n_{e_j}\}$ in A has s subsequence is converging to an element of A.

Theorem (3.8):

In intuitionistic fuzzy soft normed linear space the intersection finite numbers of open sets is open.

Proof

Let $(\tilde{X},\zeta,\eta,*,\diamond)$ be an IFSNLS and let $\{B_i: i = 1,2,3,\ldots,n\}$ be a finite collection of open set in IFSNLS,

Let $H = \{B_i: i = 1,2,3,\ldots,n\}$ we must to prove H is open set.

Let $\tilde{x}_e \tilde{\in} H \Longrightarrow \tilde{x}_e \tilde{\in} B_i, \forall i = 1,2,3,\ldots,n$ and B_i open set $\forall i$

$\Longrightarrow \exists r_i \in (0,1)\ and\ \tilde{t_i} \tilde{>} \tilde{0}\ s.t.\ B(\tilde{x}_e, r_i, \tilde{t_i}) \tilde{\subseteq} B_i, i = 1,2,3,\ldots,n$

Let $\tilde{t_k} = max\{\tilde{t_1},\tilde{t_2},\tilde{t_3},\ldots,\tilde{t_n}\}\ and\ r_k = min\{r_1,r_2,r_3,\ldots,r_n\}$

$\Longrightarrow B(\tilde{x}_e, r_k, \tilde{t_k}) \tilde{\subseteq} B_i\ for\ all\ i = 1,2,3,\ldots,n$

$\Longrightarrow B(\tilde{x}_e, r_k, \tilde{t_k}) \tilde{\subseteq} \cap B_i \quad \Longrightarrow B(\tilde{x}_e, r_k, \tilde{t_k}) \tilde{\subseteq} H$

$\Longrightarrow H$ is open set. ■

Theorem (3.9):

In intuitionistic fuzzy soft normed linear space, the union of an arbitrary collection of open sets is open.

Proof

Let $(\tilde{X},\zeta,\eta,*,\diamond)$ be an IFSNLS and let $\{G_\lambda: \lambda \in \Lambda\}$ be arbitrary collection of open set in $\tilde{X}$.

Let $G = \cup\{G_\lambda: \lambda \in \Lambda\}$ we must prove G is open set,

Now, let $\tilde{x}_e \tilde{\in} G\ \ then\ \ \tilde{x}_e \tilde{\in} G_\lambda$ for some $\lambda \in \Lambda$

Since G_λ is open set

$\Longrightarrow$there exist $r \in (0,1), \tilde{t} \tilde{>} \tilde{0}$ s.t. $B(\tilde{x}_e, r, \tilde{t}) \tilde{\subseteq} G_\lambda$ and since $G_\lambda \tilde{\subseteq} G$

$\Longrightarrow B(\tilde{x}_e, r, \tilde{t}) \tilde{\subseteq} G$

$\Longrightarrow G$ is open set. ■

Theorem (3.10):

Let $(\tilde{X},\zeta,\eta,*,\diamond)$ be an intuitionistic fuzzy soft normed linear space if A is open set in soft linear space $\tilde{X}$ and $B \tilde{\subseteq} \tilde{X}$ then $A + B$ is open setin $\tilde{X}$.

Proof

Let $\tilde{x}_e \tilde{\in} SSP(\tilde{X})$ and $\tilde{a}_{e'} \tilde{\in} A$

Since A is open set

$\Longrightarrow$there exist $r \in (0,1), \tilde{t} \tilde{>} \tilde{0}$ s.t. $B(\tilde{a}_{e'}, r, \tilde{t}) \tilde{\subseteq} A$

$\Longrightarrow B(\tilde{a}_{e'}, r, \tilde{t}) + \tilde{x}_e \tilde{\subseteq} A + \tilde{x}_e$

$\Longrightarrow B(\tilde{a}_{e'} + \tilde{x}_e, r, \tilde{t}) \tilde{\subseteq} A + \tilde{x}_e$

$\Longrightarrow A + \tilde{x}_e$is open set in $\tilde{X}\ for\ all\ \tilde{x}_e \tilde{\in} SSP(\tilde{X})$ and since $A + B = \cup\{A + b: b \in B\}$

$\Longrightarrow A + B$is open set in $\tilde{X}$. ■

Theorem (3.11):

Every convergent sequence is Cauchy sequence.

Proof

Let $\left\{\tilde{x}_{e_j}^{n}\right\}$ be a sequence in an IFSNLS $(\tilde{X}, \zeta, \eta, *, \diamond)$. Consider $\left\{\tilde{x}_{e_j}^{n}\right\}$ converges to $\tilde{x}_{e_j}^{0}$.

$\Longrightarrow$if for each $\alpha \in (0,1)$ *and* $\tilde{t} \gtrsim \tilde{0}$, there exist $n_0 \in Z^+$ such that

$\zeta\left(\tilde{x}_{e_j}^{n} - \tilde{x}_{e_j}^{0}, \tilde{t}\right) \gtrsim 1-\alpha$ and$\eta\left(\tilde{x}_{e_j}^{n} - \tilde{x}_{e_j}^{0}, \tilde{t}\right) \lesssim \alpha$ for every $n \geq n_0$.

$\Longrightarrow \zeta\left(\tilde{x}_{e_j}^{n} - \tilde{x}_{e_j}^{m}, \tilde{t}\right) = \zeta\left(\tilde{x}_{e_j}^{n} - \tilde{x}_{e_j}^{m} + \tilde{x}_{e_j}^{0} - \tilde{x}_{e_j}^{0}, \tilde{t}\right)$

$= \zeta\left(\left(\tilde{x}_{e_j}^{n} - \tilde{x}_{e_j}^{0}\right) + \left(\tilde{x}_{e_j}^{m} - \tilde{x}_{e_j}^{0}\right), \tilde{t}\right) \geqq \zeta\left(\tilde{x}_{e_j}^{n} - \tilde{x}_{e_j}^{0}, \frac{\tilde{t}}{2}\right) * \zeta\left(\tilde{x}_{e_j}^{m} - \tilde{x}_{e_j}^{0}, \frac{\tilde{t}}{2}\right)$

$\gtrsim (1-\alpha) * (1-\alpha) = 1-\alpha,$

And

$\eta\left(\tilde{x}_{e_j}^{n} - \tilde{x}_{e_j}^{m}, \tilde{t}\right) = \eta\left(\tilde{x}_{e_j}^{n} - \tilde{x}_{e_j}^{m} + \tilde{x}_{e_j}^{0} - \tilde{x}_{e_j}^{0}, \tilde{t}\right)$

$= \eta\left(\left(\tilde{x}_{e_j}^{n} - \tilde{x}_{e_j}^{0}\right) + \left(\tilde{x}_{e_j}^{m} - \tilde{x}_{e_j}^{0}\right), \tilde{t}\right) \leqq \eta\left(\tilde{x}_{e_j}^{n} - \tilde{x}_{e_j}^{0}, \frac{\tilde{t}}{2}\right) \diamond \eta\left(\tilde{x}_{e_j}^{m} - \tilde{x}_{e_j}^{0}, \frac{\tilde{t}}{2}\right)$

$\lesssim \alpha \diamond \alpha = \alpha$

$\Longrightarrow \zeta\left(\tilde{x}_{e_j}^{n} - \tilde{x}_{e_j}^{m}, \tilde{t}\right) \gtrsim 1-\alpha$ *and* $\eta\left(\tilde{x}_{e_j}^{n} - \tilde{x}_{e_j}^{m}, \tilde{t}\right) \lesssim \alpha$ *for every* $n, m \geq n_0$

and $\alpha \in (0,1)$,

Thus $\left\{\tilde{x}_{e_j}^{n}\right\}$ is a Cauchy sequence. ■

Theorem (3.12):

Limit of a sequence in an IFSNLS, if there exists is unique.

Proof

Let $\left\{\tilde{x}_{e_j}^{n}\right\}$ be a sequence in an IFSNLS $(\tilde{X}, \zeta, \eta, *, \diamond)$, such that

$\lim_{\tilde{t}\to\infty} \zeta\left(\tilde{x}_{e_j}^{n} - \tilde{x}_{e}, \tilde{t}\right) = 1$ and$\lim_{\tilde{t}\to\infty} \eta\left(\tilde{x}_{e_j}^{n} - \tilde{x}_{e}, \tilde{t}\right) = 0$ and

$\lim_{\tilde{t}\to\infty} \zeta\left(\tilde{x}_{e_j}^{n} - \tilde{x}_{e'}, \tilde{t}\right) = 1$ and$\lim_{\tilde{t}\to\infty} \eta\left(\tilde{x}_{e_j}^{n} - \tilde{x}_{e'}, \tilde{t}\right) = 0$ are two limits of the sequence $\left\{\tilde{x}_{e_j}^{n}\right\}$.

Then by definition there exists positive integers n_1, n_2 such that $\zeta\left(\tilde{x}_{e_j}^{n} - \tilde{x}_{e}, \tilde{t}\right) \gtrsim 1-\alpha$

and $\eta\left(\tilde{x}_{e_j}^{n} - \tilde{x}_{e}, \tilde{t}\right) \lesssim \alpha$ *for every* $n \geq n_1$ *and* $\alpha \in (0,1)$

And

$\zeta\left(\tilde{x}_{e_j}^n - \tilde{x}_{e'}, \tilde{t}\right) \gtrsim 1-\alpha$ and $\eta\left(\tilde{x}_{e_j}^n - \tilde{x}_{e'}, \tilde{t}\right) \lesssim \alpha$ $for\ every\ n \geq n_2\ and\ \alpha \in (0,1)$

Choose $n \geq n_0, n_0 = min\{n_1, n_2\}$

$\zeta(\tilde{x}_e - \tilde{x}_{e'}, \tilde{t}) = \zeta\left(\tilde{x}_e - \tilde{x}_{e_j}^n + \tilde{x}_{e_j}^n - \tilde{x}_{e'}, \tilde{t}\right) = \zeta\left(\left(\tilde{x}_{e_j}^n - \tilde{x}_e\right) + \left(\tilde{x}_{e_j}^n - \tilde{x}_{e'}\right), \tilde{t}\right)$

$\gtreqqless\zeta\left(\tilde{x}_{e_j}^n - \tilde{x}_e, \frac{\tilde{t}}{2}\right) * \zeta\left(\tilde{x}_{e_j}^n - \tilde{x}_{e'}, \frac{\tilde{t}}{2}\right) \gtrsim (1-\alpha) * (1-\alpha) = 1-\alpha,$

$\eta(\tilde{x}_e - \tilde{x}_{e'}, \tilde{t}) = \eta\left(\tilde{x}_e - \tilde{x}_{e_j}^n + \tilde{x}_{e_j}^n - \tilde{x}_{e'}, \tilde{t}\right) = \eta\left(\left(\tilde{x}_{e_j}^n - \tilde{x}_e\right) + \left(\tilde{x}_{e_j}^n - \tilde{x}_{e'}\right), \tilde{t}\right)$

$\lesssim \eta\left(\tilde{x}_{e_j}^n - \tilde{x}_e, \frac{\tilde{t}}{2}\right) \diamond \eta\left(\tilde{x}_{e_j}^n - \tilde{x}_{e'}, \frac{\tilde{t}}{2}\right) \lesssim \alpha \diamond \alpha = \alpha,$

$\Longrightarrow \zeta(\tilde{x}_e - \tilde{x}_{e'}, \tilde{t}) \gtrsim (1-\alpha)$ and $\eta(\tilde{x}_e - \tilde{x}_{e'}, \tilde{t}) \lesssim \alpha$

$\Longrightarrow \lim_{n\to\infty} \zeta(\tilde{x}_e - \tilde{x}_{e'}, \tilde{t}) = 1$ and $\lim_{n\to\infty} \eta(\tilde{x}_e - \tilde{x}_{e'}, \tilde{t}) = 0$

$\Longrightarrow \zeta(\tilde{x}_e - \tilde{x}_{e'}, \tilde{t}) = 1$ and $\eta(\tilde{x}_e - \tilde{x}_{e'}, \tilde{t}) = 0$

By using the conditions $(\boldsymbol{IFSN.3})$and $(\boldsymbol{IFSN.8})$, we get

$$\tilde{x}_e - \tilde{x}_{e'} = \tilde{\theta}_0 \Longrightarrow \tilde{x}_e = \tilde{x}_{e'}. \blacksquare$$

Theorem (3.13):

Let $\left\{\tilde{x}_{e_j}^n\right\}$, $\{\tilde{y}_{e_i}^n\}$ be a sequences in IFSNLS$(\tilde{X}, \zeta, \eta, *, \diamond)$ and for all $\alpha_1 \in (0,1)$ there exists $\alpha \in (0,1)$ such that $\alpha * \alpha \geq \alpha_1$ and $\alpha \diamond \alpha \geq \alpha_1$:

1) If $\tilde{x}_{e_j}^n \to \tilde{x}_e$ $then$ $\tilde{c}\tilde{x}_{e_j}^n \to \tilde{c}\tilde{x}_e, \tilde{c} \,\tilde{\in}\, K\backslash\{\tilde{0}\}(K\ is\ field)$.

2) If $\tilde{x}_{e_j}^n \to \tilde{x}_e$ and $\tilde{y}_{e_i}^n \to \tilde{y}_{e'}$ $then$ $\tilde{x}_{e_j}^n + \tilde{y}_{e_i}^n \to \tilde{x}_e + \tilde{y}_{e'}$.

Proof

1) Since $\tilde{x}_{e_j}^n \to \tilde{x}_e$

$\Longrightarrow for\ each\ \alpha \in (0,1)\ and\ \tilde{t} \gtrsim \tilde{0}, there\ exists\ n_0 \in Z^+$such that $\zeta\left(\tilde{x}_{e_j}^n - \tilde{x}_e, \tilde{t}\right) \gtrsim 1-\alpha$ and $\eta\left(\tilde{x}_{e_j}^n - \tilde{x}_e, \tilde{t}\right) \lesssim \alpha$ $for\ all\ n \geq n_0$.

put$\tilde{t} = \frac{\tilde{t}_1}{|\tilde{c}|}$ $such\ that$ $\tilde{t}_1 \gtrsim \tilde{0}, \tilde{c} \,\tilde{\in}\, K/\{\tilde{0}\}$

$\zeta\left(\tilde{c}\tilde{x}_{e_j}^n - \tilde{c}\tilde{x}_e, \tilde{t}_1\right) = \zeta\left(\tilde{x}_{e_j}^n - \tilde{x}_e, \frac{\tilde{t}_1}{|\tilde{c}|}\right) = \zeta\left(\tilde{x}_{e_j}^n - \tilde{x}_e, \tilde{t}\right) \gtrsim 1-\alpha$

and$\eta\left(\tilde{c}\tilde{x}_{e_j}^n - \tilde{c}\tilde{x}_e, \tilde{t}_1\right) = \eta\left(\tilde{x}_{e_j}^n - \tilde{x}_e, \frac{\tilde{t}_1}{|\tilde{c}|}\right) = \eta\left(\tilde{x}_{e_j}^n - \tilde{x}_e, \tilde{t}\right) \lesssim \alpha \Longrightarrow \tilde{c}\tilde{x}_{e_j}^n \to \tilde{c}\tilde{x}_e$.

2)Since $\tilde{x}_{e_j}^n \to \tilde{x}_e$

$\Longrightarrow for\ each\ \alpha \in (0,1)\ and\ \tilde{t} \gtrsim \tilde{0}\ , there\ exists\ n_1 \in Z^+$ such that $\zeta\left(\tilde{x}_{e_j}^n - \tilde{x}_e\ , \frac{\tilde{t}}{2}\right) \gtrsim 1-\alpha$

$and\ \ \eta\left(\tilde{x}_{e_j}^n - \tilde{x}_e\ , \frac{\tilde{t}}{2}\right) \precsim \alpha\ \ for\ all\ n \geq n_1$

And since $\tilde{y}_{e_i}^n \to \tilde{y}_{e'}$

$\Longrightarrow for\ each\ \alpha \in (0,1)\ and\ \tilde{t} \gtrsim \tilde{0}\ , there\ exists\ n_2 \in Z^+$ such that $\zeta\left(\tilde{y}_{e_j}^n - \tilde{y}_{e'}\ , \frac{\tilde{t}}{2}\right) \gtrsim 1-\alpha$

$and\ \ \eta\left(\tilde{y}_{e_j}^n - \tilde{y}_{e'}\ , \frac{\tilde{t}}{2}\right) \precsim \alpha\ \ for\ all\ n \geq n_2.$

Take $n_0 = min\{n_1, n_2\}\ \ for\ all\ \alpha \in (0,1)$, there exists $n_0 \in Z^+, (\alpha\ is\ arbitrary)$

$$\zeta\left(\left(\tilde{x}_{e_j}^n + \tilde{y}_{e_i}^n\right) - (\tilde{x}_e + \tilde{y}_{e'})\ , \tilde{t}\right)$$

$$= \zeta\left(\left(\tilde{x}_{e_j}^n - \tilde{x}_e\right) + (\tilde{y}_{e_i}^n - \tilde{y}_{e'})\ , \tilde{t}\right) \geqq \zeta\left(\tilde{x}_{e_j}^n - \tilde{x}_e\ , \frac{\tilde{t}}{2}\right) * \zeta\left(\tilde{y}_{e_i}^n - \tilde{y}_{e'}\ , \frac{\tilde{t}}{2}\right) \gtrsim (1-\alpha) * (1-\alpha) = 1-\alpha$$

And

$$\eta\left(\left(\tilde{x}_{e_j}^n + \tilde{y}_{e_i}^n\right) - (\tilde{x}_e + \tilde{y}_{e'})\ , \tilde{t}\right)$$

$$= \eta\left(\left(\tilde{x}_{e_j}^n - \tilde{x}_e\right) + (\tilde{y}_{e_i}^n - \tilde{y}_{e'})\ , \tilde{t}\right) \leqq \eta\left(\tilde{x}_{e_j}^n - \tilde{x}_e\ , \frac{\tilde{t}}{2}\right) \diamond \eta\left(\tilde{y}_{e_i}^n - \tilde{y}_{e'}\ , \frac{\tilde{t}}{2}\right) \precsim \alpha \diamond \alpha = \alpha$$

By taking $\lim n \to \infty$

$$\lim_{n\to\infty} \zeta\left(\left(\tilde{x}_{e_j}^n + \tilde{y}_{e_i}^n\right) - (\tilde{x}_e + \tilde{y}_{e'}), \tilde{t}\right) = 1\ and\ \lim_{n\to\infty} \eta\left(\left(\tilde{x}_{e_j}^n + \tilde{y}_{e_i}^n\right) - (\tilde{x}_e + \tilde{y}_{e'}), \tilde{t}\right) = 0$$

$\Longrightarrow \tilde{x}_{e_j}^n + \tilde{y}_{e_i}^n \to \tilde{x}_e + \tilde{y}_{e'}.$ ■

Theorem (3.14):

Let $\left\{\tilde{x}_{e_j}^n\right\}$, $\{\tilde{y}_{e_i}^n\}$ be a sequences in IFSNLS$(\tilde{X}, \zeta, \eta, *, \diamond)$ such that $\tilde{x}_{e_j}^n \to \tilde{x}_e\ \ and\ \tilde{y}_{e_i}^n \to \tilde{y}_{e'}$. $\tilde{\alpha}, \tilde{\beta} \,\tilde{\in}\, K \backslash \{\tilde{0}\}$ then $\tilde{\alpha} f\left(\tilde{x}_{e_j}^n\right) + \tilde{\beta} g(\tilde{y}_{e_i}^n) \to \tilde{\alpha} f(\tilde{x}_e) + \tilde{\beta} g(\tilde{y}_{e'})$ whenever $f\ and\ g$ are two identity function.

Proof

Since $\tilde{x}_{e_j}^n \to \tilde{x}_e$

$\Longrightarrow for\ each\ \epsilon \in (0,1)\ and\ \tilde{t} \gtrsim \tilde{0}\ , there\ exists\ n_1 \in Z^+$ such that $\zeta\left(\tilde{x}_{e_j}^n - \tilde{x}_e\ , \frac{\tilde{t}}{2|\tilde{\alpha}|}\right) \gtrsim 1-\epsilon$

$and\ \ \eta\left(\tilde{x}_{e_j}^n - \tilde{x}_e\ , \frac{\tilde{t}}{2|\tilde{\alpha}|}\right) \precsim \epsilon\ \ for\ all\ n \geq n_1$

and since $\tilde{y}_{e_i}^n \to \tilde{y}_{e'}$

$\Longrightarrow for\ each\ \epsilon \in (0,1)\ and\ \tilde{t} \gtrsim \tilde{0}\ , there\ exists\ n_2 \in Z^+$ such that $\zeta\left(\tilde{y}_{e_i}^n - \tilde{y}_{e'}\ , \frac{\tilde{t}}{2|\tilde{\beta}|}\right) \gtrsim 1-\epsilon$

$and \ \ \eta\left(\tilde{y}_{e_i}^n - \tilde{y}_{e'}, \frac{\tilde{t}}{2|\tilde{\beta}|}\right) \precsim \epsilon \ for \ all \ n \geq n_2.$

Take $n_0 = min\{n_1, n_2\} \ for \ all \ \epsilon \in (0,1)$, there exists $n_0 \in Z^+, (\epsilon \ is \ arbitrary)$

$$\zeta\left(\left(\tilde{\alpha}f\left(\tilde{x}_{e_j}^n\right) + \tilde{\beta}g(\tilde{y}_{e_i}^n)\right) - \left(\tilde{\alpha}f(\tilde{x}_e) + \tilde{\beta}g(\tilde{y}_{e'})\right), \tilde{t}\right)$$

$$= \zeta\left(\tilde{\alpha}\left(f\left(\tilde{x}_{e_j}^n\right) - f(\tilde{x}_e)\right) + \tilde{\beta}\left(g(\tilde{y}_{e_i}^n) - g(\tilde{y}_{e'})\right), \tilde{t}\right) \gtrsim \zeta\left(f\left(\tilde{x}_{e_j}^n\right) - f(\tilde{x}_e), \frac{\tilde{t}}{2|\tilde{\alpha}|}\right)$$

$$* \zeta\left(g(\tilde{y}_{e_i}^n) - g(\tilde{y}_{e'}), \frac{\tilde{t}}{2|\tilde{\beta}|}\right) = \zeta\left(\tilde{x}_{e_j}^n - \tilde{x}_e, \frac{\tilde{t}}{2|\tilde{\alpha}|}\right) * \zeta\left(\tilde{y}_{e_i}^n - \tilde{y}_{e'}, \frac{\tilde{t}}{2|\tilde{\beta}|}\right) \gtrsim (1-\epsilon) * (1-\epsilon)$$

$$= 1 - \epsilon$$

And

$$\eta\left(\left(\tilde{\alpha}f\left(\tilde{x}_{e_j}^n\right) + \tilde{\beta}g(\tilde{y}_{e_i}^n)\right) - \left(\tilde{\alpha}f(\tilde{x}_e) + \tilde{\beta}g(\tilde{y}_{e'})\right), \tilde{t}\right)$$

$$= \eta\left(\tilde{\alpha}\left(f\left(\tilde{x}_{e_j}^n\right) - f(\tilde{x}_e)\right) + \tilde{\beta}\left(g(\tilde{y}_{e_i}^n) - g(\tilde{y}_{e'})\right), \tilde{t}\right) \lesssim \eta\left(f\left(\tilde{x}_{e_j}^n\right) - f(\tilde{x}_e), \frac{\tilde{t}}{2|\tilde{\alpha}|}\right)$$

$$\diamond \eta\left(g(\tilde{y}_{e_i}^n) - g(\tilde{y}_{e'}), \frac{\tilde{t}}{2|\tilde{\beta}|}\right) = \eta\left(\tilde{x}_{e_j}^n - \tilde{x}_e, \frac{\tilde{t}}{2|\tilde{\alpha}|}\right) \diamond \eta\left(\tilde{y}_{e_i}^n - \tilde{y}_{e'}, \frac{\tilde{t}}{2|\tilde{\beta}|}\right) \precsim \epsilon \diamond \epsilon = \epsilon \text{for all } n \geq n_0,$$

Therefor $\tilde{\alpha}f\left(\tilde{x}_{e_j}^n\right) + \tilde{\beta}g(\tilde{y}_{e_i}^n) \to \tilde{\alpha}f(\tilde{x}_e) + \tilde{\beta}g(\tilde{y}_{e'})$. ■

Theorem (3.15):

An IFSNLS $(\tilde{X}, \zeta, \eta, *, \diamond)$ in which every Cauchy sequence has a convergent subsequence is complete.

Proof

Let $\left\{\tilde{x}_{e_j}^n\right\}$ be a Cauchy sequence in an IFSNLS $(\tilde{X}, \zeta, \eta, *, \diamond)$ and $\left\{\tilde{x}_{e_j}^{n_k}\right\}$ be a subsequence that converges to $\tilde{x}_{e_j}^0$.

We prove that $\left\{\tilde{x}_{e_j}^n\right\}$ converges to $\tilde{x}_{e_j}^0$.

Let $\tilde{t} \gtrsim \tilde{0}$ and $\alpha \in (0,1)$

Since $\left\{\tilde{x}_{e_j}^n\right\}$ be a Cauchy sequence, there exists $n_0 \in Z^+$ such that $\zeta\left(\tilde{x}_{e_j}^n - \tilde{x}_{e_j}^k, \frac{\tilde{t}}{2}\right) \gtrsim 1 - \alpha$

$and \ \ \zeta\left(\tilde{x}_{e_j}^n - \tilde{x}_{e_j}^k, \frac{\tilde{t}}{2}\right) \precsim \alpha \ for \ all \ n, k \geq n_0$

since $\left\{\tilde{x}_{e_j}^{n_k}\right\}$ converges to $\tilde{x}_{e_j}^0$, there is a positive integer $i_k > n_0$, such that $\zeta\left(\tilde{x}_{e_j}^{i_k} - \tilde{x}_{e_j}^0, \frac{\tilde{t}}{2}\right) \gtrsim 1 - \alpha$

$and \ \ \zeta\left(\tilde{x}_{e_j}^{i_k} - \tilde{x}_{e_j}^0, \frac{\tilde{t}}{2}\right) \precsim \alpha$

Now,

$$\zeta\left(\tilde{x}_{e_j}^n - \tilde{x}_{e_j}^0, \frac{\tilde{t}}{2}\right) = \zeta\left(\tilde{x}_{e_j}^n - \tilde{x}_{e_j}^{i_k} + \tilde{x}_{e_j}^{i_k} - \tilde{x}_{e_j}^0, \frac{\tilde{t}}{2} + \frac{\tilde{t}}{2}\right) \geqq \zeta\left(\tilde{x}_{e_j}^n - \tilde{x}_{e_j}^{i_k}, \frac{\tilde{t}}{2}\right) * \zeta\left(\tilde{x}_{e_j}^{i_k} - \tilde{x}_{e_j}^0, \frac{\tilde{t}}{2}\right)$$
$$\gtrsim (1-\alpha) * (1-\alpha) > 1-\alpha$$

And

$$\eta\left(\tilde{x}_{e_j}^n - \tilde{x}_{e_j}^0, \frac{\tilde{t}}{2}\right) = \eta\left(\tilde{x}_{e_j}^n - \tilde{x}_{e_j}^{i_k} + \tilde{x}_{e_j}^{i_k} - \tilde{x}_{e_j}^0, \frac{\tilde{t}}{2} + \frac{\tilde{t}}{2}\right) \leqq \eta\left(\tilde{x}_{e_j}^n - \tilde{x}_{e_j}^{i_k}, \frac{\tilde{t}}{2}\right) \diamond \eta\left(\tilde{x}_{e_j}^{i_k} - \tilde{x}_{e_j}^0, \frac{\tilde{t}}{2}\right) \lesssim \alpha \diamond \alpha$$
$$< \alpha$$

Therefore $\{\tilde{x}_{e_j}^n\}$ converges to $\tilde{x}_{e_j}^0$ in $(\tilde{X}, \zeta, \eta, *, \diamond)$ and hence it is complete. ■

Definition (3.16):

Let $(\tilde{X}, \zeta, \eta, *, \diamond)$ be an IFSNLS satisfying the condition **(IFSNLS.12)** and **(IFSNLS.13)**.

Define $\|\tilde{x}_e\|_\alpha = inf\{\tilde{t} \gtrsim \tilde{0}: \ \zeta(\tilde{x}_e, \tilde{t}) \geqq \alpha \ \ and \ \ \eta(\tilde{x}_e, \tilde{t}) \leqq \ 1-\alpha, \ \alpha \in (0,1)\}$.

$\|\bullet\|_\alpha$are called $\alpha - soft\ norms\ on\ SSP(\tilde{X})$ corresponding to the IFSNLS.

Theorem (3.17):

Let $\|\bullet\|_\alpha, \alpha \in (0,1)$, be defined in definition **(3.16)**, then $\{\|\bullet\|_\alpha: \ \alpha \in (0,1)\}$ is an ascending family of $\alpha - soft\ norms\ on\ SSP(\tilde{X})$.

Proof

(**1**)From definition $\|\tilde{x}_e\|_\alpha \gtrsim \tilde{0}$

And let $\|\tilde{x}_e\|_\alpha = \tilde{0}$

$\Rightarrow inf\{\tilde{t} \gtrsim \tilde{0}: \ \zeta(\tilde{x}_e, \tilde{t}) \geqq \alpha \ \ and \ \ \eta(\tilde{x}_e, \tilde{t}) \leqq \ 1-\alpha, \alpha \in (0,1)\} = \tilde{0}$

$\Rightarrow For\ all\ \ \tilde{t} \gtrsim \tilde{0}: \ \zeta(\tilde{x}_e, \tilde{t}) \geqq \alpha \gtrsim 0 \ \ and \ \ \eta(\tilde{x}_e, \tilde{t}) \leqq \ 1-\alpha \ \lesssim 1, \alpha \in (0,1)$

By condition **(IFSNLS.13)**, we get $\tilde{x}_e = \tilde{\theta}_0$.

Conversely, let $\tilde{x}_e = \tilde{\theta}_0$.

$\Rightarrow \mu(\tilde{x}_e, \tilde{t}) = 1$ and$\upsilon(\tilde{x}_e, \tilde{t}) = 0, \forall\ \tilde{t} \gtrsim \tilde{0}$

$\Rightarrow$ For all $\alpha \in (0,1)$

$inf\{\tilde{t} \gtrsim \tilde{0}: \zeta(\tilde{x}_e, \tilde{t}) \geqq \alpha \ \ and \ \ \eta(\tilde{x}_e, \tilde{t}) \leqq \ 1-\alpha, \alpha \in (0,1)\} \ = \tilde{0}$

$\Rightarrow \|\tilde{x}_e\|_\alpha = \tilde{0}$.

(**2**) For all $\tilde{c} \neq \tilde{0}$

$\|\tilde{c}.\tilde{x}_e\|_\alpha$

$= inf\{\tilde{s} \gtrsim \tilde{0}: \zeta(\tilde{c}\tilde{x}_e, \tilde{s}) \geqq \alpha \ \ and \ \ \eta(\tilde{c}\tilde{x}_e, \tilde{s}) \leqq \ 1-\alpha, \alpha \in (0,1)\}$

$= inf\left\{\tilde{s} \gtrsim \tilde{0}: \zeta\left(\tilde{x}_e, \frac{\tilde{s}}{|\tilde{c}|}\right) \geqq \alpha \ and \ \eta\left(\tilde{x}_e, \frac{\tilde{s}}{|\tilde{c}|}\right) \leqq 1-\alpha, \alpha \in (0,1)\right\}$

Let $\tilde{t} = \frac{\tilde{s}}{|\tilde{c}|} \gtrsim \tilde{0}$,

$\Longrightarrow \|\tilde{c}.\tilde{x}_e\|_\alpha$

$= inf\{|\tilde{c}|\tilde{t} \gtrsim \tilde{0} : \zeta(\tilde{x}_e, \tilde{t}) \geqq \alpha \ and \ \eta(\tilde{x}_e, \tilde{t}) \leqq 1-\alpha, \alpha \in (0,1)\}$

$= |\tilde{c}| inf\{\tilde{t} \gtrsim \tilde{0} : \zeta(\tilde{x}_e, \tilde{t}) \geqq \alpha \ and \ \eta(\tilde{x}_e, \tilde{t}) \leqq 1-\alpha, \alpha \in (0,1)\}$

$$= |\tilde{c}| \|\tilde{x}_e\|_\alpha$$

If $\tilde{c} = \tilde{0} \Longrightarrow \|\tilde{c}.\tilde{x}_e\|_\alpha = \|\tilde{0}.\tilde{x}_e\|_\alpha = \tilde{0} = \tilde{0}\|\tilde{x}_e\|_\alpha = |\tilde{c}|\|\tilde{x}_e\|_\alpha, \forall \tilde{c} \tilde{\in} K$.

$$(3)\|\tilde{x}_e\|_\alpha + \|\tilde{y}_{e'}\|_\alpha$$

$= inf\{\tilde{t} \gtrsim \tilde{0} : \zeta(\tilde{x}_e, \tilde{t}) \geqq \alpha \ and \ \eta(\tilde{x}_e, \tilde{t}) \leqq 1-\alpha, \alpha \in (0,1)\}$

$+inf\{\tilde{s} \gtrsim \tilde{0}: \zeta(\tilde{y}_{e'}, \tilde{s}) \geqq \alpha \ and \ \eta(\tilde{y}_{e'}, \tilde{s}) \leqq 1-\alpha, \alpha \in (0,1)\}$

$= inf\{\tilde{t} + \tilde{s} \gtrsim \tilde{0} : \zeta(\tilde{x}_e, \tilde{t}) * \zeta(\tilde{y}_{e'}, \tilde{s}) \geqq \alpha \text{and} \ \eta(\tilde{x}_e, \tilde{t}) \lozenge \eta(\tilde{y}_{e'}, \tilde{s}) \leqq 1-\alpha, \ \alpha \in (0,1)\}$

$\geqq inf\{\tilde{t} + \tilde{s} \gtrsim \tilde{0} : \zeta(\tilde{x}_e + \tilde{y}_{e'}, \tilde{t} + \tilde{s}) \geqq \alpha \ and \ \eta(\tilde{x}_e + \tilde{y}_{e'}, \tilde{t} + \tilde{s}) \leqq 1-\alpha, \alpha \in (0,1)\}$

$= inf\{\tilde{r} \gtrsim \tilde{0} : \zeta(\tilde{x}_e + \tilde{y}_{e'}, \tilde{r}) \geqq \alpha \ and \ \eta(\tilde{x}_e + \tilde{y}_{e'}, \tilde{r}) \leqq 1-\alpha, \alpha \in (0,1)\}$

$= \|\tilde{x}_e + \tilde{y}_{e'}\|_\alpha$.

Therefore $\|\tilde{x}_e + \tilde{y}_{e'}\|_\alpha \leqq \|\tilde{x}_e\|_\alpha + \|\tilde{y}_{e'}\|_\alpha$

Then $\|\bullet\|_\alpha$ is α-soft norms on $SSP(\tilde{X})$.

We show that for any $0 < \alpha_1 < \alpha_2 < 1$

Then $\|\tilde{x}_e\|_{\alpha_1} \leqq \|\tilde{x}_e\|_{\alpha_2}$

Since $\alpha_1 < \alpha_2$

$\Longrightarrow \{\tilde{t} \gtrsim \tilde{0} : \zeta(\tilde{x}_e, \tilde{t}) \geqq \alpha_2 \ and \ \eta(\tilde{x}_e, \tilde{t}) \leqq 1-\alpha_2, \alpha_2 \in (0,1)\}$

$\tilde{\subseteq} \{\tilde{t} \gtrsim \tilde{0} : \zeta(\tilde{x}_e, \tilde{t}) \geqq \alpha_1 \ and \ \eta(\tilde{x}_e, \tilde{t}) \leqq 1-\alpha_1, \alpha_1 \in (0,1)\}$

$\Longrightarrow inf\{\tilde{t} \gtrsim \tilde{0} : \zeta(\tilde{x}_e, \tilde{t}) \geqq \alpha_2 \ and \ \eta(\tilde{x}_e, \tilde{t}) \leqq 1-\alpha_2, \alpha_2 \in (0,1)\}$

$\geqq inf\{\tilde{t} \gtrsim \tilde{0} : \zeta(\tilde{x}_e, \tilde{t}) \geqq \alpha_1 \ and \ \eta(\tilde{x}_e, \tilde{t}) \leqq 1-\alpha_1, \alpha_1 \in (0,1)\}$

$$\Longrightarrow \|\tilde{x}_e\|_{\alpha_1} \leqq \|\tilde{x}_e\|_{\alpha_2}$$

Then $\{\|\bullet\|_\alpha : \alpha \in (0,1)\}$ is an ascending family of α-soft norms on $SSP(\tilde{X})$ corresponding IFSNLS."

Theorem (3.18):

Every intuitionistic fuzzy soft normed linear space is an intuitionistic fuzzy soft metric space.

Proof

Let $(\tilde{X},\zeta,\eta,*,\diamond)$ be an IFSNLS.

Define the intuitionistic fuzzy soft metric space by $\Delta(\tilde{x}_{e_1},\tilde{y}_{e_2},\tilde{t}) = \zeta(\tilde{x}_{e_1}-\tilde{y}_{e_2},\tilde{t})$ and $\nabla(\tilde{x}_{e_1},\tilde{y}_{e_2},\tilde{t}) = \eta(\tilde{x}_{e_1}-\tilde{y}_{e_2},\tilde{t})$for every $\tilde{x}_{e_1},\tilde{y}_{e_2} \tilde{\in} SSP(\tilde{X})$.

Then it is clear to show that IFSMS axioms are satisfied.

$(\boldsymbol{IFSM.1})$ $\Delta(\tilde{x}_{e_1},\tilde{y}_{e_2},\tilde{t}) + \nabla(\tilde{x}_{e_1},\tilde{y}_{e_2},\tilde{t}) = \zeta(\tilde{x}_{e_1}-\tilde{y}_{e_2},\tilde{t}) + \eta(\tilde{x}_{e_1}-\tilde{y}_{e_2},\tilde{t}) \leq 1$

$(\boldsymbol{IFSM.2})$ $\Delta(\tilde{x}_{e_1},\tilde{y}_{e_2},\tilde{t}) = \zeta(\tilde{x}_{e_1}-\tilde{y}_{e_2},\tilde{t}) > 0$

$(\boldsymbol{IFSM.3})$$\Delta(\tilde{x}_{e_1},\tilde{y}_{e_2},\tilde{t}) = \zeta(\tilde{x}_{e_1}-\tilde{y}_{e_2},\tilde{t}) = 1$ if and only if $\tilde{x}_{e_1}-\tilde{y}_{e_2} = \tilde{\theta}_0$ if and only if

$$\tilde{x}_{e_1} = \tilde{y}_{e_2}.$$

$(\boldsymbol{IFSM.4})$$\Delta(\tilde{x}_{e_1},\tilde{y}_{e_2},\tilde{t}) = \zeta(\tilde{x}_{e_1}-\tilde{y}_{e_2},\tilde{t}) = \zeta(\tilde{y}_{e_2}-\tilde{x}_{e_1},\tilde{t}) = \Delta(\tilde{y}_{e_2},\tilde{x}_{e_1},\tilde{t})$.

$(\boldsymbol{IFSM.5})$$\Delta(\tilde{x}_{e_1},\tilde{y}_{e_2},\tilde{t}+\tilde{s}) = \zeta(\tilde{x}_{e_1}-\tilde{y}_{e_2},\tilde{t}+\tilde{s}) = \zeta(\tilde{x}_{e_1}-\tilde{z}_{e_3}+\tilde{z}_{e_3}-\tilde{y}_{e_2},\tilde{t}+\tilde{s})$
$\tilde{\geq} \zeta(\tilde{x}_{e_1}-\tilde{z}_{e_3},\tilde{t}) * \zeta(\tilde{z}_{e_3}-\tilde{y}_{e_2},\tilde{s}) = \Delta(\tilde{x}_{e_1},\tilde{z}_{e_3},\tilde{t}) * \Delta(\tilde{z}_{e_3},\tilde{y}_{e_2},\tilde{t})$.

From definition of IFSNLS, we get $(\boldsymbol{IFSM.6})$$\Delta(\tilde{x}_{e_1},\tilde{y}_{e_2},\bullet): R(E)^* \to [0,1]$is continuous.

Similarly with respect to η, we getconditions$(\boldsymbol{IFSM.7})$, $(\boldsymbol{IFSM.8})$, $(\boldsymbol{IFSM.9})$, $(\boldsymbol{IFSM.10})$ *and* $(\boldsymbol{IFSM.11})$. Therefor $(\tilde{X},\Delta,\nabla,*,\diamond)$ *is an IFSMS*. ■

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Isolation and diagnosis of some pathogenic bacterial species contamination from red meat in shops and markets in Thi- Qar city

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Abstract

After the application of the possible purification and sterilization procedures, 100 fresh meat samples were collected from local cattle and sheep carcasses. The samples were selected in order to detect some pathogenic bacteria that might be present in the laboratory fresh red meat and categorize them in ways Microbiological methods generally applied.

56bacterial isolation Isolated from fresh sheep meat samples, classified into the following genotypes and genotypes: