Al-Mustansiriyah Journal of Science
ISSN: 1814-635X (print), ISSN:2521-3520 (online) Volume 33, Issue 4, 2022 DOI: http://doi.org/10.23851/mjs.v33i4.1197

Research Article Open Access

Special Arcs in $PG(3,13)$

Saja Makki Attook*, Emad Bakr Al-Zangana

Department of Mathematics, College of Science, Mustansiriyah University, Baghdad, IRAQ.

*Correspondent contact: sawsave2233@gmail.com

Article Info

Received
24/07/2022

Accepted
02/10/2022

Published
30/12/2022

ABSTRACT

In this paper, we constructed new arcs in the three-dimensional projective space over $GF(13)$ of degrees 2, 3, 4, 5, 6, 7, 10, 11, 14, 15, 18, 19, 23, 28, 30, 47, 55, 98 and sizes 2, 4, 5, 7, 10, 14, 17, 20, 28, 34, 35, 68, 70, 85, 119, 140, 170, 238, 340, 476, 595, 1190. These arcs are classified to complete and incomplete arcs. Also, the incomplete arcs are extended to complete arcs.

KEYWORDS: Arcs; finite field; gap; group action; projective space.

الخلاصة

في هذا البحث، نحن أنشأنا اقواس جديدة في الفضاء الاسقاطي من البعد الثالث على $GF(13)$ من الدرجات 2، 3، 4، 5، 6، 7، 10، 11، 14، 15، 18، 19، 23، 28، 30، 47، 55، 98 واحجام 2، 4، 5، 7، 10، 14، 17، 20، 28، 34، 35، 68، 70، 85، 119، 140، 170، 238، 340، 476، 595، 1190. هذه الاقواس تم تصنيفها حسب كونها اقواس تامة وغير تامة. كذلك تم توسيع الاقواس غير التامة الى اقواس تامة.

INTRODUCTION

Let $GF(13) = F_{13} = \{0,1,\tau,\tau^2,\tau^3,\tau^4,\tau^5,\tau^6,\tau^7,\tau^8,\tau^9,\tau^{10},\tau^{11}\}$ is the 13^{th}-order Galois field, where τ is the primitive element of F_{13}.

Let S be the 4×4 companion matrix over F_{13}. Then the projectivity given by the companion matrix is $T = M(S)$, where:

$$S = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \tau^5 & \tau^3 & 0 & 1 \end{bmatrix}$$

is a non-singular matrix of order $\theta(3,13) = 2380$.

As we all know, every projective space has a companion matrix $M(S)$. The points (plane's points π) of $PG(3,13)$ are found by the formula: $P(i) = P_0T^i$ $(\pi(i) = \pi_0T^i)$, where $P_0 = [1,0,0,0]$ (π_0 is the plane with 4-th coordinate's points is equal to zero) and i from 0 to $\theta(3,13) - 1$. The points of $PG(3,13)$ have a unique forms which are $[1,0,0,0]$, $[x,1,0,0]$, $[x,y,1,0]$, $[x,y,z,1]$, where $x,y,z \in F_{13}$, and the number of each form is $1, 13, 13^2, 13^3$, so we have 2380 points and planes, 31110 lines, 14 points on each line and 183 lines passing through each point [1].

The number 2380 has 22 non-trivial divisors which are: 2, 4, 5, 7, 10, 14, 17, 20, 28, 34, 35, 68, 70, 85, 119, 140, 170, 238, 340, 476, 595, 1190.

The aim of the paper is to find new arcs in $PG(3,13)$ by action of groups on $PG(3,13)$ depending on the 22 integers above, and then distinguished these arcs whether they are complete or not.

Finding arcs in finite projective space has become of interest to many researchers in projective spaces because of its close relationship to coding theory [2, 3], and since non-extension codes are important, so they also interest of complete arcs. In [4], geometric methods have used to construct theoretically special types of liner codes. In [5], Al-Mukhtar and in [6], Kareem used the frame to construct arcs in $PG(3,5)$, $PG(3,4)$ respectively. In [7] and [8], new arcs are found in $PG(3,7)$ and $PG(3,4)$ respectively using the idea of reverse. The idea of group action on spaces has been used to find new arcs in [9] and [10] in the spaces $PG(3,8)$ and $PG(3,11)$.

Basic Definitions

Let q be a prime integer or prime power integer.

Definition (1) [11]**:** A $(k;r)$-arc in PG) is a set of k points such that no$r+1$ points are collinear, at most r points of which lie in any plane. Here r is called the degree of coplanar $(k;r)$-arc.

Definition (2) [11]**:** A$(k;r)$-arc is called a complete arc if it is not contained in $(k+1;r)$-arc.

Definition (3) [11]**:** Let K be an arc of degree r, an i-secant of Kin $PG(n,q)$ is a hyperplane π such that$K \cap \pi \vee i$. The number of i-secants of K denoted by τ_i.

Let Q be a point not on the $(k;r)$-arc, the number of i-secants of K passing through Q denoted by $\sigma_i(Q)$. The number $\sigma_r(Q)$ of r-secants is called the index of Q with respect to K. The set of all points of index $i$will be denoted by C_i and the cardinality of C_i denoted by c_i. The sequence $(t_0,\cdots,t_r)$ will be represented the secant distribution and the sequences $(c_0,\cdots,c_d$ refer to the index distribution [11].

Definition (4) [11]**:** The group of projectivity of $PG(n.q)$ is called the projective general linear group, $PGL(n+1,q)$.

The elements of $PGL(n+1,q)$ are non-singular matrices of dimension $n+1$, and its cardinality is $\frac{(q^{n+1}-1)(q^{n+1}-q)\dots(q^{n+1}-q)}{(q-1)}$.

Lagrange's Theorem (5) [12]**:** If G is a finite group and H is a subgroup of G, then $H\vee$ divides $G\vee$.

As it is known in finite group, the number of elements of each orbit $G_x$divides the order of finite group G. Also, the order of each element of G divide the order of G.

The companion matrix, S is an element of $PGL(4,q)$, and it is called a cyclic projectivity; that is, it has order equal to the order of $PG(3,q)$, say $\theta(3,q)$. Therefore, for each n, a positive integer divided $\theta(3,q)$, the set $\langle S^n\rangle$ will be subgroup of the group $\langle S\rangle$ with order t such that $nt=\theta(3,q)$.

New $(k;r)$-Arcs in $PG(3,13)$

In this section, we introduced new arcs formed from the acts of cyclic subgroups of the group $\langle S\rangle$ on $PG(3,13)$.

For any divisor of the number 2380, say P, the subgroup $\langle S^P\rangle$ of $PGL(4,13)$ will has t elements where P times t is equal to 2380.

This group $\langle S^P\rangle$ will acts on the space $PG(3,13)$ and then partitioned the space into P orbits each of them has t points. In our work, we will take the first orbit, say χ_P, and the GAP programming [13] is used to execute the design algorithms to find the elements of the subgroups $\langle S^P\rangle$, to finding orbits, secant distribution and index distribution for each arc. Therefore, we have the following theorem.

Theorem (6): The 22 cyclic subgroups $\langle S^P\rangle$ of $\langle S\rangle$ acts on the projective space $PG(3,13)$ and give the following χ_P arcs:

1. The orbits from the action of $\langle S^2\rangle$ on $PG(3,13)$ are $(1190;98)$-complete arcs.
2. The orbits from the action of $\langle S^4\rangle$ on $PG(3,13)$ are $(595;55)$-complete arcs.
3. The orbits from the action of $\langle S^5\rangle$ on $PG(3,13)$ are $(476;47)$-complete arcs.
4. The orbits from the action of $\langle S^7\rangle$ on $PG(3,13)$ are $(340;28)$-complete arcs.
5. The orbits from the action of $\langle S^{10}\rangle$ on $PG(3,13)$ are $(238;30)$-complete arcs.
6. The orbits from the action of $\langle S^{14}\rangle$ on $PG(3,13)$ are $(170;14)$-complete arcs.
7. The orbits from the action of $\langle S^{17}\rangle$ on $PG(3,13)$ are $(140;23)$-complete arcs.
8. The orbits from the action of $\langle S^{20}\rangle$ on $PG(3,13)$ are $(119;19)$-complete arcs.
9. The orbits from the action of $\langle S^{28}\rangle$ on $PG(3,13)$ are $(85;10)$-complete arcs.
10. The orbits from the action of $\langle S^{34}\rangle$ on $PG(3,13)$ are $(70;18)$-complete arcs.
11. The orbits from the action of $\langle S^{35}\rangle$ on $PG(3,13)$ are $(68;10)$-complete arcs.
12. The orbits from the action of $\langle S^{68}\rangle$ on $PG(3,13)$ are $(35;11)$-incomplete arcs.
13. The orbits from the action of $\langle S^{70}\rangle$ on $PG(3,13)$ are $(34;6)$-complete arcs.
14. The orbits from the action of $\langle S^{85}\rangle$ on $PG(3,13)$ are $(28;15)$-complete arcs.
15. The orbits from the action of $\langle S^{119}\rangle$ on $PG(3,13)$ are $(20;4)$-complete arcs.
16. The orbits from the action of $\langle S^{140}\rangle$ on $PG(3,13)$ are $(17;5)$-incomplete arcs.
17. The orbits from the action of $\langle S^{170}\rangle$ on $PG(3,13)$ are $(14;14)$-complete arcs.
18. The orbits from the action of $\langle S^{238}\rangle$ on $PG(3,13)$ are $(10;4)$-incomplete arcs.
19. The orbits from the action of $\langle S^{340}\rangle$ on $PG(3,13)$ are $(7;7)$-complete arcs.
20. The orbits from the action of $\langle S^{476}\rangle$ on $PG(3,13)$ are $(5;3)$-incomplete arcs.
21. The orbits from the action of $\langle S^{595}\rangle$ on $PG(3,13)$ are $(4;3)$-incomplete arcs.
22. The orbits from the action of $\langle S^{1190}\rangle$ on $PG(3,13)$ are $(2;2)$-complete arcs.

Proof:

1. 1. $P = 2$. The orbit χ_2 ={0,2,4,6,...,2378} has 1190 points, and the numbers of intersection points between χ_2 and planes of $PG(3,13)$ are as follows: 1190 planes with 85 intersection-points, and 1190 planes with 98 intersection-points, So, χ_2 is an $(1190; 98)$-arc. The parameters c_i have the following values with respect to χ_2: $c_{85} = 1190$. Since $c_0 = 0$, thus χ_2 is a complete arc.
2. $P = 4$. The orbit χ_4 ={0,4,8,...,2376} has 595 points, and the numbers of intersection points between χ_4 and planes of $PG(3,13)$ are as follows: 595 planes with 40 intersection-points, 595 planes with 43 intersection-points, 595 planes with 45 intersection-points, and 595 planes with 55 intersection-points. So, χ_4 is an $(595; 55)$-arc. The parameters c_i have the following values with respect to χ_4: $c_0 = c_1 = \cdots = c_{39} = 0$, $c_{40} = 595$,$c_{41} = c_{42} = 0$, $c_{43} = 595$, $c_{44} = 0$, $c_{45} = 595$. Since $c_0 = 0$, thus χ_4 is a complete arc.
3. $P = 5$. The orbit χ_5 ={0,5,10,...,2375} has 476 points, and the numbers of intersection points between χ_5 and planes of $PG(3,13)$ are as follows: 1904 planes with 34 intersection-points, and 476 planes with 47 intersection-points. So, χ_5 is an $(476; 47)$-arc. The parameters c_i have the following values with respect to χ_5: $c_0 = c_1 = \cdots = c_{33} = 0$, $c_{34} = 1904$. Since $c_0 = 0$, thus χ_5 is a complete arc.
4. $P = 7$. The orbit χ_7={0,7,14,...,2373} has 340 points, and the numbers of intersection points between χ_7 and planes of $PG(3,13)$ are as follows: 340 planes with 15 intersection-points, and 2040 planes with 28 intersection-points. So, χ_7 is an $(340; 28)$-arc. The parameters c_i have the following values with respect to χ_7: $c_0 = c_1 = \cdots = c_{154} = 0$, $c_{155} = 2040$. Since $c_0 = 0$, thus χ_7 is a complete arc.
5. $P = 10$. The orbit χ_{10} ={0,10,20,...,2370} has 238 points, and the numbers of intersection points between χ_{10} and planes of $PG(3,13)$ are as follows: 2142 planes with 17 intersection-points, and 238 planes with 30 intersection-points. So, χ_{10} is an $(238; 30)$-arc. The parameters c_i have the following values with respect to χ_{10}: $c_0 = c_1 = \cdots = c_{16} = 0$, $c_{17} = 2142$. Since $c_0 = 0$, thus χ_{10} is a complete arc.
6. $P = 14$. The orbit χ_{14} ={0,14,28,...,2366} has 170 points, and the numbers of intersection points between χ_{14} and planes of $PG(3,13)$ are as follows: 170 planes with 1 intersection-point, and 2210 planes with 14 intersection-points. So, χ_{14} is an $(170; 14)$-arc. The parameters c_i have the following values with respect to χ_{14}: $c_0 = c_1 = \cdots = c_{168} = 0$, $c_{169} = 2210$. Since $c_0 = 0$, thus χ_{14} is a complete arc.
7. $P = 17$. The orbit χ_{17} ={0,17,34,...,2363} has 140 points, and the numbers of intersection points between χ_{17} and planes of $PG(3,13)$ are as follows: 2240 planes with 10 intersection-points, and 140 planes with 23 intersection-points. So, χ_{17} is an $(140; 23)$-arc. The parameters c_i have the following values with respect to χ_{17}: $c_0 = c_1 = \cdots = c_9 = 0$,$c_{10} = 2240$. Since $c_0 = 0$, thus χ_{17} is a complete arc.
8. $P = 20$. The orbit χ_{20}={0,20,40,...,2360} has 119 points, and the numbers of intersection points between χ_{17} and planes of $PG(3,13)$ are as follows: 476 planes with 6 intersection-points, 595 planes with 8 intersection-points, 595 planes with 9 intersection-points, 595 planes with 11 intersection-points, and 119 planes with 19 intersection-points. So, χ_{17} is an $(119; 19)$-arc. The parameters c_i have the following values with respect to χ_{17}: $c_0 = c_1 = \cdots = c_5 = 0$, $c_6 = 476$,$c_7 = 0$, $c_8 = 595$, $c_9 = 595$,$c_{10} = 0$, $c_{11} = 595$. Since $c_0 = 0$, thus χ_{17} is a complete arc.
9. $P = 28$. The orbit χ_{28} ={0,28,56,...,2352} has 85 points, and the numbers of intersection points between χ_{28} and planes of $PG(3,13)$ are as follows: 85 planes with no intersection-points, 85 planes with 1 intersection-point, 170 planes with 4 intersection-points, 425 planes with 5 intersection-points, 340 planes with 6 intersection-points, 340 planes with 7 intersection-points, 340 planes with 8 intersection-points, 425 planes with 9 intersection-points, and 170 planes with 10 intersection-points. So, χ_{28} is an $(85; 10)$-arc. The parameters c_i have the following values with respect to χ_{28}: $c_0 = c_1 = \cdots = c_6 = 0$, $c_7 = 170$, $c_8 = 255$, $c_9 = 0$, $c_{10} = 85$, $c_{11} = 0$, $c_{12} = 680$, $c_{13} = 0$, $c_{14} = 340$, $c_{15} = 0$, $c_{16} = 680$, $c_{17} = 0$, $c_{18} = 85$. Since $c_0 = 0$, thus χ_{28} is a complete arc.

10. $P = 34$. The orbit χ_{34} ={0,34,68,...,2346} has 70 points, and the numbers of intersection points between χ_{34} and planes of $PG(3,13)$ are as follows: 2310 planes with 5 intersection-points, and 70 planes with 18 intersection-points. So, χ_{34} is an $(70;18)$-arc. The parameters c_i have the following values with respect to χ_{34}: $c_0 = c_1 = \cdots = c_4 = 0$, $c_5 = 2310$. Since $c_0 = 0$, thus χ_{34} is a complete arc.
11. $P = 35$. The orbit χ_{35} ={0,35,70,...,2345} has 68 points, and the numbers of intersection points between χ_{35} and planes of $PG(3,13)$ are as follows: 544 planes with 2 intersection-points, 136 planes with 4 intersection-points, 544 planes with 5 intersection-points, 408 planes with 6 intersection-points, 612 planes with 7 intersection-points, and 136 planes with 10 intersection-points. So, χ_{35} is an $(68;10)$-arc. The parameters c_i have the following values with respect to χ_{35}: $c_0 = c_1 = \cdots = c_6 = 0$, $c_7 = 544$, $c_8 = 0$, $c_9 = 544$, $c_{10} = 408$, $c_{11} = 136$, $c_{12} = 0$, $c_{13} = 544$, $c_{14} = 0$, $c_{15} = 0$, $c_{16} = 136$. Since $c_0 = 0$, thus χ_{35} is a complete arc.
12. $P = 68$. The orbit χ_{68} ={0,68,136,..., 2312} has 35 points, and the numbers of intersection points between χ_{68} and planes of $PG(3,13)$ are as follows: 35 planes with no intersection-points, 420 planes with 1 intersection-point, 700 planes with 2 intersection-points, 700 planes with 3 intersection-points, 420 planes with 4 intersection-points, 35 planes with 5 intersection-points, 35 planes with 7 intersection-points, and 35 planes with 11 intersection-points. So, χ_{68} is an $(35;11)$- arc. The parameters c_i have the following values: $c_0 = 35$, $c_1 = 420$, $c_2 = 700$, $c_3 = 700$, $c_4 = 420$, $c_5 = 35, c_7 = 35$. Since $c_0 \neq 0$, so χ_{68} is incomplete arc. The large complete arc can be constructed from χ_{68} is an $(50;11)$-arc, where the set of addition points to χ_{68} is $\{52 + 68k \vee k = 0, \ldots, 14\}$.
13. $P = 70$. The orbit χ_{70} ={0,70,140,..., 2310} has 34 points, and the numbers of intersection points between χ_{70} and planes of $PG(3,13)$ are as follows: 136 planes with no intersection-points, 578 planes with 1 intersection-point, 476 planes with 2 intersection-points, 408 planes with 3 intersection-points, 544 planes with 4 intersection-points, 136 planes with 5 intersection-points, and 102 planes with 6 intersection-points. So, χ_{70} is an $(34;6)$-arc. The parameters c_i have the following values with respect to χ_{70}: $c_0 = c_1 = \cdots = c_3 = 0$, $c_4 = 136$, $c_5 = 272$, $c_6 = 340$, $c_7 = 612$, $c_8 = 68$, $c_9 = 578$, $c_{10} = 68$, $c_{11} = 0$, $c_{12} = 204$, $c_{13} = 0, c_{14} = 68$. Since $c_0 = 0$, thus χ_{70} is a complete arc.
14. $P = 85$. The orbit χ_{85} ={0,85,170,..., 2295} 28 points, and the numbers of intersection points between χ_{85} and planes of $PG(3,13)$ are as follows: 2352 planes with 2 intersection-points, and 28 planes with 15 intersection-points. So, χ_{85} is an $(28;15)$-arc. The parameters c_i have the following values with respect to χ_{85}: $c_0 = c_1 = 0$, $c_2 = 2352$. Since $c_0 = 0$, thus χ_{85} is a complete arc.
15. $P = 119$. The orbit χ_{119} ={0,119,238, ...,2261} has 20 points, and the numbers of intersection points between χ_{119} and planes of $PG(3,13)$ are as follows: 480 planes with no intersection-points, 800 planes with 1 intersection-point, 680 planes with 2 intersection-points, 180 planes with 3 intersection-points, and 240 planes with 4 intersection-points. So, χ_{119} is an $(20;4)$-arc. The parameters c_i have the following values with respect to χ_{119}: $c_0 = c_1 = \cdots = c_{12} = 0$, $c_{13} = 160$, $c_{14} = 0$, $c_{15} = 400$, $c_{16} = 80$, $c_{17} = 320$, $c_{18} = 400$, $c_{19} = 200$, $c_{20} = 200$, $c_{21} = 240$, $c_{22} = 160, c_{23} = 200$. Since $c_0 = 0$, thus χ_{119} is a complete arc.
16. $P = 140$. The orbit χ_{140} ={0,140,280, ...,2240} has 17 points, and the numbers of intersection points between χ_{140} and planes of $PG(3,13)$ are as follows: 595 planes with no intersection-points, 952 planes with 1 intersection-point, 408 planes with 2 intersection-points, 374 planes with 3 intersection-points, 34 planes with 4 intersection-points, 17 planes with 5 intersection-points. So, χ_{140} is an $(17;5)$-arc. The parameters c_i have the following values: $c_0 = 595$, $c_1 = 952$, $c_2 = 408$, $c_3 = 374$, $c_4 = 34$. Since $c_0 \neq 0$, so χ_{140} is incomplete arc. The large complete arc can be constructed from χ_{140} is an $(25;5)$-arc, where the number of addition points to χ_{140} is 8, and the points are 4, 6, 7, 8, 10, 12, 16, 22.
17. $P = 170$. The orbit χ_{170} ={0,170,340, ...,2210} has 14 points, and the numbers of intersection points between χ_{170} and planes of $PG(3,13)$ are as follows: 2366 planes with 1 intersection-points, and 14 planes with 14 intersection-points. So, χ_{170} is an $(14;14)$- arc. The parameters c_i have the following values with

ISSN: 1814-635X (print), ISSN:2521-3520 (online) Volume 33, Issue 4, 2022 DOI: http://doi.org/10.23851/mjs.v33i4.1197

respect to χ_{170}: $c_0 = 0$, $c_1 = 2366$. Since $c_0 = 0$, thus χ_{170} is a complete arc.

18. $P = 238$. The orbit χ_{238} ={0,238,476, ...,2142} 10 points, and the numbers of intersection points between χ_{238} and planes of $PG(3,13)$ are as follows: 1080 planes with no intersection-points, 850 planes with 1 intersection-point, 390 planes with 2 intersection-points, 40 planes with 3 intersection-points, and 20 planes with 4 intersection-points. So, χ_{238} is an $(10;4)$-arc. The parameters c_i have the following values: $c_0 = 390$, $c_1 = 920$, $c_2 = 700$, $c_3 = 220$, $c_4 = 120, c_6 = 20$. Since $c_0 \neq 0$, so χ_{238} is incomplete arc. The large complete arc can be constructed from χ_{238} is an $(16;4)$-arc, where the number of addition points to χ_{238} is 6, and the points are 3, 4, 9, 10, 11, 20**.**
19. $P = 340$. The orbit χ_{340} ={0,340,680, ...,2040} has 7 points, and the numbers of intersection points between χ_{340} and planes of $PG(3,13)$ are as follows: 1183 planes with no intersection-points, 1183 planes with 1 intersection-point, and 14 planes with 7 intersection-points. So, χ_{340} is an $(7;7)$-arc. The parameters c_i have the following values with respect to χ_{340}: $c_0 = 0$, $c_1 = 2366$, $c_2 = c_3 = \cdots = c_{13} = 0$, $c_{14} = 7$. Since $c_0 = 0$, thus χ_{340} is a complete arc.
20. $P = 476$. The orbit χ_{476} ={0,476, 952,1428,1904} has 5 points, and the numbers of intersection points between χ_{476} and planes of $PG(3,13)$ are as follows: 1595 planes with no intersection-points, 665 planes with 1 intersection-point, 110 planes with 2 intersection-points, and 10 planes with 3 intersection-points. So, χ_{476} is an $(5;3)$-arc. The parameters c_i have the following values: $c_0 = 990$, $c_1 = 1100$, $c_2 = 165$, $c_3 = 110$, $c_4 = 10$. Since $c_0 \neq 0$, so χ_{476} is incomplete arc. The large complete arc can be constructed from χ_{476} is an $(10;3)$-arc, where the number of addition points to χ_{476} is 5, and the points are 3,4,5,9,10.
21. $P = 595$. The orbit χ_{595} ={0,595, 1190,1785} has 4 points, and the numbers of intersection points between χ_{595} and planes of $PG(3,13)$ are as follows: 1728 planes with no intersection-points, 576 planes with 1 intersection-point, 72 planes with 2 intersection-points, and 4 planes with 3 intersection-points. So, χ_{595} is an $(4;3)$-arc.
The parameters c_i have the following values: $c_0 = 1728$, $c_1 = 576$, $c_2 = 72$. Since $c_0 \neq 0$, so χ_{595} is incomplete arc. The large complete arc can be constructed from χ_{595} is an $(10;3)$-arc, where the number of addition points to χ_{595} is 6, and the points are 3,4,5,6,7,8.
22. $P = 1190$. The orbit χ_{1190} ={0,1190} has 2 points, and the numbers of intersection points between χ_{1190} and planes of $PG(3,13)$ are as follows: 2028 planes with no intersection-points, 338 planes with 1 intersection-point, and 14 planes with 2 intersection-points. So, χ_{1190} is an $(2;2)$-arc.
The parameters c_i have the following values with respect to χ_{1190}: $c_0 = 0$, $c_1 = 2366, c_2 = c_3 = \cdots = c_{13} = 0$, $c_{14} = 12$. Since $c_0 = 0$, thus χ_{1190} is a complete arc.

CONCLUSIONS

1. Let $\#_C$ denote the number of complete caps, $\#_{Inc}$ denote the number of incomplete caps, and the fraction $\frac{n}{m}$ denote the size of extension complete cap, n over the size of incomplete cap, m.
Table 1 gave brief summary of compete and incomplete caps which are appear in this research.

Table 1: Numerical summary of caps in $PG(3,13)$

Degree	$\#_C$	$\#_{Inc}$
	Size	$\frac{n}{m}$
2	1	0
	2	
3	0	2
		$\frac{10}{5}\frac{10}{4},$
4	1	1
	20	$\frac{16}{10}$
5	0	1
		$\frac{25}{17}$
6	1	0
	34	
7	1	0
	7	
10	2	0
	85,68	
11	0	1
		$\frac{50}{35}$
14	2	0

	170,14	
15	1	0
	28	
18	1	0
	70	
19	1	0
	119	
23	1	0
	140	
28	1	0
	340	
30	1	0
	238	
47	1	0
	476	
55	1	0
	595	
98	1	0
	1190	

2. The action of the subgroup $\langle S^{170}\rangle$ on $PG(3,13)$ partitions the space into 170 lines.
3. The orbit χ_{170} is just line, and it is common set of 14 planes: $\pi(0)$, $\pi(170)$, $\pi(340)$, $\pi(510)$, $\pi(680)$, $\pi(850)$, $\pi(1020)$, $\pi(1190)$, $\pi(1360)$, $\pi(1530)$, $\pi(1700)$, $\pi(1870)$, $\pi(2040)$, $\pi(2210)$, where these planes are covers the space. For this reason, this orbit is complete.
4. The 14 planes in 3 are meets the orbits χ_2, χ_5, χ_{10}, χ_{17},χ_{34}, χ_{85} in number of points equal to the orbit's degree, so these orbits are complete.
5. The orbit $\chi_{1190} = \{1,1191\}$ is a complete since $\chi_{1190} \subset \chi_{170}$.
6. The orbit χ_{340} is a complete since $\chi_{340} \subset \chi_{170}$.

ACKNOWLEDGMENT

The authors would like to thanks the Mustansiriyah University, college of science, department of mathematics for their motivation and support.

Disclosure and conflict of interest: The authors declare that they have no conflicts of interest.

REFERENCES

[1] J. W. P. Hirschfeld, "Finite projective spaces of three dimensions," New York: Ox-ford Mathematical Monographs, The Clarendon Press, Oxford University Press, 1985.

[2] S. Ball and J.W.P. Hirschfeld, "Bounds on $(n;r)$-arcs and their application to linear codes", Finite Fields and Their Applications, 11, 2005, pp. 326-336. https://doi.org/10.1016/j.ffa.2005.04.002

[3] J. W. P. Hirschfeld and J.A. Thas," Open problems in finite projective spaces", Finite Fields and Their Applications, 32, 2015, pp. 44-81. https://doi.org/10.1016/j.ffa.2014.10.006

[4] Y. Kageyama and T. Maruta, "On the geometric constructions of optimal linear codes", Designs, Codes and Cryptography. 81(3), 2016, pp. 469-480. https://doi.org/10.1007/s10623-015-0167-2

[5] A. Sh. Al-Mukhtar,"Complete arcs and surfaces in three dimensional projective .space over Galois filed", PhD Thesis, University of.Technology, Iraq, 2008

[6] F. F. Kareem," The construction of complete $(k;n)$-arcs in 3-dimensional projective space over Galois field $GF(4)$", Mustansiriyah Journal for Sciences and Education, 1, 2013, pp. 183-196.

[7] A. A. A. Abdulla and N. Y. Kasm Yahya," Complete arcs and surfaces in three dimensional projective space PG(3,7)", Open Access Library Journal, 7(4), 2020, pp. 1-15. https://doi.org/10.4236/oalib.1106071

[8] N. Y. Kasm Yahya, and A.E.M. Sulaimaan," The reverse construction of complete $(k;n)$-arcs in three dimensional projective space $PG(3,4)$. J. Phys.: Conf. Ser. 2020, pp. 1591 012078. https://doi.org/10.1088/1742-6596/1591/1/012078

[9] N. A. M. Al-seraji, A.J. Al-Rikabi, and E.B. Al-Zangana," New arcs in $PG(3,8)$ by Singer group", Al-Mustansiriyah Journal of Science, 33(2), 2022, pp.70-76. https://doi.org/10.23851/mjs.v33i2.1128

[10] J. Sh. Radhi, and E.B. Al-Zangana," Construction of complete $(k;r)$-arcs from orbits in $PG(3,11)$", Al-Mustansiriyah Journal of Science, 33(3), 2022. To appear. https://doi.org/10.23851/mjs.v33i3.1147

[11] J. W. P. Hirschfeld, "Projective geometries over finite fields. 2nd edn., New York: Ox-ford Mathematical Monographs, The Clarendon Press, Oxford University Press, 1998.

[12] J. A. Gallian, "Contemporary abstract algebra. 9th edn., Cengage Learning, USA, 2017.

[13] The GAP Group," GAP. Reference manual. Version 4.11.1 released on .March 2021. [Online] 02 /https://www.gap-system.org

How to Cite

S. M. Attook and E. B. . Al-Zangana, "Special Arcs in PG(3,13)", *Al-Mustansiriyah Journal of Science*, vol. 33, no. 4, pp. 86–91, Dec. 2022.