The Optimality Effectiveness of Signal Amplitude on Synchronizations Using Cascading of Chaotic System

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Abstract:

Cascading chaotic of a non linear system with high Order has been used to increase the security and to prevent eavesdropper of knowledge for processor stage number increasing. The new system contains five variation parameters and exhibits Lorenz and Rossler like attractors in numerical simulations. The basic dynamical properties of the new system are analyzed by means of equilibrium points, eigenvalue structures. The use of high dimensional chaotic system like Lorenz (Transmitter and Receiver) Rossler (Controller) system will give a more complex structure, more system variables, and parameters. A new kind of dual-chaos encryption algorithm which takes Logistic mapping as chaos model has been designed and analyzed. A sequence is adopted for mapping chaos of integer space in high order to solve limited precision expression problem, dual-chaos system expend control parameters and increase complexity of chaos behavior to resisted attraction of chaos reconstruction.

***Keywords:** generalized synchronization, different fractional-order chaotic systems, Chaotic masking.*

التأثير الأمثل على قيم الإشارات المتزامنة باستخدام المتعدد الفوضوي

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الخلاصة :

استخدام النظام الغير خطي للمتعدد الفوضوي ذو الرتب العالية يزيد من أمنية الاتصال وكذلك يمنع المتصنت من استراق المعلومات والتجسس عليها وذلك بزيادة عدد مراحل المعالج وقد استخدم في هذا البحث نوعين من الأنظمة للمعالجات سويتا وعلى شكل متوازي وهي لورينز (مرسل ومستقبل) و روسيلر (منظم) الذي يعطيان أكثر تعقيد في بناء النظام والمتغيرات المستخدم هوقد تم تصميم نوع جديد من ثنائي الفوضى خوارزمية التشفير التي تأخذ رسم الخرائط ونموذج الفوضى وتحليلها. تم اعتماد تسلسل لرسم خرائط فوضى الفضاء صحيحا في الترتيب العالي لحل مشكلة محدودة دقة ألتعبير، إن النظام ثنائي الفوضى المستخدم يسيطر على المتغيرات ويزيد من تعقيد السلوك الفوضى.

1. Introduction

The communication systems using chaotic waveforms have been studied using parameter modulation of chaotic oscillators for transmitting an information signal and a synchronous subsystem augmented with a nonlinear filter for detecting and recovering the transmitted information. Specifically, claim that the construction of nonlinear filter is a new development that may offer improved performance for practical chaotic communication systems [1].Chaos is a common nonlinear phenomenon. It seems chaos, but it is not really messy, and is a class of phenomena with inherent structure.
In fact, chaos has some good characteristics:

- The sensitive dependence on initial conditions;
- The semi-stochastic property;
- Periodicity.

The third characteristic makes chaos variables go through all states in certain ranges without repetition [2].

2. Lorenz System

It is a continuous time nonlinear system exhibiting chaotic trajectories for specific values of system parameters. Atmospheric scientist E. Lorenz [3] proposed this system (1963) as a set of three ordinary differential equations. Lorenz system is described as: The

$$\left.\begin{aligned}\dot{x} &= \sigma(y - x)\\ \dot{y} &= -xz + \rho x - y\\ \dot{z} &= xy - \beta z\end{aligned}\right\} \quad (1)$$

Where [x y z] are state vector and σ, ρ, and β are constant parameters.

2.1 Properties of Lorenz System Equations

Some important basic features of this system are [4]

1- The equations involve only first order time derivatives, so the evolution depends only on the values of x, y, and z at the time. Due to the terms of xz and xy in the second and third equations, the system is non-linear.

2- The system is dissipative when the following inequality holds:

$$\nabla f = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} = -\sigma - 1 - \beta < 0 \quad (2)$$

Since parameters σ and β, are positive, the inequality always holds and, thus, solutions are bounded.

3- The system is symmetric, with respect to the z axis, which means it is invariant for the coordinate transformation :(x, y, z) → (−x, −y, z).

2.2 Chaotic Lorenz System Behavior

In order for the Lorenz system to give rise to chaotic dynamics, the Lorenz parameters (σ, ρ and β) must satisfy [5,6]:

$$\boldsymbol{\sigma}\ ,\boldsymbol{\rho}\ , \qquad\qquad > \ 0 \tag{3}$$

$$\boldsymbol{\sigma}\ >\ \beta \tag{4}$$

$$\boldsymbol{\rho} > \frac{\boldsymbol{\sigma}\,(\boldsymbol{\sigma}\ +\ \boldsymbol{\beta}\ +\ \mathbf{3})}{((\boldsymbol{\sigma} - \boldsymbol{\beta} - \mathbf{1})\ )} \tag{5}$$

$$\boldsymbol{\rho_c}\ =\ \frac{(\boldsymbol{\sigma}\,(\boldsymbol{\sigma} + \boldsymbol{\beta} + \mathbf{3}))}{((\boldsymbol{\sigma} - \boldsymbol{\beta} - \mathbf{1})\ )} \tag{6}$$

The variable $\rho_c$in Equation (6) is the critical value for ρ.
To illustrate this formula choosing any value of (β) will be greater than zero.
Let $\beta = 8/3$; Choose $\sigma = 10$;because$\sigma > \beta$ to get chaotic state.

$$\boldsymbol{\rho}$$

The results of testing these parameters in Lyapunov Exponents are shown in the following **Figure (1)** below.

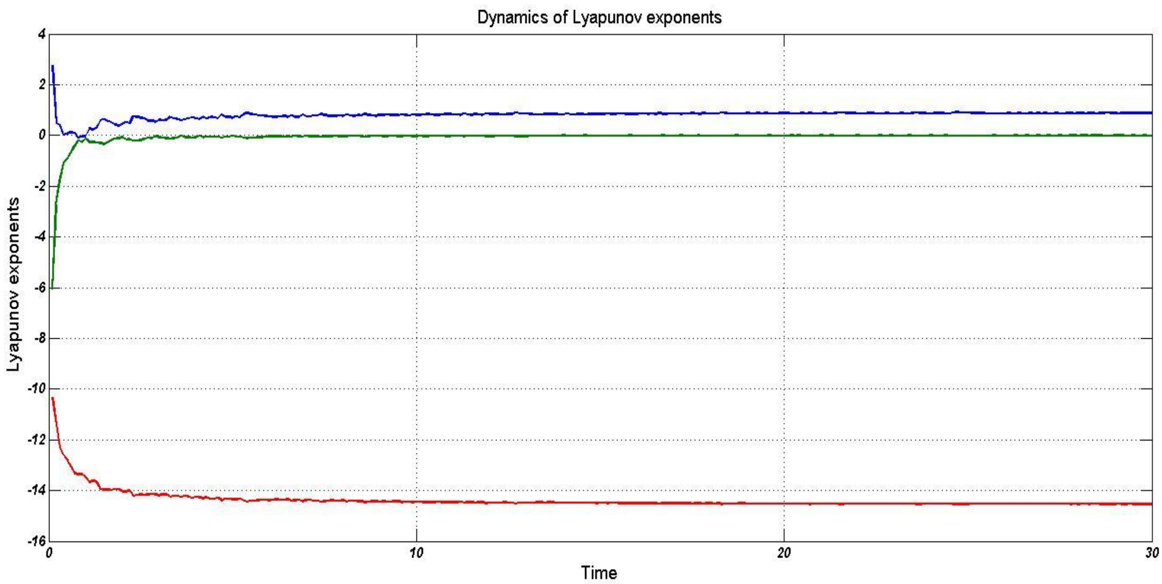

Fig .(1) Lyapunov Exponents Spectrum of the Chaotic Lorenz System for σ= 10,ρ = 28 and β= 8/3

2.3 Sensitivity to Initial Conditions

One feature of the Lorenz System is that, a small difference in initial conditions would yield a large difference in results; **Figure (2)** illustrates this phenomenon in the Lorenz System. Two trajectories begin with very close initial conditions; in particular, $x_a(0) = x_b(0) = 10$, $y_a(0) = y_b(0) = 0$, $z_a(0) = 10$ $z_b(0) = 10.00000000001$.For the first 14 time units, the two trajectories seem identical. However, beyond 15 time units, they seem completely unrelated to each other. It is this property of physical systems that

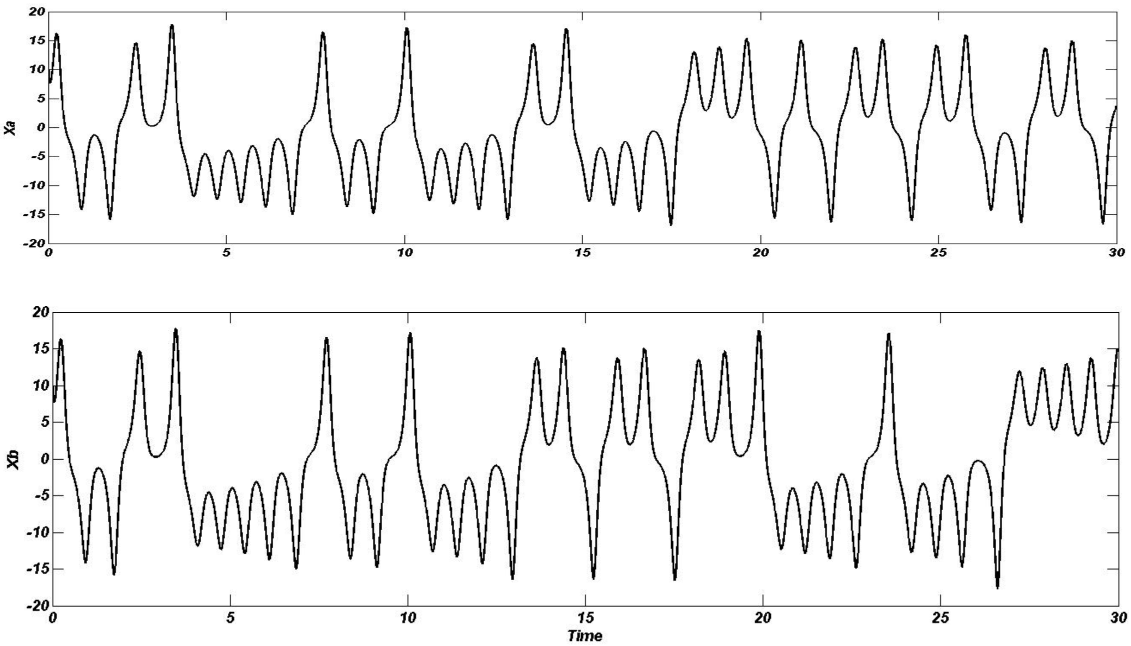

Fig .(2):Two Numerical Solutions of the Lorenz System Showing Sensitivity to Initial Conditions

2.4 Sensitivity to Parameters

Two identical Lorenz systems (*a* & *b*) are taken with the same initial conditions but starting from different parameters (nearly identical however). The difference in parameters taken between two Lorenz variables x_a and $x_b$is chosen to be 10^{-6}. **Figure (3)** depicts the time series of variables x_a and x_b for two Lorenz systems. After some period, the two variables quickly diverge from each other even though they started from identical parameters. This means a long term prediction of chaotic systems is not possible since the slightest error in the parameters will result in an exponential increase in the error. [7]. In particular, $\sigma_1 = \sigma_2 = 10$, $\rho_1 = 28$ $\rho_2 = 28.000001$ $\beta_1 = \beta_2 = 8/3$, $x_a(0) = x_b(0) = 10$, $y_a(0) = y_b(0) = 0$, $z_a(0) = z_b(0) = 10$

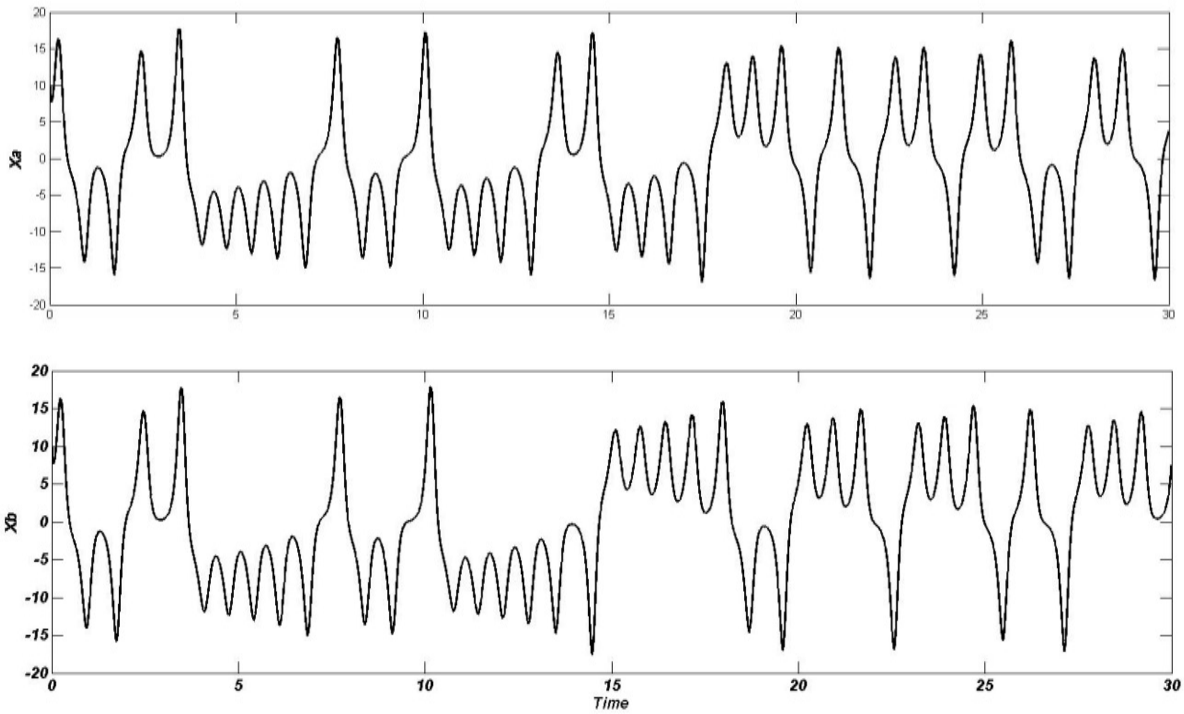

Fig .(3):Two Numerical Solutions of the Lorenz System Showing Sensitivity to Parameters

3. Rossler system

The Rossler system can be concludedaccording to linear methods such as eigenvectors, but the principal features of the system need to non-linear methods such as Poincaré maps and bifurcation diagrams. The Rossler attractor was purposed to behave in the same way to the Lorenz attractor, but also become facile to analyze specifically. An orbit within the attractor follows an external helical close to the x, y plane about an unstable steady point. Once the graph helix out enough, a second steady point affects the graph, causing anincrease and twist in the z-dimension. In the time domain, it is clear that although each variable is changing within a certain range of values, the vacillations are chaotic. This attractor has partial similarities to the Lorenz attractor, but is easier and has only one various. Rossler controller designed the Rossler attractor in 1976, [8] but the sourcely theoretical equations were recently found to be beneficial in modeling balance in chemical reactions.

3.1 Synchronization in Chaotic Systems:

The concept of using synchronization methods in communications schemes is based on the idea that two similar circuits or state space systems, one at the transmitter and the other at the receiver, can have at any particular time, the same dynamical state. It might seem that chaotic synchronization is impossible to achieve in chaotic systems since they are very sensitive to initial conditions and the slightest difference in the initial conditions will ultimately lead to totally different trajectories. But after the seminal was work done by Pecora and Carroll (PC) [9] they showed that, it is however possible to synchronize two chaotic systems starting from different initial conditions under certain conditions. However they showed subsequently that, if two chaotic systems are linked together by a common signal, it is possible to obtain chaotic synchronization regardless of the initial conditions. The earliest and the simplest form of synchronization is a Complete Synchronization. It occurs in coupled identical systems and is also referred to as a conventional synchronization or an identical synchronization. Two continuous-time chaotic systems:

$$\dot{\boldsymbol{x}}(\boldsymbol{t}) = \boldsymbol{F}(\boldsymbol{x}(\boldsymbol{t})) \qquad (7)$$

And

$$\dot{\hat{x}}(t) = F(\hat{x}(t)) \qquad (8)$$

are said to obtain Complete Synchronization if

$$\lim_{t\to\infty} [\hat{\boldsymbol{x}}(\boldsymbol{t}) - \boldsymbol{x}(\boldsymbol{t})] = \boldsymbol{0} \qquad (9)$$

i.e., for any combination of initial conditions $x(0)$ and $\hat{x}(0)$, the nature of coupling can have two possibilities. When the evolution of one of the coupled system is unaffected by the coupling mechanism, then this is unidirectional coupling or a drive-response coupling. However, when both systems are connected to each other such that the evolution of both affects each other, then this type of coupling is called bi-directional coupling mechanism [10].

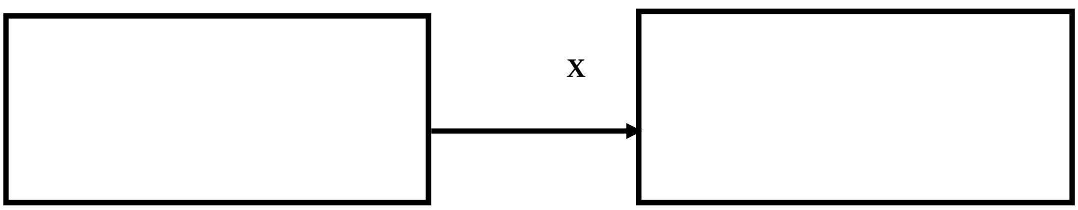

Fig .(4): Mechanism of Complete Synchronization.

4. Chaotic Masking

Chaotic masking (CM) is one of the earliest chaotic communication techniques. It is based on the principles of PC synchronization. It primarily involves the transmission of analog signals [11].

4.1 Principles of Chaotic Masking

Chaotic masking involves the addition of a message signal m to a chaotic carrier signal x, before the transmission of the sum of the two signals takes place. The block diagram illustrating the principles of chaotic masking is shown in **Figure(5)**

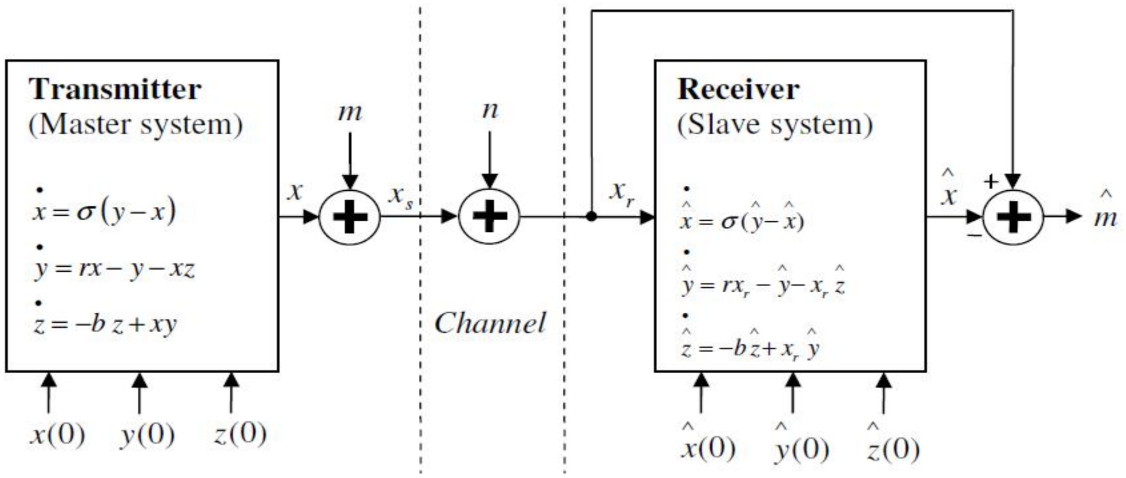

Fig .(5) The general block diagram of the chaotic communication System based on the chaotic masking concept.

In **Figure (5),** n denotes the additive white Gaussian noise (AWGN) component introduced by the channel and x_r denotes the received signal affected by AWGN. The slave system of the receiver generates a signal $\hat{x}$ which is expected to besynchronized with the corresponding master signal x of the transmitter. Assuming that the AWGN component is near zero, and that sufficient amount of time has passed for x and $\hat{x}$ to synchronize. The transmitted message (m)can be recovered in the form $\hat{m}$

$$\hat{m} = x_r - \hat{x} = (m + x) - \hat{x} \approx \mathbf{m} \qquad \textbf{(10)}$$

The requirement of a chaotic masking scheme is for the power of the information signal to be significantly lower than the power of the chaotic carrier [12].

4.2 Chaotic Masking within the Lorenz master-slave System

Chaotic masking within the Lorenz master-slave system has been demonstrated in[13]. The system has been designed using the Lorenz x signal as the driving signal. Stability Lyapunov's direct method has been used in [13] to show that using the x signal as the driving signal the master-slave system synchronizes. It has been shown that by adding a small amplitude speech signal onto the chaotic carrier one is able to recover the speech signal at the receiver. The communication system is based on chaotic masking, while implementing the Lorenz master-slave system, is shown in **Figure (6).**

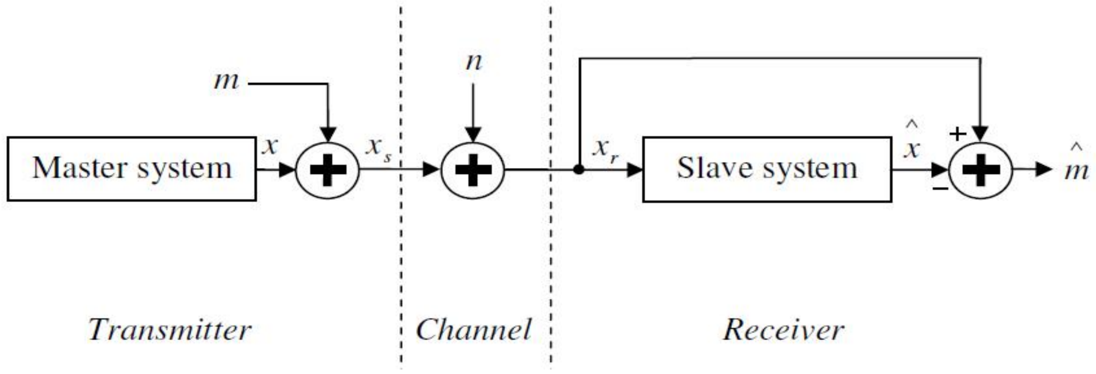

Fig .(6)The Lorenz based communication system implementing chaotic masking.

The recovering of the transmitted information is demonstrated under noiseless conditions in **Figure (7)** below, by processing and comparing the top and bottom graphs through the system.

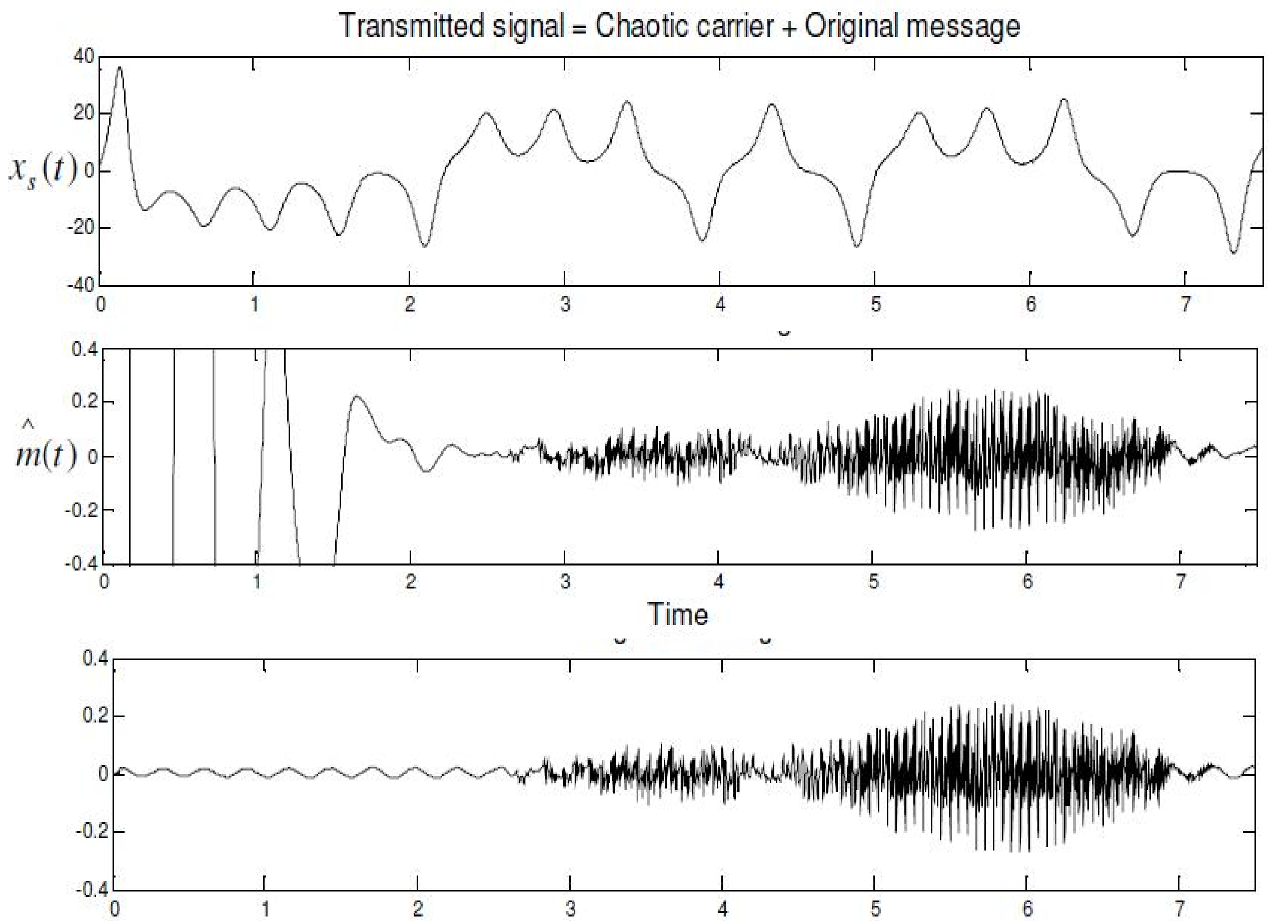

Fig .(7):The chaotic parameter values of the system have been: σ = 16, *r* = 45.6 and *b* = 4.

In the case of **Figure (7),** the chaotic parameter values of the system have been set to: $\sigma = 16$, $r = 45.6$ and $b = 4$. An evident difference in power between the chaotic carrier and the speech signal can be observed in **Figure (7).** The transmitted signal has been plotted in phase space in **Figure (8).** The small ripple, observed on the strange attractor of **Figure (8),** is caused by the message m embedded within it.

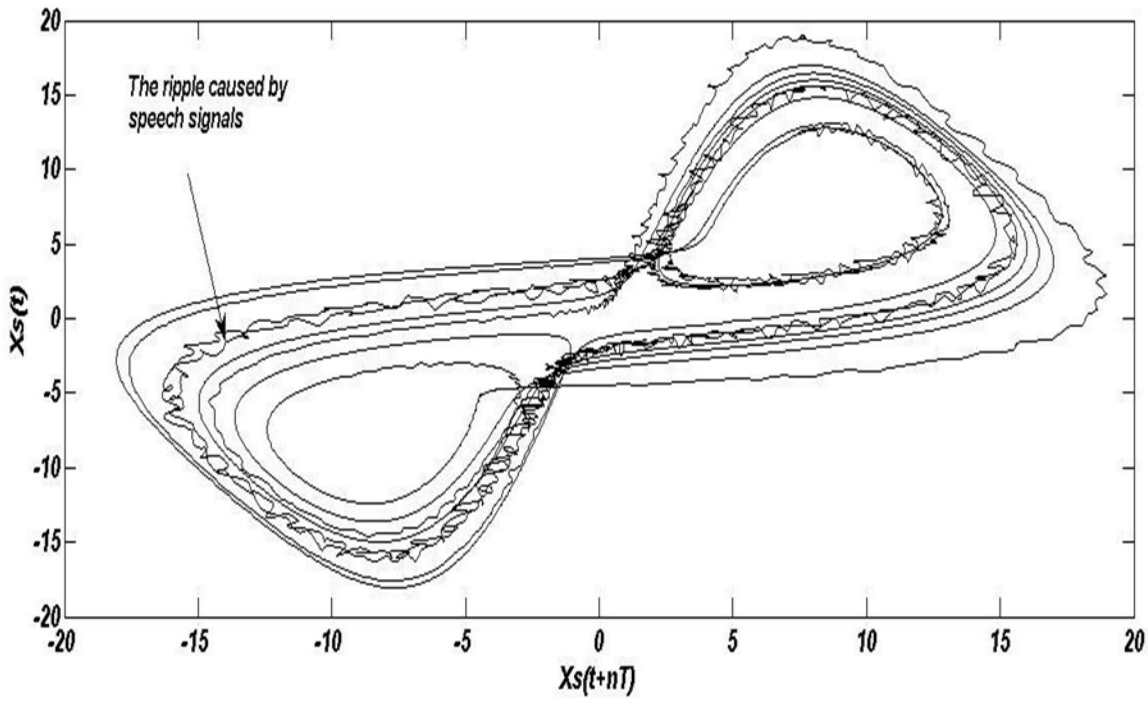

Fig .(8):The transmitted signal $x_s(t)$plotted in phase space.

5. Chaotic Parameter Modulation within the Lorenz and Rossler controller Systems

The concept of parameter modulation is now demonstrated on the Lorenz chaotic system [13,14].The binary message is used to alter the parameter b of the transmitter Lorenz chaotic system between 4 and 4.4 depending on transmission. However, at the receiver side the parameter b is fixed at 4 for all time. Thus, the synchronization either occurs or does not, depending on the state of the parameter b at the transmitter side. The parameters σ and r are fixed at 16 and 45.6, respectively. For these parameter values the system is chaotic. In order to implement the CPM scheme the authors of [15] have scaled the Lorenz chaotic system to allow for the limited dynamic range of the operational amplifiers. The generalization of synchronization Lorenz and Rossler controller Parameter this system, based on the PC synchronization concept, is presented in **Figure (9)** that demonstrates the limits of the requirements and we get a messy process synchronization of non-linear system using MATLAB Simulink (R2011a) software.

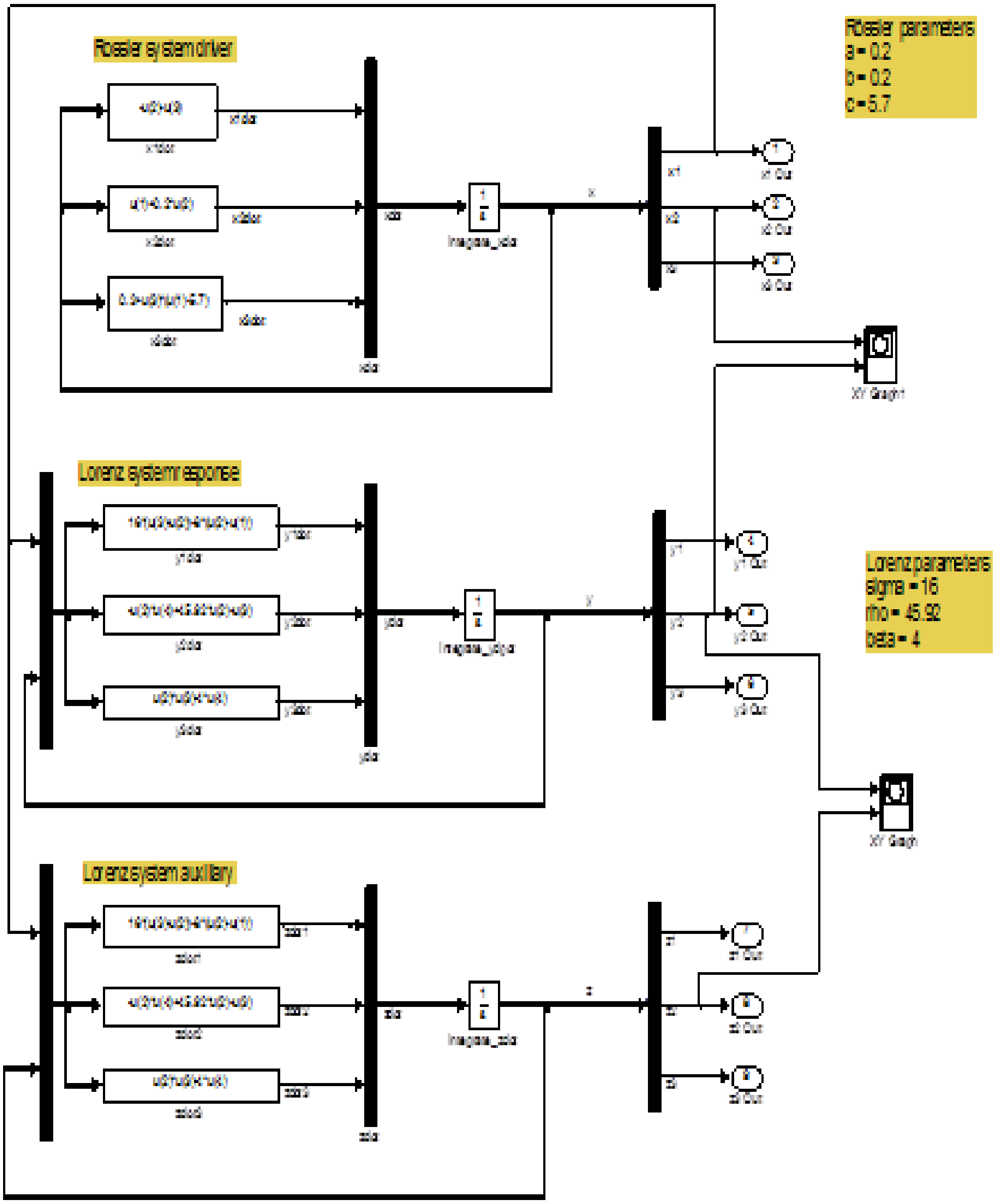

Fig .(9):The geniralzation Of synicrolazation Lorenz and Rossler Parameter.

The upper limits of the requirements of chaotic signals with specific limits mathematical calculations can be made and the extent of their vulnerary ability initial conditions which is shown in **Figure (10)** below.

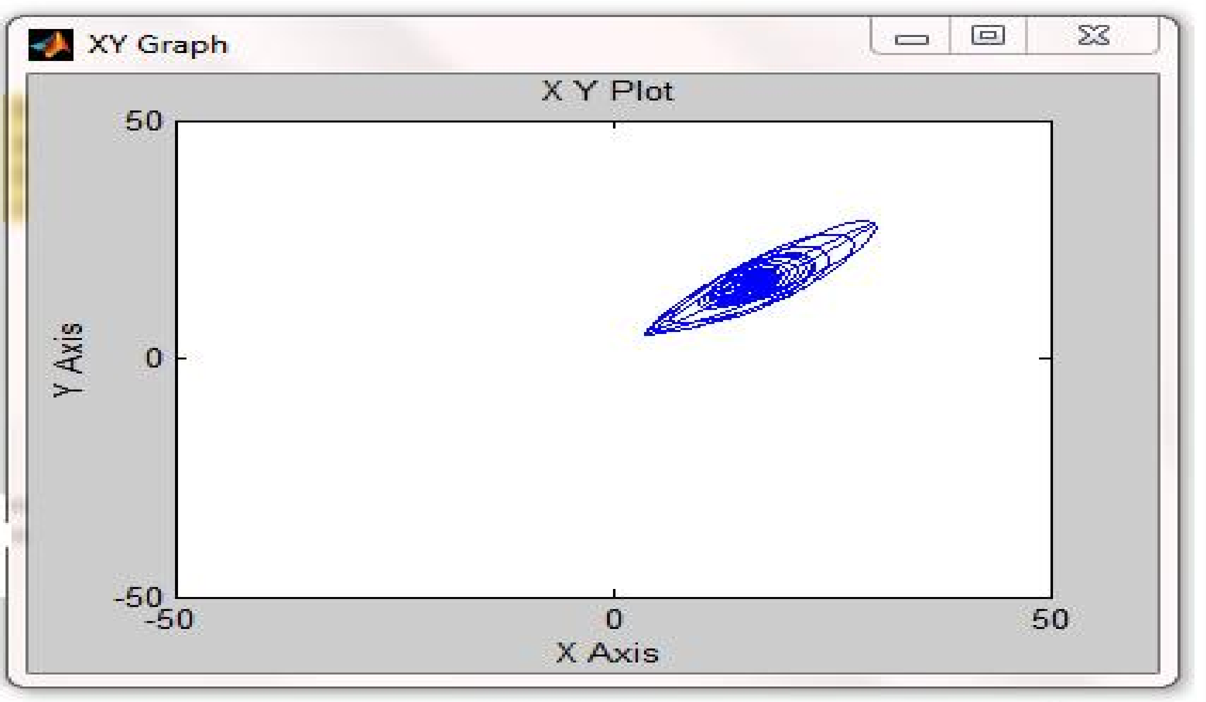

Fig .(10) The Chaiotic Signal with Lorenz Parameter α=16 , r=45.6 ,and b=4

The chaotic signal between Lorenz Parameter and Rossler controller parameter shows the effect of the initial conditions of chaotic signals at a=0.2, b=0.2 and c=5.7 which is displayed in **Figure (11)** below.

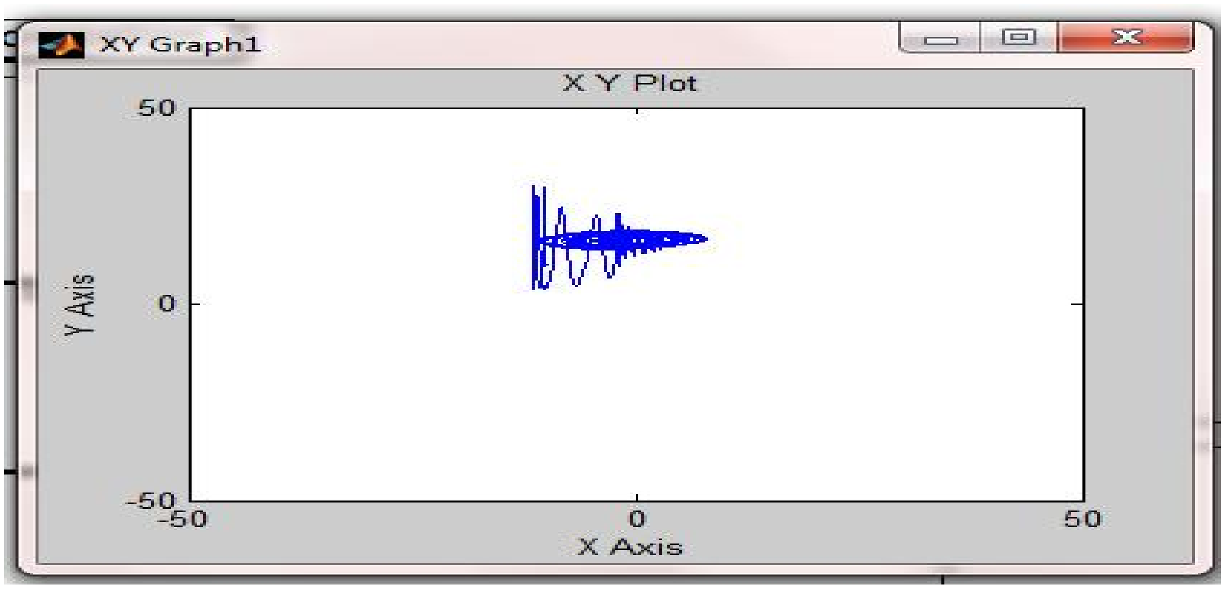

Fig .(11)The Chaiotic Signal between Lorenz Parameters and Rossler Parameter a=0.2 ,b=0.2, and c=5.7.

The intial conditions of synchronization guess for the linear system of Chaiotic Signal between Lorenz Parameters and Rossler controller are displayed and shown in **Figure (12)** below.

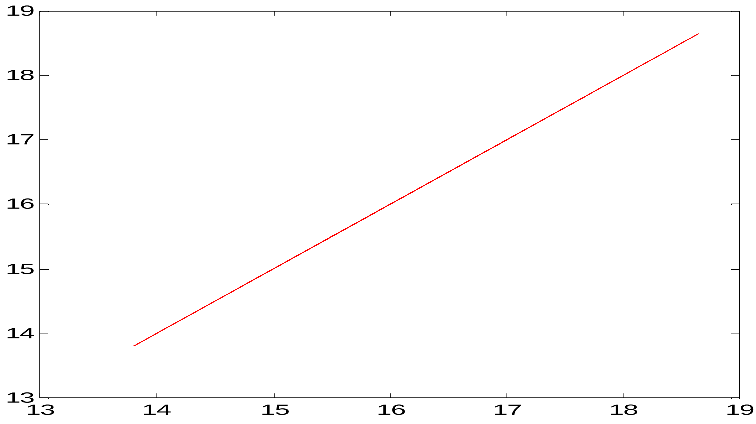

Fig .(12) The Linear Synchronization output with Lorenz and Rossler Cascading Parameters

6. Conclusions

Chaos synchronization between two linearly coupled systems have been studied and analyzed. Some sufficient conditions for global synchronization using Cascading of chaotic masking will cancel the effects of signal amplitude on synchronization. Several chaotic communication systems with receiver based on chaotic synchronization have been described. In two-channel transmitted method, a faster synchronization can be achieved, and high nonlinear encryption function can be used. These include the chaotic communication schemes of chaotic masking, chaotic modulation and the new chaotic communication scheme of initial condition modulation. A new method for analyzing the stability of synchronization solution of coupled system has been introduced to investigate the stability of synchronization solution for the classical Lorenz and Rossler controller system. The synchronization between the Lorenz system and the Rossler controller system is given to illustrate the effectiveness of the proposed scheme**.**

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