2F-Planar mappings In Riemannian spaces $\overset{3}{F} = 0$

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Abstract:

In this paper we consider 2F- planar mappings onto Riemannian spaces, which have studied by [4]-[11]. The new investigation with $\overset{3}{F} = 0$ give as general aspects of the F- planar mappings onto Hermitian and Riemannian spaces **.**

1. Introduction

2F- planar mappings is the generalization of equations of Geodesic, Holomorphically- projective and F- planar mappings with almost complex structure.

In this paper, we present some new results for 2F- planar mappings onto Riemannian spaces

For the beginning we shall give definitions.

A curve L: $x^h = x^h(t)$ is said to be 2F- planar (Raad J.k. and Kurbatova [4],[5]) if , under the parallel translation along it , the tangent vector $l^h \overset{def}{=} dx^h / dt$ lies in the tangent plane form made by the tangent vector and its conjugate $F_a^h l^a$, i.e

$$l^h(t)_{,a}\, l^a(t) = l^a(t) \sum_{I=0}^{P} \underset{I}{r}(t)\, \overset{I}{F}{}_a^h(x) \quad , \qquad (1)$$

where $\underset{I}{r}$ function of the parameter t , " , " denotes covariant differentiation and $\overset{P}{F} = (F)^P$ the affinor structure that define in both spaces $V_n , \overline{V}_n$ and a diffeomorphism is said to be an 2F- planar mappings [5] if , under this mapping , any 2F – planar curve V_n passes into the 2F- planar curve $\overline{V}_n$.

Under this condition Rank $\left\| F_i^h - r\, d_i^h \right\| > 2$ the mapping of V_n into $\overline{V}_n$ is 2F- planar if and only if the conditions

$$\text{(a)}\ \overline{\Gamma}_{ij}^h(x) = \Gamma_{ij}^h(x) + \sum_{I=0}^{2} \underset{I}{j}_{(i}\, \overset{I}{F}{}_{j)}^h \qquad (2)$$

(b) $\overline{F}_i^h = a F_i^h + b d_i^h$,

where $\overline{G}, G$ objects of affine connections , $\underset{I}{j}$ are covectors ,

(i, j) denoted a summarization of indices , $a(x)$ and $b(x)$ are functions in the coordinate system x which is general with respect to the mapping and

$$\overset{3}{F} = o, \overset{0}{F} = d \ .$$

2F- planar mappings generalize F-planar (if $\underset{2}{j} = 0$ *or* $\overset{2}{F} = 0$), quasigeodesic, holomorphically projective and almost geodesic of the type of p_2 mappings [1],[3].

2. Fundamental equation of 2F- planar mapping in Riemannian spaces

From (1), with the condition $F_{i,j}^h = F_{i/j}^h = 0$, $(\overset{1}{F} = F)$ where

$$F_a^a = \overset{2}{F}{}_a^a = 0 \ , \quad \text{and}$$

$$F_{i,j}^h \ , F_{i/j}^h \ \text{into} \ V_n \ \text{and} \ \overline{V}_n \ .$$

We can obtaining

$$F_{i/j}^h = F_{i,j}^h + F_i^a G_{ja}^h - F_a^h G_{ij}^a = 0 \ , \qquad (3)$$

or

$$F_{i/j}^h = F_{i,j}^h + \underset{0}{j}_{\bar{i}} d_j^h + (\underset{1}{j}_{\bar{\bar{i}}} - \underset{0}{j}_i) F_j^h - (\underset{2}{j}_{\bar{i}} - \underset{1}{j}_i) \overset{2}{F}{}_j^h \ , \qquad (4)$$

where $\overline{G}_{ij}^h$, G_{ij}^h are Christoffel symbols $\overline{V}_n$ and V_n respectively , d_j^h Kronecker symbol .

By using that $F_{i/j}^h = 0$, so , therefore

$$\underset{0}{j}_{\bar{i}} = 0 \ , \qquad \underset{1}{j}_{\bar{i}} - \underset{0}{j}_i = 0 \quad \text{and} \quad \underset{2}{j}_{\bar{i}} - \underset{1}{j}_i = 0 \ . \qquad (5)$$

The contraction (2-a) by h , j are

$$\overline{G}_{ia}^a = G_{ia}^a + (n+1) \underset{0}{j}_i + \underset{1}{j}_{\bar{i}} + \underset{2}{j}_{\bar{\bar{i}}} \ , \qquad (6)$$

where $\quad j_{\bar{i}} = j_a F_i^a \quad , \quad j_{\bar{\bar{i}}} = j_a \overset{2}{F}{}_i^a \quad .$ With (6)

$$\overline{G}_{ia}^{a} = G_{ia}^{a} + (n+3)\underset{0}{j}_{i} \quad .$$

If $(n+3) \ne 0$ then

$$\underset{0}{j}_{i} = \frac{1}{n+3}(\overline{G}_{ia}^{a} - G_{ia}^{a}) \quad . \qquad (7)$$

Return to (6) , rewriting another form of (2-a) by contraction it with $\overset{2}{F}{}_{k}^{j}$ by index j , obtaining :

$$\overline{G}_{ia}^{a} = G_{ia}^{a} + \underset{0}{j}_{(i} \overset{2}{F}{}_{j)}^{h} \quad .$$

with (7) , getting final form

$$\overline{\overset{1}{T}}{}_{ij}^{h} = \overset{1}{T}{}_{ij}^{h} \quad , \quad \text{where}$$

$$\overset{1}{T}{}_{ij}^{h} = G_{i\bar{j}}^{h} \quad \frac{1}{n+3} \overset{2}{F}{}_{(i}^{h} G_{j)a}^{a} \quad . \qquad (8)$$

Therefore we get the following result:

<u>*Theorem 1*</u>. [5 – 11]

Geometry form (8) by $\overset{3}{F} = o, \overset{0}{F} = d$, $F_{a}^{a} = \overset{2}{F}{}_{a}^{a} = 0$ and $F_{i,j}^{h} = F_{i/j}^{h} = 0$,
are invariant form dependent on 2F- planar mapping affine connection when the structures are fixed.

Note : It is clearly from (8) that $\overset{1}{T}{}_{ij}^{h}$ is not tensor form .

3. Riemannian tensors form between Riemannian spaces

The relationship between covariant derivative of Riemannian tensors $V_n \rightarrow \overline{V}_n$ is given by the form

$$\overline{R}_{ij}^{h} = R_{ij}^{h} + P_{ik,j}^{h} - P_{ij,k}^{h} + P_{ik}^{a} P_{aj}^{h} - P_{ij}^{a} P_{ak}^{h} \; , \qquad (9)$$

where

$$P_{ij}^{h} = \sum_{I=0}^{2} j_{I\,(i} \overset{I}{F}{}_{j)}^{h} \quad ,$$

$$P_{ij,k}^{h} = \sum_{I=0}^{2} j_{I\,i,k} \overset{I}{F}{}_{j}^{h} \quad .$$

Hence $\overset{3}{F} = o, \overset{0}{F} = d$, with (5) , obtained

$$\overline{R}^{h}_{ijk} = R^{h}_{ijk} + \underset{0}{j}_{i[j}\, d^{h}_{k]} + \underset{1}{j}_{i[j}\, F^{h}_{k]} + \underset{2}{j}_{i[j}\, \overset{2}{F}{}^{h}_{k]} \;, \qquad (10)$$

where the symbol [ ] defined as antisymmetric indices , and

$$\underset{0}{j}_{ij} = \underset{0}{j}_{i,j} - \underset{0}{j}_{i}\, \underset{0}{j}_{j} \;,$$

$$\underset{1}{j}_{ij} = \underset{1}{j}_{i,j} - \underset{0}{j}_{(i}\, \underset{1}{j}_{j)} \;,$$

$$\underset{2}{j}_{ij} = \underset{2}{j}_{i,j} - \underset{0}{j}_{(i}\, \underset{2}{j}_{j)} \,. \qquad (11)$$

Farther contraction (10) with indices h and k

$$\overline{R}_{ij} = R_{ij} + (n-1)\underset{0}{j}_{ij} + \underset{1}{j}_{i\bar{j}} + \underset{2}{j}_{i\bar{\bar{j}}} \;, \qquad (12)$$

by using (11) :

$$\overline{R}_{ij} = R_{ij} + n\,\underset{0}{j}_{i,j} - 3\,\underset{0}{j}_{i}\,\underset{0}{j}_{j} + \underset{1}{j}_{i,\bar{j}} + \underset{2}{j}_{i,\bar{\bar{j}}} \;, \qquad (13)$$

contraction with $F^{a}_{j}\,,\overset{2}{F}{}^{a}_{j}$ by index j , obtained

$$\overline{R}_{i\bar{j}} = R_{i\bar{j}} + n\,\underset{0}{j}_{i,\bar{j}} + \underset{1}{j}_{i,\bar{\bar{j}}} \;,$$

$$\overline{R}_{i\bar{\bar{j}}} = R_{i\bar{\bar{j}}} + n\,\underset{0}{j}_{i,\bar{\bar{j}}} \;,$$

where $\overline{R}_{ij}\,, R_{ij}$ Ricci tensor , if $n \,^{1}\, 0$, then

$$\underset{0}{j}_{i,\bar{\bar{j}}} = \frac{1}{n}\left(\overline{R}_{i\bar{\bar{j}}} - R_{i\bar{\bar{j}}}\right). \qquad (14)$$

From the above dissuasion , we get the following theorem :

2.Theorem

Tensors form (14) by $\overset{3}{F} = o\,, \overset{0}{F} = d$, $F^{a}_{a} = \overset{2}{F}{}^{a}_{a} = 0$ and $F^{h}_{i,j} \,^{1}\, 0$, $F^{h}_{i/j} \,^{1}\, 0$,are invariant form dependent on 2F- planar mapping Riemannian spaces when the structures are fixed .

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