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TORSIONAL BEHAVIOR OF NORMAL-STRENGTH REINFORCED CONCRETE BEAMS

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Abstract:

Recent methods for torsional design of reinforced concrete beams tend to the use of space truss analogy, instead of the earlier skew bending theory. A total of (101) rectangular beams made of normal strength concrete (NSC) that failed under pure torsion are considered in this work. NSC is defined as having the cylinder compressive strength f'c ≤ 40.1 MPa. These have been taken from the literature.Regression analysis was performed on the results to obtain two representative equations to predict: cracking torsional moment (Tcr) and torsional resistance moment (Tr). The first equation is based on (3) major parameters that include concrete compressive strength (f'c) and sectional dimensions, while the second one is based on (7) major parameters which include the quantification of the influence of both transverse and longitudinal reinforcement.When the existing code design methods were applied, they gave a coefficient of variation (COV) value ranges between (20.9-33.1) percent for the ratio of tested / calculated torsional strength (Tu-test / Tr-calc.). In contrast, the proposed equation has led to a COV of (12.9) percent**.**

Keywords: *beams; cracking torsional moment; normal strength concrete; longitudinal reinforcement; torsional resistance moment; transverse reinforcement.*

سـلوك اللي في العتبـات الخرسـانية المسلحـة المصنوعـة من خرسـانـة إعتيـاديـة المقاومـة

الخلاصة:

تعتمد الطرق الحديثة لتصميم اللي في العتبات الخرسانية المسلحة على نظرية المسنم الفضائي (Space Truss) بدلا من الإنثناء المائل (Skew Bending). تمت دراسة (١٠١) عتبة خرسانية مسلحة مستطيلة المقطع مصنوعة من خرسانة إعتيادية المقاومة تراوحت بين (١٤,٣-٤٠,١) نيوتن/مم² فشلت تحت تأثير اللي الخالص مأخوذة من بحوث سابقة.حللت النتائج بطريقة التحليل الإرتدادي للحصول على معادلتين لحساب عزم اللي الذي يسبب التشقق (T_{cr}) ومقاومة اللي التصميمية (T_r) . تعتمد المعادلة الأولى على ثلاثة معاملات رئيسية هي مقاومة إنضغاط الخرسانة وأبعاد المقطع، بينما تعتمد المعادلة الثانية على سبعة معاملات رئيسية تشتمل على التقييم الكمي لتأثير حديد التسليح الطولي والعرضي للعتبات.عندما طبقت طرق التصميم للمدونات تم الحصول على معامل تغاير تراوحت قيمته بين (٢٠,٩-٣٣,١)% لنسبة مقاومة اللي العملية / المقاومة التصميمية (T_{u-test} / T_{r-calc}). وبالمقارنة، فأن تطبيق المعادلة المقترحة نتج عنه معامل تغاير مقداره (١٢,٩%).

1. Introduction

While not all reinforced concrete members are subjected to torsion, several cases of significant torsional effects occur in practice. Torsion can become a predominant action in structures such as eccentrically loaded box beams, curved girders, spandrel beams, structures of irregular shapes, and spiral staircases [1, 2].

In 1969 the then ACI Committee 438 published its report recommending torsional design based on the skew bending theory [3]. The ACI 318 Committee used this theory starting from the 1971 Code [4] which continued up to the 1989 Code [5]. BS-85[6] and BS-97[7] Code versions also used the same approach.

The most recognized theoretical model of pure torsion in reinforced concrete is the space truss model. Based on post-doctoral research published by MacGregor and Ghoneim[8], the ACI Code in 1995[9] accepted this model. This is now included in the latest ACI 318-08 Code [10]. The Canadian [11], AASHTO-LRFD [12], and European [13] Codes also use space truss analogy for torsional design.

This new theory is based on a thin-walled tube, space truss analogy. In this theory, the torsional concrete contribution (T_c) was eliminated. This contrasts with the approach of skew bending theory where (T_c) is included in the calculation of torsional capacity of reinforced concrete beams.

There is a number of more accurate but more complex design procedures in the literature [14, 15], but they are not considered in this work.

2. Research significance

The design provisions of torsion have been substantially revised using thin-walled tube analogy. Therefore, more researches are considered valuable for explaining the associated design provisions.

This paper reviews the torsional design equations for the case of pure torsion given by (7) different code approaches: 2 using skew bending theory (ACI 318M-89[5] and BS-97[7]); plus 5 using space truss analogy (ACI 318M-99[16], ACI 318M-05[17], Canadian [11], AASHTO-LRFD [12], and EURO [13]). In addition, a number of equations adopted by some researchers to predict T_{cr} value are included. The calculations of the previous (7) methods are checked against (101) tests of torsional failure of tested beams available in the literature. Two proposed equations which are based on regression analysis are also introduced. The first one estimates the cracking torsional moment (T_{cr}) of NSC beams, while the second one predicts the torsional resistance moment of such beams.

3. Experimental results

At this stage of work, all available tests of failures under pure torsion obtained from the literature are used. Table (1) gives the ranges of the variables of these (101) rectangular solid section beams using the main significant parameters: concrete compressive strength f'_c, aspect ratio$(\frac{y}{x})$, sectional area (A_{cp}), nominal stirrup strength ($\rho_v.f_{yt}$), and nominal longitudinal steel strength ($\rho_\ell.f_{y\ell}$). These beams include 1, 48, 11, 5, 10, 4, 3, 1, 1, 5, 1, 1, 2, and 8 specimens from the references 1, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, and 30 respectively.

Table 1- Ranges of the variables for the 101 tested beams.

Detail	f'_c *(MPa)*	$\frac{y}{x}$	A_{cp} *(mm²)*	$\rho_v.f_{yt}{}^{*}$ *(MPa)*	$\rho_\ell.f_{y\ell}{}^{**}$ *(MPa)*
Low	14.340	1.000	7225	0.883	1.264
High	40.130	3.250	175000	7.441	22.142
High/Low	2.798	3.250	24.221	8.427	17.517

*ρ_v = stirrups ratio = $2A_t/(b.s)$.

**ρ_ℓ = longitudinal steel ratio = $A_\ell/(b.h)$.

3. Evaluation of experimental results

Cracking Torsional Moment Equations

Following are the methods considered in this work to estimate the cracking torsional moment (T_{cr}) of the beams:

3.1. ACI 318M-89 Code[5] method

$$T_{cr} = \frac{1}{6}\sqrt{f_c'}\,.\sum x^2\,.\,y \tag{1}$$

Where:

f'_c = cylinder compressive strength of concrete.

x = the shorter side of the cross section.

y = the longer side of the cross section.

3.2. ACI 318M-05 Code [17] method:

$$T_{cr} = 0.33\sqrt{f_c'}\left(\frac{A_{cp}^2}{P_{cp}}\right) \tag{2}$$

Where:

A_{cp} = area enclosed by outside perimeter of concrete cross section.

P_{cp} = outside perimeter of concrete cross section.

3.3. Canadian-94 Code[11] method:

$$T_{cr} = 0.4\phi_c \sqrt{f'_c}\left(\frac{A_{cp}^2}{P_{cp}}\right) \tag{3}$$

Where ϕ_c = 0.6

3.4. Hsu and Mo's [14] method:

$$T_{cr} = 0.5\sqrt{f'_c}\left(\frac{A_{cp}^2}{P_{cp}}\right) \tag{4}$$

3.5. Koutchoukali and Belarbi's[1] method:

$$T_{cr} = 0.46\sqrt{f'_c}\left(\frac{A_{cp}^2}{P_{cp}}\right) \tag{5}$$

3.6. Fang and Shiau's[30] method:

$$T_{cr} = 0.095\sqrt{f'_c}\; x^2\, y \tag{6}$$

4.Torsion Design Equations:

(7) Methods of existing design codes are included in this study to predict the torsional resistance moment of the beams. To compare between design methods, torsional resistance ($T_{r\text{-}calc.}$) will be used instead of nominal ($T_{n\text{-}calc.}$) throughout (e.g. $T_{r\text{-}calc.}$ = 0.85 $T_{n\text{-}calc.}$ per ACI 318M-89 Code [5] method).

The design code methods are based on two approaches:

a. Skew Bending Theory:

Beam torsional strength is composed of two parts: the concrete contribution (T_c) and the reinforcement contribution (T_s).

1.a. ACI 318M-89 Code[5] method:

$$T_{r-calc} \leq T_{r\,ACI-t-89} = 0.85\left[\frac{\sqrt{f'_c}}{15}\sum x^2.y + \alpha_t\,.\frac{A_t.x_1.y_1.f_{yt}}{S}\right] \qquad (7.1)$$

$$T_{r-calc} \leq T_{r\,ACI-\ell-89} = 0.85\left[\frac{\sqrt{f'_c}}{15}\sum x^2.y + \alpha_\ell\,.\frac{A_\ell.x_1.y_1.f_{y\ell}}{P_h}\right] \qquad (7.2)$$

Where:

$T_{r\,ACI-t-89}$ = torsional resistance moment provided by concrete and stirrups, calculated by ACI-89 method.

α_t = $0.66+0.33(y_1/x_1) \leq 1.5$

A_t = area of one leg of closed stirrup resisting torsion within spacing S.

x_1 = shorter centre-to-centre dimension of closed rectangular stirrup.

y_1 = longer centre-to-centre dimension of closed rectangular stirrup.

f_{yt} = specified yield strength of transverse reinforcement.

S = spacing of transverse torsional reinforcement in direction parallel to longitudinal reinforcement.

$T_{r\,ACI-\ell-89}$ = torsional resistance moment provided by concrete and longitudinal torsion reinforcement, calculated by ACI-89 method.

A_ℓ = area of longitudinal reinforcement required for torsion.

$f_{y\ell}$ = specified yield strength of longitudinal reinforcement.

P_h = perimeter of centerline of outermost closed transverse torsional reinforcement.

2.a. BS 8110-97 Code[7] method:

$$T_{r-calc} \leq T_{r\,BS-t} = 0.0375\,x^2\left(y-\frac{x}{3}\right)\sqrt{f_c'} + \frac{1.6\,A_t.x_1.y_1\left(0.95\,f_{yt}\right)}{S} \qquad (8.1)$$

$$T_{r-calc} \leq T_{r\,BS-\ell} = 0.0375\,x^2\left(y-\frac{x}{3}\right)\sqrt{f_c'} + \frac{1.6\,A_\ell.x_1.y_1\left(0.95\,f_{y\ell}\right)}{P_h} \qquad (8.2)$$

Where it is assumed that $f'_c = 0.8\, f_{cu}$,

$T_{r\,BS-t}$ = torsional resistance moment provided by concrete and stirrups, calculated by BS-97 method.

$T_{r\,BS-\ell}$ = torsional resistance moment provided by concrete and longitudinal torsion reinforcement, calculated by BS-97 method.

b. Space Truss Analogy:

This new method is considerably simpler to understand and apply than the previous one. It can also be used for prestressed concrete loaded in torsion; a case not covered by the ACI 318M-89 Code [5]. It involves assuming that the concrete contribution T_c=0. In this model, the beam cross section is idealized as a tube. After cracking, the tube is idealized as a space truss consisting of closed stirrups, longitudinal bars in the corners, and concrete compression diagonals approximately centered on the stirrups. The diagonals are at an angle θ to the member longitudinal axis.

The most significant difference between the torsion provisions of the ACI Codes and the AASHTO-LRFD [12] specifications is the specified value of θ. For non prestressed sections, the ACI Code recommends (45) degrees, while the AASTHO [12] provisions permit a value of about (36) degrees (based on the longitudinal strain at mid-span of the section) [31, 32]. The methods adopted this analogy are:

1.b. ACI 318M-99 Code[16] method:

$$T_{r-calc} \leq T_{r\,ACI-t-99} = 0.85\left[\frac{1.7A_{oh}.A_t \cdot f_{yt}}{S}\right] \qquad (9.1)$$

$$T_{r-calc} \leq T_{r\,ACI-\ell-99} = 0.85\left[\frac{1.7A_{oh}.A_\ell \cdot f_{y\ell}}{P_h}\right] \qquad (9.2)$$

Where:

$T_{r\,ACI-t-99}$ = torsional resistance moment provided by stirrups, calculated by ACI -99 method.

$T_{r\ ACI\text{-}\ell\text{-}99}$ = torsional resistance moment provided by longitudinal torsion reinforcement, calculated by ACI-99 method.

A_{oh} = area enclosed by centerline of outermost closed transverse torsional reinforcement.

2.b. ACI 318M-05 Code[17] method:

$$T_{r-calc} \le T_{r\,ACI-t-05} = 0.75\left[\frac{1.7A_{oh}.A_t.f_{yt}}{S}\right] \tag{10.1}$$

$$T_{r-calc} \le T_{r\,ACI-\ell-05} = 0.75\left[\frac{1.7A_{oh}.A_\ell.f_{y\ell}}{P_h}\right] \tag{10.2}$$

Where:

$T_{r\ ACI\text{-}t\text{-}05}$ = torsional resistance moment provided by stirrups, calculated by ACI-05 method.

$T_{r\ ACI\text{-}\ell\text{-}05}$ = torsional resistance moment provided by longitudinal torsion reinforcement, calculated by ACI-05 method.

3.b. Canadian -94 Code[11] method:

$$T_{r-calc} \le T_{r\,Can.-t} = 0.85\left[\frac{1.7A_{oh}.A_t.f_{yt}}{S}\right] \tag{11.1}$$

$$T_{r-calc} \le T_{r\,Can.-\ell} = 0.85\left[\frac{1.7A_{oh}.A_\ell.f_{y\ell}}{P_h}\right] \tag{11.2}$$

Where:

$T_{r\ Can.\text{-}t}$ = torsional resistance moment provided by stirrups, calculated by Canadian Code method.

$T_{r\ Can.\text{-}\ell}$ = torsional resistance moment provided by longitudinal torsion reinforcement, calculated by Canadian Code method.

It can be seen that the Canadian Code [11] method is symmetric with the ACI 318M-99 Code [16] method.

4.b. AASHTO-LRFD-98 Bridge Design Specifications[12] method

$$T_{r-calc} \leq T_{r\,AASHTO-t} = 0.85\left[\frac{1.7A_{oh}.A_t \cdot f_{yt}}{S}.Cot\,\theta\right] \quad (12.1)$$

$$T_{r-calc} \leq T_{r\,AASHTO-\ell} = 0.85\left[\frac{1.7A_{oh}.A_\ell \cdot f_{y\ell}}{0.9P_h}.\tan\theta\right] \quad (12.2)$$

Where:

$T_{r\,AASHTO\text{-}t}$ = torsional resistance moment provided by stirrups, calculated by AASHTO method.

$T_{r\,AASHTO\text{-}\ell}$ = torsional resistance moment provided by longitudinal torsion reinforcement, calculated by AASHTO method.

θ = angle of inclination of compression diagonals to the member longitudinal axis, equal to 36 degrees.

5.b. EURO-89 Code[13] method

$$T_{r\,EU} = 1.7\,A_{oh}\sqrt{\frac{A_t}{S}\cdot f_{yt}\cdot\frac{A_\ell}{P_h}\cdot f_{y\ell}} \quad (13)$$

Where:

$T_{r\,EU}$ = torsional resistance moment calculated by EURO method.

5.Statistical Evaluation of Existing Methods

Table (2) indicates the results of the cracking torsional moment of (28) specimens (out of 101 tested beams- not all the values of T_{cr} are included in the references). The comparison between these results and predicted values ($T_{cr\text{-}test}$ / $T_{cr\text{-}calc.}$) leads to a range of (0.810-1.894) for the mean of this ratio. It can be seen that the ACI 318M-89 Code [5] method is the one with the greatest amount (26 specimens) of unacceptable predictions- based on the value of ($T_{cr\text{-}test}$ / $T_{cr\text{-}calc.}$) < 1. The lowest ratio for this code is (0.614).

In contrast, the Canadian[11], and Fang and Shiau[30] methods lead to good predictions with no results of the previous ratio < 1, but the Fang and Shiau[30] method seems to be the better due to the lowest values of low and high of the ratio ($T_{cr\text{-}test}$ / $T_{cr\text{-}calc.}$). The ACI 318M-05 Code [17] method also gives a good prediction with only one specimen of the indicated ratio < 1.

The coefficient of variation (COV) gives a good indication of the relevance of the prediction method for the ratio ($T_{cr\text{-}test}$ / $T_{cr\text{-}calc.}$). It can be seen that the difference in COV values of all methods is very small (ranging between 15-15.4 percent). Therefore this coefficient does not indicate which method is the best.

Table 2- Statistical analysis of the ratio ($T_{cr\text{-}test}$ / $T_{cr\text{-}calc.}$) for 28 tests.

Detail	*ACI-89*[5]	*ACI-05*[17]	*Canadian*[11]	*Reference (14)*	*Reference (1)*	*Reference (30)*	*Proposed Eq. (14)*
$\bar{x}$	0.810	1.377	1.894	0.909	0.988	1.421	1.461
S.D.	0.125	0.207	0.284	0.136	0.148	0.220	0.177
COV %	15.444	15.004	15.004	15.004	15.004	15.444	12.098
Low	0.614	0.931	1.280	0.614	0.668	1.078	1.015
High	1.059	1.989	2.735	1.313	1.427	1.858	1.839
High/Low	1.724	2.137	2.137	2.137	2.137	1.724	1.811
Number<1	26	1	0	23	15	0	0

Table (3) shows the values of the results of the (101) tested beams, compared with the predicted strength ($T_{u\text{-}test}$ / $T_{r\text{-}calc.}$). The range of the mean of this ratio is (0.963-1.514). Based on the value of ($T_{u\text{-}test}$ / $T_{r\text{-}calc.}$) < 1, the EURO **[13]** method leads to unsafe predictions (58 specimens). The lowest ratio for this code is (0.356). On the other hand, the ACI 318M-89 Code **[5]** method is the most conservative of the existing methods with only (8) results with the previous ratio < 1. From table (3) it can be seen that the ACI 318M-99**[16]**, Canadian **[11]**, and ACI 318M-05 Code **[17]** methods lead to the least relevant prediction with a high COV of (33.102) percent for each one of them. From this point of view, the best COV is (20.883) percent for the ACI 318M-89 Code **[5]** method. The COV values are (27.339, 31.685, and 24.807) percent for BS **[7]**, AASHTO **[12]**, and EURO **[13]** methods, respectively.

Table 3- Statistical analysis of the ratio ($T_{u\text{-}test}$ / $T_{r\text{-}calc.}$) for 101 tests.

Detail	*ACI-89*[5]	*BS*[7]	*ACI-99*[16] *and Canadian*[11]	*ACI-05*[17]	*AASHTO*[12]	*EURO*[13]	*Proposed Eq. (15)*
$\bar{x}$	1.269	1.091	1.335	1.514	1.412	0.963	1.343
S.D.	0.265	0.298	0.442	0.501	0.447	0.239	0.174
COV %	20.883	27.339	33.102	33.102	31.685	24.807	12.921
Low	0.635	0.538	0.591	0.670	0.464	0.356	0.992
High	2.231	2.395	3.318	3.760	3.168	1.526	1.783
High/Low	3.514	4.452	5.610	5.610	6.829	4.280	1.798
Number<1	8	39	20	12	20	58	1

6. Regression Analysis of Test Results

Using regression analysis, the (28) and (101) test results of cracking and resistance moment, respectively were analyzed by computer. The aim is to obtain simple and conservative equations to predict cracking torsional moment and torsional resistance moment of NSC rectangular section beams under pure torsion, that give the lowest possible COV values of the ratios ($T_{cr\text{-}test}$ / $T_{cr\text{-}calc.}$) and ($T_{u\text{-}test}$ / $T_{r\text{-}calc.}$). This has led to the following prediction equations:

$$T_{cr-proposed} = 0.35\left(f'_c\right)^{0.35} . x^{1.85} . y \quad (14)$$

$$T_{r-proposed} = 6.2\frac{A_{0h}^{1.23}}{S^{0.43}}\left(\frac{A_t . A_\ell}{P_h}\right)^{0.27}\left(f_{yt} . f_{y\ell}\right)^{0.2} \quad (15)$$

Equation (14) is based on the (3) main parameters f'_c, x, and y, while equation (15) is based on the (7) main parameters A_{oh}, S, A_t, A_ℓ, P_h, f_{yt} , and $f_{y\ell}$. Tables 2 and 3 show the summary of statistical evaluation of the proposed methods. The proposed equation (14) which estimates T_{cr} gives the best COV value of (12.098) percent among all other methods with no result having the ratio of ($T_{cr-test}$ / $T_{cr-calc.}$) < 1 (Table 2).

As shown in Table (3), when the proposed equation (15) [that predicts T_r] was applied, it led to much safer prediction with only one specimen (out of 101) having the ratio of (T_{u-test} / $T_{r-calc.}$) < 1. Among all the pre-indicated methods, the proposed equation (15) gives the best COV value of (12.921) percent. In addition, the value of high/low of the previous ratio was (1.798) for this equation, while the range of this ratio was (3.514-6.829) for all other methods.

To illustrate the relevance of the proposed method – equation (15), the ratio of (T_{u-test} / $T_{r-calc.}$) has been compared by this method with that of the latest available ACI 318M-05 Code [17] procedure (which is the same as the procedure of the ACI 318-08 Code [10] the metric version is not published yet). These are shown in Figs. 1,2,3,4, and 5.

The comparison in Fig.1 between the ACI 318M-05[17] method and the proposed equation (15) shows clearly that for the range of f'_c (14.34 – 40.13) MPa, the proposed method shows much less scatter in the results. In addition, the number of unsafe results (T_{u-test} / $T_{r-calc.}$) < 1 is greater for the ACI 318M-05 Code [17] method, despite the fact that this ratio is high in several cases (up to 3.76). It is to be noted that there is a tendency toward greater safety with rising f'_c values for both methods.

Similar conclusions regarding the much greater scatter and the number of unsafe results by the ACI 318M-05 Code [17] method can be seen in Fig.2 (influence of the aspect ratio $\frac{y}{x}$), 3 (influence of the sectional area A_{cp}), 4 (influence of the nominal stirrup strength $\rho_v.f_{yt}$), and 5 (influence of the nominal longitudinal steel strength $\rho_\ell.f_{y\ell}$). For the ACI 318M-05 Code [17] method, there is an increase in the factor of safety with rising value of $\frac{y}{x}$, while this increase is less for the proposed method – equation (15) [Fig.2]. The influence of A_{cp} is indicated in Fig.3 which shows that for ACI 318M-05[17] method, the factor of safety decreases with increasing A_{cp} value. In contrast, the safety factor of the proposed method is approximately constant with variation of A_{cp} value.

Figs. 4 and 5 show clear trends for the overestimation of the influence of the nominal steel strength ($\rho_v.f_{yt}$ and $\rho_\ell.f_{y\ell}$) by the ACI 318M-05[17] method. On the other hand, the proposed method shows no variation in the safety factor with rising value of $\rho_v.f_{yt}$ (ranging between 0.883-7.441) MPa, and $\rho_\ell.f_{y\ell}$ (ranging between 1.264-22.142) MPa.

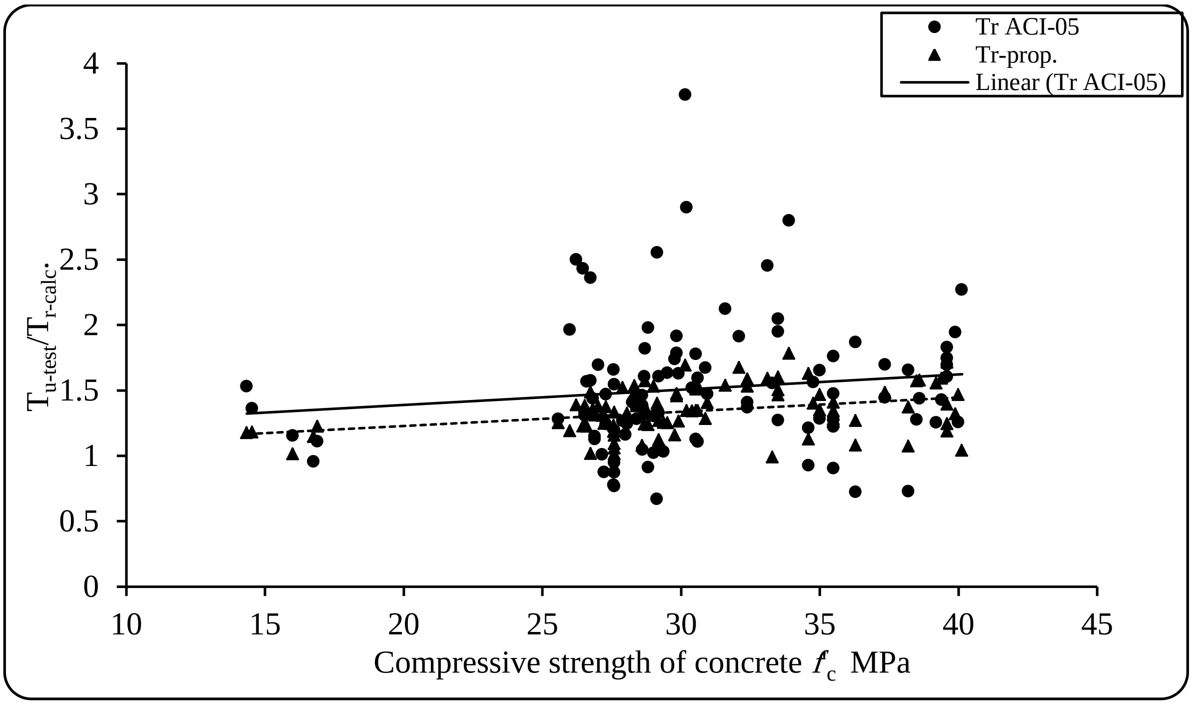

Fig. 1- Influence of compressive strength of concrete f'_c on test results

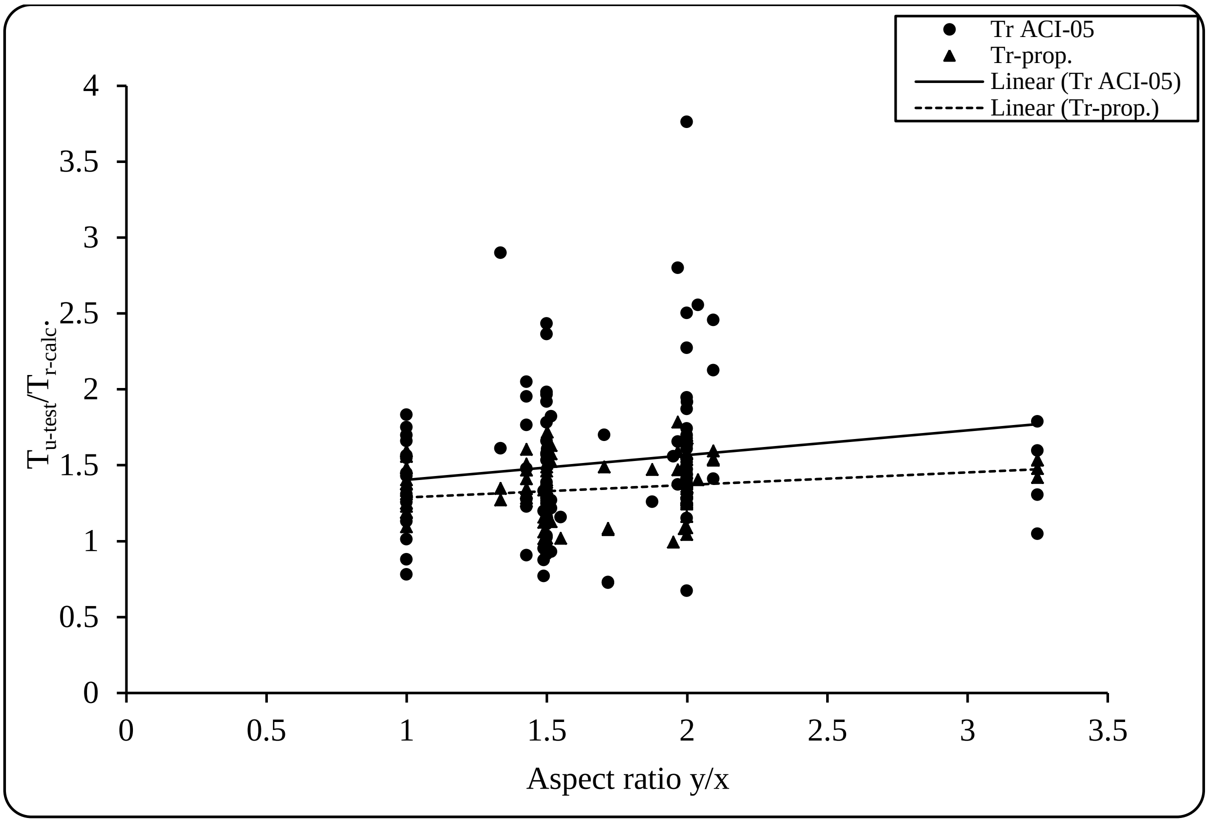

Fig. 2- Influence of aspect ratio *y/x* on test results

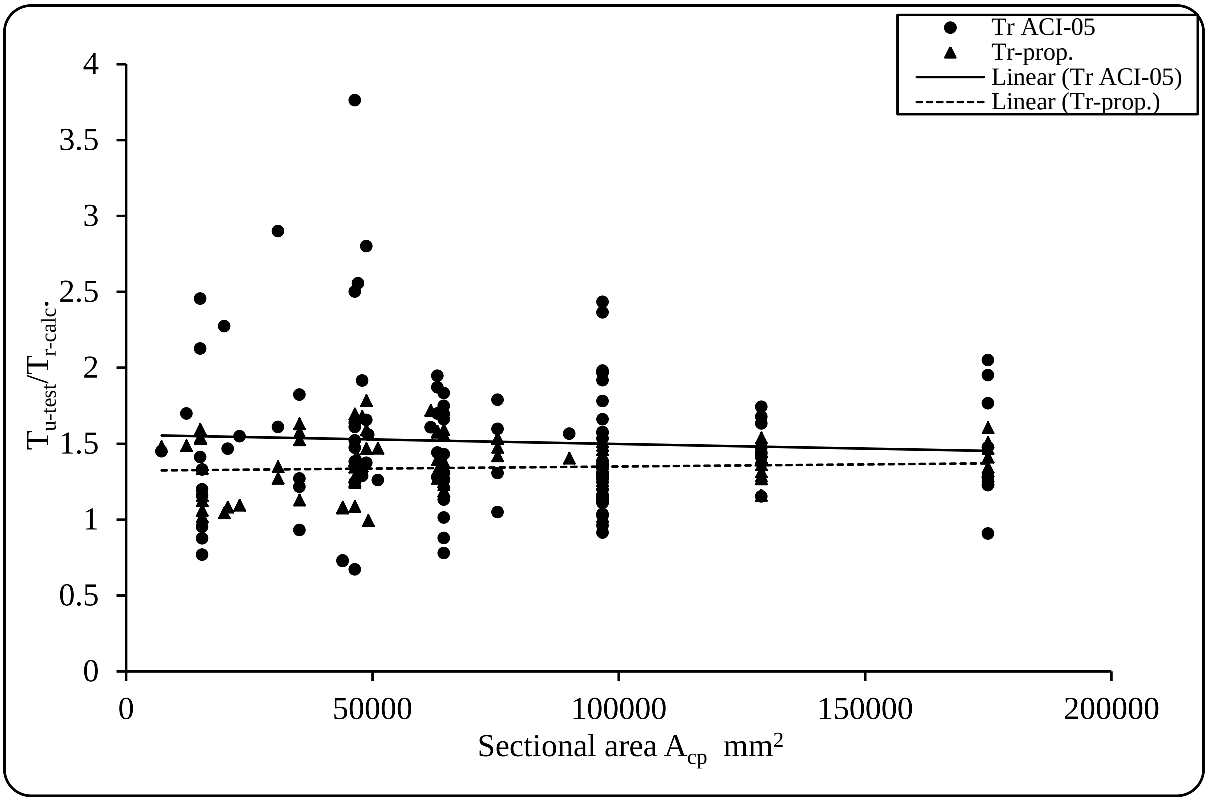

Fig. 3- Influence of sectional area A_{cp} on test results

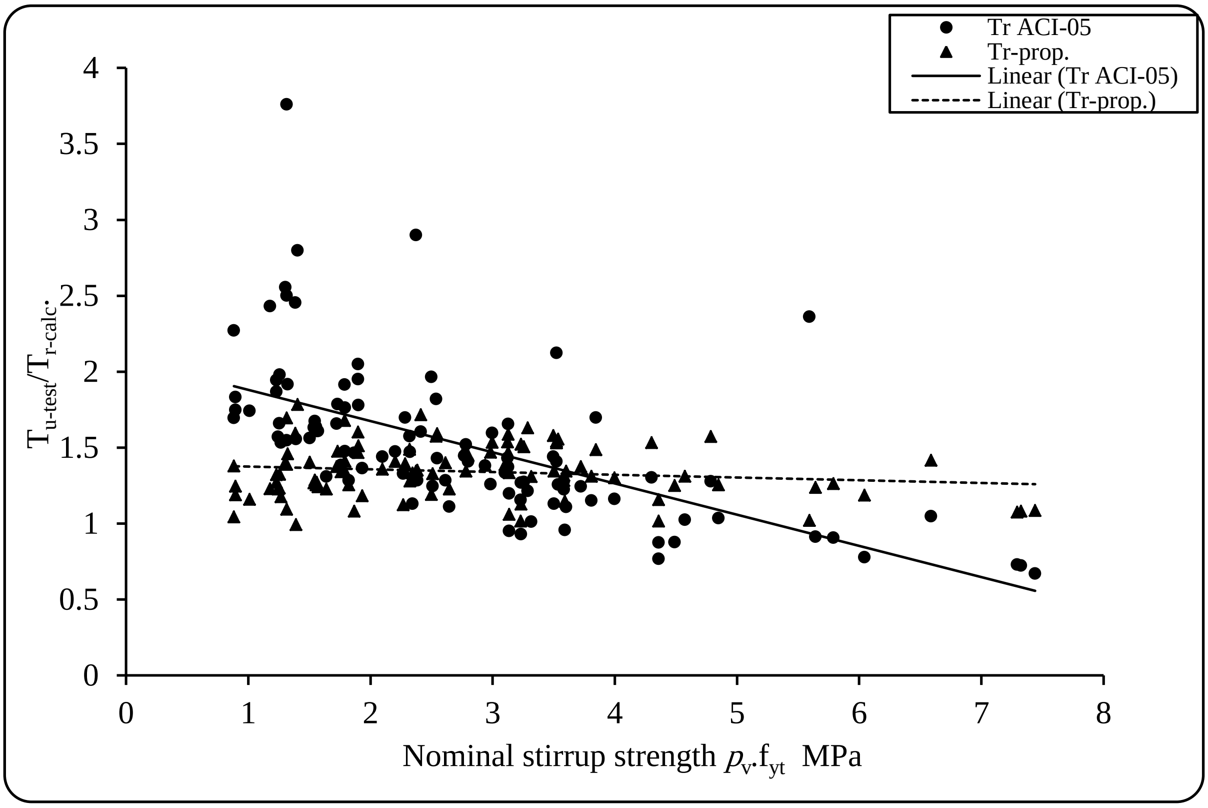

Fig. 4- Influence of nominal stirrup strength $\rho_v.f_{yt}$ on test results

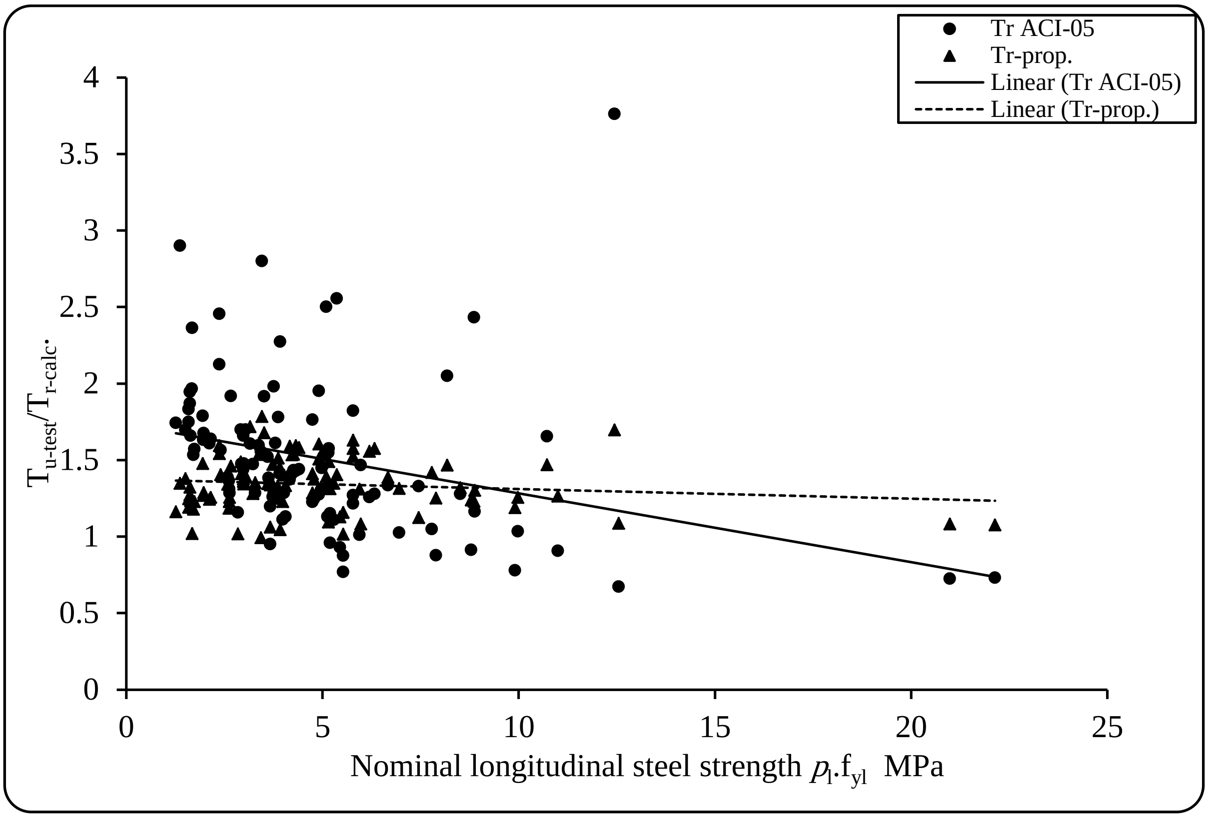

Fig. 5- Influence of nominal longitudinal steel strength $\rho_\ell.f_{y\ell}$ on test results

7. Conclusions :

Based on this work, the following conclusions are made:

1. A simple equation (14) is presented to estimate cracking torsional moment (T_{cr}) in NSC rectangular section beams.
2. Another equation (15) is suggested for predicting torsional resistance moment (T_r) of such beams. This method agrees with the recent trend of space truss analogy of shear flow that bases strength only on the contribution of reinforcement- as in ACI 318M-95[9] and later ACI Code versions, Canadian[11], AASHTO-LRFD[12], and EURO[13] methods.
3. The existing methods [1, 5, 11, 14, 17, 30], give COV values between (15-15.4) percent for the ratio ($T_{cr-test}/T_{cr-calc}$), while the proposed method leads to a COV value of (12.1) percent for this ratio.
4. The COV value of the existing code design methods [5, 7, 11, 12, 13, 16, 17] ranges between (20.9-33.1) percent for the ratio (T_{u-test}/T_{r-calc}). On the other hand, the proposed equation (15) leads to the best value of COV-(12.9) percent for this ratio.
5. The proposed method – equation (15) is similar to the EURO [13] one- equation (13), with one major difference. Equation (15) uses powers of values less than (0.5) for (4) parameters: A_t, A_ℓ, f_{yt}, and $f_{y\ell}$. Therefore, for the ratio (T_{u-test}/T_{r-calc}), equation (15) gives low value of (0.992) with only one result below 1 (Table 3). In contrast, EURO [13] method has respective values of (0.356 and 58).

6. The latest code design method (ACI 318M-05) [17] has the highest mean value (at 1.51) of all the other methods [5, 7, 11, 12, 13, 16] and the highest ratio of T_{u-test}/T_{r-calc} (at 3.76). Despite this, ACI 318M-05 Code [17] method leads to 12 unsafe ratios, with a low value of (0.67).
7. For a range of f'_c between (14.34-40.13) MPa, the proposed equation (15) gives safe prediction (Fig. 1), as well as a rising factor of safety with increasing value of f'_c. The same conclusion can be noticed for the ACI 318M-05 Code [17] method regarding the safety factor.
8. Fig. 2 indicates that the increase in the factor of safety with rising value of $\frac{y}{x}$ is greater for the ACI 318M-05 Code [17] method than the proposed eq. (15).
9. Figs. 3, 4, and 5 show that the factor of safety of the proposed equation (15) is not influenced by rising values of $A_{cp}, \rho_v . f_{yt}$ and $\rho_\ell . f_{y\ell}$. On the other hand, the safety factor of the ACI 318M-05 Code [17] method decreases with rising values of the indicated variables.

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