



## LABORATORY INVESTIGATION OF SHEAR REPAIR OF REINFORCED LIGHTWEIGHT AGGREGATE CONCRETE BEAMS WITH STIRRUPS

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**Abstract :** The structural rehabilitation of reinforced lightweight aggregate concrete beams in shear was studied with resin injection. A series of model beams, reinforced in shear, were loaded in the laboratory until failure, repair and again loaded to failure. The variables in the test program were shear span-to-depth ratio 4.0, 3.0, 2.5, 2.0, 1.5 and 1.0 ; and shear reinforcement index  $\rho_v f_y = 0.493, 0.704$  and  $0.896$  MPa. Experimental failure loads are compared with predictions using ACI 318M-02 code and the modified compression field theory by the program of "Response 2000". After testing, the beams failed due to new diagonal cracks. The ratio of ultimate load to crack load varies from 1.5 for  $a/d > 2$  to 2.27 for  $a/d \leq 2$  (deep beams). Test results indicate the ACI 318M-02 is conservative.

**Key words :** shear repair, lightweight aggregate, repair concrete, reinforced beams

### الفحوصات المختبرية للقص لاعادة تأهيل العتبات الكونكريتية المسلحة الخفيفة الوزن بوجود تسليح الاتاري

**الخلاصة :** المنشآت الخرسانية المسلحة الخفيفة الوزن المهذمة بأجهادات القص تم دراسة إعادة التأهيل بواسطة استخدام طريقة الحقن. عدة مجاميع من العتبات المسلحة بالقص تم تحميلها بالمختبر لغاية الفشل ثم تم تصليحها وتحميلها للفشل. تم دراسة المتغيرات التالية: نسبة فضاء القص الى العمق  $1,0,1,0,2,0,2,0,3,0,4,0$  وكذلك دليل تسليح القص وكانت القيم  $0,493, 0,704, 0,896$  MPa تم مقارنة النتائج المختبرية مع قيم معادلة الكود الامريكي وبرنامج (Response 2000) بعد اجراء الفحوصات تبين أن العتبات تفشل بتشقق قطري جديد وأن نسبة الحمل الأقصى الى حمل التشقق يتغير من 1.5 عندما  $a/d > 2$  الى القيمة 2,27 عندما  $a/d \leq 2$  للعتبات العميقة. النتائج تبين ان القيم المستخرجة من استخدام طريقة ACI 318M-02 هي اكثر دقة.

## 1. Introduction

During the last decades, incidence of failure of RC structures has been seen of wide occurrence for many reasons, such as increasing service loads and / or durability problems. It may be more economical to accept the need for repair at suitable intervals than to attempt to build a structure that will be maintenance-free under severe conditions for a long period [1].

A laboratory study was conducted on a set of reinforced lightweight aggregate concrete (LWAC) beams loaded in flexure to understand the behavior of beams repair by resin injection. The influence on final repair strength of damage across the repair plane was emphasized.

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## 2. Research Significance

Information regarding repair of the shear behavior and shear capacity of porcelinite LWAC is practically nonexistent. The present study has focused on the repair of reinforced LWAC beams with stirrups in shear. The failure mechanisms of repair beams have been studied.

## 3. Resin Injection

The first practical application of epoxy resin took place in Germany and Switzerland in the 1930s. Limited production of epoxy resins started in the late 1940s and commercially produced epoxy resin adhesives become available in the early 1950s [2,3].

Epoxy injection has been successfully used in the repair of cracks in building, dams and other types of concrete structures [4].

Resin properties are paramount to the successful application of this technique. Such properties as viscosity, curing, dimensional stability, elastic modulus and adherence to a wet or damp highly alkaline interface require careful consideration. A two part, solvent-free low viscosity, named Conbextra EP10 epoxy injection resin is used for the repair of the beams. This low viscosity allows adequate penetration into the finest of cracks [5].

Shear repair of RC members by resin injection has been the focus of laboratory studies by *Chung and Lui, 1977* [6]; *Hewlett and Morgan, 1982* [7]; *Brondum-Nielsen, 1978* [8] and *Collins and Roper, 1990* [9]. *Chung and Lui's* work focused on eight concrete push off specimens. The limited test results indicated a complete restoration of shear resistance within repaired joints. *Hewlett and Morgan, 1982*, studied shear repair of RC beams subjected to unidirectional static loading. Test results indicated structural reinstatement of beams with diagonal tension cracks. Repair beams were found to be stiffer and stronger than the original beams. *Brondum-Nielsen, 1978*, recorded strength increases with laboratory-scaled models of field-size reinforced concrete beams with circular web openings. Specimens were few and results widely scattered. *Collins and Roper, 1990*, tested of 20 reinforced concrete beams in shear without stirrups. The shear-repair techniques were examined include resin injection, post-tensioning in shear, bar bonding and stitching. Test results indicated the repair strength achieved for resin-injected reinforced concrete beams was shown to be influenced by the condition of the major diagonal crack after initial damage.

## 4. "Response 2000" Program

"Response 2000" program is a non-linear sectional analysis for the analysis of reinforced concrete elements subjected to shear based on the Modified Compression Field Theory [10]. These programs were written over the years 1996-1999 by *Evan Bentz*, PhD candidate at the University of Toronto under the supervision of Professor *M. P. Collins* [11]. They were to allow fast checking for errors in input and fast interpretation of results with ample graphics. They were to provide stable, state-of-the-art analysis techniques and, finally, they were designed to leave the user knowing more about the real behavior of concrete rather than less, as some computer programs seem to do [11].

Response-2000 allows analysis of beams and columns subjected to arbitrary combinations of axial load, moment and shear. It also includes a method to integrate the sectional

behavior for simple prismatic beam-segments .The assumptions implicit in the program are that plane sections remain plane, and that there is no transverse clamping stress across the depth of the beam .For sections of a beam or column a reasonable distance away from a support or point load, these are excellent assumptions .These are the same locations in beams that are usually the critical locations for brittle shear failures .

## 5. Laboratory Investigation of Model Beams

### 5-1 Model reinforced concrete beam specimens

A total of 12 reinforced LWAC beams with web reinforcement were cast and tested by *M.K.Medhlom* ,PhD candidate at the University of Technology ,Iraq, under the supervision of professor *K.F.Sarsam* in 2005 [12] ."Table 1" lists the materials for  $f_c' > 20$  MPa mixes and 12 mm maximum size aggregate were chosen for the mixes [12]. Three groups of beams with shear reinforcement index ( $\rho_v f_y$ ) of 0.493, 0.704 and 0.896 MPa (stirrups at 100, 70 and 55 mm respectively) were loaded symmetrically with two equal concentrated loads .These beams had  $a/d$  ratios of 4.0 ,3.0 ,2.5 ,2.0 ,1.5 and 1.0 .Smooth wire (2.5 mm diameter ,yield and ultimate strength of 512 and 588 MPa respectively) were used in fabricating the stirrups . "Fig. 1" and "Table 2" give all the specimens details .Each specimen is identified by a label  $S_{xyz}$  where  $x$  is a number denoting the longitudinal steel ratio ,  $y$  denotes  $a/d$  and  $z$  representing the amount of shear reinforcement .

Table 1. Proportion, Slump and Air Density of LWAC for Mixes [12].

No.	Quantities, kg/m <sup>3</sup>			W/C	Hardened Density kg /m <sup>3</sup>	$f_c'$ , MPa, 28 days	Slump mm	Admixture % weight of cement
	Cement	Natural sand	Porcel. Aggreg.					
1	480	480	658	0.313	1820	22.6	101	4%* or 2%**
2	550	500	520	0.32	1930	23.4	108	4%* or 2%**

\* Melment L10

\*\* Sp<sub>3</sub>

Table 2 . Details of Specimens

Beam specimens	Ln mm	a mm	$\rho_w$ %	$a/d$	$\rho_v f_y$ MPa	Spacing (mm)
S 341	1638	684	2.593	4.0	0.493	100
S 331	1296	513	2.593	3.0	0.493	100
S 321	1130	430	2.593	2.5	0.493	100
S 311	960	345	2.593	2.0	0.493	100
S 312	960	345	2.593	2.0	0.704	70
S 313	960	345	2.593	2.0	0.896	55
S 3Ö1	790	260	2.593	1.5	0.493	100
S 3Ö2	790	260	2.593	1.5	0.704	70
S 3Ö3	790	260	2.593	1.5	0.896	55
S 301	612	171	2.593	1.0	0.493	100
S 302	612	171	2.593	1.0	0.704	70
S 303	612	171	2.593	1.0	0.896	55

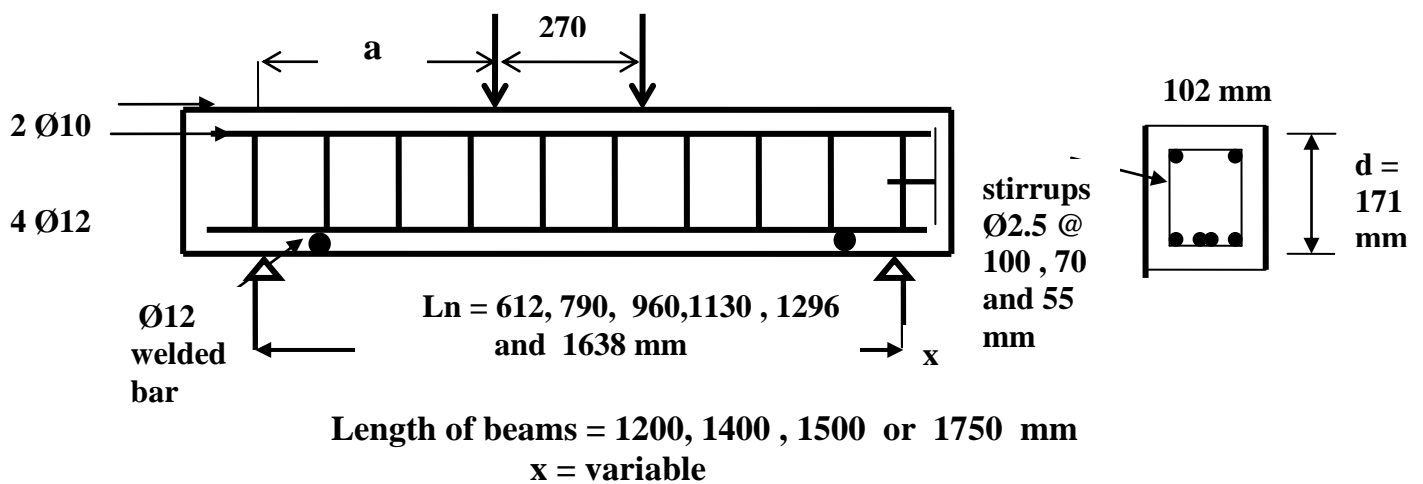


Fig. 1. Specimen Details for Test Beams

### 5-2 Epoxy resin

A two part, solvent-free low viscosity, named Conbextra EP 10 injection resin is used for the repair of the beams .It has many advantages such as suitability for hot climates ,excellent bond to concrete, and no-shrinkage .The properties of Conbextra EP 10 (according to the manufacturer editions) are listed in"table 3" .

Table 3 . Properties of Conbextra EP 10 .

Property	Typical results
Compressive strength *	70 MPa @ 20 <sup>0</sup> C 90 MPa @ 35 <sup>0</sup> C
Tensile strength *	26 MPa @ 35 <sup>0</sup> C
Flexural strength *	63 MPa @ 35 <sup>0</sup> C
Young modulus in compression	16 GPa
Specific gravity	1,0 ±
Mixed viscosity	1 poise @ 35 <sup>0</sup> C

\* at 7 days

### 5-3 Test procedure

After repairs became effective ,beams were tested in a 300 ton hydraulic testing machine (MFL) ,as shown in "Fig.2" , and loaded in increments of 4 kN until failure .At each load stage deflection reading and the development crack pattern was marked on the beam surface .The testing sequence of the 12 model specimens is summarized in "Table 4" .



Fig. 2 : Test Machine

Table 4 : Summary of Test Results

Beams without stirrups [12]			Beams with stirrups [12]		Repair beams		VrR / VcR	VrR / Vro	VrR / Vrw		
a/d	Vcw kN	Vrw kN		Vco kN	Vro kN	VcR kN	VrR kN				
4.0	24	28	S 341	0.493	17	33	17	22	1.30	0.66	0.79
3.0	18	24	S 331	0.493	22	39.5	18	28	1.55	0.71	1.17
2.5	22	36.5	S 321	0.493	20	44.5	20	31	1.55	0.70	0.85
2.0	24	42	S 311	0.493	28	63	14	32	2.3	0.51	0.76
2.0			S 312	0.704	30	68.5	18	38	2.11	0.55	
2.0			S 313	0.896	35	77	20	43	2.15	0.56	
1.5	27	62	S 301	0.493	36	71	24	55	2.3	0.77	0.88
1.5			S 302	0.704	37	79	30	63	2.1	0.80	
1.5			S 303	0.896	44	81	27	66	2.45	0.82	
1.0	37	69	S 301	0.493	37.5	89	33	72	2.2	0.81	1.05
1.0			S 302	0.704	35	92	30	76	2.54	0.83	
1.0			S 303	0.896	50	108	37	85	2.3	0.80	

#### 5-4 Repair Techniques

The following steps are followed in the epoxy injection repair process for each failed beam:

1. After failure, cracks and their neighbor areas are cleaned from dust, debris and other contaminants by applying a compressed air using electrical blower to ensure good penetration of the resin and proper bond of the crack paste.

2. Surface ports are then fixed along the considered crack. The port has an opening at the top for the epoxy to enter and a flange at the bottom that is bonded to the concrete. The ports are placed 100 – 150 mm apart. The port is fixed in its proper position by applying an epoxy paste to the flange portion of the port taking care not to cover hole, and then tacking it in place.
3. Epoxy paste is then used to seal over the surface ports and exposed cracks. The paste is extended 20-30mm on either sides of the crack with 2-3mm thickness to prevent resin seepage. The beam is left after this stage for 30-45 minutes to ensure complete curing of the paste.
4. The two components of epoxy resin are then mixed in a metal batch using a mechanical stirrer at a proportion of 1 (base): 3 (hardener) according to the manufacturer's instructions.
5. A mechanical injection gun is fed with the mixed epoxy and the injection process starts. The injection process begins by pumping epoxy into the lowest port until the epoxy begins to flow from the port above it. The first port is then plugged with a cap ,and the process is repeated until the crack has been completely filled and all ports have been capped .Low pressure is used in injecting epoxy into cracks .A curing period of about 24 hours is provided to the injected epoxy .
6. After the injected epoxy has cured, the ports are removed by striking with a hammer and the surface seal is chipped. Fig.(3) shows the injection process.



Fig. 3 . Epoxy Injection Process

## 6. Discussion of Test Results

### 6-1 General behavior

Prior to discussing the test results, it is helpful to discuss the general behavior of the repaired beams .All 12 beams tested failed in shear. Photographs of test specimens that show typical observed cracked patterns and failure mode are shown in "Fig. 4" .In this study the diagonal cracking force was defined as the shear force at the time when the critical diagonal crack became inclined and had crossed the mid-depth [13].

In those beams with  $a/d$  ratios of 4.0 ,3.0 and 2.5 (stirrups at 100 mm spacing) ,the crack pattern indicates that all the beams failed in diagonal tension .The ultimate shear capacity of beams in the  $a/d$  ratios 4.0 ,3.0 and 2.5 exceeded the inclined cracking capacity by an amount of 29.5 ,27.8 and 55 percent ,respectively .

The general cracking performance was similar for all the beams with  $a/d$  ratios of 2.0 and 1.5 (deep beams as defined by ACI 318M-02) [14] having  $p_v f_y$  values of 0.493 ,0.704 and 0.896 MPa . The crack pattern development indicating that these beams failed in shear-compression failure except the beam S313 failed by crushing of concrete at the support .The

average ultimate shear capacity of the beams in the  $a/d$  ratio of 2.0 and 1.5 exceeded the average inclined cracking capacity by 130 and 136 percent , respectively .

The beams with  $a/d$  ratio of 1.0 are deep beams having  $\rho v f_y$  values of 0.493 , 0.704 and 0.896 MPa .These beams developed arching action and ,usually, a significant amount of load could be added after the major inclined crack had developed .The average increase in load after inclined cracking is 132 percent .

After testing the repaired beams, the repaired diagonal cracks in all beams do not reopen except the beams S302 and S302 .The beams failed due to new diagonal cracks development with approximately the same formation sequence as the diagonal cracks in the original beams.The new diagonal crack is developed adjacent to or near the repair crack.

At failure, longitudinal splitting along the main reinforcement was prevalent in all beams.

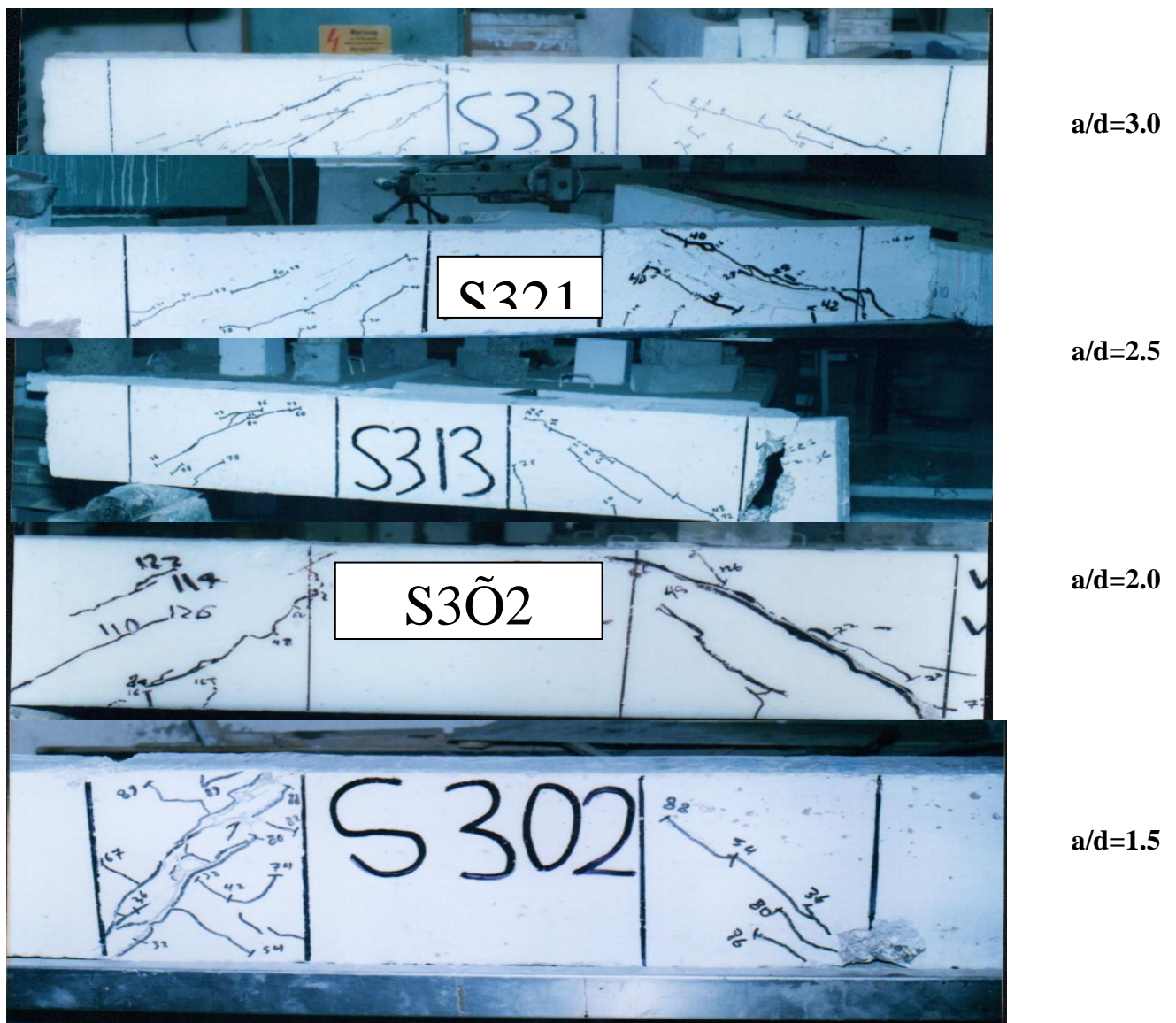


Fig.4 . Crack Pattern Development for Typical Beams



## 6-2 Effect of shear span-to-depth ratio

The variations in diagonal cracking and ultimate shear forces for original and repaired beams with increasing  $a/d$  are shown in "Table 4". The diagonal cracking forces were lower than the measured ultimate shear forces. Generally, the variation in  $V_{cR}$  and  $V_{co}$  with increasing  $a/d$  is small,  $V_{cR}$  and  $V_{co}$  seems independent of  $a/d$ . In fact, if extrapolation is allowed both  $V_{cR}$  and  $V_{rR}$  would converge at higher  $a/d$ .

In a slender beam with a long shear span the exact location of the initiating flexure crack is not too important, because the inclined crack can adjust its path as it extends towards the load. In a deep beam the initial inclined crack develops suddenly along almost its entire length as shown in "Fig.4". The initial crack location is critical and helps determine if much arching action can develop leading ultimately to crushing of arch rib (as in all deep beams except the beam S313 which failed in crushing over the support).

Test results indicate that the ratio of  $V_{rR} / V_{cR}$  varies from 1.5 for  $a/d > 2.0$  to 2.27 for  $a/d \leq 2.0$  (deep beams). This ratio is a measure of the reserve strength beyond diagonal cracking.

For  $\rho_v f_y$  equal to 0.493 MPa; when  $a/d$  decreased from 4.0 to 1.0 the measured ultimate shear forces increased about 227 percent for repaired beams and 170 percent for original beams.

For  $\rho_v f_y$  equal to 0.704 and 0.896 MPa; When  $a/d$  decreased from 2.0 to 1.0 (deep beams) the measured ultimate shear forces increased about 100 and 97 percent respectively for repaired beams and 34 and 40 percent respectively for original beams.

From the above calculations, the increase in strength for repaired beams is shown greater than for original beams when  $a/d$  decreased. It is now generally recognized that the shear span-to-depth ratio parameter has a very important influence on the shear strength of repaired and original beams.

From Table 4, the results show that the behavior of repaired beams with stirrups are same as the behavior of beams without stirrups because that the stirrups after failure had reached yield in all beams therefore the repaired beams lost the advantage of stirrups in resisting the shear. To increase the strength, it must be added beside the resin injection the carbon fiber reinforced polymer (CFRP) this needs more research.

Experimental failure loads for the repaired and original beams with stirrups are compared with predictions using *ACI 318M-02 code* [14] and the modified compression field theory (MCFT) by the program of "Response 2000" [11] as shown in Table 5 and Fig.5 through Fig.7. In this clause, the *ACI 318M-02 code* analysis the deep beams using the strut-and-tie model to predict values of shear capacity of D-regions in such members. These regions are idealized as trusses consisting of concrete compressive struts and reinforcing steel tension ties interconnected by nodal zones of multi-directionally compressed concrete. The truss shall be capable of providing internal load paths for all the forces acting on the regions. For more information see Appendix B in *ACI318M-02 code* and Appendix C in PhD theses in reference [12].

Table (5) : Comparison of Measured and Calculated Shear Forces

Beam number	ACI 318-M-02			Response 2000 (MCFT)**		
	V <sub>r</sub> -ACI kN	V <sub>r</sub> R / V <sub>r</sub> -ACI	V <sub>ro</sub> / V <sub>r</sub> -ACI	V <sub>r</sub> - Response kN	V <sub>r</sub> R / V <sub>r</sub> -Response	V <sub>ro</sub> / V <sub>r</sub> -Response
S 341	14.5	1.52	2.27	30.7	0.72	1.07
S 331	15.3	1.83	2.58	33.6	0.84	1.17
S 321	15.4	2	2.88	34.2	0.91	1.3
S 311	22.75*	1.4	2.77	36.2	0.88	1.75
S 312	20.2*	1.88	3.4	39.2	0.97	1.75
S 313	21*	2	3.66	46	0.94	1.67
S 3O1	21.42*	2.57	3.31	46.9	1.17	1.52
S 3O2	21.66*	2.9	3.65	52.4	1.2	1.5
S 3O3	21.54*	3.1	3.76	60.7	1.1	1.34
S 301	32.2*	2.23	2.76	65.4	1.1	1.36
S 302	32.72*	2.32	2.82	78.6	0.97	1.17
S 303	31.82*	2.67	3.4	90.9	0.94	1.19
Mean		2.2	3.1		0.97	1.4
Standard deviation		0.53	0.485		0.141	0.236
Coefficient of variation %		24	15.6		14.5	16.8

\* Deep beams (method of strut-and-tie model)    \*\* Modified compression field theory

Test results indicate that the ratio of  $V_rR / V_r\text{-ACI}$  and  $V_rR / V_r\text{-Response}$  has an average of 2.2 and 0.97 respectively. The results shown in "Fig.5" through "Fig.7" indicate that the ACI 318M-02 code is conservative compare to the experimental results and program of "Response 2000". The results demonstrate that the program of "Response 2000" un conservative for LWAC repaired beams ,because the stirrups in repaired beams were yielded. The program gives conservative predictions for original beams .

It is clear that the ACI 318M-02 code has the largest standard deviation (SD) of 0.55 and the program of "Response 2000" has the lowest SD of 0.141 for repaired beams .

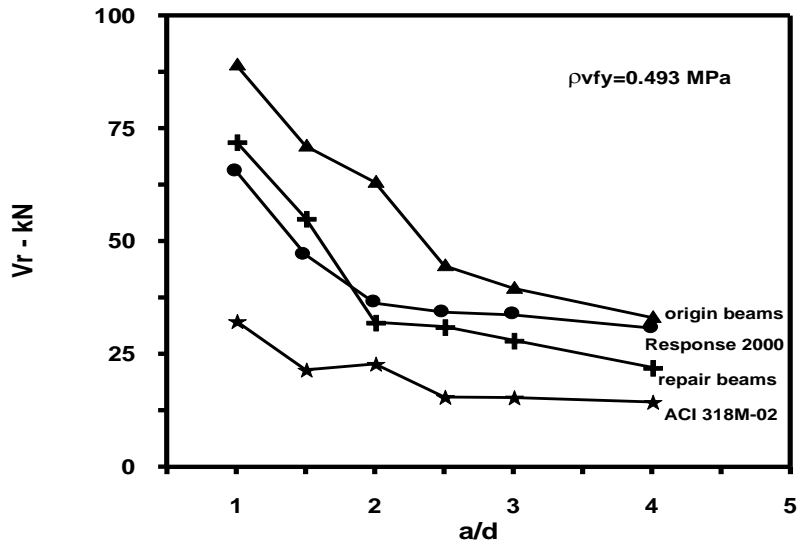


Fig.5 :Effect of shear span to depth ratio ,  $\rho_{vfy} = 0.493$

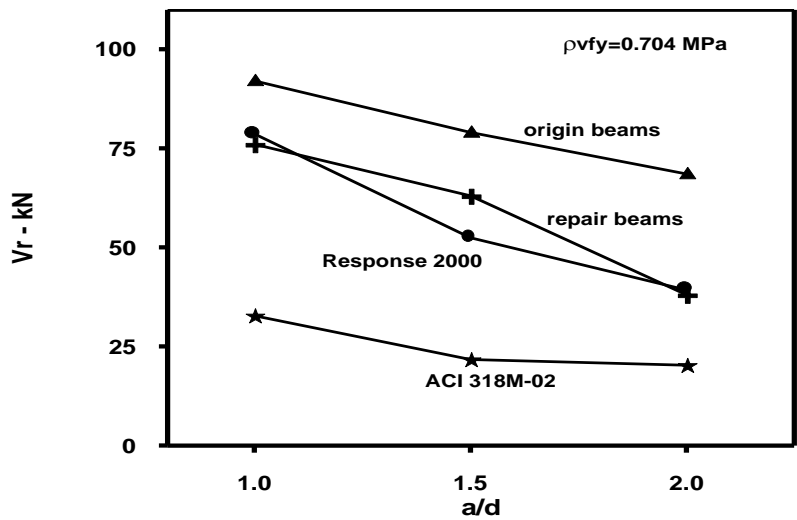


Fig.6 :Effect of shear span to depth ratio ,  $\rho_{vfy} = 0.704$

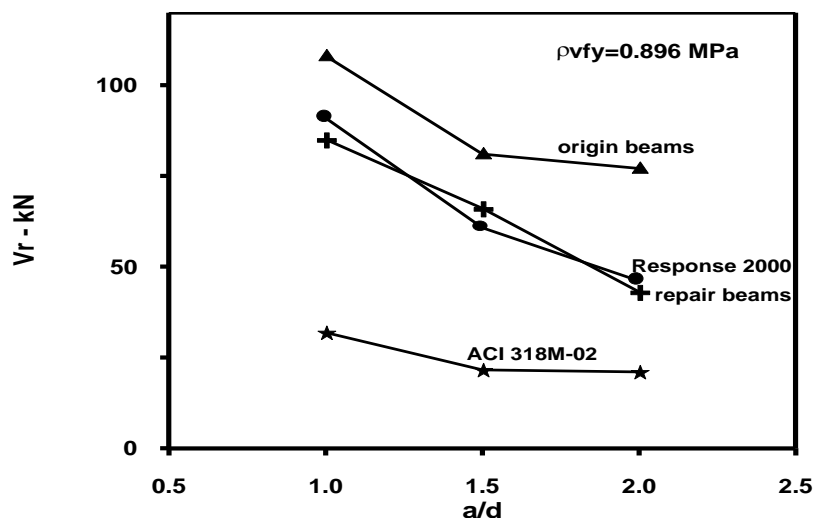


Fig.7 :Effect of shear span to depth ratio ,  $\rho_{vfy} = 0.896$

### 7. Deflection

Deflection of reinforced LWAC beams is an important design consideration because of the relatively low modulus of elasticity of this material . "Fig.8" shows the applied load (twice the shear ) versus the span center deflection of all the original and repair beams .The maximum deflection at failure was not recorded .All the original beams had both linear and non-linear portions in their load-deflection curves .The non-linear part was due to ductility provided by the web reinforcement .This non-linear part did not found in the repair beams .The stiffness and ductility of repaired beams are less than the original beams .

However, deflections of repaired beams were generally higher for corresponding load levels compared with the original beams with the same a/d. The mid-span deflection increases with higher a/d, as expected .For the beams with a/d > 2.0, the increase in central deflection is not as significant as for those with lower a/d .Formation of the first major inclined cracks significantly reduced beam stiffness, with this effect more evident in beams with higher a/d .

If the deflection of repaired and original beams are compared ,the deflection of the repaired beams are from 10.7 to 145.1 percent greater than those of the original beams .However, mid-span deflection at failure was greater than  $L_n /200$  for original and repaired beams .

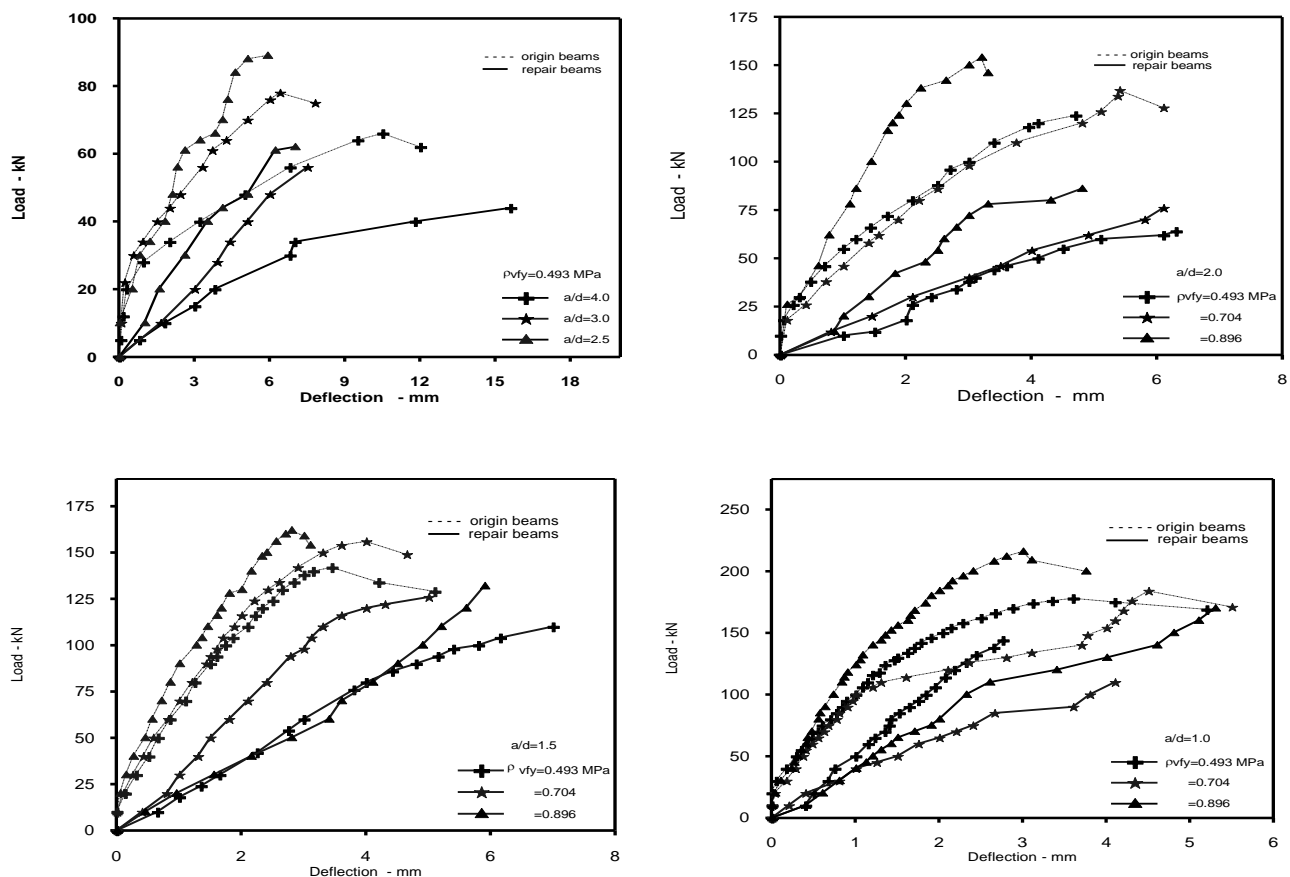


Fig.(8) :Load-Deflection Curves for Original and Repaired Beams

## 8. Conclusions

The limited test results have led to the following conclusions concerning the effectiveness of epoxy injection in repairing LWAC beams in shear:

- The shear resistance of the repaired beam is less than that of the original beam ( $V_{rR} / V_{ro}$ )<sub>avr.</sub> = 0.71 .
- The repaired beams can sustain at least the same magnitude of deformation as the original beams.
- The epoxy repaired cracks did not reopen in the tests of the repaired structures; new cracks tended to develop in the concrete adjacent to the repaired cracks.
- The resin injection method produced considerable strength increases. The final failure mechanism was characterized by a ductile flexural failure.
- The failure of repaired beams with stirrups are similar to the beams without stirrups , ( $V_{rR} / V_{rw}$ )<sub>avr.</sub> = 0.917 .
- Test results indicate that the ratio of  $V_{rR} / V_{cR}$  varies from 1.5 for  $a/d > 2.0$  to 2.27 for  $a/d \leq 2.0$  .
- Test results indicate that the ratio of  $V_{rR} / V_{r-ACI}$  and  $V_{rR} / V_{r-Response}$  has an average of 2.2 and 0.97 respectively .The results indicate that the ACI code is conservative for prediction the strength of repair beams.
- To increase the strength of repair beams with stirrups, it must be add beside the resin injection the carbon fiber reinforced polymer (CFRP) .

## 9. Notation

a	shear span; distance between concentrated load and face of support ,mm
bw	web width , mm
a/d	shear span to depth ratio
d	effective depth of beam , mm
Ln	distance between two supports, mm
$\rho_w$	longitudinal steel ratio
$\rho_v f_y$	shear reinforcement index
$V_{cw}$	test shear force at crack load for beams without stirrups, kN
$V_{rw}$	test shear force at ultimate load for beams without stirrups, kN
$V_{co}$	test shear force at crack load for beams with stirrups, kN
$V_{ro}$	test shear force at ultimate load for beams with stirrups, kN
$V_{cR}$	test shear force at crack load for repair beams, kN
$V_{rR}$	test shear force at ultimate load for repair beams, kN
avr.	Average

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