



ORTHOGONAL CHAOTIC VECTORS BASED ON RIKITAKE SYSTEM FOR CODE DIVISION MULTIPLE ACCESS APPLICATIONS

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Abstract:

The use of chaotic sequences which have low cross-correlation can be useful for spreading the direct sequence code division multiple access (DS-CDMA) system. In this paper, a chaotic multiple access communication system based on Orthogonal Chaotic Vector (OCV) generated from Rikitake system has been proposed. The simulation results proved that the performance of the proposed system is almost the same regardless of the number of users in Additive White Gaussian noise AWGN channel. The results also showed that for single user transmission and at bit-error-rate of 10^{-3} , the proposed system can achieve signal-to-noise ratio gains of 7 dB and 9 dB in over traditional DS-CDMA in AWGN and Rayleigh fading channels respectively. While for 4 users transmission and at bit-error-rate of 10^{-3} , the proposed system can achieve signal-to-noise ratio gains of 7 dB and 9 dB in over traditional DS-CDMA in AWGN and Rayleigh fading channels respectively.

Keywords: Chaos Theory; CDS- CDMA; Rikitake system; OCV .

المتجهات الفوضوية المتعامدة مستندة على نظام دالة ريكيتاك لتطبيقات تقسيم التردد الوصول المتعددة

الخلاصة:

ان استعمال السلاسل الفوضوية التي تمتلك مستوى واطى من الارتباط يكون مفيدا لنشر رمز سلسلة طيف نظام الوصول المتعدد المباشرة-DS (CDMA) لقد تم في هذا البحث اقتراح نظام اتصالات وصول متعدد مستند على متجه فوضوي متعامد متولد من نظام ريكيتاك. ولقد أثبتت النتائج التي تم الحصول عليها بأن النظام المقترح متميز في ادائه وانه لا يتأثر بتغير عدد المستخدمين عندما تكون قناة الارسال ضوضائية من نوع كاوس. ولقد بينت النتائج ايضا أن النظام المقترح يمكن ان يحقق ارباحا مقدارها 7 ديسيبل و 9 ديسيبل في نسبة الإشارة الى الضوضاء مقارنة بنظام تقسيم رمز الوصول المتعدد التقليدي في قناتي كاوس الضوضائية و رايلي التوهينيه على التوالي عند نسبة خطأ مقدارها 3-10 لالارسال المنفرد. بينما لالارسال ذو أربعة مستخدمين وعند نسبة الخطأ نفسها فان النظام المقترح يمكن ان يحقق ارباحا مقدارها 7 ديسيبل و 9 ديسيبل في نسبة الإشارة الى الضوضاء مقارنة بنظام تقسيم رمز الوصول المتعدد التقليدي في قناتي كاوس الضوضائية و رايلي التوهينيه على التوالي.

1. Introduction

The recent work [1-6] has shown the effectiveness of applying chaotic maps' signals/sequences in multiple access digital communication systems to replace the quasi-orthogonal binary code sequences. For CDMA systems, sequences with low cross-correlation properties are desired. Such sequences can be produced by chaotic systems due to its sensitivity to initial conditions. A concrete proposal for the chaotic CDMA system could already be found as early as 1992 [7], and similar proposals published later [8-10]. Mazzini et al. [11] presented the performance analysis of the chaotic CDMA system. They showed that chaos-based sequences can outperform m-sequences & Gold codes.

The publications on the performance comparison of chaotic CDMA based on Orthogonal Chaotic Vector (OCV) with traditional CDMA based on orthogonal spreading codes are unfortunately rare. Probably because it is commonly recognized that orthogonal spreading codes will generally outperform chaos map spreading codes. Furthermore, the little existing works considered the use of chaotic maps for CDMA although the chaotic flows like Rikitake are easier to be generated practically from physical electronic circuit.

In this work, direct sequence DS-CDMA digital communication system based on the OCV Rikitake flow sequences (x , y , and z) has been proposed and simulated. A random time-invariant multipath channel is used for BER simulations. The receiver is a RAKE receiver with enough fingers to capture all the energy from the multipath. Perfect channel estimation is assumed, and only downlink multiuser cases are simulated. The performance of the proposed system has been compared with traditional CDMA system.

The rest of the paper is organized as follows: Section II describes the architecture of the proposed system. Section III describes the Rikitake system and section IV describes the orthogonal chaotic vector generation. The Performance of the proposed system over AWGN and fading channels are given in section V and section VI respectively. Finally, the simulation results and conclusions are presented in sections VII and VIII respectively.

2. Architecture of the proposed Rikitake chaotic Multiple Access system

The block diagram of the proposed DS-CDMA based on Chaotic Rikitake system (CDS-CDMA) is shown in Fig.1. In the transmitter for each user (say j th user) of the N_u users, the data $d^{(j)}$ is generated and modulated to $d_m^{(j)}$ using BPSK modulator (QPSK or other digital modulation schemes are also applicable). The modulated data $d_m^{(j)}$ of each user are spread

using a unique Rikitake chaotic sequence. The Rikitake sequences are generated by taking one of the outputs of Rikitake system (i.e. x-sequence, y-sequence, or z-sequence) and applying it to an orthogonal chaotic vector (OCV) circuitry that generates N_u Rikitake sequences orthogonal to each other for n different users. The spreading operation is simply a multiplication process between the one of orthogonal chaotic sequences and the BPSK modulated signals. The spread signals are added to each other and sent through the transmission channel. At the receiver, each of the N_u received signals is de-spread by multiplying it by exactly similar replica of chaotic signal at transmitter of the corresponding user. An integration and dump operation is performed to detect the energy produced by the de-spread signal and finally the recovered signal is demodulated to obtain the transmitted data.

The difference between the proposed system and the traditional DS-CDMA used for wireless applications is that the digital Walsh-Hadamard sequence is replaced by Rikitake OCV analogue sequence to improve the system performance and capacity

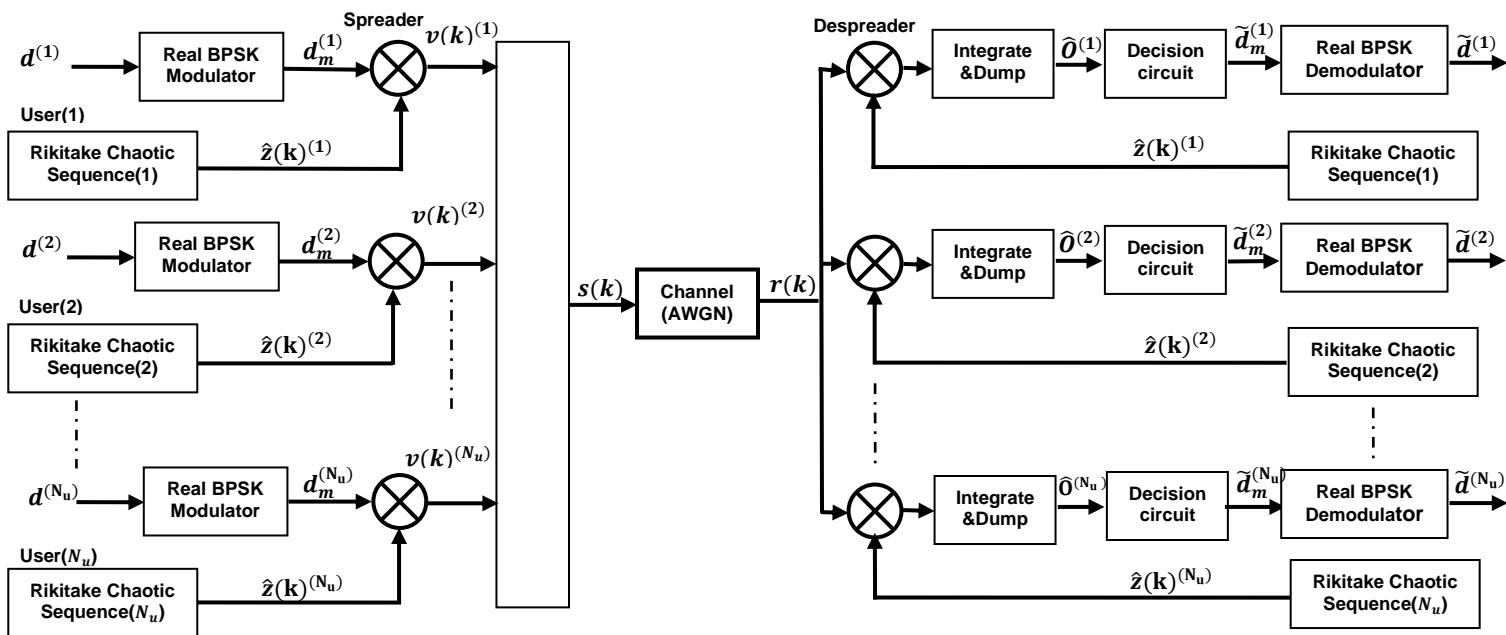


Fig.1: The proposed chaotic DS-CDMA system based on Rikitake model.

3. The Rikitake chaotic system

The Rikitake chaotic dynamical system is a model which attempts to explain the irregular polarity switching of the earth’s geomagnetic field [12]. The system exhibits Lorenz-type chaos and orbiting around two unstable fixed points. This system describes the currents of two coupled dynamo disks. Following nonlinear autonomous ordinary differential equations are the Rikitake chaotic system [13]:

$$\dot{x} = -\mu \cdot x + z \cdot y \quad (1)$$

$$\dot{y} = -\mu \cdot x + (z - a) \cdot x \quad (2)$$

$$\dot{z} = 1 - x \cdot y \quad (3)$$

Where $[x \ y \ z]$ is the output state vector while a, μ are parameters which we will assume to be nonnegative. The Rikitake system can lead to very complicated behavior on changing the parameter values. Choosing the standard parameter values $\mu = 2, a = 5$ [13], the Fig.2, shows the simulation results of the Rikitake Attractor.

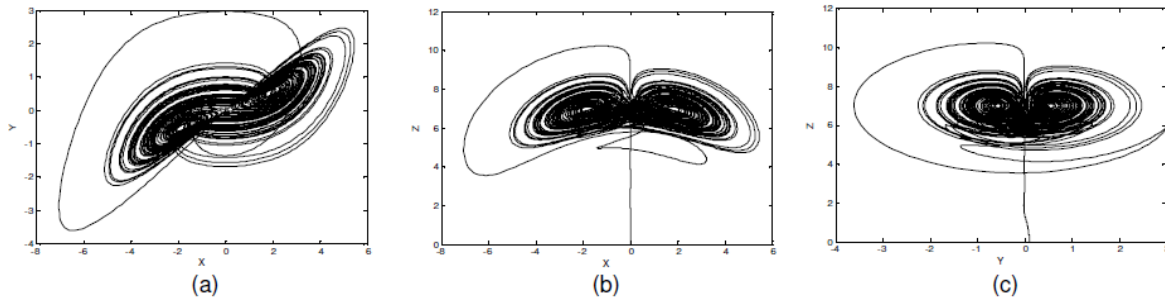


Fig.2. Phase portraits of the Rikitake attractor when $\mu = 2, a = 5$ initial conditions $[x_0 \ y_0 \ z_0] = [0 \ 0.1 \ 0]$.

5. The Orthogonal Chaotic Vector algorithm:

The non-zero values of cross-correlation between mutual chaotic sequences ($x, y,$ and z) of Rikitake system in chaotic CDMA system result in Multiple Access Interference (MAI) between users. The effect of MAI increases as the number of users increases. In order to eliminate the effect of problem (MAI), we apply rule of Gram-Schmidt's ortho-normalization process [14] on flow chaotic sequence vectors to obtain flow orthogonal chaotic vectors OCV codes (x, y and z) respectively. Gram-Schmidt ortho-normalization process for N_u orthogonal chaotic signal generators is given by [14]:

$$\hat{z}(k)^{(p)} = \frac{z(k)^{(p)} - \sum_{q=1}^{p-1} \left[\sum_{k=1}^{\beta} z(k)^{(p)} \hat{z}(k)^{(q)} \right] \hat{z}(k)^{(q)}}{\sqrt{\sum_{k=1}^{\beta} \left[z(k)^{(p)} - \sum_{q=1}^{p-1} \left[\sum_{k=1}^{\beta} z(k)^{(p)} \hat{z}(k)^{(q)} \right] \hat{z}(k)^{(q)} \right]^2}} \quad (4)$$

Where $p = 2, 3 \dots N_u$. For $p = 1$ (i.e. single user)

$$\hat{z}(k)^{(1)} = \frac{z(k)^{(1)}}{\sqrt{\sum_{k=1}^{\beta} [z(k)^{(1)}]^2}} \quad (5)$$

Where $\hat{z}(k)^{(p)}$ is the chaotic carrier for p^{th} user, and β is the number of chaotic samples used to transmit single binary bit (i.e. spreading factor). These OCVs are used as spreader to spread message bits, and to increase the numbers of active users. The orthogonal chaotic sequences can be generated from the different outputs of Rikitake chaotic system ($x, y,$ and z)

with similar or different initial conditions. Fig.3. shows the orthogonal chaotic sequences generated using the output z of Rikitake system.

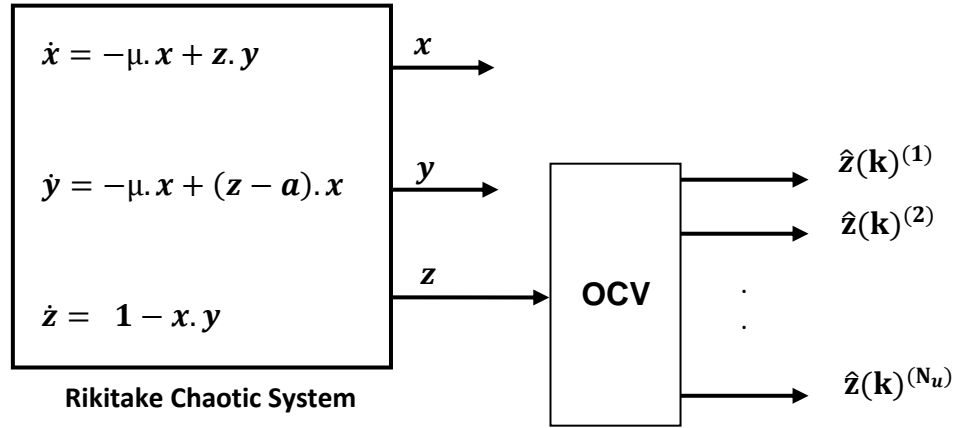


Fig.3: Generation of orthogonal chaotic sequences from Rikitake system.

The mean value of chaotic carrier is made equal to zero, in order to avoid unwanted dc power transmission. As it was mentioned in section II, the chaotic sequences are multiplied by baseband modulated BPSK data sequence $d_1\{-1, +1\}$ to obtain the spread vector $v(k)^{(j)}$. The transmitted signal $s(k)$ is the sum of modulated orthogonal chaotic vectors of each user and can be represented as:

$$s(k) = \sum_{i=1}^{N_u} v(k)^{(j)} \quad (6)$$

6. Bit-Error Probability in AWGN Channel

Assuming that the signal is corrupted only due to AWGN, the received signal $r(k)$ can be represented as:

$$r(k) = \sum_{i=1}^{N_u} V(k)^{(i)} + \xi(k) \quad (7)$$

Where $\xi(k)$ is the additive white Gaussian noise with zero mean and $N_0/2$ variance. At the receiver, it is assumed that a similar replica of spreading sequence is available and it is exactly synchronized with the transmitted one. The m^{th} decoded symbol for the j^{th} user, denoted by $\hat{d}_m^{(j)}$, is determined according to the rule [15]:

$$\hat{a}_m^{(j)} = \begin{cases} +1, & \text{if } \hat{O}_m^{(j)} = \sum_{k=1}^{\beta} r(k)\hat{z}(k)^{(j)} > 0 \\ -1, & \text{if } \hat{O}_m^{(j)} = \sum_{k=1}^{\beta} r(k)\hat{z}(k)^{(j)} \leq 0 \end{cases} \quad (8)$$

Without the loss of generality, we consider the probability of error for the first symbol. Omitting the subscripts of the variables $\hat{d}_m^{(j)}$ and $\hat{o}_m^{(j)}$ for the sake of brevity, the decision parameter of the j^{th} user is given by:

$$O^{(j)} = d^{(j)} \sum_{k=1}^{\beta} [\hat{z}(k)^{(j)}]^2 + d^{(j)} \sum_{i=1, i \neq j}^{N_u} \sum_{k=1}^{\beta} (\hat{z}(k)^{(i)} \hat{z}(k)^{(j)})^2 + \sum_{k=1}^{\beta} \xi(k) \hat{z}(k)^{(j)} \quad (9)$$

Since, chaotic vectors used for each user is ortho-normal to each other, the second term in eq. (9) causing MAI will be equal to zero. Assuming, that $O^{(j)}$ has a Gaussian distribution, the BER for j^{th} user can be written as [15]:

$$BER^{(j)} = \frac{1}{2} \operatorname{erfc} \left(\frac{E(O^{(j)} | d^{(j)} = +1)}{\sqrt{(2 \operatorname{var}(O^{(j)} | d^{(j)} = +1))}} \right) \quad (10)$$

Where, mean value of $(O^{(j)} | d^{(j)} = +1)$ is given by:

$$E(O^{(j)} | d^{(j)} = +1) = \beta E \left[(\hat{z}(k)^{(j)})^2 \right] = E_b \quad (11)$$

Where E_b is energy per bit. The variance is given by:

$$\operatorname{Var}(O^{(j)} | d^{(j)} = +1) = \operatorname{var} \left[\sum_{k=1}^{\beta} [\hat{z}(k)^{(j)}]^2 \right] + \beta \frac{N_o}{2} E \left[(\hat{z}(k)^{(j)})^2 \right] = \frac{E_b N_o}{2} \quad (12)$$

Substituting equations (11) & (12) in equation (10), we get:

$$BER^{(j)} = \frac{1}{2} \operatorname{erfc} \left(\sqrt{\frac{E_b}{N_o}} \right) \quad (13)$$

From equation (13) it can be concluded that BER performance of the proposed system is independent on the number of users and spread factor which is a unique and strong feature not exist in other multiple access systems.

7. Bit-Error Probability in Rayleigh Fading Channel

Assuming that the channel is a slow Rayleigh fading channel, let α is a Rayleigh distributed random variable denoting fading gain. Then it can be shown that the BER of j^{th} user in symbol duration is:

$$BER_{\alpha}^{(j)} = \frac{1}{2} \text{erfc}(\gamma) \quad (14)$$

Where

$$\gamma = \frac{\alpha E_b}{N_o} \quad (15)$$

Since α is Rayleigh-distributed random variable, γ (the received instantaneous signal to noise ratio per bit) will be chi-square distributed and has the form [15],

$$f_{\text{Rayleigh}} = \frac{1}{\bar{\gamma}} e^{-\frac{\gamma}{\bar{\gamma}}}, \gamma \geq 0 \quad (16)$$

Where

$$\bar{\gamma} = E[\gamma] = E[\alpha] \cdot \frac{E_b}{N_o} \quad (17)$$

Therefore, the average BER for j^{th} user is:

$$BER_{\text{Rayleigh}} = \int_0^{\infty} BER_{\alpha}(\gamma) f_{\text{Rayleigh}}(\gamma) d\gamma, \quad (18)$$

The last equation shows that in case of Rayleigh fading channel, there is no clear relation whether the BER performance of the proposed system is independent on the number of users and spread factor or not. Therefore, we will investigate performance's results by using computer simulation.

8. Simulation Results

A complete simulation model for the proposed OCV Rikitake based CDS-CDMA system shown in Fig.1 has implemented using MATLAB package 7.13.0.564 (R2011b). From other hand, another simulation model for traditional CDMA system based on orthogonal Walsh-Hadamard code with the same simulation parameters has also implemented for the purpose of performance comparison. In all simulation results the spreading factor is chosen to be 64 in both systems. The parameters of the generated Rikitake sequence were $\mu=2$, and $a=5$, while the initial conditions were $[x_0 \ y_0 \ z_0] = [0.255 \ 0.45 \ 0.255]$. The number of users has changed from single user to a maximum of 16 users.

• Simulation results in AWGN channel:

Fig.4 shows the BER performances of CDMA based on OCVs generated from individual outputs of Rikitake system (x, y, and z) for single user transmission in AWGN channel together with traditional CDMA system. It can be noticed from this figure that the performances of OCVs generated using different outputs of Rikitake system is not the same. The OCV associated with output z has the best performance. Therefore, it will be selected for all the remaining simulation results. Also it is obvious in this figure that OCVs outperforms the traditional CDMA. For instant, at $BER=10^{-3}$ a gain of 7 dB in E_b/N_0 can be obtained when we use OCV (z-output) as compared with the traditional CDMA. Fig.5 depicts the BER performances when the number of users is 4. Once again, the OCV based CDMA outperforms the traditional CDMA. The improvement starts to occur when SNR more than -10 dB. At $BER=10^{-3}$, a gain of 7 dB in E_b/N_0 is obtained. The improvement can be attributed to good combination of auto and cross-correlation properties of Rikitake OCVs. In other word, OCV sequences has ideal auto-correlation properties and acceptable cross-correlation ones while Walsh sequences has ideal cross-correlation properties but worse auto-correlation ones.

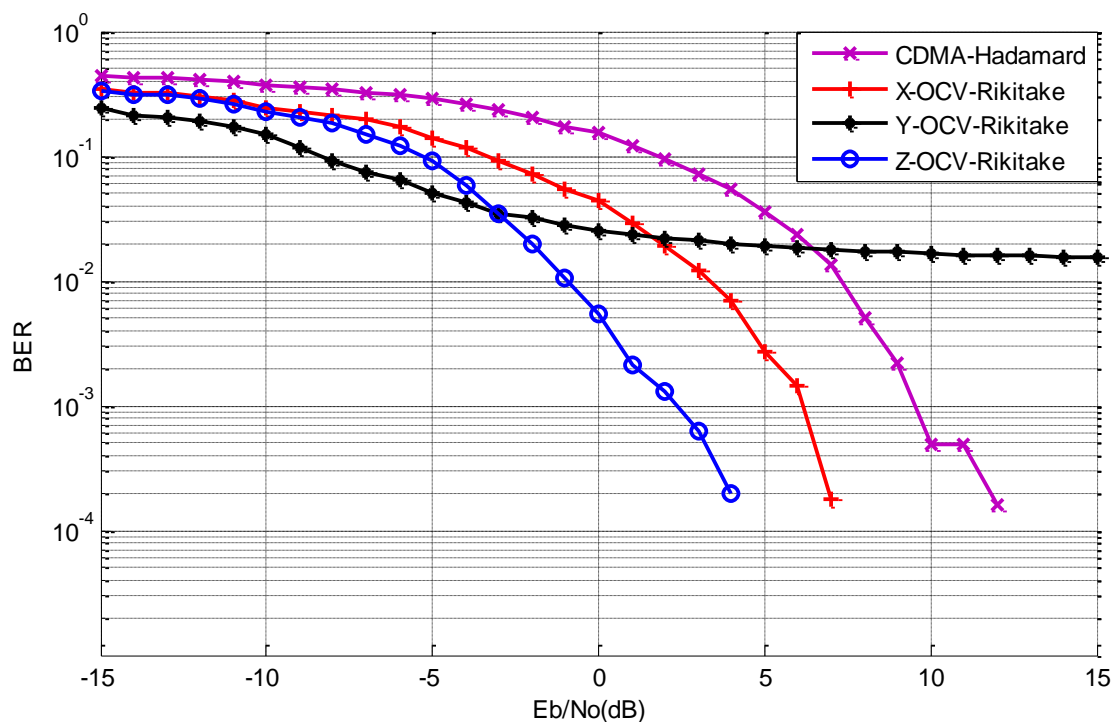


Fig.4: Performance of OCV of Rikitake chaotic sequences x, y and z, and the orthogonal Walsh-Hadamard sequence for single user transmission in AWGN channel

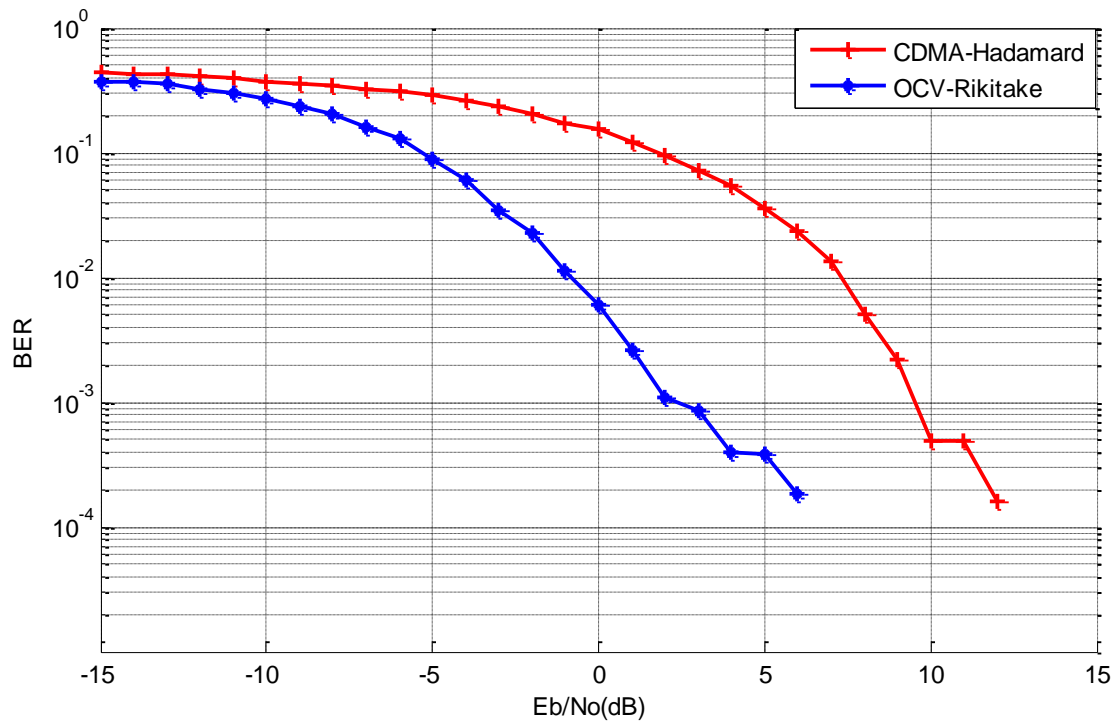


Fig.5: Performance of OCV of Rikitake based CDMA (sequence z) and traditional CDMA for 4 users' transmission in AWGN channel

Fig.6 shows the performance of the proposed OCV of Rikitake based CDMA with different number of users. It is obvious from this figure that the performance is almost the same regardless of the number of users. This result confirms the conclusion mentioned early in section V upon the derivation of equation (13).

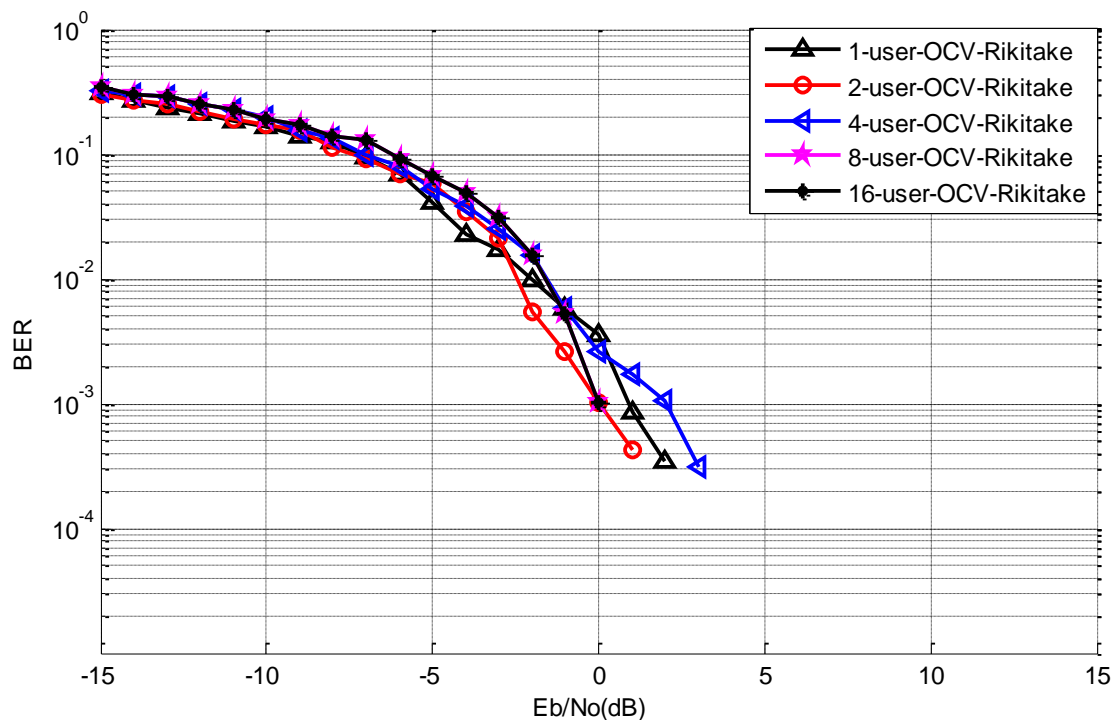


Fig.6: Performance of OCV of Rikitake based CDMA for different number of users in AWGN channel

• Simulation results in Rayleigh fading channel:

Fig.7 shows the BER performances of CDMA based on OCVs generated from individual outputs of Rikitake system (x, y, and z) for single user transmission in 3-tap random time-invariant Rayleigh multipath channel together with traditional CDMA system. It can be seen in this figure that all OCVs generated from different Rikitake system outputs performs the traditional CDMA. However, the best performance results are obtained in the case of (output z). It is worth noted that the improvement in the case Rayleigh fading channel is more than AWGN case. For instant, at $BER=10^{-3}$, a gain of 9 dB in SNR is obtained Rayleigh fading channel while it was 7 dB in AWGN channel. Fig.8 shows the BER performance comparison of both systems when the number of users is 4. Here, it can be easily noticed that the OCV Rikitake based CDMA outperforms the traditional CDMA. Once again, the improvement in the case Rayleigh fading channel is more than AWGN case. For instant, at $BER=10^{-3}$, a gain of 9 dB in SNR is obtained in Rayleigh fading channel while it was 7 dB in AWGN channel. Finally, Fig.9 depicts the performance of the proposed OCV Rikitake based CDMA with different number of users. It is obvious from this figure that the performance is improved as the number of users is decreased which is the similar case in traditional CDMA systems. However, this was not the case in AWGN channel discussed in Fig.6 where the performance was the same regardless the number of users. But in all cases the performance of the proposed scheme is better than the traditional CDMA for the similar number of users as we have seen for an example case $N_u=4$ in the previous figure.

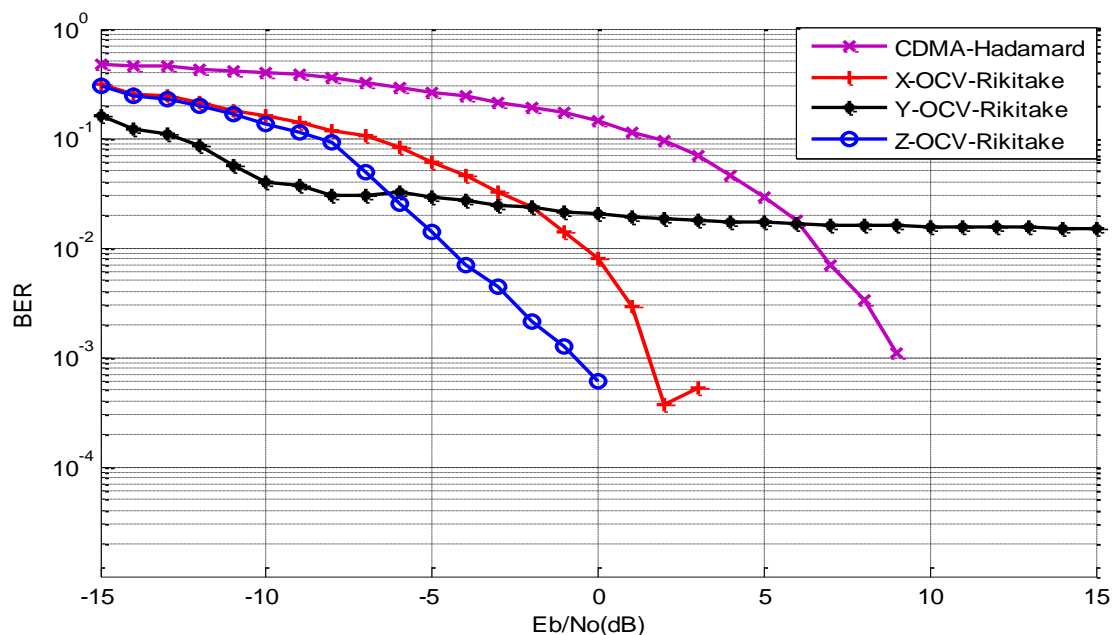


Fig.7: Performance of OCV of Rikitake chaotic sequences x, y and z, and the orthogonal Walsh-Hadamard sequence for single user transmission in Rayleigh fading channel.

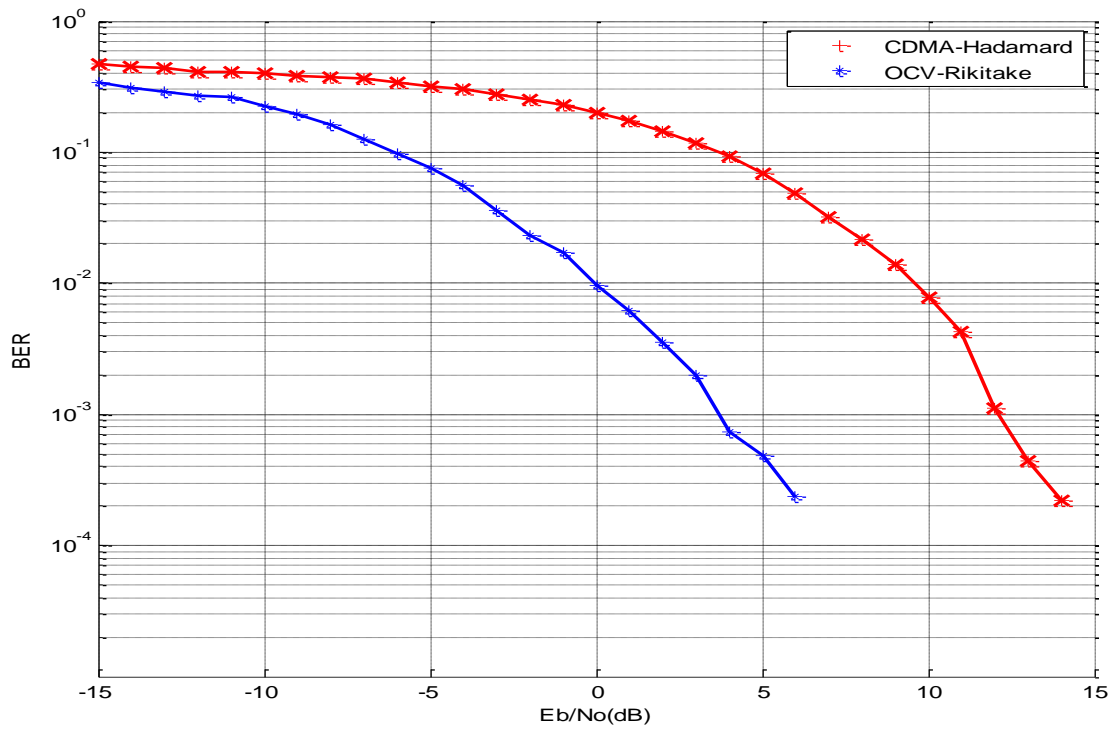


Fig.8: Performance of OCV of Rikitake based CDMA (sequence z) and traditional CDMA for 4 users' transmission in Rayleigh fading channel

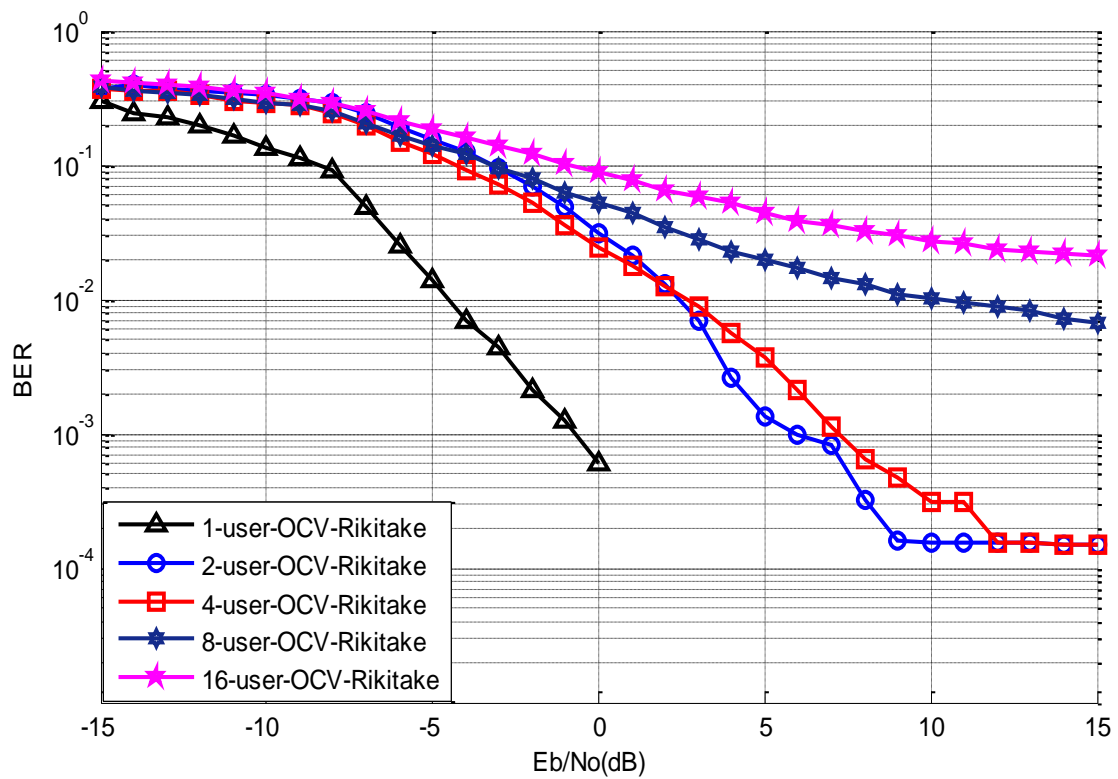


Fig.9: Performance of OCV of Rikitake based CDMA for different number of users in Rayleigh fading channel

9. Conclusions

The performance of CDMA system can be significantly improved by the use of orthogonal chaotic vector generated from Rikitake chaotic system using Gram-Schmidt orthonormalization process as a spreading sequence. The orthogonal chaotic vector has ideal auto-correlation properties and acceptable cross-correlation ones. This good combination is the reason behind its good performance in both AWGN and fading channels as compared with the traditional Walsh-Hadamard sequence which has ideal cross-correlation properties but worse auto-correlation ones. The orthogonal chaotic vectors generated from different Rikitake system outputs have different performances; so, the one of the best performance should be selected to obtain the best improvement. Finally, as the theoretical analysis and simulation results show, the orthogonal chaotic vectors have almost the same performance regardless of the number of users or the spreading factor in AWGN which is a unique feature as compared with other multiple access techniques.

10. References

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